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Spatial reasoning in mathematics standards: identifying how early elementary educators are systematically supported to teach spatial skills

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Spatial reasoning is critical to early mathematics learning, but it is unclear how early elementary educators learn to teach and are supported in teaching its comprising skills. One view of the available supports can be found by examining the alignment of spatial reasoning skills and mathematics education standards, as standards provide the content of the intended curriculum children are expected to learn at each grade level. This study used content analysis methods to investigate how spatial reasoning might be taught through broadly adopted early elementary education standards in the United States, the Kindergarten through Grade 2 Common Core State Standards for Mathematics. The paper describes the frequency and degree of explicitness with which 38 spatial reasoning skills are therein represented. Findings indicate that most standards implicitly relate to some form of spatial reasoning through a pedagogical reach of teaching expertise, but few standards contain explicit spatial linkages. The implications and limitations of this analysis are discussed in relation to teaching spatial reasoning in early elementary grades and students' opportunities to learn these critical skills.

KEYWORDS

spatial reasoning, early childhood, mathematics education, content standards, content analysis, curriculum

1 Introduction

Spatial reasoning, or the ability to visually recognize and mentally manipulate objects' physical properties and the spatial relations between them (Bruce et al., 2017), is a hard-to-define construct comprised of skills critical to early mathematics learning (National Council of Teachers of Mathematics, 2006). Included in the category of content knowledge entitled "space and shape" on the Program for International Student Assessment (PISA), spatial reasoning skills involve considering objects' orientations, positions, properties, and representations, as well as navigation, visualization, and dynamic interactions and movements (Organization for Economic Co-operation and Development, 2023a). Spatial reasoning is one type of content knowledge that supports mathematical learning and literacy (Mix and Cheng, 2012; Hawes and Ansari, 2020; Organization for Economic Co-operation and Development, 2023a). While overall mathematics performance in OECD participating countries declined from 2018 to 2022 (Organization for Economic Co-operation and Development, 2023b), performance on the space and shape subscale was lower than all others for students in the

United States (U.S.) as well as some other countries with similar socioeconomic statuses (e.g., France, the United Kingdom; [Organization for Economic Co-operation and Development, 2023b](#)). Given the predictive nature of early mathematics experiences on later academic achievement ([Duncan et al., 2007](#)) and links between spatial reasoning and mathematics learning ([Mix and Cheng, 2012](#)), understanding how to systematically support educators in teaching spatial reasoning in early grades is crucial to bolstering students' success ([Sorby and Panther, 2020](#)).

Specifically, early opportunities to develop spatial reasoning skills can support students' later academic achievement and their interest in science, technology, engineering, and mathematics (STEM) careers ([Guay and McDaniel, 1977](#); [Lord and Rupert, 1995](#); [Duncan et al., 2007](#); [Wai et al., 2009](#); [Clements and Sarama, 2011](#)). However, number sense is often the primary focus of early mathematics instruction, and spatial reasoning is de-emphasized within both classroom instruction and written curricula ([Copley, 2010](#); [Clements and Sarama, 2011](#); [Bruce et al., 2012](#); [Gilligan-Lee et al., 2022](#); [Parviainen et al., 2023](#)). While early elementary educators (EEEs) [i.e., those teaching kindergarten through grade 2 (K-2)] are the best-positioned professionals to integrate spatial reasoning into young children's mathematics instruction, they receive highly variant pre- or in-service professional development and support to teach spatial reasoning through mathematics ([Ginsburg et al., 2006](#); [Clements and Sarama, 2011](#); [Gagnier et al., 2022](#); [Gilligan-Lee et al., 2022](#); [Bufasi et al., 2024](#)). One way to promote EEEs teaching spatial reasoning is by supporting them in integrating specialized teaching practices into their current mathematics instruction ([Hawes et al., 2017](#)).

Education standards offer a viable pathway to integrate spatial reasoning in mathematics as they represent the intended curriculum ([Porter et al., 2011](#); [Boda et al., 2022](#)); however, scholars have differing views about the role of education standards and how spatial reasoning appears therein. The present study builds upon the extant bodies of research that document education standards as the intended curriculum in the U.S. ([Porter et al., 2011](#)) and call for the integration of spatial reasoning skills into early elementary mathematics education (c.f., [Davis et al., 2015](#); [Hawes et al., 2017](#)). While there is no national curriculum or standards in the U.S., the Common Core State Standards for Mathematics (CCSS-M; [Common Core State Standards \(CCSS\) Initiative, 2010](#)) were initially adopted by 46 states plus the District of Columbia and continue to be used in part or whole by many states (see [Edgerton, 2020](#)). Researchers have taken up similar work in a European context (see [Bufasi et al., 2024](#)) where national curricula exist; this work proposes a compromise to the opposing viewpoints about standards in the U.S. context. Specifically, [Boda et al. \(2022\)](#) identified implicit relations between the CCSS-M and spatial reasoning, while [Gilligan-Lee et al. \(2022\)](#) asserted that curricular and policy changes are necessary to support EEEs in teaching spatial reasoning. This research posits that standards can support teaching practice, as they specify the content of the intended curriculum, and explores the relations between spatial reasoning and the K-2 CCSS-M.

2 Literature review

Spatial reasoning is a set of cognitive skills that allows humans to interact with the physical world and supports our making sense of the world mathematically ([Freudenthal, 1973](#); [Clements and](#)

[Battista, 1992](#); [National Research Council, 2009](#)). It includes skills that clearly relate to mathematics (e.g., composing and decomposing, transforming; [Davis et al., 2015](#)) and skills that seem more practical in nature and less directly aligned with mathematics education (e.g., mapping, perspective-taking; [Muir and Cheek, 1986](#); [Liben and Downs, 1989](#); [National Council of Teachers of Mathematics, 2006](#); [Sarama and Clements, 2009](#)). Nonetheless, skills across this nebulous construct support learning and achievement in other mathematics domains ([Hawes and Ansari, 2020](#); [Gilligan-Lee et al., 2022](#)).

Historically, spatial reasoning was an integral part of early mathematics instruction (e.g., Freubel's model of kindergarten), wherein children were presented with learning opportunities on numbers, geometry, and spatial thinking together ([Ginsburg et al., 2008](#)). However, spatial reasoning has been all but eliminated from early grades mathematics instruction in most industrialized countries over the last 175 years ([Sinclair and Bruce, 2015](#)). The spatial reasoning content that does remain in curricula relates mainly to geometry and measurement domains, like transforming shapes and understanding length, area, and capacity ([Gilligan-Lee et al., 2022](#)).

Despite knowledge of strong linkages between spatial reasoning and mathematics learning ([Mix and Cheng, 2012](#); [Hawes and Ansari, 2020](#)), defining spatial reasoning and identifying its comprising skills represents a long-standing challenge for psychology and mathematics education researchers ([Lohman, 1979](#); [McGee, 1979](#); [Chu and Kita, 2011](#); [Uttal et al., 2013](#); [Bruce et al., 2017](#)). Whereas the study of spatial reasoning skills over time has been taken up frequently through a psychological approach and focused on spatial ability, spatial orientation, and spatial visualization ([Uttal et al., 2013](#)), models exist in the mathematics education landscape as well, including the van Hiele levels of geometric thought ([Fuys et al., 1984](#)) and [Uttal et al.'s \(2013\)](#) classification of spatial reasoning skills. More recently, [Davis et al. \(2015\)](#) introduced an emergent conceptualization of spatial reasoning that illustrates the multifaceted nature of the construct in a way intended to support the spatialization of school mathematics. This study uses an adapted form of [Davis et al.'s \(2015\)](#) model to illustrate the granularity of skills comprising spatial reasoning. See [Table 1](#) for the skills individuals use when reasoning spatially, the structure of which will be explained in Section 2.2.

2.1 Spatial reasoning in the United States education standards

Mathematics education standards in the U.S. were derived from curricular plans that originated in the 19th century, which magnified number sense and computation while excluding spatial reasoning ([Davis et al., 2015](#)). Further, the U.S. instructional model intends to tightly align curriculum and assessment with those standards ([Porter et al., 2011](#); [Pak et al., 2020](#)), which should influence what content educators teach. [Rivera \(2011\)](#) asserted that the tightly aligned system of education standards, instruction, and assessment in the U.S. has limited how teaching occurs and that common mathematics instructional practices de-emphasize visualization and spatial reasoning. However, the perceived role of education standards in teaching and learning differs between policymakers and practitioners. Whereas a policy-oriented view says that standards represent the intended curriculum ([Porter et al., 2011](#)), a practitioner-oriented view

TABLE 1 Spatial reasoning skills: list adapted from Davis et al. (2015) emergent representation of spatial reasoning.

Overarching skills	Elements	Subelements
Transforming and Understanding	Moving	balancing, reflecting, rotating, translating
	Altering	dilating/contracting and scaling, distorting/morphing, folding, shearing
	Situating	dimension-shifting, intersecting, locating, mapping, orienting, pathfinding
	de/re/Constructing	de/re/arranging, de/re/composing and sectioning, un/re/packing and fitting
	Interpreting	comparing, designing, diagramming, modeling, relating, symmetrizing
	Sensating	imagining, perspective-taking, projecting, propriocepting, tactilizing, visualizing

indicates that standards may narrow teachers' instructional practice (Kaufman et al., 2018).

This research approaches the current policies around what should be taught at each grade level through the lens that standards *can* support EEEs in teaching mathematics skills that are not explicit in their standards *if* they know how and have the tools. I herein take an asset view of education standards as supporting EEEs' practices by providing affordances to teach skills represented within those standards, even if unspecified or implicit. In that, there is a need for teachers to understand the connections between spatial reasoning skills and their mathematics standards to integrate spatial reasoning into their teaching practices, as most standards do not explicitly specify teaching these skills (Gilligan-Lee et al., 2022; Pinilla, 2023). It is essential to understand both the standards as written and the openings through which spatial reasoning can be taught (Boda et al., 2022).

2.1.1 Education standards as the intended curriculum

If we consider education standards as specifying the "content of the intended curriculum" (Porter et al., 2011, p. 103) and primary informers of EEEs' written curricula (i.e., the materials, resources, and guides teachers use for teaching; Remillard, 2005), we can identify teachers' opportunities to teach spatial reasoning. Using this argument, it logically follows that written curricula would provide the content of the learning targets and suggested instructional practices on which EEEs can layer pedagogical practices to teach toward the standards. This layering of intended and written curricula plus pedagogy should increase students' opportunities to learn specified skills. However, there is variation in intended and written mathematics curricula in the U.S. based on state policy and local resources.

While the National Council of Teachers of Mathematics (2000, 2006) outlined standards for what mathematics content and processes should be taught at each grade level in U.S. schools, they are not the same as mandated education standards. Instead, education standards adopted at the state level are what teachers are held accountable for teaching, and no national intended or written curriculum exists. Further, there is debate on how well-aligned standards, written curricula, and instructional practices are in the classroom (Rivera, 2011; Polikoff, 2012; Pak et al., 2020). While the provision of written curricula has been proposed as a support for teaching practice (Whitehurst, 2009; Chingos and Whitehurst, 2012), written curricula in the U.S. are locally selected and, therefore, have less transferability to alternative contexts (Polikoff, 2018). This means that the intended curricula should be studied to identify ways for EEEs to broadly incorporate spatial reasoning into their teaching practices. Further,

due to the predictive nature of early mathematics experiences on later academic achievement (Duncan et al., 2007), it is critical to better identify how EEEs can incorporate spatial reasoning teaching practices into their intended curricula through pedagogical practice without detracting from children's other mathematics learning, and how this can be done for all children, not just a few.

2.1.2 Content of early elementary mathematics standards

Numeric relational reasoning and spatial reasoning are two primary constructs of mathematical thinking and development that warrant instructional emphasis in K-2 (National Council of Teachers of Mathematics, 2006; National Research Council, 2009; Perry, 2016). However, spatial reasoning skills are seldom referenced explicitly within intended curricula, such as the CCSS-M (Gilligan-Lee et al., 2022). The current layering of the intended and written curricula in the U.S. generally underrepresents spatial reasoning (Gilligan-Lee et al., 2022); where (i.e., in what content standards) and how (i.e., implicitly, optionally, or explicitly) the skills are included varies, meaning many students may not be receiving opportunities to learn to reason spatially. Boda et al. (2022) found implicit opportunities to teach spatial reasoning within the K-2 CCSS-M, but those connections were mainly within the Measurement and Data or Geometry domains (Common Core State Standards (CCSS) Initiative, 2010). Given the small number of standards in those domains compared to the overall number of mathematics standards at each grade (e.g., three out of 26 second-grade standards are in the Geometry domain), spatial reasoning's representation is considered relatively sparse.

Despite few standards related to spatial reasoning, Fowler et al. (2019) asserted that spatial reasoning can be taught through overarching standards. Davis et al. (2015) supported that idea, claiming that most spatial reasoning skills in their emergent conceptualization (see also Table 1; Pinilla, 2024) could be taught using standard mathematics curricula. This means that by intentionally planning instruction using mathematics education standards (e.g., the CCSS-M), EEEs could provide children with opportunities to learn spatial reasoning skills without interrupting the scope and sequence of their existing instruction. However, few EEEs are prepared to supplement their given mathematics instructional tools (Ginsburg et al., 2006), and adopted, commercially produced curricular packages do not typically offer spatially relevant connections (Kalyankar, 2019). Given the minimal explicit representation of spatial reasoning in common education standards (i.e., CCSS-M) and knowledge that recommended curricular reforms (Gilligan-Lee et al., 2022) may prove difficult, this work seeks to locate

existing connections between the standards and spatial reasoning skills.

2.2 Conceptual framework

This study uses spatially specific definitions (see [Appendix A](#); [Pinilla, 2024](#)) and the K-2 CCSS-M ([Common Core State Standards \(CCSS\) Initiative, 2010](#)) to identify potential opportunities for EEEs to teach spatial reasoning through mathematics. This work was informed by [Davis et al.'s \(2015\)](#) emergent conceptualization of spatial reasoning skills that aimed to capture the interconnectedness and emergent nature of skills used when reasoning spatially. While [Davis et al. \(2015\)](#) offered their conceptualization, and some researchers have used the named skills to identify spatial reasoning occurring in classrooms (e.g., [McFeetors et al., 2022](#)), the skills themselves were not explicitly defined. To better assess the relations between the skills and the standards, I adopted or refined the words [Davis et al. \(2015\)](#) presented to create a conceptual framework, then defined each term to be spatially specific (see [Appendix A](#); [Pinilla, 2024](#)).

The conceptual framework (see [Figure 1](#); [Pinilla, 2024](#)) is structured with two overarching skills that include six elements, each containing four to six subelement skills (see [Figure 1](#)). The terms within the conceptual framework (see [Table 1](#)) name spatial reasoning skills that are used throughout this study in specific ways; it is important to call attention to this because many included words have multiple, nonspatial meanings (e.g., *moving*, *understanding*, etc.). When any of the 40 included terms are used as the spatial reasoning skill, the word will be presented in italics. Overarching skills (i.e.,

Transforming and *Understanding*) will be called out as such; they are nebulous and not assigned into nested structures. There are also six spatial reasoning elements, which all contain subelements; they appear far more frequently and are demarcated with elements capitalized in italics and subelements in all lower case (e.g., *Interpreting* contains *comparing*, *diagramming*, etc.). While the skills are inherently interrelated, subelements comprise an element, and elements come together to fill out the overarching skills.

2.3 Objectives and research questions

While it is well-established that spatial reasoning holds promise to support student learning in STEM domains and enhance STEM career interests ([Mix and Cheng, 2012](#); [Hawes and Ansari, 2020](#)), less is known about how EEEs can integrate this type of learning into their classrooms. There is a gap between calls to spatialize school curricula ([Davis et al., 2015](#)) and the implicit relations between spatial reasoning and current education standards ([Boda et al., 2022](#)). Although some relations may be apparent to educators with appropriate training and pedagogical understandings, well-defined supports for teaching spatial reasoning through mathematics are needed to systematically support EEEs in teaching these skills. To make the relations between spatial reasoning skills and one set of early elementary mathematics standards clear, this study investigated the extent to which spatial reasoning is represented in mathematics content standards of the CCSS for the early elementary grades (i.e., K-2 CCSS-M). The study sought to answer the questions:

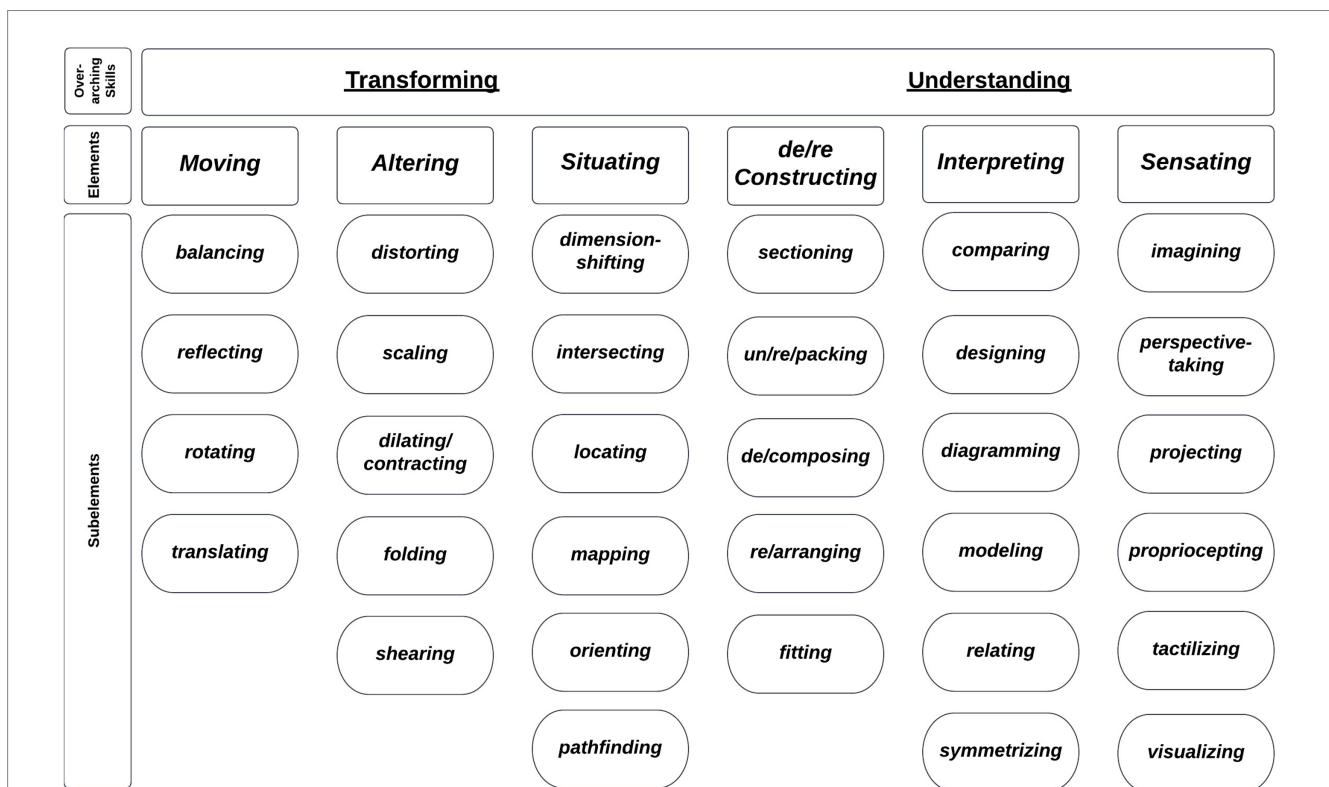


FIGURE 1 Spatial reasoning conceptual framework: terms adapted from [Davis et al. \(2015\)](#) emergent conceptualization of spatial reasoning. See [Appendix A](#) and [Pinilla \(2024\)](#) for details on the framework and its definitions..

1. How frequently is spatial reasoning implicitly represented in the K-2 CCSS-M?
2. To what extent is spatial reasoning represented explicitly and optionally in the K-2 CCSS-M?

The first question provided a structure to examine the frequency of opportunities to teach spatial reasoning in the standards, whereas the second question sought to specify the degree of explicitness with which spatial reasoning skills were represented. By responding to these questions, this study meaningfully adds to the literature by identifying openings in a common intended curriculum for early grades mathematics in the U.S. through which EEs can include spatial reasoning in their typical mathematics instruction.

3 Methods

As part of a larger project seeking to understand EEs' spatial reasoning teaching practices, this study used an abridged form of content analysis (Krippendorff, 2019) to assess the frequency and extent of spatial reasoning's explicitness in a set of K-2 mathematics content standards (i.e., CCSS-M; [Common Core State Standards \(CCSS\) Initiative, 2010](#)). Using 38 of the skills from a conceptual framework derived from [Davis et al.'s \(2015\)](#) emergent conceptualization of spatial reasoning (see [Figure 1](#)) and the K-2 CCSS-M as the two sources of meaningful matter from which to make relational inferences (Krippendorff, 2019), standards were coded for the frequency with which spatial reasoning was implicitly represented in the standards and then the explicitness of their relation. The analysis steps included first-cycle independent coding, establishing intercoder reliability through consensus conversations, second-cycle coding, and narrating the findings quantitatively and qualitatively.

3.1 Coding

3.1.1 First cycle

I developed an *a priori* coding scheme using Krippendorff's (2019) content analysis components and followed Saldaña's (2016) protocol coding methods. Whereas [Davis et al.'s \(2015\)](#) emergent conceptualization of spatial reasoning included 40 skills, I used 38 as *a priori* categories to deductively code all K-2 CCSS-M standards. Two skills named in [Davis et al.'s \(2015\)](#) conceptualization were considered overarching skills (i.e., transforming and understanding); they were too broad to make meaningful connections with standards when initially coding. I abductively inferred (Krippendorff, 2019) the presence of the 38 skills in each standard using constant comparison ([Glaser and Strauss, 1967](#)) to decide whether each spatial reasoning skill was implicit in a standard. I assigned the skills dichotomous values of 1 for present or 0 for not present. Coding in this phase was utilitarian; if there was any relation, the skill was coded as present.

3.1.2 Consensus coding

I next collaborated with an experienced educator to establish intercoder reliability, which is fundamental in enhancing the transparency of methods and quality of content analyses ([O'Connor and Joffe, 2020](#)). I hired the educator as a project consultant and trained them on the coding scheme by providing a project overview

and time to independently review and give written feedback on the definitions of the 38 spatial reasoning skills (see [Appendix A](#) for definitions). In their feedback, the consultant documented their understandings and sought clarity between some terms that were close in meaning. For example, they questioned the difference between some inner-element subelement skills, like *packing* and *fitting*, which are subelements of *Constructing*. Due to the close meanings and anticipated challenges in differentiating them meaningfully within standards, we condensed three pairs of inner-element subelements into groups. Within the *Altering* element, *scaling* was grouped with *dilating/contracting*. Within the *Constructing* element, *packing* and *fitting* were made into one group, while *sectioning* was grouped with *de/re/composing*. While the structure of the conceptual framework (see [Figure 1](#)) remained, we coded from thereon with the grouped codes, reducing the number of codes to 35.

I assigned the consultant a proportional stratified random sample of 20% of the K-2 CCSS-M to double-code for the presence of spatial reasoning skills. Our initial agreement was 83.44% across spatial reasoning skills. While there is no definitive guideline, [Miles et al. \(2014\)](#) recommended that coders reach 85 to 90% overall agreement; more granularly, [Miles and Huberman \(1994\)](#) suggest that coders establish 80% agreement across 95% of the codes for 20% of the data to establish intercoder reliability. To reconcile differences, we engaged in consensus conversations and discussed the nine spatial reasoning skills on which our agreement was less than 80%. Through these conversations, we came to negotiated understandings of how those skills appeared in standards and reached an overall agreement of 95.25%. I applied the updated meanings for those spatial reasoning skills about which we had the consensus conversations to all independently coded standards. I calculated descriptive statistics to narrate how spatial reasoning is implicitly represented across the K-2 CCSS-M.

3.1.3 Second cycle coding

I extended the analysis from dichotomously identifying the implicit presence of spatial reasoning in the standards to applying a four-point scale to determine the extent to which spatial reasoning was represented explicitly, optionally, implicitly, or absent in the standards. While initial results indicated that spatial reasoning is somewhat frequently represented within the K-2 CCSS-M, the degree of inference required to see those connections required tacit knowledge of the skills and pedagogy. I engaged in second-cycle coding to answer the second research question and determine how explicitly the skills were represented in the standards. I recoded all spatial reasoning skills marked as present in a standard as explicit, optional, or implicit.

Standards explicitly connected to spatial reasoning name one or more skills (e.g., *composing*) in the standards themselves. For example, *composing* is present in standard K.G.6 ([Common Core State Standards \(CCSS\) Initiative, 2010](#), p. 12), which explicitly calls for children to *compose* shapes. Skills coded as optional were those given as optional representations in a standard; *diagramming* is named in standard 2.OA.1 ([Common Core State Standards \(CCSS\) Initiative, 2010](#), p. 19), wherein a child *could* draw a diagram to solve a problem, but it is not required to demonstrate that skill to master the standard. Implicitly represented skills are those within pedagogical reach by an experienced teacher but not named explicitly or as an optional representation in the standard. For instance, K.CC.4.b requires

children to understand that the last number said when counting objects is the total regardless of the arrangement, which implicitly relates to the *rearranging* skill. Finally, spatial reasoning skills with no connection to a standard are those that could not be incorporated when teaching toward a standard without significant planning and alterations to the teaching practice; they were those coded as not present in the first cycle. All K-2 CCSS-M were coded using this more granular scheme, and findings were narrated using descriptive statistics and graphically.

3.2 Narrating findings

To communicate findings, I generated descriptive statistics (i.e., frequencies) to reduce the data and narrated the aggregate counts of how many times and to what degree of explicitness each spatial reasoning skill was represented within the K-2 CCSS-M overall, by grade, and by spatial reasoning element and subelement (Krippendorff, 2019). This quantification allowed the comparison of how well spatial reasoning skills were represented in the standards. Further, by aligning the K-2 CCSS-M content standards to the spatial reasoning conceptual framework, the standards served as an alternate analytic construct to conjecture the extent to which students have opportunities to learn spatial reasoning via how intentionally their teachers could teach each skill, given this mapping as a tool.

4 Findings

The extent to which spatial reasoning skills are represented in the K-2 CCSS-M varies greatly by skill and degree of explicitness. Despite disagreement in the extant research on the presence of skills (e.g., Boda et al., 2022 vs. Gilligan-Lee et al., 2022), this study found that some type of spatial reasoning is implicitly present in most examined standards. In this section, I first describe the breadth of spatial reasoning's representation within the K-2 CCSS-M as opportunities to teach spatial reasoning skills. I then characterize the degree of explicitness with which those skills are present in the standards. Notably, while 40 skills are named in the conceptual framework (see Figure 1) as adapted from Davis et al. (2015), results are reported on only 35 skills. This is because the two overarching skills (i.e., *Transforming* and *Understanding*) are too broad to be associated with individual standards in a meaningful way, and the number of codes was further reduced from 38 to 35 during coding due to close meanings.

4.1 Implicit representation of spatial reasoning in education standards

In response to the first research question, I found that spatial reasoning skills are implicitly represented in the K-2 CCSS-M frequently, but that frequency varies between skills. Table 2 shows each spatial reasoning element's and subelement's implicit representation by grade. Representation by element was calculated based on the aggregate representation of its comprising subelements. Across elements, *Constructing*, *Interpreting*, and *Sensating* were each implicitly represented in more than 60% of standards, whereas *Altering*, *Moving*,

and *Situating* were each represented in less than 40%. While *Moving* subelements were conjectured to relate to Geometry standards closely, fewer standards called for transformations as grade levels increased, which translated to lower representation. Conversely, *Constructing* subelements were often represented in standards that included numeric compositions and decompositions, which are numerous and meant higher overall representation.

The better-represented elements, *Constructing*, *Interpreting*, and *Sensating*, were considered initially to generally align with the overarching skill of *Understanding* (i.e., realizing and making sense of spatial relationships; see Appendix A). For example, subelements of *Interpreting* were implicitly represented in 21 of the 25 kindergarten standards, 21 of the 24 Grade 1 standards, and 22 of the 28 Grade 2 standards (see the *Interpreting* row in Table 2). This high degree of representation emerged from the number of standards across mathematics domains calling for students to engage in *comparing*, *modeling*, and *relating* that offer students opportunities to make sense of how objects relate spatially.

However, there was variation within elements, wherein some subelement skills were implicitly present in more standards than others. Many subelements within the well-represented elements were referenced infrequently or not at all. For instance, *symmetrizing* is a subelement of *Interpreting* (the best-represented element) but was only implicitly represented in a single kindergarten standard. Likewise, two *Sensating* subelements were ill-represented; *propriocepting* was represented in no standards, and *perspective-taking* was represented in less than one standard per grade level, on average ($M = 0.67$). Of note, *propriocepting* (i.e., receiving stimuli via physical senses regarding spatial awareness or movement; see Appendix A) is a term that was found infrequently in extant literature but appears closely related to *tactilizing* skills. However, finding representation in the K-2 CCSS-M for such a nuanced skill proved elusive in the coding process.

Like those elements with more representation, there was variation between the elements and subelements less represented in the standards. Specifically, *Altering*, *Moving*, and *Situating*, which were thought to generally align with the overarching skill of *Transforming* [i.e., (Mentally) changing, moving, creating, or removing objects, and imagining outcomes of those changes; see Appendix A], were each implicitly represented in 11 to 39% of the standards on average across grades (see Table 2). *Transforming* skills focus on making or imagining physical changes, which can be taught through concrete *modeling*, (i.e., an *Interpreting* subelement) a commonly named activity in the CCSS-M that allows students to demonstrate their knowledge of a standard.

4.2 Varying degrees of representation

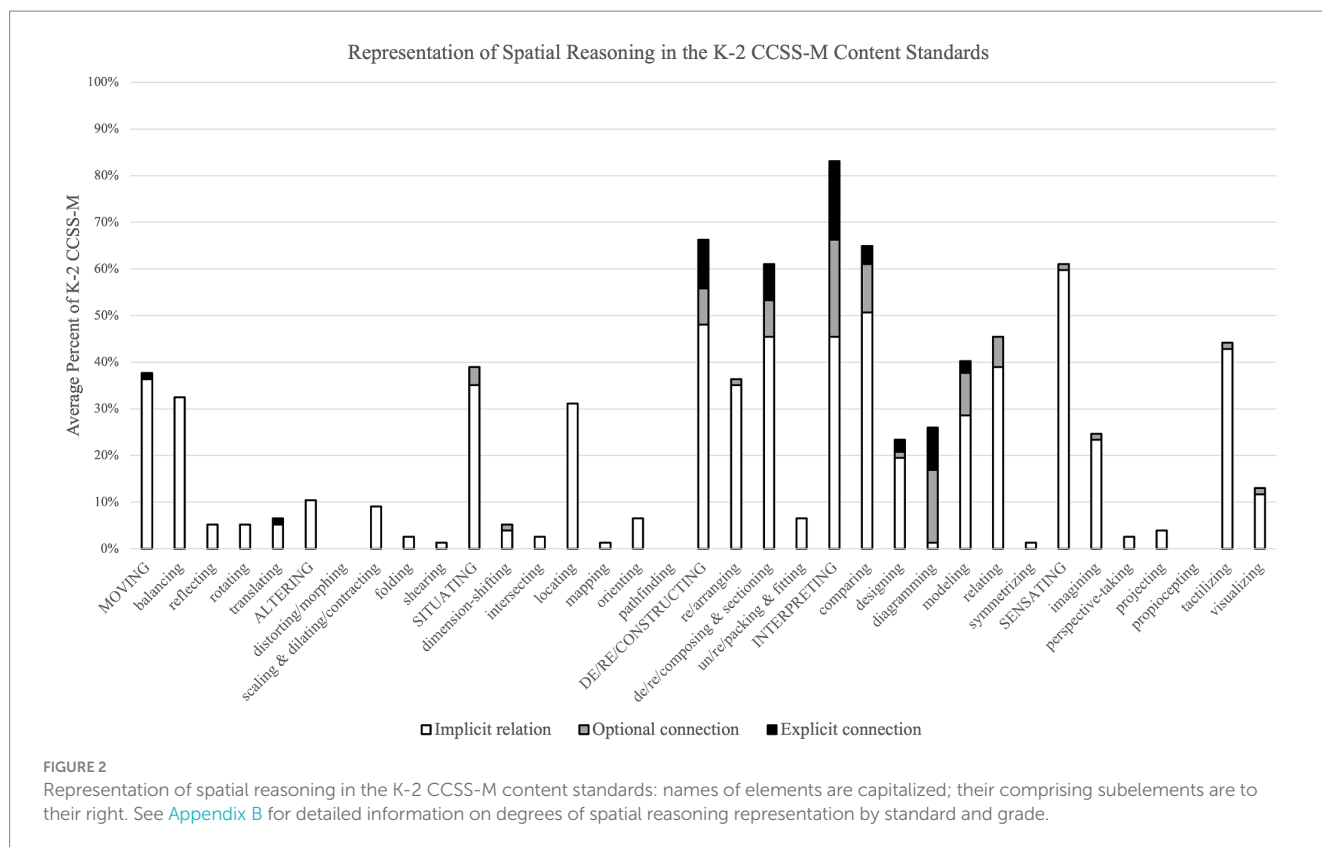
Whereas 32 of the 35 coded skills were found to be implicitly represented in the K-2 CCSS-M, findings for research question 2 indicate that few skills are represented explicitly or optionally. Only six skills were explicitly represented in any standard, and only 11 were ever represented as options. Figure 2 illustrates the average percent of standards across grade levels that explicitly, optionally, and implicitly include each skill, with the elements (i.e., *Moving*, *Altering*, *Situating*, *Constructing*, *Sensating*, and *Interpreting*) aggregating the presence of their subelements. All explicit representations fall within the *Constructing* and *Interpreting*

TABLE 2 Implicit spatial reasoning representation in mathematics standards by grade: The number of standards for which skills are implicitly present at the element level includes each subelement; more than one subelement may be represented in any given standard.

Spatial reasoning term	Number of standards in which the skill is implicitly present						Average per grade	Overall representation
	Kindergarten		Grade 1		Grade 2			
	<i>n</i>	%	<i>n</i>	%	<i>n</i>	%	<i>n</i>	%
Elements and subelements								
Moving	9	0.360	12	50.00	8	0.286	9.67	0.377
balancing	7	0.280	10	41.67	8	0.286	8.33	0.325
reflecting	3	0.120	1	4.17	0	0.000	1.33	0.052
rotating	3	0.120	1	4.17	0	0.000	1.33	0.052
translating	3	0.120	2	8.33	0	0.000	1.67	0.065
Altering	3	0.120	1	4.17	4	0.143	2.67	0.104
distorting/morphing	0	0.000	0	0.00	0	0.000	0.00	0.000
dilating/contracting and scaling	3	0.120	1	4.17	3	0.107	2.33	0.091
folding	1	0.040	0	0.000	1	0.036	0.67	0.026
shearing	0	0.000	0	0.000	1	0.036	0.33	0.013
Situating	15	0.600	6	0.250	9	0.321	10.00	0.390
dimension-shifting	3	0.120	1	0.042	0	0.000	1.33	0.052
intersecting	0	0.000	1	0.042	1	0.036	0.67	0.026
locating	12	0.480	4	0.167	8	0.286	8.00	0.312
mapping	1	0.040	0	0.000	0	0.000	0.33	0.013
orienting	4	0.160	1	0.042	0	0.000	1.67	0.065
pathfinding	0	0.000	0	0.000	0	0.000	0.00	0.000
de/re/Constructing	15	0.600	18	0.750	18	0.643	17.00	0.662
de/re/arranging	10	0.400	10	0.500	6	0.214	9.33	0.364
de/re/composing and sectioning	15	0.600	16	0.667	16	0.571	15.67	0.610
un/re/packing and fitting	0	0.000	3	0.125	2	0.071	1.67	0.065
Interpreting	21	0.840	21	0.875	22	0.786	21.33	0.831
comparing	14	0.560	17	0.708	19	0.679	16.67	0.649
designing	4	0.160	6	0.25	8	0.286	6.00	0.234
diagramming	4	0.160	7	0.292	9	0.321	6.67	0.260
modeling	13	0.520	10	0.417	8	0.286	10.33	0.403
relating	13	0.520	11	0.458	11	0.393	11.67	0.445
symmetrizing	1	0.040	0	0.00	0	0.000	0.33	0.013
Sensating	18	0.720	14	0.583	16	0.571	15.67	0.610
imagining	4	0.160	6	0.25	10	0.357	6.33	0.2468
perspective-taking	2	0.080	0	0.00	0	0.000	0.67	0.026
projecting	1	0.040	1	0.042	1	0.036	1.00	0.039
propriocepting	0	0.000	0	0.00	0	0.000	0.00	0.000
tactilizing	15	0.600	9	0.375	10	0.357	11.33	0.442
visualizing	3	0.120	3	0.125	4	0.143	3.33	0.130

elements. Specifically, the *Constructing* subelement *composing* mostly appears within standards relating to composing and decomposing numbers. The *Interpreting* subelements *comparing*, *diagramming*, and *modeling* appear within standards relating to measurement and geometry. For greater specificity on the relations,

see [Appendix B](#) for the standards coded by spatial reasoning skills with the degree of connection represented as heatmaps. Standards with no relation to a skill are blank on the map. Those with implicit representation show an empty circle, optional representations show a half-filled circle, and explicit representations show a completely



filled circle. Overall, few spatial reasoning skills are explicitly represented in the K-2 CCSS-M, and there is slightly higher optional representation.

Based on this more nuanced analysis, I found that of the six subelements explicitly represented (i.e., *translating*, *de/re/composing* and *sectioning*, *comparing*, *designing*, *diagramming*, and *modeling*), four were from the *Interpreting* element (see subelements shaded black at the top of their bars in Figure 2). Of the 11 subelements connected to standards as an option for how a student could demonstrate that intended learning target (i.e., *dimension-shifting*, *re/arranging*, *de/re/composing & sectioning*, *comparing*, *designing*, *diagramming*, *modeling*, *relating*, *imagining*, *tactilizing*, and *visualizing*) five were elements that were also explicitly represented (see subelements shaded grey in the middle or top of their bars in Figure 2). The greatest representations of optional connections were clustered in the *Constructing* and *Interpreting* elements. To explicate these differences and better highlight the response to the overarching research question, I next review each element and provide descriptive statistics to indicate the extent to which each subelement skill is implicitly, optionally, and explicitly represented in the standards. Frequencies of implicit representations can be found in Table 2, and details about explicit and implicit relations are located in Appendix B.

4.2.1 Moving

On average, almost 10 standards per grade level ($M=9.67$) implicitly related to *Moving* skills (i.e., changing the position of something by using spatial transformations; see Appendix A). Of *Moving's* subelements, *balancing* (i.e., bringing into proportion or visual equilibrium by creating equivalence or sameness; see Appendix A) was represented most frequently ($M= 8.33$), with

standards from the Operational and Algebraic Thinking domain of the CCSS-M across grades often found to relate with the skill implicitly. For example, standard 2.OA.3 reads, “Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends” (Common Core State Standards (CCSS) Initiative, 2010, p. 19). Given the example of pairing objects in the standard, I inferred that students would work with concrete manipulatives when pairing and counting by 2s and use the equals sign to symbolize balance when writing the equation. This inference was supported when discussing *balancing* with the EEE consultant engaged in the coding; she referenced using a pan balance when teaching about the equals sign. While the example illustrates a high-inference representation, we agreed on the skill’s implicit presence in the standard, which gives credence to the idea that pedagogy and shared understandings of the tools used in mathematics instruction are within the knowledge and practices of experienced educators and researchers.

However, the representation of other *Moving* subelements tended to decrease as grade level increased. Whereas there were implicit representations of *reflecting*, *rotating*, and *translating* in three standards each at kindergarten, there was no representation of those skills by second grade (see details in Appendix B). This difference is associated with how the Geometry domain standards are written across the grades; kindergarten standards state that children will be *composing* shapes and *modeling* compositions (i.e., K.G.4, K.G.5, and K.G.6), yet second-grade standards focus on partitioning shapes into fractional pieces (i.e., 2.G.2 and 2.G.3), and the *Moving* subelements were not found to be represented therein.

Connections to *Moving* skills were generally limited to those implicit opportunities. No standards across grade levels provided optional opportunities to teach any *Moving* subelements, but one standard at kindergarten did explicitly connect to the *translating* skill. CCSS-M.K.G.6 says, “Compose simple shapes to form larger shapes. For example, ‘Can you join these two triangles with full sides touching to make a rectangle?’” (Common Core State Standards (CCSS) Initiative, 2010, p. 12), which requires students to translate shapes, thus forming the single explicit connection between *Moving* skills and the K-2 CCSS-M.

4.2.2 Altering

Altering (i.e., modifying or changing something’s appearance; see Appendix A) was the least represented element from the spatial reasoning conceptual framework in the K-2 CCSS-M. Its comprising subelements were found to implicitly relate with fewer than three standards per grade level ($M=2.67$). Of *Altering*’s subelements, *dilating/contracting* and *scaling* skills were found most frequently. In kindergarten standards, the Geometry domain contained references to these skills. However, at the first- and second-grade levels, any representation was within the Number and Operations in Base Ten domain. Specifically, those standards that call upon children to “bundle” ones into 10s or 10s into 100s (e.g., 1.NBT.2.a and 2.NBT.1.a) implicitly relate to *scaling*, or understanding the correspondence between unit sizes (see Appendix A). However, other *Altering* subelements (i.e., *folding* and *shearing*) were found to have very little representation across grades, with one or fewer standards implicitly related to each skill per grade (see Appendix B). Further, no standards offered optional or explicit connections to *Altering* skills.

4.2.3 Situating

Situating subelements were implicitly represented in 10 standards per grade level, on average. *Locating* skills were identified most often in an average of eight standards per grade. Specifically, standards in the Measurement and Data domain (e.g., 2.MD.1 or 2.MD.2) tended to relate to *locating*. For example, when comparing an object to a measurement device (e.g., measuring the height of a shelf with a yardstick), students locate the corresponding number on that measurement device that tells the object’s size. Additionally, time-telling standards (e.g., 1.MD.3 or 2.MD.7) require students to locate the numbers on an analog clock and determine their meaning. However, these connections are all implicit.

Surprisingly few standards were related to *dimension-shifting* (i.e., moving between two and three dimensions; see Appendix A) across grades ($M=1.33$). Some standards call upon students to recognize shapes as being two- or three-dimensional (i.e., K.G.3) or to compose two- or three-dimensional shapes (i.e., 1.G.2) but not work across dimensions. The *intersecting*, *orienting*, and *pathfinding* subelements were either infrequently represented or not at all. *Orienting* (i.e., understanding and operating on the relationships between the positions of objects in space with respect to one’s own position; see Appendix A) was best represented in kindergarten Geometry domain standards (i.e., K.G.1, K.G.2, K.G.3) in which students were to recognize objects regardless of their orientation. *Intersecting* (i.e., meeting or crossing at a point or in a plane; see Appendix A) was only represented in first- and second-grade standards that called upon students to represent data (i.e., 1.MD.4 and 2.MD.9). No standards contained a reference to *pathfinding*.

Similarly to skills in the *Moving* element, *Situating* skills were most often implicit within standards. One kindergarten standard offered an optional connection to *dimension-shifting* (i.e., moving between two and three dimensions; see Appendix A) but there were no other optional or explicit connections between this set of skills and the K-2 CCSS-M. Of note, optional connections were found to the *Situating* element in two kindergarten standards, but they did not connect similarly with any of its comprising subelements.

4.2.4 Constructing

On average, the *Constructing* element was implicitly represented in 17 standards per grade level. Of those, *de/re/composing* and *sectioning* (i.e., putting together parts to make a whole, taking wholes apart or cutting into parts, and putting together again in a different way; see Appendix A) was the group of subelements most frequently represented. On average, 15.67 standards per grade level required students to compose, though most often, those compositions were of numbers; the implicit connections to spatial reasoning emerged through the examples of *how* students would demonstrate mastery of a standard. For example, 1.OA.1 reads,

Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem (Common Core State Standards (CCSS) Initiative, 2010, p. 15).

This standard includes examples of concrete objects and drawings, which provide optional connections to the *de/re/composing* subelement; on average, two standards per grade level contained such optional connections, and two standards per grade level explicitly connected to them (see Table 2).

The subelement *de/re/arranging* is also relatively well represented implicitly ($M=9.33$) through standards that specify the use of concrete representations in their examples. However, there was only one standard at kindergarten that offered an optional connection to that subelement (i.e., K.CC.4.B). Conversely, the *un/re/packing* and *fitting* subelements were infrequently represented in the standards ($M=1.67$). Implicit relations were found within the Measurement and Data domain (e.g., 1.MD.2 asks for students to lay multiple copies of a comparator end to end when measuring an object, thus determining how many of those pieces *fit* into the same length) and the Geometry domain (i.e., 2.G.2 and 2.G.3 address partitioning shapes in different ways). No optional or explicit connections were found between standards and *un/re/packing* and *fitting*.

4.2.5 Interpreting

The *Interpreting* element was better represented in the standards than all others, with at least one of its subelements appearing implicitly in an average of 21 standards across grade levels ($M=21.33$). For example, 13 of the 25 kindergarten standards called upon students to model their solutions, which implicitly supports the *modeling* subelement (i.e., constructing [scale or dimension-shifted] representations of real spaces to simplify problems when interpreting information; see Appendix A). That high frequency of representation was slightly lower in grades 1 and 2, wherein *modeling* was represented

implicitly in nine standards each. When assessing optional representations, there were connections to two standards per grade level on average, and *modeling* was only explicitly described in a single standard in each kindergarten and grade 2 (see [Figure 2](#) and [Appendix B](#)).

Conversely, the *comparing* subelement (i.e., judging sameness or difference by distinguishing between forms using appearance-based relational reasoning; mapping correspondences between two or more forms; see [Appendix A](#)) was implicitly represented in standards with greater frequency as grade levels increased. Whereas 14 kindergarten standards implicitly called upon students to use *comparing* skills, 19 of the 28 grade 2 standards did the same (see [Table 2](#)). However, the number of optional and explicit representations of *comparing* were found to be similar to that of *modeling*. I located optional practices to support *comparing* with increasing frequency across grades, which averaged almost three standards per grade ($M=2.67$) and explicit connections to one standard per grade on average. For example, 1.MD.1 is “Order three objects by length; compare the lengths of two objects indirectly by using a third object” ([Common Core State Standards \(CCSS\) Initiative, 2010](#), p. 16). The standard states that students should compare the length of physical objects, which is a spatial representation and, therefore, explicitly connects to the *comparing* subelement.

Relating skills [i.e., showing or establishing connections between two or more (things) to make sense of how (things) are spatially organized; see [Appendix A](#)] were often implicitly represented in the same standards as *comparing*, though less frequently because *relating*'s definition focuses on the sensemaking of spatial organization. Between one to two standards offered optional connections to *relating* per grade level ($M=1.67$), but no explicit connections were found. *Diagramming* and *designing* were also often implicitly represented, with an average of almost seven standards per grade relating to *diagramming* ($M=6.67$) and 6 relating to *designing* ($M=6$). The degree of representation varied between the elements, however. I located optional connections to *diagramming* in 4 standards per grade on average and explicit connections to just over 2 ($M=2.33$). For *designing*, I found an optional and explicit connection to less than one standard per grade level on average ($M=0.33$ and $M=0.67$, respectively).

The outlier *Interpreting* subelement was *symmetrizing* (i.e., interpreting and/or explaining balanced proportions through equivalent structures, often as a bilateral reflection; see [Appendix A](#)). I initially assessed standards for its presence similarly to how I examined them for representing *balancing*; however, the words “equivalent structure” in the *symmetrizing* definition led to me looking more closely for ideas related to an identical image rather than just balanced proportions. After clarifying its meaning, I found that only one kindergarten CCSS-M content standard (i.e., K.G.4) implicitly related to *symmetrizing* because it calls on students to compare spatial structures.

4.2.6 Sensating

Sensating was implicitly represented in almost 16 standards per grade level on average ($M=15.67$), with less overlap between subelements than in other elements. *Tactilizing* (i.e., making perceptible by touch, or tangible; see [Appendix A](#)) was the best-represented *Sensating* subelement with over 11 standards per grade level on average relating to the skill implicitly ($M=11.33$). This prevalence was attributed to the numerous standards that call for children to use concrete representations (e.g., moving objects or counting fingers); if children use tangible materials, they are physically

engaging and, thus, may be using *tactilizing* skills. However, that potential connection is implicit, at best, as merely using a physical representation does not necessitate engaging in reasoning spatially. The second best implicitly represented subelement was *imagining* ($M=6.33$), followed by *visualizing* ($M=3.33$). Some keywords in determining if a standard related to *imagining* or *visualizing* included stating the use of “mental images.” For example, K.OA.1 is “represent addition and subtraction with objects, fingers, mental images, drawings, sounds [e.g., claps], acting out situations, verbal explanations, expressions, or equations” ([Common Core State Standards \(CCSS\) Initiative, 2010](#), p. 11). Given that students could use mental images, it is a reasonable conclusion that the *Sensating* subelements of *imagining* (i.e., forming mental images; see [Appendix A](#)) and *visualizing* (i.e., imagining and mentally transforming spatial representations in space; see [Appendix A](#)) are connected as not just implicit, but also optional representations. However, that standard was the only one across all three grade levels that offered an optional connection, and none explicitly connected to the skills.

Additionally, the final three *Sensating* subelements (i.e., *perspective-taking*, *projecting*, and *propriocepting*) were almost or entirely absent from the standards. *Projecting* was implicitly represented in one standard per grade level in the Measurement and Data or Operations and Algebraic Thinking domains (see [Supplementary Figures B1–B3](#) in [Appendix B](#) for details).

4.3 Summary of spatial reasoning within the K-2 CCSS-M

Most CCSS-M have some implicit connection to one or more spatial reasoning skills (see [Table 2](#)). While these findings illustrate avenues through which teachers could include spatial reasoning skills in their mathematics instruction, few explicit or optional connections were provided in the K-2 CCSS-M content standards (see [Figure 2](#) and [Appendix B](#)). Given that the connections were found to be primarily implicit, the challenge remains that EEEs may need learning opportunities or curricular tools to know when or how to insert these crucial skills in their instructional practices, given only implicit relations.

5 Discussion

This research connects a broad set of spatial reasoning skills and common mathematics content standards for early elementary grades in the U.S. as intended supports for EEEs to teach spatial reasoning within their mathematics instruction. Given spatial reasoning's criticality to students' mathematics learning and STEM career interests ([Guay and McDaniel, 1977](#); [Lord and Rupert, 1995](#); [National Council of Teachers of Mathematics, 2006](#); [Wai et al., 2009](#); [Clements and Sarama, 2011](#); [Sorby and Panther, 2020](#)), children need opportunities to learn and practice using spatial reasoning skills. A viable avenue to provide those opportunities is supporting EEEs in teaching spatial reasoning through their typical instruction, which findings from this research could do if paired with opportunities for EEEs to *learn* to teach spatial reasoning.

Findings reflect the [Common Core State Standards \(CCSS\) Initiative's \(2010\)](#) intention for the content standards to require a

balanced understanding of mathematics content and procedural skills. Although this analysis did not explore the relations between spatial reasoning and the standards for mathematical practice, authors of the CCSS-M ([Common Core State Standards \(CCSS\) Initiative, 2010](#)) specified their utility in connecting practices with content. Due to the frequency with which implicit, optional, and explicit representations of spatial reasoning were located in the K-2 CCSS-M, this research offers a compromise between the implicit connections [Boda et al. \(2022\)](#) identified, and [Gilligan-Lee et al. \(2022\)](#) call for standards revisions by providing a continuum of explicitness with which the skills relate to the standards.

When examining implicit relations between the K-2 CCSS-M and the spatial reasoning skills, I found that 32 of the 35 skills, or groups of skills, coded were implicitly represented in some way. However, I found that overt connections were sparse when digging deeper to examine how explicit those connections were. Overall, spatial reasoning skills' representation in the K-2 CCSS-M varied widely (see [Table 2](#) and [Appendix B](#)). Whereas *Constructing*, *Interpreting*, and *Sensating* skills were each implicitly represented in over half of the standards across grade levels, some of their comprising subelements had little to no representation (e.g., *perspective-taking*, *symmetrizing*, and *un/re/packing* or *fitting*). Meanwhile, *Altering*, *Moving*, and *Situating* were less represented in the standards. This finding was surprising because the Geometry domain in the CCSS-M would seem a natural avenue through which to teach *Moving* skills. However, few connections were found to *reflecting*, *rotating*, and *translating* in the standards. Although coding standards for skills illuminated the variation in their representation in the K-2 CCSS-M, more work is needed to identify what teaching practices could support those spatial reasoning skills' inclusion in the classroom when the standards themselves may not change to include them explicitly and vary between contexts.

5.1 Limitations and implications

Despite the documented representation of spatial reasoning skills within the K-2 CCSS-M, limitations emerged, which may be attributed to the study design and context. First, the spatial reasoning framework used represents only one way to define spatial reasoning and its skills. Aligning the standards with another model, like one more focused on psychological constructs (e.g., the 2×2 classification of spatial skills presented by [Uttal et al., 2013](#) and refined for early education by [Đokić and Vorkapić, 2024](#)) or the developmental progression of skill acquisition (e.g., [Sarama and Clements, 2009](#)) may have resulted in different findings. Additional research is needed to ascertain if this model best supports EEEs and other educators teaching these skills.

Second, a limitation emerged from the methodological decision to code standards using grouped skills. This decision was documented and reasoned, but potentially increased the extent to which we saw spatial reasoning represented in the CCSS-M. The *Altering* subelements *dilating/contracting* and *scaling* were grouped, as well as two groups of *de/re/Constructing* subelements (*de/re/composing & sectioning* and *un/re/packing & fitting*). There may be a need to refine the conceptual framework itself, including definitions for the terms (see [Appendix A](#)). Initial informal feedback from mathematics education researchers indicated that the number of terms could be too great, and their definitions could be too granular to have utility for

researchers and practitioners alike. There were also questions about how some skills might overlap and be more easily captured through singular terms and definitions. Given the decision to group skills during coding, it follows that some skills in the framework may be too similar to parse from one another meaningfully. After revising the conceptual framework, additional examination of the alignment will need to follow for the tools to work together closely as potential supports for EEEs to teach spatial reasoning.

Also, although standards coding was found to be reliable through careful coder training and consensus conversations, some of the initial relations between the spatial reasoning skills and standards identified in this study required a high degree of inferencing. For example, Standard 2.OA.3 (i.e., “Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2 s; write an equation to express an even number as a sum of two equal addends”; [Common Core State Standards \(CCSS\) Initiative, 2010](#), p. 19) involves determining if a group of objects is odd or even, with suggested strategies including matching and pairing or writing equations; it was found to connect to *balancing*, yet that coding was based upon both coders recalling a specific representation (i.e., a pan balance). Because the standard does not mandate using a pan balance, a teacher with a different pedagogical lens may not see that connection. Therefore, having a group of trained practitioners code standards for the skills may enhance the findings' external validity and the utility of findings to practitioners.

Relatedly, these findings may be challenging to transfer to situated contexts in the U.S. or beyond as pre- and in-service teacher professional learning opportunities may not adequately prepare or support EEEs to teach spatial reasoning through mathematics ([Ginsburg et al., 2006](#); [Clements and Sarama, 2011](#); [Gagnier et al., 2022](#); [Gilligan-Lee et al., 2022](#); [Bufasi et al., 2024](#)). While reforms to policy and standards have been recommended ([Gilligan-Lee et al., 2022](#)), changes to teacher preparation and professional development programs are also needed to give EEEs opportunities to learn to teach spatial reasoning. Such reforms can occur more unilaterally in countries with national curricula that value spatial reasoning (e.g., Australia; see [Resnick and Lowrie, 2023](#)), but changes are dependent upon individual programs and practices in countries such as the U.S. Supporting EEEs in learning to identify opportunities to teach spatial reasoning with and without national curricular supports represents an additional area for future research. Further, many states are repealing or replacing the CCSS-M with other education standards ([Edgerton, 2020](#)), so future research should examine updated standards systems.

Finally, this research relied on a key assumption that EEEs use education standards to guide their instruction by specifying the content of the intended curriculum ([Porter et al., 2011](#)). How closely educators' instruction follows the education standards is debatable ([Polikoff, 2012](#)). Nevertheless, assuming that standards specify the content to be included in enacted curricula means that showing connections would be helpful, if not necessary, to facilitate EEEs teaching spatial reasoning skills, given controversies in existing research.

5.2 Conclusion

[Gilligan-Lee et al. \(2022\)](#) asserted that curricular change is needed to help teachers enact spatial reasoning teaching practices,

which supports Davis et al.'s (2015) call to radically spatialize the school mathematics curriculum and is characterized within Bufasi et al.'s (2024) account of the enablers and barriers to teaching spatial reasoning in the early grades. However, Boda et al. (2022) introduced a more conservative approach by locating implicit opportunities within the K-2 CCSS-M to teach spatial reasoning. For EEEs to be adequately supported in teaching spatial reasoning, providing a direct mapping that indicates the implicit, optional, and explicit connections between specific standards and specific skills, as was done in this study, might better aid EEEs in teaching spatial reasoning through mathematics until there are changes in the content of the intended curriculum (Porter et al., 2011).

To make use of the connections as identified, EEEs will need opportunities to understand the underlying construct and contents of the framework. To learn whether identifying the presence of spatial reasoning skills within the CCSS-M will change practice, EEEs need to interact with the skills within their intended curriculum both through professional learning and in the classroom. While findings from this study provide clear descriptions of the skills and specific standards through which they are implicitly represented, it will be important to investigate if this provision could enhance EEEs' teaching of these vital skills and how findings may transfer to other contexts.

Data availability statement

The original contributions presented in the study are included in the article/Supplementary material, further inquiries can be directed to the corresponding author.

Author contributions

RKP: Conceptualization, Data curation, Formal analysis, Funding acquisition, Investigation, Methodology, Writing – original draft, Writing – review & editing.

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Conflict of interest

The author declares that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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Supplementary material

The Supplementary material for this article can be found online at: <https://www.frontiersin.org/articles/10.3389/feduc.2024.1407388/full#supplementary-material>

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