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Embodied and dis-embodied affordances in mathematics education

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In this paper, we discuss a two-way mechanism for acquiring mathematical knowledge: dis-embodied-to-embodied and embodied-to-dis-embodied shifts. Shifting from an embodied representation to a dis-embodied representation enables mathematics learners to discover general rules and patterns that can be applied to a wide range of mathematical problems, while shifting from a dis-embodied representation to an embodied representation allows mathematics learners to acquire a more graspable knowledge of mathematics. After discussing these two shifts in mode of processing, we describe a two-way mechanism through which a parallel employment of these two modes of thought allows mathematics learners to acquire a grounded and deep knowledge of mathematics. Therefore, any mathematics education program should engage learners in both modes of processing. Finally, we conclude that it is not possible to have an efficient education program by focusing on just one mode of mathematical thought.

KEYWORDS
dis-embodied representation, embodied representation, mathematics education, affordances, education program

1 Introduction

Presenting and discussing mathematical concepts and constructs in terms of abstract symbols has a long history in textbooks and mathematics education. Abstract mathematical symbols can stand for any mathematical object; they can represent numbers, matrices, vectors, spaces, sets, groups, operations, etc. Since abstract mathematical symbols can stand for any mathematical object, they are used to express general rules and represent general constructs that apply to a wide range of situations. Although the process of teaching and learning mathematics through abstract symbols and abstract representations of mathematical concepts and constructs has been an old tradition in mathematics education, in recent years there has been a growing shift to embodied mathematics education. By embodied or body-based mathematics education, we mean an educational program in which mathematical concepts are represented in terms of body movements or other physical materials in the world. In this sense, embodied representations of mathematical concepts are bodily and fleshed (physical objects representing mathematical concepts) representations that can easily be perceived through human sensory channels. Therefore, our use of the term *embodied representation* includes both body and flesh in Roth's (2010) terminology. A shift from a dis-embodied mode of mathematics education to an embodied mode has been really successful in solving many problems in mathematics education. Within the framework of an embodied education, a tangible and easily understandable picture of mathematical concepts is presented to students, which makes mathematics more digestible (Anderson, 2018; Boonstra et al., 2023; Delafield-Butt and Adie, 2016; Farsani, and Mendes, 2021; Farsani et al., 2022; Khatin-Zadeh et al., 2022, 2024, 2025; Macedonia, 2019). It is an undeniable fact that a body-based approach to mathematics education has helped us solve some problems in mathematics education and enhance the process of mathematics teaching and learning. But, it is equally undeniable that some crucial issues in mathematics education—particularly those related to higher-order abstract mathematical thought—have not been tackled by a strongly body-based approach to mathematics education. Some important aspects of mathematics cannot be learned by relying on embodied representations of mathematical concepts and constructs. They need the employment of a higher-order mode of thinking that is largely detached from sensorimotor experiences. This is particularly the case with those aspects of mathematical knowledge that involve abstract or higher-order mathematical structures. Such structures constitute a base for developing general rules that can be applied to a wide range of mathematical problems in various conditions.

Techniques for transforming dis-embodied representations of mathematical representations into embodied representations such as visual representations, gestural representations, and motion-based representations have been widely discussed in the literature in recent years (e.g., Rosa and Farsani, 2021; Rosa et al., 2020; Khatin-Zadeh et al., 2023a, 2023b). Such techniques are highly reliant on concrete objects that are used as mediatory tools for the process of transformation. For example, when the algebraic symbolic representation of a function is transformed into a graphic representation in the Cartesian coordinate system, visual tools and the visual system are actively employed to process the original algebraic representation in terms of its visual representation. In this process, some additional features (especially visual features) come into play and extra sensorimotor resources (such as the visual system) are actively employed. To take another example, the mathematical notion of derivative can be represented in terms of mathematical symbols ($\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$). This can be seen as the symbolic or dis-embodied representation of this mathematical notion. This notion has many concrete realizations or concrete representations in the real-world situations, such as instantaneous speed or instantaneous acceleration of a moving object. These concrete real-world representations of this mathematical notion can offer students a tangible understanding of mathematical notion of derivative. In fact, finding concrete examples of how this notion can be practically used the real situations is a technique for giving students a tangible picture of this fundamental notion in mathematics. Slope of the tangent line is also a visual representation of this mathematical notion, which gives a visual representation of derivative. This visual representation of derivative shows that the derivative of a function at a certain point is the slope of the tangent line at that point (see Figure 1). While $\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$ is a symbolic or dis-embodied representation, slope of the tangent line in a Cartesian coordinate system is an embodied (visual) representation of this mathematical notion.

On the other hand, the process of transforming embodied representations into disembodied representations has a different nature. In this process, since concrete or sensory-perceivable features of concrete representations are suppressed, the activation of sensorimotor systems may be reduced. The de-activation of sensorimotor systems can mean that the cognitive system shifts toward a different mechanism of processing. The cognitive system may de-activate structurally-irrelevant features and instead focus on some features that are independent of sensorimotor properties. This mechanism is a combination of suppressing irrelevant sensorimotor features and maintaining a selected structural features that are shared by a set of concrete representations. These non-sensorimotor or abstract structural features are largely the product of a suppressive-oriented mechanism. In other words, while the process of transforming dis-embodied representations into embodied representations may be the result of employing extra sensorimotor resources, the process of transforming embodied representations into dis-embodied representations can be the result of de-activating or de-employing sensorimotor resources.

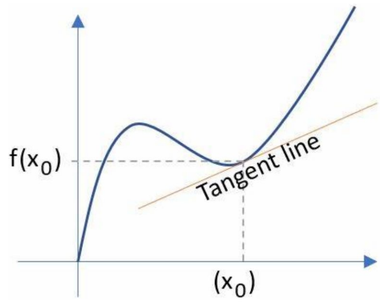

FIGURE 1
Embodied representation of the mathematical notion of derivative.

As mentioned, there has been a rising interest in techniques for transforming dis-embodied mathematical representations into embodied representations. But, it seems to us that techniques for transforming embodied representations into dis-embodied representations, which we believe can be equally important in enhancing mathematical thought, have been disregarded in the literature of mathematics education. Therefore, in this paper, we intent to emphasize why an over-emphasis on body-based education and embodied mode of thought can weaken mathematical thought. In the following two sections, we reflect on the cognitive mechanism of suppression, a process through which the individual shifts from an embodied mode of thinking to a dis-embodied mode. This is done by examples from abstract algebra. Then, we discuss the need for employing the two modes of mathematical thought: dis-embodied-to-embodied (body-based approach to education) and embodied-to-dis-embodied (abstract symbol-based approach to education).

2 Embodied to dis-embodied mode of thought

Drawing on an example from elementary mathematics, Roth (2010) argues that material bodies are not sufficient resources and tools for emerging mathematical thought and acquiring a deep mathematical knowledge. Here, we intend to emphasize that not only material bodies but also fleshed mathematical representations are incapable of helping us acquire a deep mathematical knowledge without some sort of dis-embodied mode of thinking. We do this by discussing an example from abstract algebra. Even prior to the emergence of embodiment theories in the context of mathematics education, Pimm (1995) extensively discussed the notion of manipulation in mathematics classroom. Manipulation or transformation is in fact a strategic tool to shift between mathematical representations to enhance the process of acquiring new knowledge. Algebraic group is the basic concept that is studied in abstract algebra. An algebraic group consists of a set of elements and an operation between elements of the set. The elements of a group can be any mathematical object such as numbers, functions, matrices, vectors, ordered pairs, and even shapes. The key point is that a set of superficially different groups can be isomorphic at a deep or structural level. That is, while they seem to be different in appearance, they are the same at an abstract structural level. In fact, it can be said that the embodied representations of a set of isomorphic groups may seem to be very different from one another. For example, a group consisted of matrices can be isomorphic with a group of geometric shapes. Although there is no easily-observable similarity between these two groups, they can be isomorphic. Both of them can be isomorphic with a group consisted of abstract symbols. In fact, the groups consisted of abstract symbols is the abstract representation of both of them. An over-reliance on an embodied mode of thought and dis-regarding a higher-order structure-oriented mode of thought may hinder the process of discovering the abstract structural similarity between isomorphic groups. In this example, groups consisted of matrices and shapes can be regarded as embodied representations (or more embodied representation) and the group consisted of abstract symbols can be seen as dis-embodied representation (or less embodied representation).

The process of discovering and extracting the dis-embodied representation or abstract symbol-based representation of a set of isomorphic groups is a shift from a body-based mathematical thinking to an abstract structure-oriented mode of processing. This is in fact a shift from embodied mathematical thought, which involves the active role of sensorimotor systems, to a higher-order dis-embodied mode of processing/reflecting, which is oriented to discovering abstract structures underlying concrete constructs. In other words, the dis-embodied mode of thought helps the individual discover difficult-to-observe structures that underlie superficially or concretely different constructs. In the case of algebraic groups, while the embodied mode of processing helps learners see how a set of elements and an operation defined between elements constitute a group, the dis-embodied mode of thought helps them discover how a set of groups are isomorphic and structurally similar. Therefore, we can talk about two levels of thinking about algebraic groups: a lower-level embodied and a higher-level dis-embodied modes of thought. When an individual thinks about groups and processes them to solve an algebraic problem, s/he may strategically shift between these two levels to tackle different aspects of the problem. As Pirie and Kieren (1989, p. 8) argue in their recursive theory of mathematical understanding, "mathematical understanding can be characterized as leveled but non-linear. It is a recursive phenomenon and recursion is seen to occur when thinking moves between levels of sophistication." The individual may shift between a lower level of sophistication (embodied level and concrete) and a higher-level of sophistication (dis-embodied level and abstract) to find a solution for an abstract algebraic problem or to acquire new knowledge. Therefore, as Pirie (1988) has suggested, putting a clear line between different kinds of mathematical learning and describing each one as an independent process is inadequate. Mathematical understanding is a complex process that can involve several levels of sophistication in thinking. A question that is raised here is how this shift from an embodied mode to a dis-embodied mode takes place. In the next section, we reflect on this question.

3 Embodied to dis-embodied through suppression

Embodied representations of mathematical concepts carry a lot of sensorimotor information. This information is associated with sensorimotor features of embodied representations. To shift from an embodied representation to a dis-embodied representation, this sensorimotor information should be suppressed (El Hakim and Farsani, 2024; Khatin-Zadeh et al., 2025). The result of this shift is a dis-embodied representation that does not have any sensorimotor association (or have less sensorimotor associations). In the case of algebraic groups, abstract structural similarity between isomorphic groups, which is largely a dis-embodied construct, is not easily noticeable. It is not easily perceivable because it is detached from sensory associations of groups. To extract this dis-embodied construct from a set of isomorphic groups, the irrelevant features of groups, which are superficial and associated with sensory features of isomorphic groups, should be suppressed. In fact, suppression is a process through which irrelevant information is disregarded and relevant information (related to abstract structures) is maintained (Khatin-Zadeh et al., 2025). The point that we want to emphasize here is that the process of suppression, which is the base of embodied-to-dis-embodied shift, is as much as important as dis-embodied-to-embodied shift. Here, we discussed algebraic group as an example to prepare the ground for presenting the main point of the paper. Embodied representations of mathematical concepts help the learners acquire a grounded knowledge of mathematics. This grounded knowledge is acquired by the mediation of sensorimotor systems. In fact, sensorimotor systems are actively employed to ground mathematical concepts in the concrete environment. On the other hand, a dis-embodied mode of thinking may help the learners discover and extract deep structures in mathematical constructs. This is done by suppressing information that is not relevant to the structures of mathematical constructs. Working with embodied representations through an embodied mode of thought involves structurally-irrelevant information. This may hinder the process of discovering abstract mathematical structures and general rules, as it can overload the cognitive system and push the attention to irrelevant information. The important point is that both modes of thinking are needed to acquire a grounded and deep knowledge of mathematics. While the embodied mode of thinking (working and practicing with embodied representations) makes mathematical knowledge grounded, the higher order dis-embodied mode of thinking (working and practicing with dis-embodied representations) offers a broad or general picture of a wide range of mathematical concepts.

It should be noted that when we are talking about embodied and dis-embodied representations of mathematical concepts, we dot mean absolutely embodied or absolutely dis-embodied representations. An embodied representation may have some degree of dis-embodied or abstract structural elements and a dis-embodied representation may have some degree of embodied associations. In other words, being embodied or dis-embodied is a graded characteristics and may vary across a range. In the same way that degree of abstractness/concreteness is not an absolutely binary relationship but a graded one (Crutch and Jackson, 2011), being embodied or being dis-embodied is a matter of degree. Some mathematical representations are strongly embodied or strongly dis-embodied; between these two extreme ends of being strongly embodied and being strongly dis-embodied, there is a wide range of possibilities. For example, the embodied representation of an algebraic group may contain not only concrete features but also some degree of abstract structural features as every algebraic group is a set of elements and an abstract structure that connects the elements together at a deep level. Similarly, the dis-embodied representation of an algebraic group includes not only an abstract structure but also some degree of concrete features, as symbols representing elements of an algebraic group have some degree of concrete features. Symbols are visual although they may not have any similarity to their referents. Therefore, transforming an embodied representation to a dis-embodied representation does not mean a shift from an absolutely embodied representation to an absolutely dis-embodied representation. This is in fact a process of transforming a more embodied or more concrete representation to a more dis-embodied or more abstract representation.

To take another example of algebraic groups, a group consisted of matrices and an operation between matrices can have an isomorphic relationship with a group consisted of ordered pairs with an operation between ordered pairs. At the first look, no similarity may be observed between the superficial features of these two groups. However, these two groups share the same structural features. When these apparently different groups are represented in terms of abstract symbols and abstract operations, the abstract structural similarity between them becomes evident. The more embodied or more concrete representation of the first group consists of matrices, and the more embodied or more concrete representation of the second group consists of ordered pairs. To extract the dis-embodied representations of these groups, the unshared concrete features of the two concrete representations should be suppressed. However, as mentioned above, the group that consists of abstract symbols and an operation between abstract symbols has some degree of embodied or concrete associations as symbols are represented by written characters. Yet, these concrete associations are much smaller than concrete associations that are attached to embodied representations of the two original groups. In the following section, we look at the process of transforming mathematical representations from radical constructivism.

4 Embodied to dis-embodied from a radical constructivist perspective

From the perspective of radical constructivism (e.g., Confrey, 1995; Steffe and Kieren, 1994; von Glasersfeld, 1994), using abstract mathematical symbols is a way for forming deep abstract mathematical representations and constructing higher-order mathematical knowledge that can be applied across a wide range of concrete mathematical representations. From this view, abstract mathematical structures or constructs do not exist in the concrete environment. They are constructed in the mind (see Krause and Farsani, 2022). Where it is said that a set of concrete mathematical representations or constructs are isomorphic (e.g., a set of isomorphic groups), they may be very different in their concretely-perceivable features. But, they share an abstract structural similarity at a level beyond easily-perceivable sensory features. This abstract structural similarity does not exist in the concrete environment. It is constructed in the mind and is represented by abstract symbols. The mind associates this system of abstract symbols to a set of superficially/concretely different (but structurally similar) representations. In fact, abstract structure of a set of isomorphic representations is the product of mental processes. In essence it does not have any sensorimotor features; it does not exist in the concrete environment; but, it can represent a wide range of isomorphic concrete representations.

Abstract mathematical symbols are mediatory tools for deriving deep similarities among a set of concretely-different mathematical representations. This is done by suppressing concrete dissimilarities and structurally-irrelevant features of concrete representations. Once the deep abstract representation has been formed in the mind, it can serve as a higher-order referential representation for all concrete representations that share it as their abstract structure. Here, although abstract symbols do not have much sensorimotor features, they form a structure that can be realized in a large number of isomorphic concrete representations. In fact, a shift from a number of isomorphic concrete representations to a shared abstract representation is a way for constructing abstract mathematical knowledge. From the perspective of radical constructivism (von Glasersfeld, 1994), abstract representations consisted of abstract mathematical symbols become meaningful when they are associated with their concrete representations through mental operations that take place in the mind of the individual. An abstract representation or abstract structure does not have any meaning in itself. It is just a set of symbols that may seem to be unrelated to each other. However, when this apparently meaningless representation of abstract symbols serves as a referential tool for representing the structure of a concrete representation, it becomes fully meaningful. This is done through mental processes that lead to constructing deep mathematical knowledge. These mental processes are mechanisms through which deep mathematical knowledge is constructed and developed.

As mentioned, a deep mathematical knowledge enables the individual to process a wide range of concretely different representations that share an abstract structural similarity. A question that is raised here is how the acquired deep mathematical knowledge is employed to deal with concrete mathematical representations. To answer this question, we suggest that deep mathematical knowledge may be constructed through a dis-embodied mode of thought that is suppressive-oriented in its early stages. Once the deep mathematical knowledge has been constructed through the formation of abstract representations, it should be maintained and then activated to deal with new situations. In this stage, the constructed abstract representation can be applied to new concrete representations that share the same abstract structure. This is an embodied mode of thinking, as the constructed abstract structure is embodied or realized in new concrete situations. The process of matching an abstract structure or abstract representation against newly-faced concrete representations is in fact an exploration into new concrete situations to see whether the constructed abstract structure/representation can be applied to them or not. Therefore, it can be said that the process of knowledge development is a combination of dis-embodied and embodied thinking processes. The first set of processes are suppressive and lead to the construction of abstract structures that are detached from sensorimotor features; the second set of processes are 'receptive' and lead to the construction of a set of concrete structures out of a single abstract structure. Here, by 'receptive' process, we mean a process through which concrete features are added to the abstract structure to see whether a new concrete mathematical representation matches the abstract structure or not. Our proposal is consistent with Pirie and Kieren's (1989) recursive theory of mathematical understanding. As mentioned, this theory holds that mathematical thinking is a matter of shifting between various levels of sophistication.

Therefore, from this perspective, mathematical thought is a combination of suppressive-oriented and receptive-oriented modes of thinking. These two modes can be in operation at the same time, each one tackling certain aspects of a problem. In the following section, we reflect on the pedagogical implications of laying a balanced emphasis on embodied and dis-embodied modes of thinking.

5 Pedagogical implications in mathematics education

In an ideal or near-ideal mathematics education system, both embodied and dis-embodied modes of thinking need to be attended to, as both of them are needed in a good mathematics education program. Emphasizing one mode and de-emphasizing the other would be a de-service to our mathematics education system. In other words, both modes need to be emphasized in parallel and in a well-balanced manner. Although emphasizing embodied representations and embodied mode of thinking has helped mathematics educators solve some problems in our mathematics education system, an over-emphasis on it in favor of under-emphasis on dis-embodied mode of thinking could harm the process of mathematics education. We feel that a well-balanced emphasis on embodied and dis-embodied modes of thought is not observed in our mathematics education programs (at least not in Iran, UK, Chile, Brazil, Norway, and China). In traditional mathematics classrooms, there has been an over-emphasis on dis-embodied representations and the dis-embodied mode of thinking. On the other hand, in many modern classes of recent years, this over-emphasis has been shifted to embodied representations and embodied mode of thought (see Breda et al., 2021; Seckel et al., 2022). It seems to us that an optimal balance between these two has not been properly observed. Each mode of thought can tackle certain aspects of mathematics education. We believe if we focus on one and dis-regard the other, the process of mathematics education would be imbalanced and non-efficient. It cannot be denied that the shift to embodied education has had very positive results in improving the quality of mathematics education. But, this should not lead to completely ignoring dis-embodied representations and dis-embodied mode of mathematical thinking. We think that this imbalance between the two modes of thought has been one of the main reasons behind the weak performance of our mathematics education system in some areas. Therefore, now we need to reconsider our approach to these two modes of mathematical thought and create a balanced state between them in the process of mathematics education.

To make this point clear, let us return to the example of group theory. As mentioned, group theory is one of the most abstract subjects in pure mathematics. Because of high abstractness of abstract algebraic concepts, it is sometimes very difficult to offer a concretely-perceivable representation of concepts and constructs in this area of mathematics. Some attempts have been made to present visual representations of concepts in group theory (e.g., Carter, 2009). In these works, groups and associated concepts are discussed in the form of visual representations that can be perceived and processed by the support of the visual system. Although it cannot be denied that this way of presenting group theory can offer an embodied and easily-understandable picture of key concepts in this area of mathematics, it may not make a great contribution to the process of learning in highly advanced levels of mathematics. Visual presentation of group theory is really helpful in the early stages of learning abstract algebra. But, advanced levels of mathematical thought involve a mode of thinking that is largely detached from even visual features. At such advanced levels of mathematical understanding and thinking, the abstract structures of algebraic groups are extracted from concrete representations and then applied to new concrete representations. The latter concrete representations may be very different from the former concrete representations. The key feature of high levels of mathematical thinking it that it enables the individual to apply abstract representations or abstract structures across a wide range of structurally-similar but superficially diverse concrete representations. That is, it enables the individual to see deep isomorphic relationship between concretely-diverse representations. Full reliance on concrete mathematical representations (e.g., visual representations of groups) cannot give such ability to mathematics learners. Mathematics learners should go beyond absolute reliance on concrete representations, because it may prevent them to achieve the ability to apply abstract representations across concretely diverse situations. In other words, both embodied and dis-embodied modes of thought need to be attended to in the process of teaching and learning the group theory and other subjects of mathematics that deal with structural features of mathematical constructs.

Creative mathematical thinking involves a combination of embodied and dis-embodied modes of thought. In a given situation, a creative mathematical thought goes beyond the concretely-perceivable representations and discovers deep structures that underlie mathematical constructs. In another situation, it can apply the extracted structure to newly-faced concrete representations. A creative mathematical thought can even manipulate the extracted deep structure for constructing new developed structures and to solve new problems. If we intend to train creative mathematical thinkers, we have to attend to both embodied and dis-embodied aspects of mathematical thinking, as disregarding one of them and over-emphasizing the other cannot be an ideal or near-ideal way to teach and to learn mathematics.

6 Final remarks

Efficient mathematical thought is a combination of embodied and dis-embodied modes of thought. Shifting from an embodied representation to a dis-embodied representation enables mathematics learners to discover general rules and patterns that can be applied to a wide range of mathematical problems, while shifting from a dis-embodied representation to an embodied representation allows mathematics learners to acquire a more graspable knowledge of mathematics. In other words, shifting from an embodied representation to a dis-embodied representation helps mathematics learners grasp underlying rules and patterns that form the essence of mathematical thought. To summarize, mathematics understanding and the process of acquiring mathematical knowledge involve a two-way mechanism: dis-embodied-to-embodied and embodied-to-dis-embodied shifts. A set of factors such as nature of the mathematical problem and level of learner's knowledge determine which direction of this two-way mechanism is more important in a specific situation. Even for solving a certain problem, both dis-embodied-to-embodied and embodied-to-dis-embodied shifts may be needed. The key point is to use each one in the proper place to respond to a specific aspect of the problem. A parallel and balanced employment of these two modes of thought allows the individual to acquire a grounded and deep knowledge of mathematics. It is not possible to have an efficient mathematics education program by fully relying on one mode and disregarding the other one. Acquiring mathematical knowledge is a multidimensional process. Certain dimensions of this process can be addressed by certain modes of thinking. But, in an efficient mathematics education program, all of them should be addressed together. It is the job of mathematics educators, teachers, textbook writers, and syllabus designers to pay attention to both modes of mathematical thought, because we cannot have an efficient mathematics education program by focusing on just one mode of mathematical thought.

Data availability statement

The original contributions presented in the study are included in the article/supplementary material, further inquiries can be directed to the corresponding author.

Author contributions

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