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Comparing different types of instructional videos in a flipped proof-based classroom

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Background: Proofs are a key component in undergraduate mathematics, but understanding presented proofs and constructing proofs is a challenge for many students. Flipped undergraduate mathematics classrooms often employ instructional videos, yet little is known about their potential to help students understand and construct proofs.

Objective: This study investigates the potential of video-based proof presentations on student learning. We compared a video that presented the proof construction process (proof video); a video that heuristically presented the proof construction process, which modeled key decisions and named the phases of proof construction and activities (heuristic proof video); and a video that offered prompts during the proof construction process, where self-explanation prompts guided students through these phases and activities (prompted proof video).

Methods: A between-subjects design was employed, involving 177 mathematics (teacher) students in a first-semester proof-based linear algebra course. Data were collected on students' comprehension of the presented proof, their knowledge for proof construction, and their evaluative perceptions. Statistical analyses were performed using ANOVA (proof comprehension) and MANOVA (evaluative perceptions) to compare the groups. Qualitative content analysis was employed to identify different facets of knowledge for proof construction and the groups were contrasted using χ^2-tests.

Results: We found that independent of the video they watched, students achieved a rather local comprehension of the presented proof. The heuristic proof video showed potential for offering meta-knowledge of how to approach proof construction and knowledge on process-related activities that support individual phases of proof construction but required more time. Yet, while students perceived all videos positively, they liked the heuristic proof video best.

Conclusion: The results provide insights into the design of instructional videos, suggesting that, in the early stages of learning about proofs, a heuristic proof video may help address the challenges students face.

KEYWORDS

instructional videos, proof videos, heuristic examples, proof comprehension, proof construction, linear algebra, undergraduate mathematics students

1 Introduction

Undergraduate mathematics courses often use instructional videos to supplement face-to-face classes or even replace lectures (Kay, 2012; Lo et al., 2017), as videos offer various advantages, such as self-paced content (students can watch or rewatch videos at their leisure) and the ability to combine explanations with dynamic visualizations (Kay and Kletskin, 2012). Instructional videos are frequently used in flipped classrooms (Cevikbas and Kaiser, 2023), in which most information transfer-related teaching takes place out of the classroom, such that in-class time can be spent on active learning (Abeysekera and Dawson, 2015). Because flipped classrooms reserve in-class time for active learning, they have demonstrated better student performance compared to traditional teaching models (Strelan et al., 2020). However, the effectiveness of flipped classrooms depends on thoughtfully designed instructional videos that are used for class preparation and their alignment with the course's overall objectives (Lo et al., 2017).

While some research has proposed general guidelines for designing instructional videos (e.g., Fyfield et al., 2022), and studies in mathematics education have focused on designing videos for algorithmic problems (e.g., Kay, 2014), much less is known about how to design instructional videos that focus on proofs, which are a key component in undergraduate mathematics education (Clark and Lovric, 2009; Gueudet and Thomas, 2020).

In proof-based courses, a common teaching method is to present items of proof, such that students can learn a mathematical idea from the example proofs (Weber, 2004). However, research has shown that students often do not understand the underlying mathematical ideas (Lew et al., 2016). In general, students tend to struggle with comprehending (Conradie and Frith, 2000) and constructing proofs (Stylianides et al., 2017), and these difficulties are evident across different domains, such as abstract algebra (Weber, 2001) and linear algebra (Stewart and Thomas, 2019); in linear algebra, for example, challenges with proofs stem from understanding and using the concepts introduced in the course (Britton and Henderson, 2009). To enhance the effectiveness of presenting proof, it is possible to modify how this is done as well as how students engage with the proof itself (Hodds et al., 2014). Opposed to lectures, presenting a proof through an instructional video can allow students to pause and rewind at any time.

Consequently, given undergraduate students' common challenges in proof comprehension and construction, this paper proposes three types of instructional videos that may be useful in enhancing video-based proof presentations. We then compare these three types of videos in a flipped proof-based linear algebra classroom, where we assess how well students understand the proof presented and how well they convey knowledge needed for proof construction. In addition, we consider students' evaluative perceptions of these video types, since this may affect their willingness to engage with the resource (Engelbrecht et al., 2020). By examining which type of video best supports undergraduate students' challenges with proofs, this research aims to contribute to the design of effective instructional videos for presenting proofs in flipped proof-based mathematics classrooms.

2 Theoretical background

2.1 Proof-based courses in undergraduate mathematics education

Undergraduate proof-based courses primarily offer instruction through lectures (Melhuish et al., 2022). Commonly, lecturers engage in "chalk talk" (Artemeva and Fox, 2011), where they write on the board and simultaneously give verbal comments that provide insights into a mathematician's thought process and impart meta-comments at certain points that pan out to more global ideas. Often, proofs are presented with the objective of not only conveying factual information but also as a means to help students understand the proof and learn from it (Mejía-Ramos et al., 2012), for example through learning methods that can be used for constructing a proof (Hanna and Barbeau, 2008). However, comments that go beyond formal mathematics are rarely written on the board and, therefore, are often not recorded in students' notes (Fukawa-Connelly et al., 2017). This may be one reason why students frequently fail to comprehend the mathematical points the lecturer wants to make (Lew et al., 2016). This challenge to presenting proofs in lectures may be overcome by presenting proofs in instructional videos, because students can pause and repeat explanations. Yet, before designing instructional videos that present proofs, one must first consider the skills required for students to understand and construct proofs.

2.1.1 Proof comprehension and construction as a challenge for students

Activities concerning proofs entail a receptive component, which involves reading a given argument to comprehend it, and a productive component, which includes constructing novel arguments (Mejía-Ramos and Inglis, 2009). According to Mejía-Ramos et al. (2012), comprehending a proof requires understanding that is both local and holistic. Students show local understanding if they comprehend the meaning of terms and statements, the justification of claims, and the logical status of the proof's statements and proof framework. Students convey holistic understanding if they are able to summarize high-level ideas, identify the modular structure, transfer the general ideas or methods to another context, and understand how the proof relates to examples. Thus, students may grasp a proof on the holistic level but not understand its technical details on the local level. Correspondingly, students may understand local aspects of a proof but not comprehend the proof holistically (Mejía-Ramos et al., 2012).

Studies have shown that students' ability to comprehend proofs is linked to their prior knowledge, an important component of which is understanding key concepts (Neuhaus and Rach, 2019; Bauer et al., 2022). Proofs in linear algebra are often packed with concepts (Britton and Henderson, 2009), so students may struggle with understanding the proofs, even when they are thoroughly discussed in class, because they do not grasp the underlying concepts (Stewart and Thomas, 2019).

Besides comprehending proofs, another important activity in undergraduate mathematics is constructing proofs (Mejía-Ramos and Inglis, 2009), and, in this context, students often have to justify a statement estimated to be true (Selden A. and Selden, 2017). The product of a proof construction process is a proof (Boero, 1999) and Czocher and Weber (2020) list five properties that contribute to considering a justification as a proof, namely that the justification (1) is convincing, (2) is perspicuous, (3) is *a priori* which means that the theorem to be proven is the deductive consequence, (4) is transparent, and (5) has been sanctioned by the mathematical community. Importantly, the proof itself is not a report of the construction process (Selden J. and Selden, 2017), which may be why many undergraduate students believe that proofs are constructed linearly and, thus, give up quickly when they do not immediately know how to construct a proof (Selden and Selden, 2008).

Consequently, students need to have meta-level knowledge about the phases of proof construction and what these phases involve. In undergraduate mathematics, the proof construction process involves progressing through the phases of understanding (the statement given), identifying arguments, structuring arguments, and formulating the proof (Kirsten, 2018, 2021). Within each phase, proof construction can be guided by heuristics specified as activities to make progress on a problem (Schoenfeld, 1985, p. 15), such as drawing a diagram when working to understand the statement (Schoenfeld, 1985, p. 108). Another prerequisite for successful proof construction is strategic knowledge (Sommerhoff, 2017). Strategic knowledge involves awareness of the domain's proof techniques, which theorems are important and when they are useful, and when symbolic manipulations are sufficient. Yet, research indicates that undergraduate students have difficulty applying strategic knowledge during proof construction (Weber, 2001). Finally, another key resource important for proof construction is content-specific mathematical knowledge (Sommerhoff, 2017), but students tend to have trouble recalling and applying concepts and theorems when constructing proofs (Moore, 1994; Weber, 2001). These manifold demands that students encounter in proof construction are among the reasons they often do not know how to start a proof (Moore, 1994).

Because both comprehending and constructing proofs are important activities in university mathematics (Selden A. and Selden, 2017), developing measures that support these activities is of special interest in mathematics education (Stylianides et al., 2024).

2.1.2 Supporting students in comprehending and constructing proofs

As identified by Hodds et al. (2014), proof comprehension can be enhanced via two general methods: modifying the way a proof is presented or modifying how students engage with the proof. Approaches to modify proof presentation include using e-Proofs (Alcock and Wilkinson, 2011), which complement a text-based proof on slides with visuals and on-demand auditive explanations. In Roy et al.'s (2017) study, although students felt that e-Proofs supported their understanding, they performed worse in a delayed proof comprehension test than those studying a standard written proof. This may be because the students had difficulty integrating textual and auditive information, such that students might have performed better if they engaged with the proof before having seen the e-Proof presentation. Moreover, students may have performed better if they processed the e-Proof more actively (Roy et al., 2017); this idea corresponds to the method of modifying how students engage with a proof. As an example of this method, Hodds et al. (2014) found that students showed immediate and sustained improvements in proof comprehension over 3 weeks when they underwent self-explanation training, in which they had to explain each step of a proof, connecting it to previous knowledge and prior steps.

To overcome difficulties in constructing proofs, students should be told that proofs are usually not constructed linearly (Selden and Selden, 2008). Within the phases of proof construction, Schoenfeld (1985) proposed that mathematicians should model how heuristics are employed; after doing so, he found that students' performance improved in transfer problems. These findings contributed to the development of text-based *heuristic examples* (Reiss and Renkl, 2002), which combine Schoenfeld's approach with step-by-step solutions to problems. Thus, heuristic examples display a realistic solution process which, in the context of proofs, means that the proof is presented along with the process of proof construction. Undergraduates engaging with heuristic examples in geometry were found to perform significantly better in proof construction and to develop richer knowledge about the proving process than the control group, who studied an instructional text (Hilbert et al., 2008). Therefore, heuristic examples may contribute to acquiring knowledge for solving transfer problems (Renkl, 2017).

The above methods may represent a guide when designing instructional videos that present proofs. Such videos should offer a well-considered explanation of the proof but they modify the way the proof is presented (compared to in lectures). Hence, we combine elements of the above methods with the general benefits of instructional videos, which are described next.

2.2 Instructional videos as a resource for learning

Various digital resources are employed in mathematics learning and teaching. According to Winsløw et al.'s (2023) classification of technology use in university mathematics education, such resources can refer to *tools* or *media*, both of which can be used in a receptive or a productive way. While tools allow students to carry out mathematical steps (e.g., calculations in a computer algebra system) or are produced by students (e.g., through programming), media allows students to access or exchange mathematics. In that sense, watching an instructional video is considered "receptive media use" because it allows students to access information and knowledge.

To date, many studies have reported that instructional videos can positively affect higher education learning outcomes (Noetel et al., 2021), such as by helping students apply statistical knowledge (Lloyd and Robertson, 2012). Because instructional videos can be paused and replayed, students can adapt the videos to their pace of their learning (Kay, 2012). However, in a study by Weinberg and Thomas (2018), undergraduate mathematics students who watched statistics videos were frequently unable to monitor their understanding of the concepts and thus did not tend to pause or replay the video. Perhaps because videos are often perceived as "easy" compared to printed material (Salomon, 1984), the students did not invest the required effort to comprehend the video content. Furthermore, instructional videos' effectiveness depends on key design choices, such as whether to exclude interesting but irrelevant facts (Fiorella, 2021), how long to make the video (Guo et al., 2014), or how much control to give students regarding pausing, fast-forwarding, and rewinding a video (Fyfield et al., 2022).

Instructional videos are often perceived positively, as students have reported instructional videos to be helpful and motivating (Kay, 2012). However, several studies have highlighted that students care about the videos' quality which involve key design choices, such as the videos' structure, the length, an explanation perceived as efficient (Shoufan, 2019; Beautemps and Bresges, 2021), and the opportunity to be actively involved (Kay and Kletskin, 2012). Despite students' positive evaluative perceptions of videos for learning, some find it challenging that videos do not allow for asking questions and that watching videos requires self-discipline (Kay, 2012).

To sum up, the results on the effectiveness and perceptions toward instructional videos indicate that videos should be carefully designed and implemented in a way that considers the target group and the learning domain.

2.3 Designing instructional videos for a flipped proof-based classroom

One advantage of using instructional videos in a flipped proof-based classroom is that students receive information transfer-related teaching at home, so that in-class time can be spent on student-centered activities. Therefore, the common teaching mode of presenting proofs is outsourced to videos. The objectives of presenting a proof are (i) that students gain mathematical insight that is reflected in their local and holistic understanding of the proof in the sense of Mejía-Ramos et al. (2012) and (ii) to address knowledge needed for proof construction. To achieve these goals, different design choices may be beneficial when presenting the same mathematical proof, which we outline here.

2.3.1 Proof videos

As many proving situations in undergraduate mathematics (Selden A. and Selden, 2017), a proof video starts with a theorem or statement that is estimated to be true. Subsequently, the proof is constructed step-by-step, where the video shows all the necessary activities involved in that process, such as recalling related definitions, theorems, and concepts (see pink writing in Figure 1). As with proof presentations in lectures, the idea is that students will come to understand the proof and learn from it (Mejía-Ramos et al., 2012). At the end of the proof construction process, the result is a formal proof.

2.3.2 Heuristic proof videos

A heuristic proof video includes the same elements as a proof video but, in addition, during proof construction the presenter not only conducts the steps but also gives, as is done with heuristic examples (see Sect. 2.1.2), insight into decision-making processes. For example, when choosing a helpful theorem for constructing the proof, the presenter may address why one theorem seems better than others. Moreover, the video presenter may name the activities and their related phases of proof construction (see blue part of Figure 1), which will help break the video down into meaningful chunks, an important aspect when presenting complex material within instructional videos (Mayer, 2020, p. 262). As a basis for segmenting the video, a natural choice might use the phases of undergraduate students' proof construction process, which involve understanding (the statement given), identifying arguments, structuring arguments, and formulating the proof (Kirsten, 2018, 2021). By making explicit how proof construction is approached, such videos can address

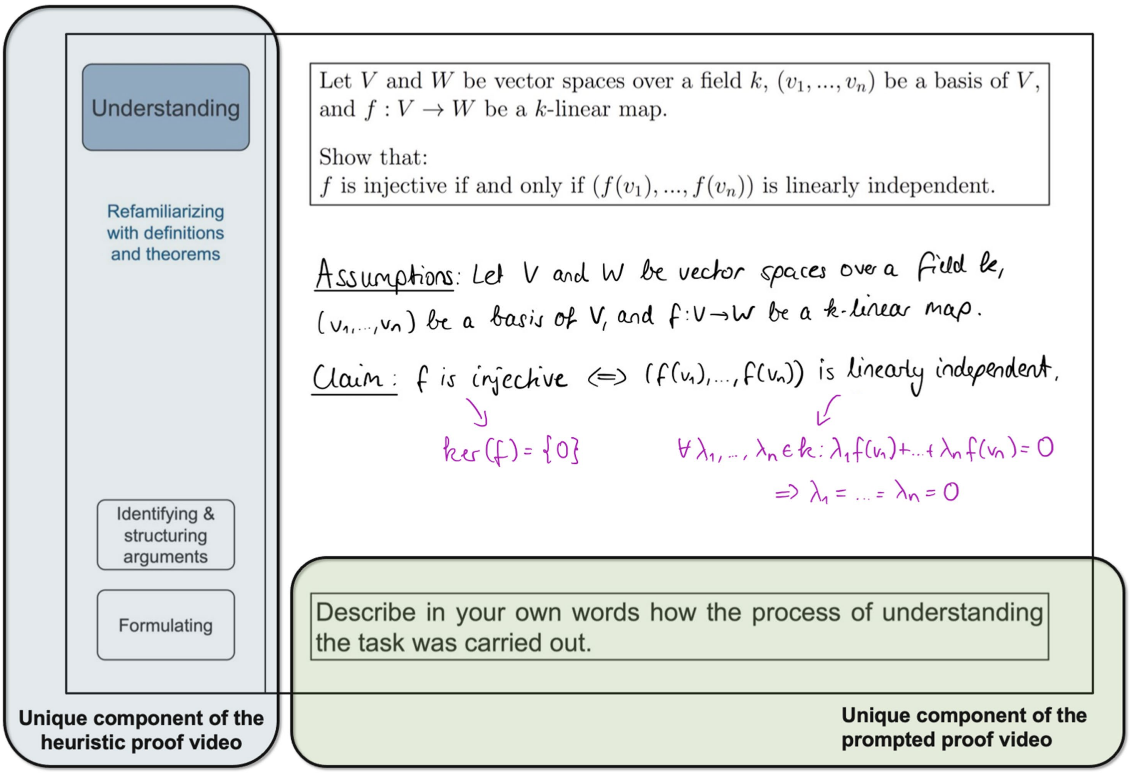

FIGURE 1
Demonstration of the unique components of the heuristic and the prompted proof videos, with the proof video in the background.

students' misconception that proofs are constructed linearly (Selden and Selden, 2008).

This additional information in a heuristic proof video thoroughly guides students through the proof construction process, but may have drawbacks. Heuristic examples target students with low knowledge regarding the focused domain (Reiss and Renkl, 2002) and studies have repeatedly shown that such students benefit particularly from this teaching approach (e.g., Hilbert et al., 2008). Presenting too much information that students already know can impede their learning according to the "expertise reversal effect" (Kalyuga, 2021). The expertise reversal effect occurs when high-guidance instruction benefits low-knowledgeable students but disadvantages students with higher prior knowledge. In this case, high-knowledgeable students may benefit more from less-guided instruction, including autonomous problem solving (Renkl, 2017). As a consequence, video producers need to consider their target audience carefully, but it will not be possible to produce an explanation that is suitable for all students. Moreover, a heuristic proof video necessarily results in a video that is longer than a proof video. Not only is an explanation that is perceived as efficient important to students (see Sect. 2.2), but the length of the video may impair student engagement. In their study, Guo et al. (2014) found that in the context of a Massive Open Online Course (MOOC), students watched videos for a median of 6 min, regardless of the total video length, and were less likely to engage in follow-up problems as the video length increased. Therefore, video producers should aim to keep videos as short as possible.

2.3.3 Prompted proof videos

A prompted proof video is identical to a proof video but automatically stops at predetermined points to present self-explanation prompts. Compared to justification prompts, which aim to direct students to generate a conceptual justification for a single step, step-focused prompts (Nokes et al., 2011) or procedural prompts (Rittle-Johnson et al., 2017) are used to focus students' attention on how to approach the proof construction process. Focusing students on such structural features of a problem allows them to generalize this knowledge to different problems (Rittle-Johnson et al., 2017), which has been evident in a study in the context of heuristic examples on proof construction: Prompting the phases of proof construction while undergraduates studied heuristic examples was shown to have a positive effect on proof construction and knowledge about the proving process (Hilbert et al., 2008). In that way, prompting students to self-explain how each phase of proof construction was conducted in a specific example offers learning opportunities for meta-knowledge on how to approach proofs and for activities that help during each phase. Compared to the heuristic proof video, students are prompted for these explanations instead of having these explanations provided for them. Such a use of prompts represents an activity that helps shift the video from being a passive-receptive medium to one that students actively engage with, which is important when designing instructional videos (Fiorella, 2021; Ploetzner, 2022). However, the effectiveness of integrating prompts within instructional videos may depend on students' prior knowledge (Bai et al., 2022) and the quality of self-explanations (Hefter et al., 2023).

2.4 Aim and research questions

Previous studies have shown that undergraduate mathematics students have difficulty comprehending and constructing proofs (Gueudet and Thomas, 2020). To support students, presenting proofs to students in lectures is a common teaching approach in undergraduate mathematics education (Melhuish et al., 2022). In recent years, more attention has been paid to the use of instructional videos for teaching and learning mathematics (Cevikbas and Kaiser, 2023; Winsløw et al., 2023). Advantages of instructional videos include that explanations can be replayed and that students find them helpful and motivating (Kay, 2012). Previous research on instructional videos in undergraduate mathematics (e.g., Kay and Kletskin, 2012; Weinberg and Thomas, 2018) has mainly focused on instructional videos in more algorithmic domains. However, there is limited understanding of how to effectively design instructional videos for presenting proofs and what role such videos play in students' learning processes.

The present study seeks to address this research gap by comparing the three proposed video types (see Sect. 2.3), which are based on research on supporting students with proof and instructional video design. Specifically, we were interested in the potential of the videos types (i) for students to gain a mathematical insight that is reflected by local and holistic proof comprehension in the sense of Mejía-Ramos et al. (2012) (see Sect. 2.1.1) and (ii) to address knowledge needed for proof construction.

Therefore, the following research questions were investigated:

> RQ1 (proof comprehension): Do students show different levels of proof comprehension after working with a proof video, a heuristic proof video, or a prompted proof video?
>
> RQ2 (knowledge for proof construction): What kind of knowledge for proof construction do students indicate after watching a proof video, a heuristic proof video, or a prompted proof video?

Although we did not have a specific expectation regarding the differences in proof comprehension, because all three video types presented the same proof, we wondered whether the additional information included in the heuristic proof video would hinder students' concentration (Guo et al., 2014), such that they would be distracted from comprehending the proof. On the other hand, due to the heuristic proof video's additional information on proof construction and because this knowledge is prompted in the prompted proof video, we expected students to indicate different kinds of knowledge for proof construction.

Since students are the targets for this measure of support, we were also interested in their evaluative perceptions of the proposed video types, because these may affect their will to engage with a digital resource (Engelbrecht et al., 2020). We therefore addressed a third research question:

> RQ3 (evaluative perceptions): Do the students' evaluative perceptions measured as the satisfaction with the video and the perceived video quality differ depending on watching a proof video, a heuristic proof video, or a prompted proof video?

3 Materials and methods

3.1 The flipped proof-based classroom

To compare the three video types, we conducted a study in a first-semester proof-based linear algebra course at one of the largest

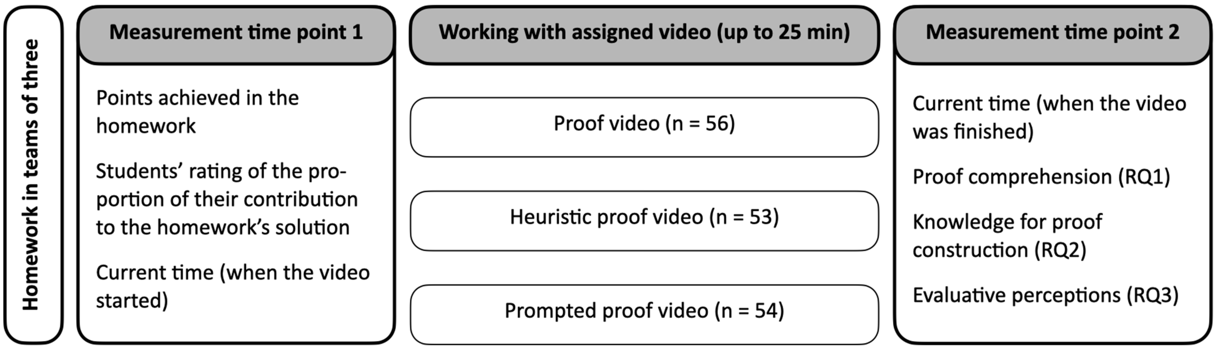

FIGURE 2
Study design.

Let V and W be vector spaces over a field k, $(v_1, \ldots, v_n)$ be a basis of V, and $f : V \to W$ be a k-linear map.

Show that:
f is injective if and only if $(f(v_1), \ldots, f(v_n))$ is linearly independent.

FIGURE 3
The proof construction task.

universities in Germany. As it is common in first year mathematics in Germany, constructing proofs and mathematical content are learnt simultaneously (e.g., Rach and Ufer, 2020). Therefore, along with learning about proof construction, on the content level students learn about the basics of linear algebra, such as vector spaces, linear mappings, matrices, and eigenvalues.

Each week of the 14-weeks course consists of two 2-h lectures; corresponding homework, in which students work on proof construction tasks in teams of three; and a 2-h tutorial that reviews the homework and the lectures taught by (under)graduate mathematics tutors. Because the original tutorial sessions had mostly been spent on a lecture-style review of the homework without much active engagement from students (Serpé, 2020), a flipped classroom was implemented. The implementation followed the principles to "(1) move most information-transmission teaching out of class (2) use class time for learning activities that are active and social and (3) require students to complete pre-and/or post-class activities to fully benefit from in-class work" (Abeysekera and Dawson, 2015, p. 3). With the introduction of the flipped classroom, the tutorial session has since been primarily used to prepare students for their homework by revising concepts taught in the lecture through small group work, classroom discussions, and student-led presentations. In this flipped classroom context, our study aimed to assess the information-transmission part of the teaching, which was provided in the instructional videos on proofs that students watched at home to review their homework.

3.2 Participants and design

Using a between-subjects design (see Figure 2), we compared three videos that presented the solution to a proof construction task. The videos were produced by a graduate mathematics tutor and corresponded to the above-described proof videos, heuristic proof videos, and prompted proof videos. The study sample was generated by convenience sampling. Students enrolled in the first-semester course described in Sect. 3.1 solved the task during week 8 of the semester as part of their weekly homework. Therefore (most) students had some exposure to the statement that was to be proven before watching the video.

The instructional videos and questionnaires were made available through the university-based learning management system. All students enrolled in the linear algebra course were randomly assigned to one of the three videos and invited to participate on a voluntary basis. The students were given up to 25 min to work with their assigned video and were instructed not to interrupt the process. After the completion of the study, all videos were made available to all students.

The study sample consisted of 177 students. The first-semester linear algebra course is compulsory for mathematics bachelor students and future upper-secondary level mathematics teachers, while it is optional for computer science, physics, or geoinformatics students. The students' mean high school GPA was 1.68 ($SD = 0.53$), with the best possible GPA in Germany for obtaining a high school-leaving certificate being 1.0 and the lowest being 4.0.

3.3 Materials

3.3.1 Proof construction task

The proof construction task that the videos reviewed is presented in Figure 3. To construct the proof, students needed to recall that it is

In the first part of the proof, the following equivalence was used:

$$\lambda_1 f(v_1) + \ldots + \lambda_n f(v_n) = 0 \Leftrightarrow f(\lambda_1 v_1 + \ldots + \lambda_n v_n) = 0.$$

Please select the statement that justifies the validity of this equivalence.

a)	$(v_1, \ldots, v_n)$ is a basis of V.
b)	The function f is a map.
c)	Because V is a vector space and therefore closed with respect to scalar multiplication.
d)	The map f is linear.
e)	I don't know.

FIGURE 4
Sample item from the proof comprehension test targeting the local dimension *justifying claims* (Mejía-Ramos et al., 2012).

often helpful to prove the implication and the converse separately (proof framework). Moreover, a common technique for proving that a k-linear map is injective is to show that the kernel consists only of the zero vector. Realizing how and why theorems like this are helpful for constructing proofs is often challenging for students (Weber, 2001). In addition, the task demands familiarity with several definitions and concepts introduced in linear algebra, particularly bases of vector spaces, k-linear maps, injective maps, and linear independence. Thus, this task included several of the aforementioned challenges that students face when constructing proofs in linear algebra.

3.3.2 Instructional videos

The three videos were produced as screencasts and constructed the same proof for the task in Figure 3. The proof video (Supplementary material Video 1) and the heuristic proof video (Supplementary material Video 2) including English subtitles are available as Supplementary material. Figure 1 shows how the different design choices concerning each video were implemented.

The proof video (background of Figure 1) conducted the mathematical steps to construct the proof by implicitly following the phases of undergraduates' proving processes (Kirsten, 2018, 2021).

The heuristic proof video, in addition, explicitly named these phases and the activities applied in each phase (blue part of Figure 1). In the beginning, it gave an overview of the steps (minutes 0:22–1:24 in the heuristic proof video) and it provided insight into the decision-making process when required (e.g., minutes 4:04–5:43 in the heuristic proof video).

The prompted proof video was identical to the proof video but automatically stopped after each of the (implicitly followed) three phases of the proving process. For example, after the phase of understanding (minute 1:47 in the proof video), students were asked to "Describe in your own words how the process of understanding the task was carried out" (green part of Figure 1). Prompts for the phases of identifying and structuring arguments (minute 4:31 in the proof video) as well as formulating the proof (minute 7:29 in the proof video) were devised accordingly. Each prompt was displayed until students were satisfied that they had sufficiently answered a prompt and continued the video by pressing play.

The length of the proof video was 7:34 min, and that of the heuristic proof video was 13:39 min.

3.4 Measures

Various measures were assessed at two time points (see Figure 2). Prior to watching the assigned video, the students filled in the points they had achieved in the proof construction homework task (up to 8 points) and were asked to rate how much they contributed to the solution relative to their two homework partners (as a percentage). Both pieces of information were used as a measure of prior knowledge and to conduct a randomization check. We assessed the time points right before and after they watched their assigned videos to determine whether students had exceeded 25 min and to assess differences in the time students spent watching their assigned video. Of particular interest was the influence of the prompts, and because of these additional elements, it was assumed that students would spend more time watching the prompted proof video than the proof video. After watching the videos, we measured students' proof comprehension, knowledge for proof construction, and evaluative perceptions of the videos.

3.4.1 Proof comprehension

Similar to other studies (e.g., Roy et al., 2017), we employed a short test based on the assessment model for proof comprehension by Mejía-Ramos et al. (2012) with three items targeting the local level and two targeting the holistic level of proof comprehension (see Supplementary materials Data Sheet 1 for all five items). To validate the test, two mathematicians evaluated the suitability of the multiple-choice items regarding the presented proof; further, we ensured items were understandable via conducting cognitive interviews with four undergraduate students. On the test, each item had four answer options, of which one or several were correct, and each item also had an "I don't know" option (see Figure 4). To account for the complexity of the items, we used a partial credit scoring system (0, 0.5, and 1 point). Each item was used to assess different facets of knowledge and understanding.

3.4.2 Knowledge for proof construction

To assess the knowledge for proof construction students indicated, we used the following open item: "In the video, different strategies were mentioned (explicitly or implicitly) on how to approach proving tasks. Please name all the strategies you can remember." We used an open item to allow for a wide range of responses (including unanticipated ones). The objective was to gain nuanced insights into aspects that students considered to be strategies for approaching proof construction tasks.

3.4.3 Evaluative perceptions

To measure students' evaluative perceptions of the videos, we assessed *satisfaction with the video* and *perceived video quality*. The characteristic values of the individual scales are shown in Table 1.

To measure the students' satisfaction with their assigned video, we used an adapted version of the satisfaction with taught content scale (ZSI; Schiefele and Jacob-Ebbinghaus, 2006). Students evaluated the learning opportunity relative to their expectations and demands.

To assess the video quality, we used three items from the item pool for instructional quality based on the PaLea scale (Kauper et al., 2012). Students rated how comprehensible they found their assigned video. Three additional items were developed according to the design choices to determine whether the students thought the inserted information, the structure, and the written comments were helpful.

3.5 Data analysis

To conduct the preliminary analyses and to answer RQ1 and RQ3, we used parametric tests [one-way analysis of variance (ANOVA) and multivariate analysis of variance (MANOVA)]. The homogeneity of variances was tested using Levene's test ($p > 0.05$). Kolmogorov–Smirnov tests revealed non-normal score distributions for some measures, but because of the sample size and because there were no outliers in the distributions, parametric tests could still be employed (cf. Rasch and Guiard, 2004). For the multivariate analyses, the homogeneity of variance–covariance matrices was tested using Box's M test ($p > 0.001$; Verma, 2015).

To identify the different facets of knowledge for proof construction that students had extracted from their assigned videos (RQ2), their answers were segmented and coded. The data-driven development of categories followed the principles of qualitative content analysis (Kuckartz, 2019). The related themes that emerged were summarized as categories, and these categories were continuously systemized and organized. The final category system is presented in Table 2. All data were coded by the first and second author, yielding a very good interrater reliability (Cohen's κ 0.91–0.96; Landis and Koch, 1977).

4 Results

First, we checked whether students had exceeded the time limit of 25 min for watching their assigned video; 14 students exceeded this limit and were excluded, leaving 163 students included in the analysis (see Figure 2).

Second, as a check for randomization, we compared the extent to which students had engaged with the proof construction task prior to the video (assessed by the points they received in the homework and the proportion they said they contributed to the solution) with a MANOVA in which the assigned video was the factor. The results showed no significant differences between the video groups [$F(4,122) = 0.58$, $p = 0.68$, Wilks's $\Lambda = 0.96$]. Table 3 presents the descriptive statistics for the three video groups and shows, in particular, that the proportion students said they contributed to the solution varied largely in all video groups.

Third, we used an ANOVA to check whether the time spent watching the video differed between the groups. As such, the dependent variable was the time spent watching the video, and the factor was the assigned video. We found a significant difference between the groups [$F(2,160) = 10.84$, $p < 0.001$, partial $\eta^2 = 0.12$], corresponding to a medium effect (Cohen, 1988). *Post hoc* tests using the Bonferroni correction revealed that the heuristic proof video group ($M = 14.51$, $SD = 4.08$) spent significantly more time watching the video than the proof video group ($M = 11.05$, $SD = 3.91$, $p < 0.001$) and the prompted proof video group ($M = 12.24$, $SD = 3.79$, $p = 0.01$). No significant difference was found between the proof video and the prompted proof video groups ($p = 0.34$).

TABLE 1 Overview of the characteristic values of scales on students' evaluative perceptions.

	Sample item	Value	# Items	α
Satisfaction	"I am so satisfied with the video that I would watch more such videos."	1–4	4	0.86
Video quality	"I can easily follow the line of thought in the video at any time." "The structure of the video supports my understanding."	1–4	6	0.84

TABLE 2 Categories related to the students' stated knowledge for proof construction.

Category	Description
Knowledge on proof methods	
Applicable	Students named a proof method that was applied in the task.
Non-applicable	Students named a proof method that was not applied in the task.
Meta-knowledge	Students indicated on a meta-level how proof construction tasks can be approached.
Knowledge on process-related activities	Students referred to an activity that supported one phase of proof construction.
Domain-specific knowledge	Students described domain-specific knowledge that was helpful for proof construction.
Not indicated	Students indicated that they did not know.

TABLE 3 Means and standard deviations (in parentheses) of the variables included in the preliminary analysis.

	Proof video	Heuristic proof video	Prompted proof video
Achieved points (0–8)	5.56 (1.90)	5.59 (1.95)	5.80 (1.76)
Own proportion (%)	43.93 (34.15)	37.17 (29.57)	40.74 (30.71)
Time spent (0–25)	11.05 (3.91)	14.51 (4.08)	12.24 (3.79)

TABLE 4 Means and standard deviations (in parentheses) for each item of the proof comprehension test based on the assessment model by Mejía-Ramos et al. (2012).

	Item of proof comprehension	Proof video	Heuristic proof video	Prompted proof video
Local	(1) Justifying claims	0.63 (0.49)	0.60 (0.50)	0.68 (0.47)
	(2) Understanding the meaning of terms and statements	0.19 (0.28)	0.28 (0.38)	0.31 (0.40)
	(3) Justifying claims	0.48 (0.50)	0.54 (0.50)	0.55 (0.50)
Holistic	(4) Summarizing the high-level idea/transferring a method	0.19 (0.31)	0.20 (0.34)	0.15 (0.30)
	(5) Summarizing the high-level idea/transferring a method	0.28 (0.45)	0.34 (0.48)	0.40 (0.49)
		1.78 (1.09)	1.96 (1.13)	2.09 (1.34)

4.1 Proof comprehension

We excluded six students from the proof comprehension analysis due to missing values in the proof comprehension test. To explore whether differences in proof comprehension appeared across the video groups, we conducted an ANOVA; it revealed no significant differences between the video groups [$F(2,154) = .90$, $p = 0.41$]. The mean score for proof comprehension was rather low ($M = 1.94$, SD = 1.19). However, the descriptive analysis revealed that the solution rates for the local level of proof comprehension tended to be higher than those for the holistic level (see Table 4).

4.2 Knowledge for proof construction

When asked to name all the strategies on how to approach proving tasks that had been (explicitly or implicitly) mentioned in the videos, students referred to various aspects (see Table 2). *Knowledge on proof methods* refers to proof methods that were either applicable (direct proof or demonstrating the equivalence by showing the implication and converse) or were not applied within the video, such as mathematical induction or proof by contradiction.

Meta-knowledge consists of knowledge on how the proof construction task was approached on a global level, for example:

> Student: Understand, look up theorems, definitions, etc., and consider which are applicable and then apply. Then formulate the proof.

When referring to *knowledge on process-related activities*, students stated activities that had been helpful in a certain phase of proof construction; for instance, in the phase of understanding, a stated activity might be taking notes on the premise and the claim, which could help the student identify the goal of the task. For both understanding and identifying arguments, a helpful activity might be familiarizing oneself with the definitions and concepts that emerged from the task, as the following answer illustrates:

> Student: For initial understanding, you should also take notes on any helpful theorems, examples, etc., from the lecture that might fit here.

Concerning *domain-specific knowledge*, students referred to the theorem that a k-linear map is injective if and only if the kernel consists of the zero vector. Using this theorem is a common proof technique within the domain.

Comments stating that the students did not know were categorized as *not indicated*.

All in all, students' answers contained 259 references to aspects indicating knowledge for proof construction. The students in the proof video group referred to, on average, 1.5 aspects, those in the heuristic proof video group to 1.8 aspects, and those in the prompted proof video group to 1.7 aspects. Students in all groups gave responses related to all aspects, although students in the heuristic proof video group did not refer to domain-specific knowledge (see Figure 5).

Overall, we found a significant association between the video group and the types of knowledge [χ^2 (10) = 74.88, $p < 0.001$]. Note that the category *domain-specific knowledge* was excluded from this and the following tests because a cell frequency was zero. *Post hoc* pairwise tests with the Bonferroni correction indicated significant pairwise differences between the heuristic proof video group and the other two groups [χ^2 (5) > 39.93, $p < 0.001$].

A significant association was found between *knowledge on applicable proof methods* and the video group [χ^2 (2) = 9.25, $p = 0.01$], suggesting that the heuristic proof video group referred less frequently to applicable proof methods. Moreover, a significant association was found between *meta-knowledge* and the video group [χ^2 (2) = 50.84, $p < 0.001$], indicating that the heuristic proof video group stated meta-knowledge more often than the other two groups. No significant associations were found between the video group and other types of knowledge.

Over 40% of the answers in the proof video group could be considered evasive, since students referred to non-applicable proof

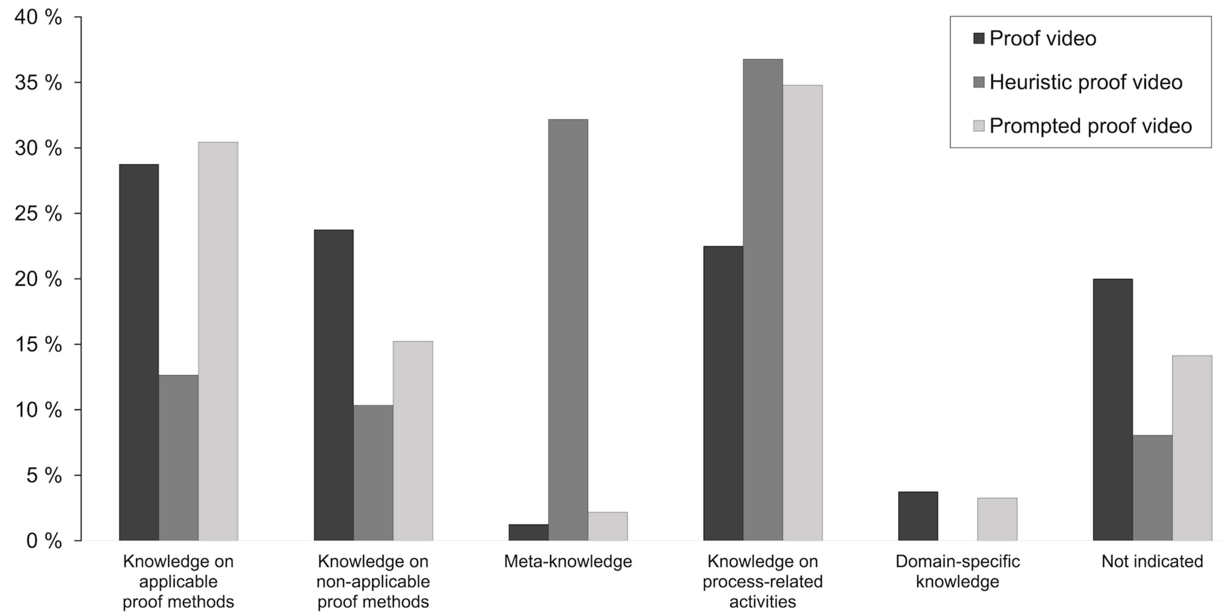

FIGURE 5
Type of knowledge corresponding to the three video groups (relative to the number of coded segments within each group).

TABLE 5 Means and standard deviations (in parentheses) of students' evaluative perceptions in terms of satisfaction and rated video quality.

	Proof video	Heuristic proof video	Prompted proof video
Satisfaction (1–4)	2.77 (0.65)	3.21 (0.65)	2.81 (0.65)
Video quality (1–4)	3.07 (0.58)	3.42 (0.48)	3.09 (0.51)

methods or indicated nothing. Other than that, students in this group offered applicable proof methods most frequently.

Besides the frequently mentioned meta-knowledge in the heuristic proof video group, knowledge on process-related activities was an important theme. This was also the most frequently mentioned type of knowledge in the prompted proof video group. By contrast, domain-specific knowledge was only mentioned to a small extent by students in the proof video and the prompted proof video groups (and not at all in the heuristic proof video group), thus it was not prevalent.

4.3 Evaluative perceptions

On the scales for assessing students' evaluative perceptions (satisfaction with the video and perceived video quality), eight students had missing values and were excluded from the analysis. To determine whether students perceived their assigned video differently measured as the satisfaction with the video and the perceived video quality, we conducted a MANOVA with the assigned video as the factor. We found a significant difference between the groups [$F(4,302)=4.06$, $p=0.003$, partial $\eta^2=.05$, Wilks's $\Lambda=0.90$], corresponding to a small effect (Cohen, 1988). *Post hoc* tests using the Bonferroni correction revealed that students in the heuristic proof video group were more satisfied with their assigned video ($M=3.21$, $SD=0.65$) than those in the proof video group ($M=2.77$, SD $=0.65$, $p=0.002$) or the prompted proof video group ($M=2.81$, SD $=0.65$, $p=0.006$). Moreover, students who watched the heuristic proof video rated the video quality higher (M$=3.42$, SD $=0.48$) than those watching the proof video ($M=3.07$, SD $=0.58$, $p=0.003$) or the prompted proof video ($M=3.09$, SD $=0.51$, $p=0.007$). No significant differences were found between the proof video and the prompted proof video groups in terms of satisfaction ($p=0.99$) and video quality ($p=0.99$). Table 5 presents the descriptive statistics and highlights that, despite the differences, students rated satisfaction and video quality relatively highly in all the groups.

5 Discussion

Despite the widespread use of instructional videos in flipped undergraduate classrooms, little is known about how to design videos that present proofs in ways that support students in comprehending and constructing proofs. Therefore, we theoretically derived three video types by combining research on how to support students with proofs and research on instructional videos. We then investigated their potentialities regarding proof comprehension, knowledge for proof construction, and evaluative perceptions of the videos in a flipped proof-based linear algebra classroom.

5.1 Proof comprehension

The results on proof comprehension revealed no statistically significant differences between the three video groups. This finding is encouraging because it suggests that additional elements targeting

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knowledge on proof construction did not hinder concentration in such a way that students were distracted from comprehending the proof. In particular, we were concerned that the length of the heuristic proof video, which was approximately twice as long as the other videos and well over 6 min, might have impeded engagement (Guo et al., 2014), but this was not the case.

While in this study proof comprehension can be treated as a one-dimensional construct, overall, we found that students scored better on the local level of proof comprehension than on the holistic level. This finding may have occurred because all videos showed a step-by-step explanation that focused on the local level of the proof. Moreover, the audio explanation and some visual explanations to retrospectively retrace individual steps were transient, such that students might not have had an overview of the complete proof. As in other studies on instructional videos in undergraduate mathematics (Weinberg and Thomas, 2018), students may have been unable to monitor their understanding, potentially because they perceived the proof presentation as "easy" (Salomon, 1984), which is one risk when using videos.

As students scored rather low overall on the proof comprehension test, measures of support are necessary to support proof comprehension. For example, integrating pauses at key steps might help students handle the transient information (Spanjers et al., 2012; Biard et al., 2018), and a review at the end might help them gain an overview of the proof presented. Additionally, the students might have needed to be more actively involved to comprehend the proof because, particularly for the holistic level of proof comprehension, they had to make inferences by themselves. Although the students had already been working with video proof presentations for 8 weeks, they might not yet have possessed beneficial strategies for profiting from this format. Thus, such strategies could be discussed during the in-class component, offering a self-explanation training that aims to have students explain each step of the proof (Hodds et al., 2014). Finally, the prompts used in this study focused students on the steps of proof construction. In order to focus students more on comprehending the proof, justification prompts, such as prompting students to identify the justification for different steps, could be used (Nokes et al., 2011). Similarly, the heuristic proof video focused more on explaining the different steps of proof construction and what those steps entail, and less on comprehending the product of that process. Therefore, it may need a heuristic proof video on comprehending a presented proof, if proof comprehension and not learning about proof construction is the predominant goal of a video.

All in all, these results must be interpreted with caution because the proof comprehension test was quite short. However, this limitation was helpful to keep the test duration short and motivate students to participate.

5.2 Knowledge for proof construction

One of our research questions was to determine what knowledge for proof construction students indicated after watching their assigned video. This was of particular interest because the heuristic proof video explicitly stated key knowledge for proof construction and focused on the decision-making process, while the prompted proof video aimed to have students reflect on that knowledge through prompts.

Students' answers were categorized as different types of knowledge, namely *knowledge on applicable and non-applicable proof methods*, *meta-knowledge*, *knowledge on process-related activities*, and (although not often mentioned) *domain-specific knowledge*.

The results suggest that students who watched the heuristic proof video indicated knowledge about the proving process (*meta-knowledge*). Meta-knowledge can be considered important for proof construction because knowing that different phases of proof construction exist addresses the misconception that proofs are linearly constructed (Selden and Selden, 2008), and knowledge about the different phases may guide the process of proving. In line with studies on text-based heuristic examples (Hilbert et al., 2008), heuristic examples presented as videos can also offer knowledge about the proving process.

Students' indication of *knowledge on process-related activities*, which are helpful in different phases of the proving process (Schoenfeld, 1985), was comparable in the heuristic and the prompted proof video groups. Yet, students in the prompted proof video group spent less time watching the video than students in the heuristic proof video group (and a similar amount of time as students in the proof video group), such that the prompted proof video may offer advantages when the central learning goal is to offer knowledge on process-related activities.

Overall, it is important to consider that our study included no pre-test on knowledge for proof construction, so we cannot provide information about knowledge gains but we can describe differences in students' indicated knowledge according to their assigned video. Further, to fully capture the influence of the video types on students' knowledge for proof construction, additional studies will need to investigate the extent to which students can apply that knowledge in proof construction tasks.

5.3 Evaluative perceptions

We found that students generally perceived all the videos positively. This is encouraging, because students need to positively perceive a learning resource to engage with it (Engelbrecht et al., 2020). This is especially important in flipped classrooms, because not engaging with the out-of-class component may hinder a student's ability and performance during the in-class component (Cevikbas and Kaiser, 2023).

Regarding whether students' evaluative perceptions differed for the three video types, we found that those in the heuristic proof video group had a more positive perception of their assigned video, as measured by their satisfaction and evaluation of video quality. This result is notable given that the heuristic proof video was almost twice as long as the other two videos, and students generally prefer videos to be as short and efficient as possible (Shoufan, 2019). Therefore, despite its longer length, students seemed to appreciate that this video made strategic elements explicit, and they did not find this content redundant.

5.4 Limitations

This study represents a first step toward enhancing knowledge about three different video formats that can be used to present proofs, especially in the context of a flipped proof-based classroom. Because we do not know how students actually engaged and worked with their assigned videos, additional qualitative studies would help to develop a fuller picture of each video type. Such studies would be required to examine students' study habits (e.g., pausing, rewinding, and taking notes). Particular attention should be paid to the extent to which students engage with the prompts and to the quality of their self-explanations, especially since students did not spend significantly more time watching the prompted proof video than the proof video. In addition, in this flipped classroom design the in-class component is (currently) used to prepare students for solving their homework (see Sect. 3.1), but some of this in-class time might be better spent by providing support measures to help students comprehend proofs. Specifically, more in-class time could be used to discuss and practice strategies that students can apply while watching proof presentation videos to gain the most benefit from them.

All in all, when interpreting the data, it is important to consider that our study examined instructional videos that presented only one proof. This allowed us to thoroughly investigate students' proof comprehension, knowledge for proof construction, and evaluative perceptions of the videos for a characteristic problem in undergraduate linear algebra. To better understand the use of the proposed video types, further studies will need to investigate problems on other concepts or domains. Such studies could build on the proposed video design. The proof construction phases and the activities displayed in the video were used in the context of linear algebra, but are not specifically limited to this domain. Moreover, it is important to bear in mind that convenience sampling was used, considering a relatively small sample of first-semester students from a German university. The chosen sampling procedure carries limitations in terms of generalizability. However, teaching about proofs through presenting proofs is a common teaching method in mathematics courses internationally (Melhuish et al., 2022). In addition, the difficulties related to proofs that the videos used in this study addressed are widely reported (Stylianides et al., 2017; Gueudet and Thomas, 2020). Therefore, similar results may be expected for different student populations. Overall, the study conducted provides initial insights into the design of instructional videos that address students' difficulties, and further studies can build on the present study to examine such videos in their respective settings.

6 Conclusion

Overall, we found that students in a flipped proof-based linear algebra classroom perceived proof videos, heuristic proof videos, and prompted proof videos positively.

The heuristic proof video group showed encouraging results regarding meta-knowledge on proof construction and knowledge on process-related activities, indicating that this video type, though longer than the others, may be beneficial, especially at the beginning of proof-based courses. This claim is further supported by the fact that students perceived this video type more positively than the other two types.

Along with the heuristic proof video, the prompted proof video also focused students' attention on process-related activities. If knowledge on these activities is the goal of the video, then the prompts used, which were easy to implement, seem to give this video type an advantage over the proof video, which required a comparable amount of time, and the heuristic proof video, which required more time. Thus, the prompted proof video might be suitable at later steps of proof-based courses.

However, regardless of the video type used, particular attention should focus on providing students with learning opportunities to apply their knowledge for proof construction and to discuss and practice strategies that foster proof comprehension. When instructional videos presenting proofs are used in flipped proof-based classrooms, additional support could be implemented as an in-class component.

Data availability statement

The raw data supporting the conclusions of this article will be made available by the authors, without undue reservation.

Ethics statement

Ethical review and approval was not required for the study on human participants in accordance with the local legislation and institutional requirements. Written informed consent for participation was not required for this study in accordance with the national legislation and the institutional requirements.

Author contributions

LW: Conceptualization, Data curation, Formal analysis, Investigation, Methodology, Visualization, Writing – original draft, Writing – review & editing. KK: Conceptualization, Data curation, Formal analysis, Investigation, Methodology, Writing – review & editing. CS: Conceptualization, Data curation, Investigation, Writing – review & editing. GG: Supervision, Writing – review & editing.

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Conflict of interest

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

Publisher's note

Supplementary material

The Supplementary material for this article can be found online at: https://www.frontiersin.org/articles/10.3389/feduc.2024.1438355/full#supplementary-material

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