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Lesson plan design for teaching mathematics in multicultural classrooms

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This paper describes a mathematics professional development experience designed from an intercultural perspective. This resulted in a hybrid Lesson Study cycle conducted online over two half-school years (autumn 2023 and spring 2024). The first phase of this professional development program aimed to explore the challenges of teaching and learning mathematics in multicultural and multilingual contexts—an emerging issue for the Italian school system. In the subsequent phase of professional development, teachers, supported by researchers, approached the Lesson Study, creating teaching materials from an intercultural perspective, emphasizing cultural aspects. A key finding during the online Lesson Study phase was the need for a preliminary phase before developing a Lesson Plan. This necessity, which is shaped by the flexibility and challenges of online settings, aligns with the theoretical foundations of our professional development program. We named this preliminary stage the *Lesson Plan Design* phase. Here, the teachers discussed not only the content but also the structure of the Lesson Plan. Starting with an example provided by researchers, teachers worked on a *design* level and collaboratively co-constructed a shared *Lesson Plan Template*, integrating the key factors highlighted during the initial professional development phase. The shift from merely using or filling out a given Lesson Plan to designing a structure of Lesson Plan is explored through a specific case study. This case study, focused on the design phase, demonstrates how cultural themes addressed at the beginning of professional development influenced the structure and content of the Lesson Plan. The aim of this new phase is to establish an online community of teachers from the bottom. This step aligns with the standard Lesson Study approach, which assumes the existence of such a community from the outset. After this phase, of creating a cohesive community of teachers, the Lesson Study cycle, as described in the literature, can proceed.

KEYWORDS

Lesson Study, Lesson Plan, interculture, cultural transposition, multicultural classroom

1 Introduction

This paper presents a research contribution relevant to two fields: the study of mathematics teaching and learning from an intercultural perspective (Beacco et al., 2016) and teacher education research. Specifically, it discusses Lesson Study (LS) as an approach to teachers' professional development in multicultural contexts.

In the first part of the paper, we outline the needs of teachers working in multicultural and multilingual contexts. To address these implicit needs, we designed an online professional development (PD) path to provide theoretical insights and practical support for teaching in these complex environments. We describe the structure and methodology of this PD path, which aims to equip teachers with tools to meet this new educational challenge of the 21st century. The next step of the PD path consisted of an online LS phase (Nickerson et al., 2014;

Capone et al., 2022), where teachers and researchers (as teachers' trainers) worked collaboratively. This collaboration was guided by a clear pedagogical focus and shared goals: *inclusive teaching for each student*. We explain our choice of LS as a flexible methodology (Stigler and Hiebert, 1999) for creating a supportive and ongoing teacher-training environment (Bartolini Bussi and Ramploud, 2018). This approach is aimed to build a community of teachers by continuing the work that was started in the first phase of PD. Focusing on mathematics teaching, we selected an LS model inspired by Learning Study (Lo and Marton, 2012) and certain LS approaches (Huang and Shimizu, 2016). This model emphasizes the careful selection of topics and materials for collaborative work. *Teaching with variation*, drawn from the Chinese mathematics tradition, became a guiding principle (Sun, 2011) for analyzing existing teaching resources *and* creating new ones tailored for multicultural and multilingual Italian classrooms (since we focused on the Chinese mathematics tradition). The concept of variation was shared between researchers and teachers to foster a concrete *Cultural Transposition* (Mellone et al., 2019). This process raised teachers' cultural and disciplinary awareness by highlighting the differences, opportunities, and insights that can be brought about by the coexistence of diverse cultural backgrounds within the same classroom. This concrete declination of Cultural Transposition aligns with a broader intercultural perspective, shifting the focus from abstract "cultural heritage" to the lived experiences of students. Thus, the first part of the paper describes the outline of the PD path, both as a response to teacher needs and as a foundation to approach LS.

The second part of this paper focuses on the research aspects of LS as a training methodology. When working with teachers from various institutions (geographical locations, cultural contexts, school levels) and disciplines (mathematics and science) to create a new teaching community, we found it necessary to revisit and clarify implicit educational purposes and principles. This need for shared understanding led us to rethink the starting point of LS, particularly for participants from diverse teaching contexts. The main practical finding was the need for teachers and researchers to deconstruct the usual Lesson Plan structure, which is typically used freely at the classroom or school level. This involved thematizing daily practices based on the national curriculum, existing Lesson Plans, and new insights from the theoretical phase of the PD. We elaborate on this outcome by describing the *Lesson Plan Design* phase, which bridges, on a meta-level, the first (theoretical) and second (LS) phases of the PD. A case study supports the importance of this design phase as a necessary link between the two. Unlike the traditional Italian approach to LS (Bartolini Bussi and Ramploud, 2018), which assumes a pre-existing Lesson Plan, our perspective aligns with the Japanese approach of *Jugyo Kenkyuu* (Fujii, 2014), treating the Lesson Plan as an open research proposal and, thus, a fruitful research setting.

2 Macro and micro needs: Italian schools and teachers' perspectives

To understand the Italian context, particularly the intersection of cultures, traditions, and languages in mathematics classrooms (Spagnolo and Di Paola, 2010), it is crucial to analyze the available data [ISTAT (Italian National Institute of Statistics), Eurostat (European Statistical Office), OECD (Organization for Economic Cooperation and Development)]. These data highlight teachers' needs and support our decision to focus on multicultural and multilingual themes as strategic for the future of society and school systems (2030 Agenda, Goal 10).

The data reveal that approximately 10% of the student population in Italy has a non-native Italian background (Figure 1). However, there is a significant drop in the number of non-native Italian students at the high school level, nearing the end of mandatory education. This trend underscores students' vulnerability and the education system's inability to value these students effectively. Figure 2 compares the dropout rate in Italy to EU-27 (European Union-27) countries, showing that although the drop rate has decreased, the rate remains too high, especially for non-native students, highlighting systemic inequities. This is a key standpoint for valuing how inclusive and fair the Italian school system is. While progress is visible compared to earlier data (2004) and aligns with broader European trends, the challenge of inclusion remains pressing, even across Europe. This will be a topic of discussion, work, and research by public opinion, schools, universities and education institutions even more in future. Thus, the road to *an* (one of the many possible) inclusive "model" (or approach) of teaching and learning—especially in mathematics—is still long and just at the beginning. In Italy, the inclusive approach for non-native students is guided by the concept of *Interculture* (Beacco et al., 2016). This idea holds that the dialog and dynamic relationship between cultures are positive values; cultures are not fixed, and people are not bounded by their culture, as would be in the traditional multicultural perspective. In line with the *personalization* perspective assumed by the Italian Education Ministry, the focal point is to switch from culture to the individual (MIUR, 2012, 2018), without erasing the students' cultural heritage in favor of the host culture, as would be in an assimilative approach. The shift moves from a generic "for everyone" to a three-dimensional "for each one." Nevertheless, due to the lack of ministry guidelines involving the intercultural perspectives, as a pedagogical point of view, on particular disciplines, especially STEM (Science, Technology, Engineering, and Mathematics) ones, we argue that the most efficient answer to the nowadays changing situation is a *bottom-up approach*, focusing on in-service teacher training for (mathematics) teachers (Bianco et al., 2023). This led to the creation of an online training environment (including an LS circle) in which to dialog, discuss ideas and issues, and design and share teaching practices. The strategic role of these two steps is exactly to focus on designing and rethinking, as a community, new teaching practices with an inclusive aim, responding especially to the needs of the teachers who teach and students who live in multicultural and multilingual contexts (Di Paola, 2016). The findings from the Teaching And Learning International Survey (TALIS, 2014, 2018) underscore these needs.

The TALIS (2014) data (Table 4.7, p. 63) tell exactly this: except for the professional development concerning the use of technologies and the study of classroom context, the professional development labeled as "didactics in multilingual and multicultural contexts," which is included in the broader "teaching to special needs students," is for the Italian teachers the most pressing need and ranks as twice as high for Italian teachers compared to the TALIS average. On the other hand (Table 4.6, pp. 61–62), training sessions on this topic are infrequent and rank second to last in frequency, despite high appreciation from attendees. These data point out how the teaching community shows a clear need but not a subsequent awareness: due to the limited training proposals and/or the practical priorities of the teachers (this is a

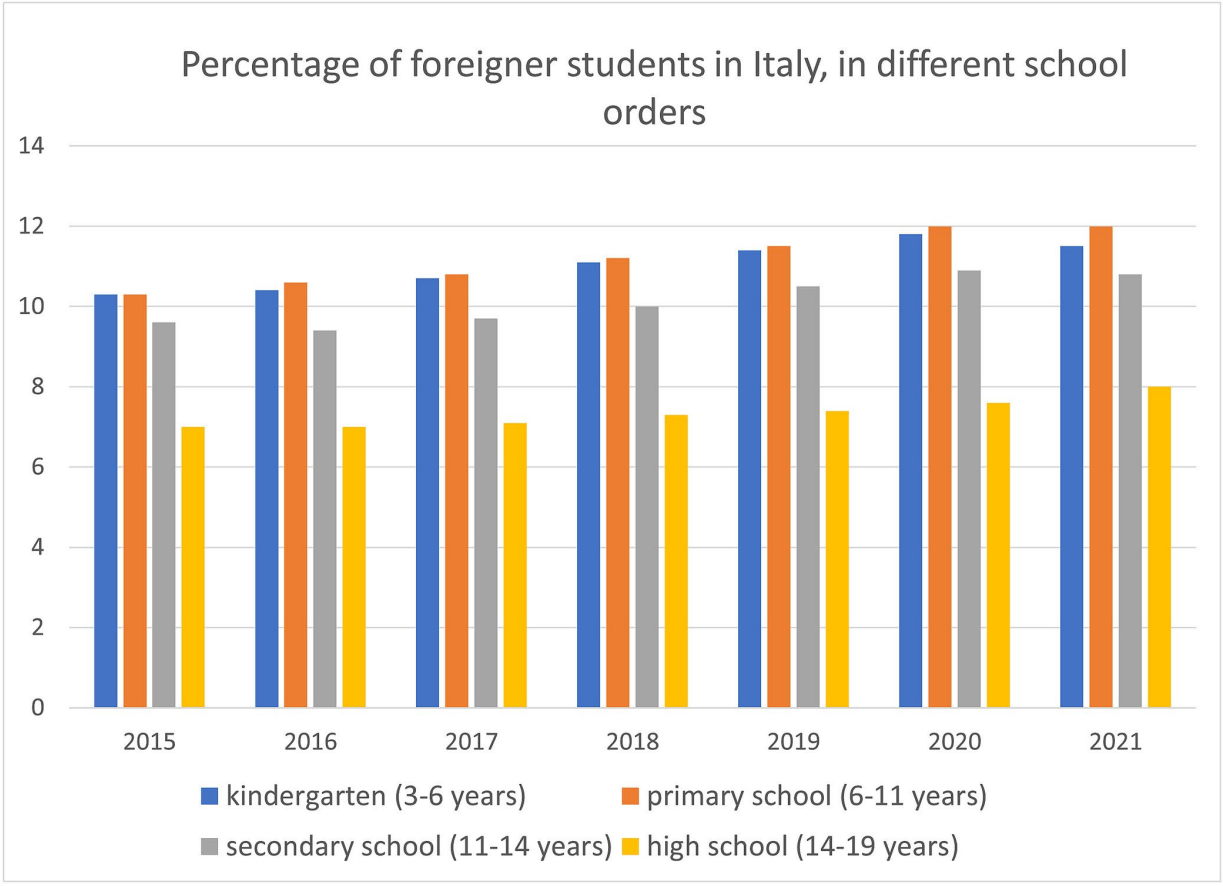

FIGURE 1
Non-native students in Italian school. Elaboration based on Eurostat Data Browser and ISTAT data.

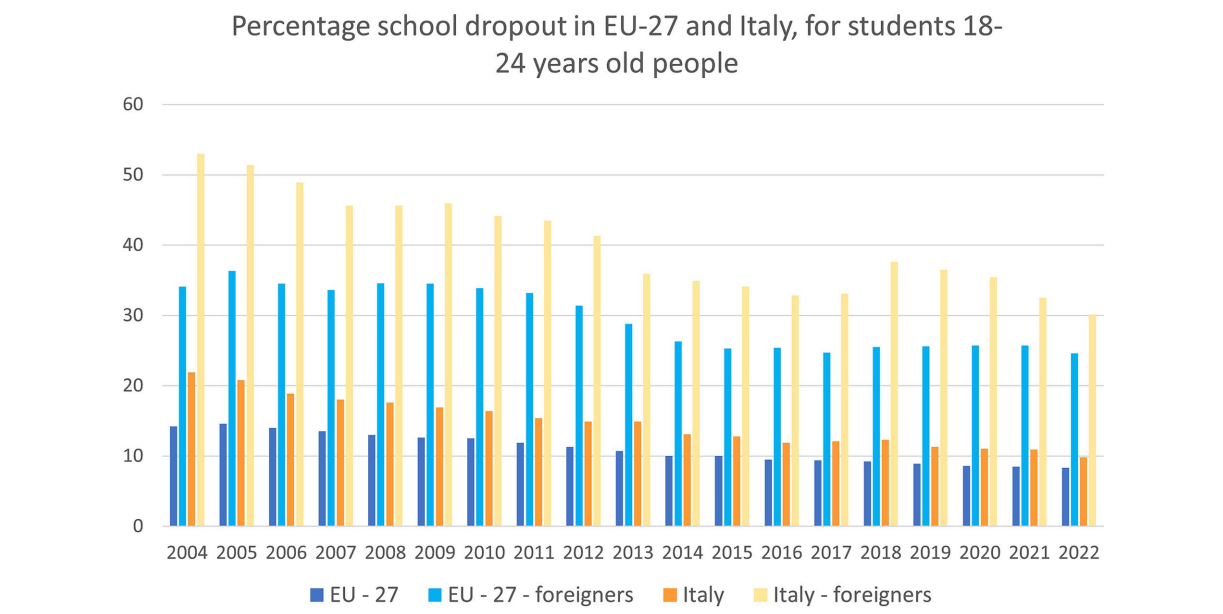

FIGURE 2
Non-native students' dropout. Elaboration based on ISTAT data.

cause-effect chain), professional development courses focused on the topic of cultures and languages are not popular. Therefore, this need for training and more in-depth study/work does not raise any clear questions to address, so there is no draft answer. The latest TALIS (2018) data, which focused on middle school teachers, highlighted the same trend: only 28% of teachers reported having received training on "didactics in multilingual and multicultural contexts" as part of their professional development (Table 9.b, p. 12). Similarly, only 26% mentioned having received such training during their formal education (Table 2.a, p. 2). Exactly as in the previous survey, "didactics in a multilingual and multicultural context" is the fourth highest needed professional development area; lastly, less than one fifth of the Italian teachers feel "well prepared" or "very well prepared" in this area (Table 2.b, p. 3). In all these items, Italy lags behind the average for the TALIS, OECD, and EU countries. Nevertheless, on the topic of "teaching classes with students of different cultural backgrounds" and addressing "the challenges that a multicultural classroom poses," 80% of Italian teachers felt "quite" or "very much" able, in contrast with the lower self-efficacy TALIS, the OECD, and the EU countries averages (Table 18, p. 27). This disconnection between perceived self-efficacy and the need for professional development suggests a need for targeted interventions from the bottom.

Our approach bridges this gap through two phases: the first focuses on the theoretical foundations of PD, and the second emphasizes practical outcomes through an LS structure. This combined path aims to build a shared professional awareness and competency in "teaching mathematics in multilingual and multicultural contexts." The following sections explore these meta-level aspects through the Research Problem and Questions.

3 PD path: an answer to teachers' needs

Before presenting the theoretical framework of the PD path, it is important to outline some details about the teachers involved in the PD. These details will help contextualize the discussions about the LS. Figures 3–5 illustrate key aspects: the geographical locations of the teachers, the school levels they teach, and the cultural diversity within their classrooms. These data highlight the diversity of teachers' backgrounds, needs, and goals, as well as the varied cultural and linguistic contexts of their students. Despite these differences, the teachers shared a common drive: the need to discuss, collaborate, share resources, and overcome daily classroom challenges. To support this, we adopted an online approach that maximizes flexibility, accessibility, and inclusivity. The online meetings allowed teachers from different areas (Figure 3) to participate, discuss their contexts, receive theoretical input, and give practical feedback. A key aspect of these meetings was identifying and addressing the structural training needs explicitly highlighted by the participants (Figure 6).

3.1 The online professional development path

The PD path engaged 57 teachers across all school levels, focusing on urgent topics such as inclusion and teaching practices in multicultural and multilingual contexts. This pilot PD program was conducted entirely online and involved teachers from areas with high

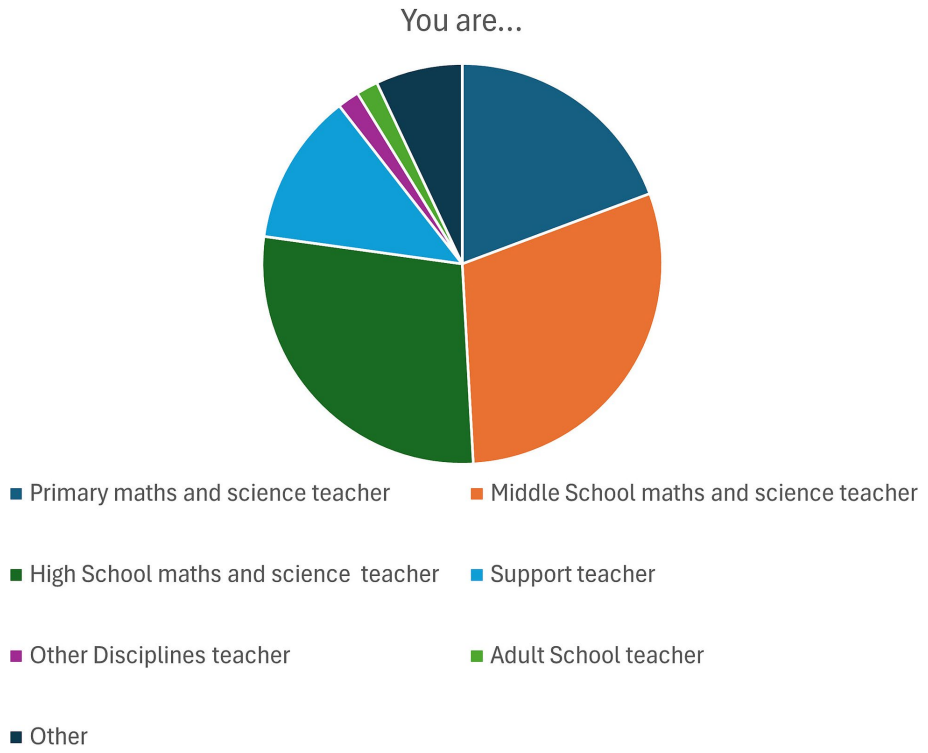

FIGURE 3
The school order of the teachers.

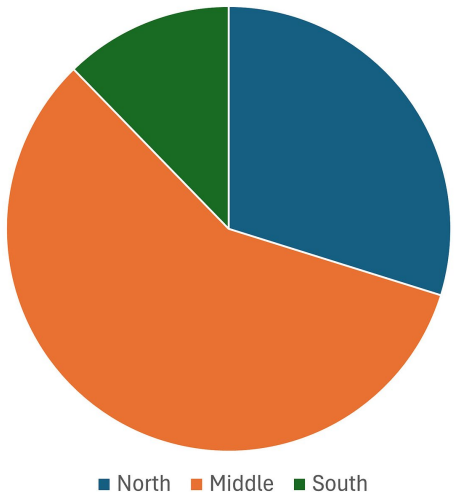

FIGURE 4
The geographical distribution of the teachers.

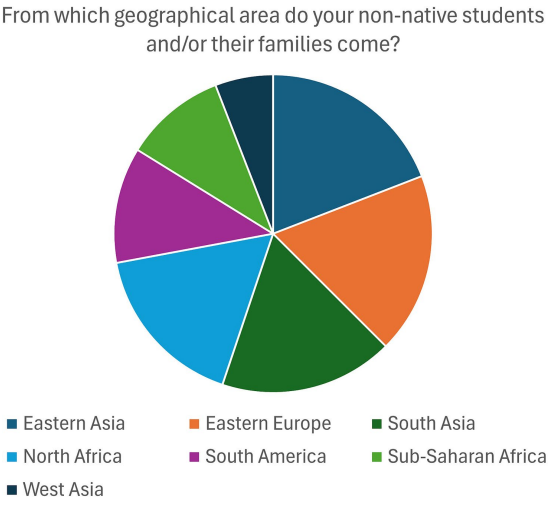

FIGURE 5
The culture present in the classrooms of the teachers.

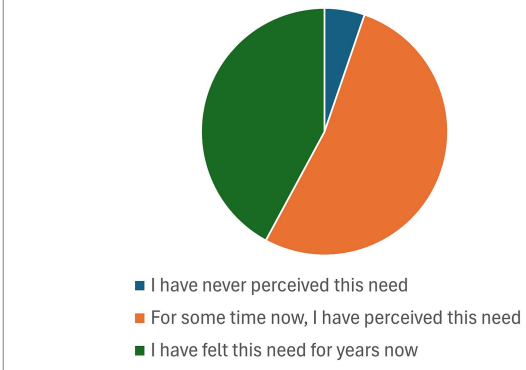

FIGURE 6
The teachers' needs and their persistence.

proportions of non-native Italian students. Teachers applied through an online form that was disseminated in autumn 2023.

The PD aimed to address open problems and share practical suggestions for teaching mathematics in multicultural and multilingual classrooms. Each training activity emphasized building cultural and linguistic sensitivity and awareness among teachers and tailored (from and) to their local contexts. The methodologies and contents proposed during the path were therefore readjusted in relation to the needs and origins (languages, family traditions) of the classroom context. The first PD sessions also addressed historical, linguistic, and cultural dimensions alongside mathematical topics. This interdisciplinary focus equipped teachers with tools to analyze their students' experiences, challenges, and expectations—whether the students had recently arrived in Italy or were born into non-native Italian families. We will examine how this interdisciplinary perspective has been integrated into the practice of one of the teachers, Sabrina. Based on the teachers' input (Figure 5), the content was divided into two cultural-geographic themes: the *Far Eastern* branch, which focused on Chinese mathematics traditions, and the *Maghreb* branch, reflecting North African contexts. These branches addressed the most prominent cultural groups represented in the classrooms of the participating teachers.

3.2 Structure of the online PD path

In this section, we briefly outline the structure of the teachers' training path, focusing on the theoretical outline of cultural aspects as an approach to LS (the first phase) and the actual LS (the second phase). The training path consisted of 25 h. The *approach to the LS* phase lasted for the first 9 h, divided as follows: 4 h dedicated to theoretical perspectives from research and schools on inclusion and multilingual/multicultural issues (meetings 1 and 2); following a brief overview of the current Italian situation (meeting 1), there was a comparison of educational systems from diverse cultural contexts, beyond Western paradigms, highlighting historical-epistemological aspects connected with teaching and learning of mathematics (meeting 3); meeting 4 included 1 hour devoted to analyzing teaching materials (especially textbooks) and school curricula from the traditions involved (e.g., Eastern tradition) for primary, middle, and high school levels. For more details, see Bianco and Di Paola (2023).

After the initial theoretical section (*approach to LS*), 16 h were allocated to the collective co-construction—aligned with the LS perspective—and shared design of inclusive materials and practices. These designs started from the Lesson Plan and aimed at multicultural and multilingual classrooms, beginning with topics suggested by participants (teachers and researchers) for various school levels. This second phase is ongoing; here, we discuss the preliminary results from the initial stages of this LS. During the collective design of teaching materials (LS phase), the authors participated in defining and structuring materials with teachers from different grades and institutions. These groups were composed of four to five people and were organized according to school level. To promote a longitudinal perspective, some meetings were open to all school levels, as in the

approach to the LS phase. Conversely, the final meetings were tailored to specific school levels to ensure that the designs were appropriately adapted.

4 Theoretical background: rethinking cultural aspects in teaching

Here, we present tools useful for addressing the Research Problem and Question, enabling both practical and theoretical engagement with the case study. The underlying theoretical assumption is that culture and context are integral to learning and teaching. Mathematics, in particular its teaching, is influenced by cultural contexts: "Even if mathematical truths are universal, that does not mean that mathematics education [and mathematics as a historical-situated discipline] should ignore the individuality of the learner, nor the social and cultural context of education" (Bishop, 1991, p. 10). This seemingly straightforward assertion deepens our understanding of teachers' choices during the LS process. Indeed, the cultural backgrounds and experiences of teachers, when shared in a group setting, can evolve (Gunnarsdóttir and Pálsdóttir, 2019). An open group environment fosters change, allowing teachers to confront new educational challenges, discuss their own assumptions, and thematize their culture.

Cultural Transposition and Interculture bridge the culture, assumptions, and values (implicit or explicit) of teachers with those of students and the broader educational context. Multicultural and multilingual contexts necessitate re-examining the culture of teachers, not only among themselves (e.g., during LS experiences) but also with their students, in the classroom. To prevent a static or generic multicultural perspective, deep reflection on factors influencing teaching and learning mathematics—such as language differences, cultural background, socio-economic status, and interpersonal relationships—is crucial. This analysis helps define and promote a truly inclusive educational environment and innovative teaching practices. Cultural Transposition and Interculture offer frameworks for understanding how teachers can explicitly incorporate cultural aspects into their evolving practices for teaching mathematics. These frameworks are applied to the case study to analyze the transition between the approach to LS and the actual LS phases in terms of cultural transposition and intercultural approaches.

4.1 Cultural transposition

The idea of Cultural Transposition (Mellone et al., 2019, 2021) is that encountering another cultural tradition can foster disciplinary awareness and improve teaching practices. Traditional problems addressed within (bounded and) localized contexts can be reinterpreted through a different cultural lens. Moreover, challenges in one context often mirror those in others. When this encounter occurs in the classroom, mediated by a student's "own culture," it becomes authentic rather than theoretical. Thus, the disorientation and re-thought given by the contact with another cultural tradition ("the other"), described by the Cultural Transposition, do not occur just on an abstract level, but thanks to the authentic culture of the students in the classroom. This real "contamination" shifts the frame from a vague Cultural Transposition to concrete intercultural engagement. While Cultural Transposition offers a clear starting point for reflection, it is insufficient for inclusive teaching aimed at engaging every student during the learning process, starting from the bottom, from the students' culture (Bianco et al., 2023).

For example, the Chinese practice of Teaching with Variation (literally: "changing-form teaching") (Sun, 2011; Gu et al., 2004) focuses on problems' variations and invariances by varying problem conditions, solution processes, or applications on similar problems, emphasizing structural understanding. Sun (2011) highlighted strategies such as "One Problem, Multiple Solutions" (OPMS), "Multiple Problems, One Solution" (MPOS), and "One Problem, Multiple Changes" (OPMC). This practice can serve as content for Cultural Transposition, where the adaptation of teaching practices from one geographical context (e.g., Eastern Asia) to another (e.g., Italy) supports teachers' professional growth. However, if the teachers' purpose is to *learn from their own students* and their respective cultural and linguistic experiences and backgrounds, a further step is necessary. Fostering deeper engagement with students necessitates shifting the focus from teachers (as central in Cultural Transposition) to students, as prioritized in *Interculture*.

4.2 Interculture

An intercultural perspective promotes dynamic practices tailored to individual contexts and cases, emphasizing inclusion "for each one" (Beacco et al., 2016). Interculture thrives on daily encounters with alterity and pivots attention from broad cultural views to the individual experiences of students. It challenges the notion of "The Culture," instead valuing how culture is uniquely lived, practiced, and adapted by each individual and differently by different people. However, it is not enough, especially if the aim is to act to value and include each diversity into the class's imaginary and activity. Culture is not an abstract concept but a daily experience, so what matters is the set of cultural experiences (familial, etc.) inside or around the students, the culture inside and lived by each person. To start from the diversities (beliefs, habits) is seen as a deep need by teachers and can lead to more inclusive teaching practices.

Highlighting various perspectives on the same topic fosters open discussion, integrating rather than merely comparing cultural viewpoints. However, this is just the starting point; understanding students' backgrounds helps optimize their learning in a new environment. Therefore, interculture becomes a cornerstone for concretely rethinking classroom practices. Cultural Transposition, as discussed above, offers a methodological approach to fostering intercultural teaching and learning practices. This approach provides a foundation for dynamic, context-sensitive pedagogy that integrates diverse cultural experiences into the educational process.

5 Lesson Study for teachers' training

This presents the necessity of adopting an LS approach to strongly support teachers involved in the online training path while also providing them with sufficient autonomy. In particular, we emphasize the importance of sustaining teachers—especially those unfamiliar with LS—while they collaborate with colleagues and researchers to

develop new teaching materials with a clear pedagogical aim: the inclusion of non-native students.

As suggested in the introduction, the double focus on the teacher as the strategic change agent and on students as the center of the classroom practice leads to teachers' training paths, in this case based on LS, and rooted in the research, according to the *research for the innovation* paradigm (Arzarello and Bartolini Bussi, 1998). In our perspective, LS connects the research on the issues of mathematics teaching/learning in multicultural classrooms—discussed during the first phase of PD—with the research on teachers' professional development. Consequently, LS is not considered here as a theoretical framework, research methodology, or research object, but rather as a professional development methodology, aligning with Xu and Pedder (2014) and Bartolini Bussi and Ramploud (2018).

The key steps and aims of this methodology, following LS literature (Huang et al., 2019), in our case have been:

1 Goal Setting: Selection of a thematic core, type of exercise, or problem by participating teachers via questionnaires, chat, or shared online documents, further developed by researchers.
2 Collective Planning: Collaborative planning and refinement (Lesson Planning) of teaching practices and Research Lesson objectives by teacher groups, based on what emerged from the experiences of each teacher, through comparisons within the group of teachers, also in relation to shared documents and the suggestions from the researchers.
3 Teaching Intervention: Proposal of an educational intervention in the classroom by a teacher (Research Lesson) in accordance with what has been agreed upon by the group. The researchers and colleagues (from the institute) attend classroom practice and take note of any critical issues related to teaching choices.
4 Observation and Review: Evaluation of the outputs of teaching practice according to different points of view: how the Research Lesson has been taught and how it has been perceived by the students. Particular attention is also paid to the learning induced in students based on semi-structured interviews and protocols designed by the group of teachers and researchers and subsequently shared with the group itself.
5 Discussion and Reflection: Review of the teaching practice and Lesson Plans by the entire group of teachers during subsequent meetings.
6 Dissemination: Sharing of teaching practices with other teachers external to the community so defined.

Steps 2–5 are cyclic, adapting to the evolving needs of participating teachers. The iterative process enables—thanks to the online dimension—a dialogical accumulation of knowledge and proposals. These diversities and similarities provide a foundation for meaningful dialog. As assumed by LS (Inprasitha et al., 2015), one of the key aspects is the phase of collective planning (2), in which the discussion between teachers leads to a shared Lesson Plan. For this reason, our research specifically focuses on this moment, rather, a moment slightly before step 2, where the dialogic nature of LS is stressed and various teaching needs emerge. In the following section, we discuss the Research Problem and how we addressed this specific moment in the process of LS.

6 Research problem and question

The Research Problem encompasses three interrelated aspects: (1) *How to assist teachers in initiating an LS cycle, particularly when approaching and using a Lesson Plan for the first time*; (2) *How to operationalize in an LS cycle the theoretical insights of a professional development path*; (3) *How to collaboratively create shared Lesson Plans within an online community of teachers from different geographical areas*. The first aspect (1) implicitly assumed the utility of providing a fixed Lesson Plan: we changed our perspective regarding this point, and this led us to reinterpret the role of Lesson Plan with the aim to make possible even (2) and (3).

Standardized Lesson Plans, tailored to culturally distinct teaching contexts, are addressed through a multi-step approach: (a) Strengthening the LS perspective with explicit attention to pedagogical assumptions; (b) Building these pedagogical reflections on theoretical insights introduced during the first phase of PD. The structure and content of the PD are shaped both by teachers' implicit needs and as a starting point for implementing changes in teaching practices through LS. The first phase of PD is seen as an *approach to LS*, thus answering the question of "How to support teachers during their initial LS experience." LS's flexibility suggests that, especially in contexts exploring new teaching perspectives, structured support (e.g., drawing from Learning Study frameworks, Lo and Marton, 2012) might be beneficial. This led us to emphasize a space and moment where Lesson Plans can be created collaboratively, within a community, rather than providing predefined ones. Our approach reflects the idea that Lesson Plans should be shared but flexible and not fixed, created through collaboration rather than merely adapted, within the community, as seen in other models (Bartolini Bussi and Ramploud, 2018; Part II in Huang et al., 2019).

6.1 Research setting

Between the stages of goal setting (Step 1) and collective planning (Step 2) of LS, an unanticipated obstacle arose: for teachers new to LS, planning a lesson collaboratively proved challenging, especially when incorporating perspectives such as students' language proficiency and cultural heritage, which had been introduced at the beginning of PD. To address this issue, the researchers developed a summary table of key and novel steps, informed by the theoretical suggestions of the PD.

We analyzed the dynamics of this preparatory stage, termed *Lesson Plan Design*, which marks the development of starting points (e.g., *Lesson Plan Templates*) for building collective Lesson Plans. The *Lesson Plan Design* is the setting for our case study. This phase provides an opportunity for teachers to reflect on lesson structures and collaboratively design a shared tool to design Lesson Plans; therefore, it means a departure from routine teaching practices. Therefore, *Lesson Plan Design* is a real opportunity for in-training teachers to develop awareness, implying a focus on new levels. This approach emphasizes new dimensions beyond subject content, such as language, culture, and interpersonal relationships, which teachers may have rarely considered explicitly. The planning or studying phase is emphasized in LS literature and beyond (Brahier, 2013): "Kyozaikenkyu (planning) is [the] most necessary part for LS" (Inprasitha et al., 2015, p. 85). Kyouzai Kenkyuu (literally,

Kyouzai—teaching materials; Kenkyuu—studying, the same term in Jugyo Kenkyuu—Lesson Study) traditionally bridges goal setting (Step 1) and collective planning (Step 2), focusing on content. In contrast, *Lesson Plan Design* centers on community-driven development of pedagogical perspectives aligned with shared goals, offering flexibility—being less content-bounded—in designing Lesson Plans and teaching resources. The two perspectives share a focus on studying and working jointly before planning. In the next sections, a deeper description of the *Lesson Plan Design* phase will be provided.

6.2 Research question

Therefore, our problem has been reframed as follows: *How to create a shared Lesson Plan that is flexible enough to be both theoretically grounded and applicable in diverse local contexts?* This is clearly a design-level shift. This pragmatic and educational problem prompted us to consider the idea and need, taken as a theoretical assumption, of the *Lesson Plan Design* phase, which will be the research setting. Therefore, instead of observing teachers working on a pre-designed Lesson Plan, we (teachers and researchers) agreed on co-constructing a shared preparatory stage before the Lesson Planning phase. For sure, LS is a privileged lens and environment from which to observe—from a research viewpoint—the changes in the teachers: "Lesson study makes various types of knowledge more visible…thereby enabling teachers to encounter new or different ideas and to refine their knowledge" (Lewis et al., 2009, p. 286). Thus, collective work is both a pragmatic opportunity for professional development and a research setting, especially during the design and planning phases (Akiba et al., 2019). Here, through the case study, we explore two dimensions linked with the three points of the Research Problem above:

Cultural aspects: *How do teachers integrate cultural considerations regarding learning and teaching of mathematics in their teaching design and planning during Lesson Study?*

Planning aspects: *How does the Lesson Plan Design phase lead to this phase?*

The aspects are addressed using the theoretical framework, as the following research question:

> *RQ:* What roles do cultural transposition and interculture play in Lesson Plan Design, during a Lesson Study focused on teaching and learning mathematics in multicultural and multilingual classrooms?

7 A crucial point for creating a Lesson Study community

7.1 From a community of practices to a community of teachers and researchers

Our initial educational goal was to foster a community of in-training teachers who were sensitive to the complexities of teaching in plural (multicultural and multilingual) classrooms. These complexities could be explored through theoretical insights from the PD or through the intuition and experience of the teachers. However, neither approach alone was sufficient to establish systematic, inclusive, and intercultural teaching practices that could be shared among teachers and adapted to various contexts. As we began working directly on the Lesson Plan, we identified a gap between the PD's theoretical suggestions, the typical teaching practices, and the ultimate goal of designing and implementing inclusive practices. Bridging this gap required collaboration within the entire community of educators during a foundational phase, prior to the creation of shared teaching materials, like Lesson Plans.

The Research Problem and Research Question emerged from teachers' need for a pragmatic tool to develop teaching materials suited to their specific contexts. The Research Question reflects the students' needs and is shaped by the themes, goals, and sociocultural contexts of their classrooms during the early stages of LS. Initially, many teachers perceived the Lesson Plan as a template to be "filled in." They found it misaligned with their needs and viewed it as an external imposition rather than a starting point for designing teaching materials. Teachers expressed the need for *a* Lesson Plan that was practical, adaptable to their varied contexts, and aligned with the theoretical and shared suggestions of the PD. Addressing this situation required not only practical solutions but also a theoretical rethinking of LS.

7.2 From teachers needs to a design perspective on Lesson Study

In LS, the Lesson Planning phase follows the goal-setting stage. However, we identified the need for a preliminary design phase that thematizes not only the Lesson Plan content but also its structure. To address this, we focus on a new LS phase (step 1.5, see Figure 7) of sharing the *Lesson Plan Template* and propose to modify the canonical LS model (near what is discussed above) by emphasizing foundational aspects aimed at fostering shared perspectives (e.g., defining inclusive practices) within the teachers' community. This additional phase creates a space fundamental to the first approach to LS, which is sometimes undervalued by the LS literature (Fernandez and Yoshida, 2004).

This phase focuses on the collective discussion of the Lesson Plan's structure and objectives before actual planning begins. Such groundwork is particularly important in challenging educational settings and training programs. The joint development of a *Lesson Plan Template* serves not only practical purposes (creating a community document) but also fosters community-building. Before step 2, we believe there is a preliminary step in which the structure and aim of the Lesson Plan should be collectively discussed. This step had not, in fact, been previously envisaged, but we consider it to be necessary in light of the teachers' difficulty in initiating a shared and online path with colleagues from other institutes and with researchers, according to a new methodology (LS), and with challenging, even if explicit, pedagogical aims. In these cases, joint work on principles and design levels is useful not just in practice: shared documents (as is in LS), which are enriched through collective reflection, are crucial for nurturing a sense of belonging among participants. The collective creation of this document (*Lesson Plan Template*) can therefore allow each teacher to feel and be(come) part of the community. This work on the *Lesson Plan Template* during the *Lesson Plan Design* phase is an occasion to focus concretely on educational aims. Thus, the *Lesson Plan Design* phase on the *Lesson Plan Template* is the premise to the *Lesson Plan work* on an *empty Lesson Plan*: before the empty Lesson Plan, there is the creation of this

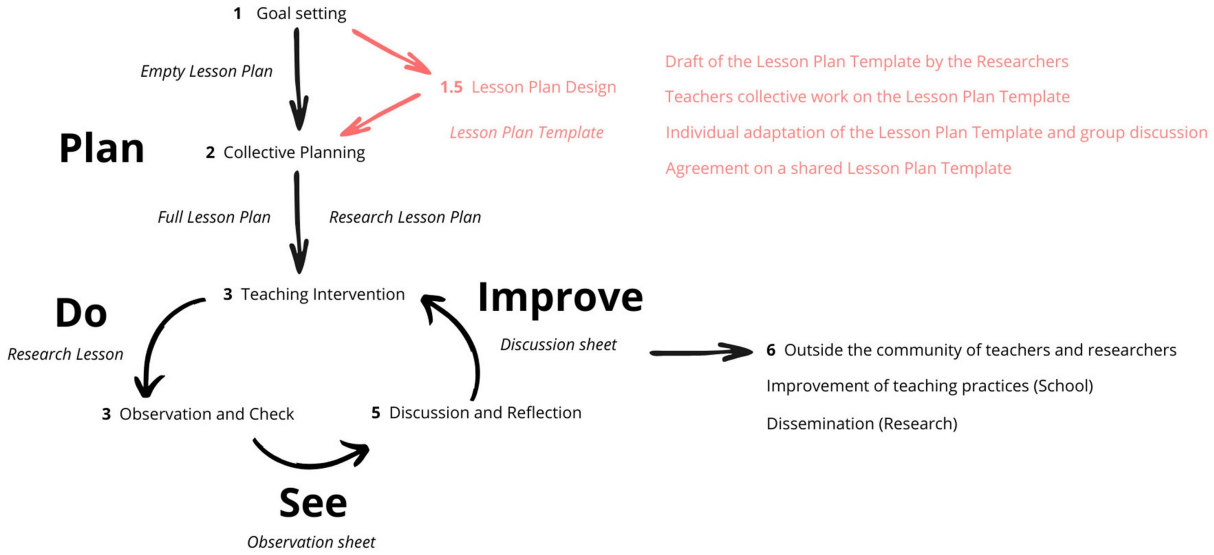

FIGURE 7
The Lesson Plan Design phase in the Lesson Study cycle.

structure, exactly as the "filled" Lesson Plan follows the empty template. The emphasis is on process, not the product (which Lesson Plan and/or which *Lesson Plan Template*)—on creating a shared disciplinary path and vision regarding educational methods and goals. This approach resonates with the principles of Collaborative Research Lessons (Takahashi and McDougal, 2019; Huang and Shimizu, 2016) but diverges in its subsequent steps, as our Lesson Plan is less structured than a typical Research Lesson *proposal*. Furthermore, adapting a culturally rooted practice like LS to new contexts—especially involving a national community of teachers rather than a single institution—raises significant challenges (Manolino et al., 2020). Here, we are not "just" moving a cultural teaching practice from one context to another (Minisola and Manolino, 2022), as is common for several LS perspectives (Ramploud et al., 2022); we are also changing the community of educators involved, from the teacher of one institution (as common for LS) to teachers from all over the country. Therefore, we are moving far from the initial assumptions and development conditions of LS as a (global) cultural practice (Fernandez and Yoshida, 2004). A fixed structure on which to develop the Lesson Plan is not suitable for this community of teachers; we need another phase, a backward step, to create a community from the sharing. We need to rethink the structure (principles, aims) and design of the Lesson Plan and explore what is behind the structure of the Lesson Plan: we (teachers and researchers) need to work on, and not just assume as given, the *Lesson Plan Template*. Teachers' attitudes are documented in the *Lesson Plan Templates* and draft of the Lesson Plan evolution through collaborative efforts and peer feedback. However, the shared process of reflection and collaboration creates a strong foundation for community-building.

7.3 The four Lesson Plan Design phases

The four steps according to which we conceived the *Lesson Plan Design* phase are:

1 First proposal by researchers of a deliberately primitive and partial *Lesson Plan Template*;
2 Group and individual work by teachers and researchers on the *Lesson Plan Template;*
3 Sharing (from individual to the group) and adaptation (from the group to the individual), according to personal choices, of this *Lesson Plan Template.*
4 Definition of a unique *Lesson Plan Template* shared by the group.

Points 2 and 3 are not qualitatively different but can be split on a pragmatic basis. This approach highlights a shift in the emergence of individual differences: in LS, these differences arise during Lesson Planning; here, they manifest earlier, during the *Lesson Plan Design* phase, reflecting the need for flexibility and adaptability in diverse educational contexts. In the following sections, we focus on the first three steps, giving value to the process of *Lesson Plan Design*, rather than the product of the final step, the *Lesson Plan Template*.

8 A case study: working on the Lesson Plan Template

8.1 Methodology to analyze the Lesson Plan Design phase

In a collegial setting, the researchers—authors of this paper—presented a *Lesson Plan Template* to the teachers' community. This template incorporates insights from the theoretical phase of PD and suggestions from the teachers. Recognized as a shared preliminary step in *Lesson Plan Design*, the template provides a foundation for teachers to create common Lesson Plans. Teachers worked autonomously, tailoring the provided templates to the chosen mathematical content. During this phase, the teachers had the

opportunity to collaborate with peers in the absence of the researchers. Subsequently, the researchers analyzed the resulting templates using the following criteria:

1 Alignment of each teacher's *Lesson Plan Template* with the shared template;
2 Coherence between the *Lesson Plan Template* and the mathematical content selected;
3 Presence in the *Lesson Plan Template* of culture- and language-centered elements, especially in terms of implementation in multicultural classrooms.

This analysis underscored the process-oriented nature of *Lesson Plan Design*, emphasizing collaborative creation over rigid adherence, which may occur using a single template and not necessarily shared with and by the teachers. This emphasizes how the stress of *Lesson Plan Design* is not the product. Each teacher's template, if aligned with these criteria, became *a* potential starting point (not *the* template) for the entire community. To explore this phase in-depth, we focused on Sabrina's construction of the *Lesson Plan Template* as a case study to deepen this phase of LS, the *LS Design* phase. Her template became *the* template for all the communities, based on their chosen goal.

8.2 A case study of a Lesson Plan Design phase

This section outlines the co-construction of a *Lesson Plan Template* through the first three *Lesson Plan Design* phases within the group of teachers and researchers. It highlights, first, how this space is suitable for a re-discussion and transition from theoretical insights to practical design (see Cultural aspects in the Research Problem); second, the pragmatic utility of an intermediary design step linking a theoretical teacher's training moment and LS (Xu and Pedder, 2014) (see Planning aspects in the Research Problem). The case study illustrates how the PD's focus on cultural aspects influenced Sabrina's design and how *Lesson Plan Design* (step 1.5) leads to Collective Planning (step 2). This shows how *Lesson Plan Design* can be interpreted as a starting point for the construction of a community of teachers and how to link the design and planning phases, according to the teachers' contexts and needs and the theoretical background discussed in Section 4.

8.2.1 Phase 1

The researchers began by analyzing teachers' needs and contexts (explored using online forms and during the online meetings), as well as ministerial documents, curricula, and school practices. They created a preliminary *Lesson Plan Template* as a table, highlighting key aspects to discuss, including characteristics of the class context, common obstacles, the topic or area of interest (e.g., Pythagoras or plane geometry), and the kind of teaching activity (e.g., workshop activity, classical lesson, etc.). To work on a design level, behind/before Lesson Plans, the common Lesson Plan tables were juxtaposed with the school programming boards, and some common aspects emerged. Again, the focus is not on the particular table or phase of the *Lesson Plan Template* co-construction; the key aspect is the passage between the different tables or stages of the *Lesson Plan Template*, toward a shared approach, but not necessarily a unique *Lesson Plan Template*. We (teachers and researchers) chose to start with an *area* of interest and thematic *topic*/nucleus; then, we added *starting points* ("level" of the classroom at the beginning of the teaching intervention), *expected goals* (according to Italian ministerial documents), and divided them into *knowledges* (what students should know) and *abilities* (what students should be able to do); these can be strictly disciplinary or general (e.g., communication). Finally, we added a little space for the *evaluation* (Table 1). These aspects are common to the structure of the *Lesson Plan Template*. To stress the theoretical perspective and approaches raised during the PD, we inserted three strategic dimensions/lenses to interpret the above categories (columns) and to teach effectively in multicultural and multilingual contexts with an intercultural purpose: *disciplinary (mathematics) level*; *linguistic level*; *sociocultural level*.

8.2.2 Phase 2

The second step involves the collective online discussion of the proposed *Lesson Plan Template*. Here, the study focused on the table provided by the researchers (Table 1) to be taken as a suggestion and not as the final structure to use to build the Lesson Plan. During the collective discussion, the *starting points* were reframed as *prerequisites*, a typical Italian name for "beginning knowledge," and the *evaluation* became *evaluation aspects and tools* (Table 2). During the online meeting, we (teachers and researchers) also stressed the role of teaching resources: school textbooks, curricula, ministry documents, and real problems. After the meeting, we reached an agreement on a less structured table, with a specific use of less commonly denoted and more evocative nouns: *context observation, area,* and thematic *topic* as starting information; *tools and teaching methods, where we start from, journey, where we want to go, expected obstacles, misconceptions and errors, feedback* (Table 3). In each of these broad categories, we put the aforementioned elements: *prerequisites* inside *where we start from*; *expected knowledge* and *abilities* inside *where we want to go; evaluation aspects, and tools* inside *feedback*. In parallel, we started to work on the content, which was beyond the pure designing. This demonstrates how the proposed *Lesson Plan Template* is a pragmatic starting point for actual work on a Lesson Plan. Workshop and group activities were the teachers' most frequent choices. Real problems were equally a

TABLE 1 Draft of the Lesson Plan Template.

Levels to care about…	Area	Topic	Starting points	Expected knowledges	Expected abilities	Evaluation
Disciplinary (mathematical) aspects						
Linguistic aspects						
Sociocultural aspects						

TABLE 2 Evolution of the Lesson Plan Template during the meeting.

Levels to care about…	Area	Topic	Prerequisites	Expected knowledges	Expected abilities	Evaluation aspects and tools
Disciplinary (mathematical) aspects						
Linguistic aspects						
Sociocultural aspects						

TABLE 3 Shared Lesson Plan Template.

Levels to care about…	Context observation	Area	Topic	Tools and teaching methods	Where we start from	Journey	Where we want to go	Expected obstacles, misconceptions, and errors	Feedback
Disciplinary (mathematical) aspects									
Linguistic aspects									
Sociocultural aspects									

topic of discussion (for instance: what does it mean to be a meaningful and "real" problem for non-native students?).

8.2.3 Phase 3

During the third phase, teachers worked on the shared *Lesson Plan Template*. We now focus on the specific choices (adaptations) made by one teacher, Sabrina. From the outset, Sabrina clearly understood the intercultural aim of the community and the needs of her students. She chose to focus on two historical problems: one from the *Liber Abaci*, a typical Western tradition, and the other from the *Nine Chapters of Mathematical Art*, the most famous ancient Chinese mathematics book, representing the Eastern tradition. The use of the history of mathematics guided her deep didactical analysis, which was supported by primary resources. Her goal was to illustrate, starting from meaningful historical problems, the analogy between the structure (both problems were real-world and word problems) and the mathematics involved in each. This approach addresses both the historical and cultural contexts. Using the shared table as a guide, Sabrina explicitly clarified her focus on the role of processes. To move in this direction, while still at a design level, she elaborated and modified the shared table, which marked the adaptation moment of Phase 3. She refined the meanings of the keywords used in the template, such as "didactical aims" (concrete outputs of the activity) and "goals" (aligned with ministerial documents), to make the vague phrase "where we want to go" more precise. Similarly, she clarified "pre-knowledge," tailoring it to the context and problems chosen, which enriched the general category "where we start from" (Table 4). In doing so, she has already begun transitioning toward lesson planning. Other previously shared aspects were recalled and made explicit in Sabrina's work: *area* (in which she wrote arithmetic and algebra), mathematical *topic* (wherein she put rational numbers, their representations, and ratios), *misconceptions, and expected errors* (left empty). Moreover, the *didactical approach* (where she wrote: participated lesson, cooperative learning, discussion in classroom), *teaching method,* and tools (she outlined: work in groups of three or four students). For each of these aspects, Sabrina transitioned from working at a design level on the table to working at a design level for the activity. By filling in the table with her choices, she began concretely planning the lesson. This transition between Step 1.5 (*Lesson Plan Design*) and Step 2 (Lesson Planning) became clear and natural. Finally, she began designing her activity by focusing on key points such as the historical/cultural context, pre-knowledge testing/prerequisites, feedback, group work, and class discussions on the problems, and finally, an explanation and collective discussion of the Rule of Three, the underlying mathematical principle common to both problems.

8.2.4 Phase 4

The template structured by Sabrina became *the Lesson Plan Template* for all group. This template will be used by the group for the actual Lesson Planning. We focused on the work of Sabrina to appreciate the dialectics between a teacher and the group during the different phases.

8.3 Analysis of Sabrina's Lesson Plan Template

Here, we analyze Sabrina's choices through the lens of the Theoretical Framework. Her work can be framed by the Cultural Transposition paradigm, which is particularly evident in her selection of teaching materials and her decision to parallel European and Chinese historical mathematical problems. This underlines her effort to connect two problems from distinct historical and cultural contexts that share a common mathematical core that can be addressed using the same mathematical tool: the Rule of Three. Using a mathematical common core as a unifying factor aligns with Teaching with Variation, particularly the OPMC (One Problem Multiple Changes) described by Sun (2011), wherein the mathematical aspect remains unchanged (invariant), while the context or shape of the problem varies (is subject to variation). The explicit and implicit insights introduced during the first phase of

TABLE 4 Sabrina Lesson Plan Template.

Levels to care about…	Area	Topic	Didactical approach	Teaching method	Pre-knowledge	Goals	Didactical aims	Misconceptions and expected errors
Disciplinary (mathematical) aspects								
Linguistic aspects								
Sociocultural aspects								

PD—variation and Cultural Transposition—emerged at the design level during LS. Variation shaped the choice of disciplinary content, while Cultural Transposition informed a broader perspective.

Moreover, the first draft of Sabrina's Lesson Plan reflected an implicit assumption of the PD: the intercultural perspective. This perspective helped Sabrina address her students' needs by bridging mathematical aspects from the curriculum with engagement and motivation aspects, centered on individual student experiences. This attitude, which Sabrina demonstrated innately, can be further valued, and strengthened through group discussions and interdisciplinary reflections.

Cultural Transposition and Interculture, which were introduced implicitly by researchers during the first phase of PD, were naturally induced and emerged as central themes during the first steps of LS. These concepts became the *fil rouges* that underpinned the development of the teachers' community. Reflecting on the joint work (phase 1) on the *Lesson Plan Template*, we see how the *disciplinary (mathematics) level* was addressed through variation, while the *sociocultural level* was framed by Cultural Transposition and Interculture. However, the *linguistic level*, which is a critical and complex component, requires further reflection.

Even if there is asymmetry in the roles of Cultural Transposition, Interculture, and variation, all three emerged in Sabrina's work. Cultural Transposition and Interculture served as guiding principles for the researchers during the first phase of PD, while variations were explicitly addressed by both researchers and teachers. This two-level approach allowed Sabrina to transcend cultural differences rooted in practices, leveraging these differences to build—on a meta-level—a mathematical dialog based on a shared mathematical core viewed from diverse yet equivalent perspectives.

9 Conclusion

For several years, schools and researchers have sought to—based on the needs of teachers, students, and their families—create forms of cooperation, moments of discussion, and support spaces centered on the theme of inclusion. These initiatives aim to structure personalized educational projects where the valorization of "other" cultures becomes a fundamental factor for the growth of the entire class group. Educational research, specifically Mathematics Education, has attempted to respond to these challenges by proposing theoretical and methodological frameworks centered on salient and strategic characteristics for effective school inclusion. Specific reflections on teachers' training regarding (inter)cultural issues have become increasingly present in the field of research (Spagnolo and Di Paola, 2010; Bartolini Bussi and Ramploud, 2018; Bianco and Di Paola, 2022) or in adjacent fields (Dotger and Burgess, 2021).

In this context, this paper offers a research contribution that we believe is of interest both to researchers addressing mathematics teaching/learning in an intercultural perspective (Beacco et al., 2016) and to those involved in teachers' education, referring specifically to LS as an approach for teachers' professional development. We designed a PD path inspired by Cultural Transposition (Mellone et al., 2019) and Interculture (Beacco et al., 2016), with a specific focus on Teaching with Variation (Gu et al., 2004; Sun, 2011). These theoretical perspectives also supported the second phase of the path, which was structured through an LS methodology (Huang et al., 2019). We worked with an LS approach to balance the training and updating needs of teachers—focused on specific teaching issues, such as the theme of cultures—with the need for less structured and more dialogic moments of discussion among colleagues.

Due to the diversity within the teacher sample, we felt the need to create a community of teachers, which is usually assumed to pre-exist in many LS studies. However, a community can only emerge when there are shared principles of practice and common goals. This culture of community needs to be built together. With this aim, we suggested an intermediary phase before Lesson Planning (step 2, as called in Section 5), during which thoughts and practices are shared to allow the community to emerge and become aware of itself. As we said, this paper is specifically focused on this moment.

The case study of Sabrina shows the reasonableness of our assumption regarding modifying the role and position of the Lesson Plan within the LS cycle. The Lesson Plan structure emerged after a collaborative work between Sabrina and the teachers' community as a product of a process. The analysis of Sabrina's work emphasizes the effectiveness of collaborative work in defining a Lesson Plan structure (such as a template) within a teaching community.

This *Lesson Plan Design* phase supports teachers in online communities and makes tangible the theoretical suggestions of PD, enabling them to be adapted to the specific contexts. Teachers belong to both a local community (geographically, as part of a school institution), the more stable one, and the online community (broader), the newer one. This dual level of teacher training could facilitate the spread of innovative teaching practices across institutions relatively quickly. This dialectic between the structured local community and the emergent online community aligns with one of the aims of LS: to innovate by reinterpreting and reconstructing the curricular and ministerial guidelines from the bottom up and in accordance with context-specific needs. In our case, the challenges shared across various contexts mean that local practices, solutions, and resources can be shared beyond the local level and improved—or built

upon—on a broader national or *glocal* level. Thus, the creation of this new step addresses our research problem. Given the flexibility of LS, this preliminary joint reflection serves as a foundation for creating a community, which is the starting point for any teaching innovation, as supported by LS principles. This process is enriched by theoretical suggestions and singular or local experiences, maintaining an openness to change without relying on fixed, immutable practices (such as a rigid structure for the Lesson Plan). In our case, we had to make explicit the various contexts and tensions and establish a common language—elements usually assumed to be shared but, in our case, needed to be built from the ground up. The *common* aspect of the community shifted from a common geographical setting and school institution to a common purpose (working interculturally toward inclusion) and educational issue (addressing multicultural and multilingual classrooms).

A change in the community leads to changes in the documents and tools used, such as the *Lesson Plan Template*. However, this does not necessitate a unified perspective among all members of the community. The preliminary open discussion does not aim to construct a single template but rather facilitates joint dialog. This flexibility makes LS adaptable for including diverse members, even within the same community, and it establishes the *Lesson Plan Template* as a tool for (re)creating or expanding teacher communities. The question of whether LS could be adjusted arose from our case study, in which the absence of a pre-existing community was evident. Here, the community had to be built through an earlier practical step. Since LS literature often assumes the existence of a homogenous community with needs (e.g., the cultures of students in a classroom), our context required work in this direction to create such a community. Moment 1.5 is both pragmatic and theoretical. Pragmatically, it creates a space for dialog—the foundation of building a community. Phase 1.5 is necessary when a community must be created entirely from scratch. The focus is not on the final product (e.g., a particular *Lesson Plan Template*) but on the process through which teachers and researchers align their perspectives. The *Lesson Plan Template* is a medium for *Lesson Plan Design*. In addition, the case study demonstrates how *Lesson Plan Design* is a suitable setting to observe the evolution of teachers' awareness and how it supports the transition to a first LS experience. This alignment facilitates the creation of documents designed for teaching practices with precise pedagogical and disciplinary goals.

The point we are drawing attention to is not the end point, which may be this or that Lesson Plan, but the process thanks to which teachers and researchers share similar perspectives, and thus how the community structures a document, a "template," for designing teaching practices with a precise pedagogical and disciplinary focus. In conclusion, shared documents remain strategically significant, in line with LS research (Dudley, 2014; Bartolini Bussi and Ramploud, 2018). However, the origin and contextual role of these documents must be thematized. In our view, these should emerge from dialog between teachers and researchers, be created collaboratively to address teaching and/or research needs, and embody the essence of LS. An instance of collaborative practice between teachers is one that Sabrina proposed and then was shared by the group. Through the example of Sabrina's *Lesson Plan Template*, this article highlights the *Lesson Plan Design* phase and the steps leading to it. The Lesson Plan so becomes both personal and communal because it originates from a shared template, chosen by the group, thanks to each contribution. Sabrina's work, while personal, is part of the community's collective efforts, offering insights into the community's evolution during the LS experience till the assumption of her work as the starting point for the next collective phases.

Data availability statement

The original contributions presented in the study are included in the article/supplementary material, further inquiries can be directed to the corresponding authors.

Ethics statement

Ethical review and approval was not required for the study on human participants in accordance with the local legislation and institutional requirements. The participants provided their written informed consent to participate in this study.

Author contributions

GB: Conceptualization, Data curation, Formal analysis, Investigation, Methodology, Project administration, Resources, Software, Supervision, Validation, Visualization, Writing – original draft, Writing – review & editing. BP: Conceptualization, Data curation, Formal analysis, Funding acquisition, Investigation, Methodology, Project administration, Resources, Software, Supervision, Validation, Visualization, Writing – original draft, Writing – review & editing.

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Conflict of interest

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

Publisher's note

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