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A critical turn in numeracy education and practice

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Research into the nature of numeracy and numeracy practice has typically been focused on the capabilities required to participate effectively within personal, civic, and work life. In this article, we document the development of numeracy theory and practice from a functional perspective through to a view that includes evaluative and decision-making capabilities now required for informed and responsible citizenship—a critical turn. We map this development through an audit of policy and curriculum documents in addition to published research, making connections to the ever-changing mathematical demands of society. These include new demands that require critical approaches to the deployment of numeracy capabilities. We argue that this *turn to the critical* is central to how citizens support their societies' responses to recent and intensifying disruptive phenomena such as COVID-19 and for ensuring equity, inclusiveness, and social justice remain a high priority in a rapidly changing world.

KEYWORDS

numeracy, mathematical literacy, critical numeracy, critical thinking, social justice

1 Introduction

A numerate citizenry has been an educational goal internationally since at least the mid-20th century (e.g., Ministry of Education, 1959; Bolstad, 2023; Coffey and Sharpe, 2023). This goal is seen as a key underpinning of a STEM capable workforce—essential for current and future national prosperity (e.g., Maass et al., 2019); personal wellbeing and financial security (e.g., Tout, 2020); and informed, participatory, and critical citizenship (e.g., Geiger et al., 2023a). While these reasons for promoting a numerate society have well-established histories in research literature (e.g., Frankenstein, 1990; Steen, 2001; D'Ambrosio, 2003; Jablonka, 2003; Zevenbergen, 2004; Geiger et al., 2015), it is the latter that has received increasing attention during recent disruptive times. These times are marked by the emergence or intensification of significant disruptive phenomena that are impacting nearly all aspects of life on the planet—environmental, economic, and societal. Disruptive phenomena, such as global warming and health crises, have been identified in the United Nations Sustainability Development Goals (SDGs; United Nations General Assembly, 2015) as influences that are placing pressure on the health of the planet and, as a consequence, global cooperation, and the social cohesion of nations. The impact of such disruptions has been brought into stark relief by phenomena such as the COVID-19 pandemic in 2020–2022 (see for example, Geiger, in press a). The ways in which the nature of these disruptions, and associated predictions about their progress, are reported to the public has served to highlight the need for citizens to possess interpretive and evaluative capabilities that inform the decisions that they make for themselves, their families and society at large.

While there has been long-term interest in the notion of numeracy, a variety of terms are used internationally to identify differently nuanced constructs. Goos et al. (2011), for example, claim that "numeracy is a term used in many English-speaking countries, such as the UK, Canada, South Africa, Australia and New Zealand, in the USA, and elsewhere, it is more common to speak of quantitative literacy or mathematical literacy" (p. 131).

Consistent with this view, Steen (1999) considers the terms quantitative literacy, a term most often used by US scholars, and numeracy as synonymous, a view also supported by Vacher (2017). There are also different "facets" of numeracy (Geiger et al., 2020) which represent specific foci within the broader construct, for example, adult numeracy (e.g., Yasukawa et al., 2017), statistical literacy (e.g., Gal, 2004), and financial literacy (Sawatzki and Sullivan, 2017).

Within this broad church, we argue that there is an increasing focus on a critical perspective, especially on how an understanding of and capacity to use mathematics can support citizens' empowered agency when contributing to society—a turn to the critical. This perspective has been explored by scholars who have generated different ways of thinking about the notion of criticality in mathematics education, including critical orientation (Goos et al., 2014), critical mathematical numeracy (Frankenstein, 2009), critical mathematical thinking (Gutstein et al., 1997), mathemacy (Skovsmose, 1998), and matheracy (D'Ambrosio, 1999). While these represent different ways of thinking about criticality in mathematics education, at their center is a concern for the development of informed, participatory, and critical citizenship.

A critically numerate citizenry is central to informed and responsible decision-making at personal and collective levels within society. This requires the capacity to understand mathematics-based descriptions of the ways in which the world is changing, and the ability to interpret and evaluate information from ever increasing and diversifying sources. Such sources include official channels (e.g., government agencies), traditional media (news items [in print and online]), and more recently social media (see for example, Gal and Geiger, 2022). While the source of information is still a key indicator of the trustworthiness of a claim or position, different sources of information cannot be simply categorized as reliable or unreliable, as each can include commentary, by both expert and non-expert commentators, that is fuzzy, contradictory, or even misleading. In some cases, reporting may include misinformation (unintentional) or disinformation (intentional), which can disrupt confidence in institutions that the public relies on for accurate and reliable information (e.g., Lewandowsky et al., 2017), leading to important messages that need to be received by the public being questioned or ignored. This may include, for example, connection between working with asbestos and black lung disease, questioning the connection between climate change and human activity, and the dangers of vaccination (e.g., the injection of mind controlling transmitters) (Zhou and Shen, 2022).

The purpose of this article is to outline the evolution of what it means to be numerate, and to describe how criticality within mathematics education has become increasingly important within the context of a society that is attempting to respond to the impacts of disruption, both natural and human. We will address this issue by first discussing the development of numeracy over time. Second, we present a summary of important contributions to the idea of critical thinking in mathematics education. Third, different kinds of citizenship and their connection to criticality are discussed. Fourth, illustrative examples are provided that point to greater recognition of critical aspects of mathematics education in policy documents and international assessment programs. Fifth, connections between criticality, research in mathematics education, and disruption are discussed. Finally, we point to new directions in research and practice in numeracy education.

2 Development of the notion of numeracy

2.1 Initial developments

The first mention of numeracy in English speaking countries is generally attributed to the Crowther Report 15–18 (Ministry of Education, 1959), which was instigated to identify the mathematics needed for continuing participation in further and higher education within the UK. The key motivation for this initiative was a demand for a mathematics/science-capable population that was seen as essential for post-war economic development. In this report, numeracy was described as the "mirror image" of literacy:

> *On the one hand is an understanding of the scientific approach to the study of phenomena—observation, hypothesis, experiment, verification. On the other hand is a need in the modern world to think quantitatively, to realise how far our problems are problems of degree even when they appear as problems of kind* (Ministry of Education, 1959, p. 270).

The changing demands of the workplace in the UK during the 1970s led to criticism of the mathematical preparedness of school leavers for tasks that were required by business and industry. In response, Mathematics Counts (Cockcroft, 1982) was commissioned—a review that examined and made recommendations for how to address the issue. A major focus of the report was the identification of the mathematical capabilities acquired during schooling that were needed for participation in further and higher education, employment, and adult life generally. The review also included a revised perspective on what it meant to be numerate:

> *We would wish "numerate" to imply the possession of two attributes. The first of these is an "at-homeness" with numbers and an ability to make use of mathematical skills which enable an individual to cope with the practical mathematical demands of his everyday life. The second is ability to have some appreciation and understanding of information which is presented in mathematical terms, for instance in graphs, charts or tables or by reference to percentage increase or decrease* (Cockcroft, 1982, p. 11).

The statement reiterates the connection between numeracy and the capabilities needed for managing the mathematical demands of everyday life. Further, the term "at-homeness" introduces the notion of a positive disposition toward mathematics, perhaps for the first time, as a key aspect of being numerate. Reference to a "second" ability also heralds the need to develop competence with an emerging challenge—the availability and accessibility of large quantities of data and information and how to make sense of them.

2.2 Addressing changing demands

Cockcroft's foreshadowing of a society in which information was increasingly presented to the public through the use of mathematical ideas and devices was seen as coming to fruition by Steen (1999) in what he described as an increasingly "data drenched world." He termed the capability required to meet these demands as quantitative literacy. In Steen (1999)'s view, quantitative literacy consisted of seven dimensions: confidence with mathematics; appreciation of the nature and history of mathematics and its significance for understanding issues in the public realm; logical thinking and decision-making; use of mathematics to solve practical everyday problems in different contexts; number sense and symbol sense; reasoning with data; and the ability to draw on a range of pre-requisite mathematical knowledge and tools (Steen, 2001). These capabilities represented a far more sophisticated perspective on what had previously been seen as required to be numerate—far beyond that of mere functionality—as higher-order thinking capacities such as reasoning and problem-solving were included. Steen also drew attention to affective aspects of numeracy, arguing that individuals must feel confident in their ability to use mathematics to solve real-world problems, in addition to having relevant knowledge, if they are to apply it to unfamiliar situations in personal, civic, and work life.

More recently, Goos et al. (2014) proposed a Model of Numeracy for the 21st Century. The model was generated via a synthesis of research literature and validated through a series of research projects (e.g., Geiger, 2019; Goos et al., 2019). The model consists of four key dimensions, contexts, mathematical knowledge, tools, and dispositions, which are integrated and activated by an analytical and evaluative capability, a critical orientation—represented in Figure 1 and described in Table 1. While initially conceived as a guide for effective teaching and learning practice in numeracy in response to curriculum requirements (a cross-curricular general capability), the model has also been used as the basis for the design of numeracy and interdisciplinary STEM tasks (e.g., Geiger, 2016, 2018; Geiger et al., 2018); informing initial teacher education instruction in numeracy (Goos et al., 2019) and as an embedder-of-numeracy identity (e.g., Bennison, 2016).

In this section, we have traced the development of numeracy as a construct, through illustrative examples, over time, from the mathematics needed to contribute to the workforce and gain entry into higher education, to the need for sophisticated reasoning capabilities in order to interpret and evaluate data-based claims, and more recently, to a conceptualization in which critical capabilities are viewed as essential for informed and responsible citizenship. Each of these developments represents progress toward a more critical perspective on numeracy—an increasing turn to the critical. We argue in the following section that such change is still taking place, particularly in relation to the role of criticality.

3 Critical thinking in mathematics education

Jablonka (2020) points out that critical thinking in mathematics education is viewed from two quite different perspectives. First, as higher-order thinking and problem-solving capabilities within mathematics itself, including the selection of relevant techniques when seeking a solution to a specific problem, the deployment of strategies, and approaches to the validation of a proposed solution. Second as the practice of mathematics within a socio-political milieu with a focus on the empowerment of citizens—foregrounded and backgrounded by consideration for equity, inclusiveness, and social justice. It is this second perspective we have adopted when referring to a critical turn in numeracy education and practice.

This second perspective has a number of branches, generated almost at the same time within different cultural contexts across the world. These draw on two principal influences on their development: (1) Freire (1968) pedagogy of the oppressed, which gave rise to his conceptualization of a critical pedagogy in which teachers and students are in the process of *becoming*—learning from each other and blurring the lines between the oppressed and oppressor, eventually leading to equity and a lack of oppression; and (2) critical theory from the Frankfurt School, based on social critique aimed at bringing about sociologic change and intellectual emancipation—giving rise to the notion of praxis as action informed by tradition and an understanding of moral, social, and political consequences (see Kemmis et al., 2014). These perspectives have informed key contributions to critical thinking in mathematics education including: critical mathematics education, critical mathematical thinking, critical numeracy/mathematical literacy, and ethno-mathematics.

3.1 Critical mathematics education

Skovsmose (1990, 1994) set about challenging orthodoxies in mathematics education and citizenship education by drawing on the work of the Frankfurt School to conceptualize Critical Mathematics Education (CME). CME brought forward two key constructs—mathemacy and formatting. Mathemacy, which can be considered as a form of critical numeracy, is defined by three components: (1) mathematical knowing, or the skills developed in traditional mathematics classrooms, (2) technological knowing, or the ability to build models with mathematics, and (3) reflective knowing, or competency in evaluating applications of mathematics. It is the third of these components that brings a critical lens to the practice of mathematics as "Mathemacy can be used for the purpose of empowerment, because it can be a means to organize and reorganize interpretations of social institutions, traditions and proposals for political reforms" (Skovsmose, 1994, p. 39). Skovsmose (1994) further argued that "Mathematics produces new inventions in reality, not only in the sense that new insights may change interpretations, but also in the sense that mathematics colonizes part of reality and reorders it" (p. 42). He referred to this colonization and reordering of reality as *formatting*—the power to shape individual's and society's perceptions of reality including the messages those in authority distribute to citizens. The ways in which formatting with mathematics might be used to shape opinions and sway judgment make its recognition a key element of critical numeracy and therefore critical citizenship.

3.2 Critical mathematical thinking

Eric (Rico) (e.g., Gutstein et al., 1997) used the expression critical mathematical thinking to identify mathematical capabilities he was seeking to promote in disadvantaged schools in the USA and South America.

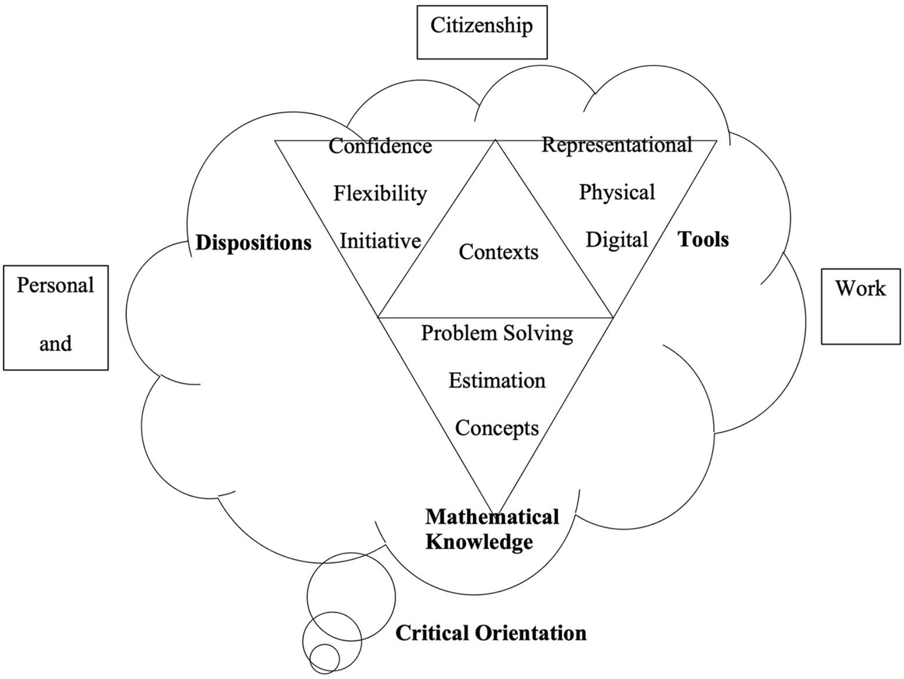

FIGURE 1
A model for numeracy in the 21st century (Goos et al., 2014).

TABLE 1 Descriptions of the dimensions of the Model of Numeracy for the 21st Century (Goos et al., 2014).

Mathematical knowledge	Mathematical concepts and skills; problem solving strategies; estimation capacities.
Contexts	Capacity to use mathematical knowledge in a range of contexts, both within schools and beyond school settings
Dispositions	Confidence and willingness to use mathematical approaches to engage with life-related tasks; preparedness to make flexible and adaptive use of mathematical knowledge.
Tools	Use of material (models, measuring instruments), representational (symbol systems, graphs, maps, diagrams, drawings, tables, ready reckoners) and digital (computers, software, calculators, internet) tools to mediate and shape thinking
Critical orientation	Use of mathematical information to: make decisions and judgements; add support to arguments; challenge an argument or position.

> *A major focus of this article is to examine ways in which culturally relevant mathematics educators extend the notion of critical mathematical thinking to critical approaches to knowledge in a broad sense beyond mathematics, and we reflect on how these approaches may help students develop critical literacy* (Gutstein et al., 1997, p. 714).

In his work (e.g., Gutstein et al., 1997; Gutstein, 2003, 2005), school students were challenged to use mathematics when grappling with the challenges associated with topics such as socio-economic disadvantage, unnecessary waste, poverty, race, and prejudice. This *grappling* requires *reading the world with mathematics*, a concept developed by Freire and Macedo (1987, p. 35), that is, "using mathematics to understand relations of power, resource inequalities between different social groups and explicit discrimination" (Gutstein, 2003, p. 45). To change the world, however, requires action—*writing the world*. He argued that bringing about change in education requires both reading and writing the world (Gutstein, 2005) and that for this emancipatory approach to bear fruition for minority students or those living in the margins of society, culturally relevant mathematics teaching must be adopted (Gutstein et al., 1997). Thus, in Gutstein's view, mathematics is a tool for social agency that empowers the user to understand and interrogate unjust structures in their lives and society at large.

3.3 Critical numeracy/mathematical literacy

Frankenstein (1983, 1990, 2009) also drew on the work of Freire to develop the concept of critical numeracy/mathematical literacy through her research with working-class adults in urban contexts in the USA, described in her article *Critical mathematics education: An application of Paulo Freire's epistemology*. Her work in critical mathematical literacy is aimed at disrupting mathematical practices that she argues support hegemonic ideologies (Frankenstein, 1983). This involved teaching practices which used mathematics to highlight the inequalities and disadvantages that existed in society—particularly through the analysis of statistical data. She argues that a critical understanding of numerical data prompts individuals to question taken-for-granted assumptions about society structures that maintain systematic marginalization and disadvantage, disproportionately among non-dominant groups (women, people of color, lower income families), enabling an informed position that can lead to change. Frankenstein draws on Freire's notion of "banked knowledge," in which teaching is based on memorisation and repetition, to explain the development of mathematics anxiety, which in turn limits individual capacity to question the mathematics-based justifications used to maintain current social, political, and industrial structures.

3.4 Ethno-mathematics

The research program known as ethnomathematics which emerged from Brazil during the 1980s was initiated by D'Ambrosio (1985, 1989, 1999). This work drew on Freire's philosophy to develop a response to the use of Eurocentric models for teaching mathematics that were not sympathetic to other cultural contexts. His aim was to identify and further develop culturally aligned teaching practices that recognize that all cultures have used, created, and innovated with mathematics across time. D'Ambrosio (1999, 2001)'s grand ambition for ethnomathematics was to develop a framework in which "mathematics can help to fulfill the commitment to children and to promote equity and democracy, dignity, and peace for all of humankind" (p. 131). He argued that this approach would (1) enhance creativity, and (2) facilitate the achievement of *complete* citizenship, which includes the capacity to make responsible decisions.

In developing the underpinnings of ethnomathematics, D'Ambrosio (1999) conceptualized the trivium curriculum composed of *literacy*, *matheracy*, and *technoracy*. Matheracy is the ability to interpret and analyse signs and codes necessary to develop models aimed at finding solutions to problems in daily life (D'Ambrosio and D'Ambrosio, 2013). This goes beyond the use of mathematics to solve real-world problems as it also empowers individuals to articulate their beliefs, traditions, myths, symbols, and scientific and mathematical knowledge. In this way, mathematical and cultural knowledge are elements of a holistic perspective on mathematics and how it is used.

While D'Ambrosio's objective was the development of an ethnomathematical curriculum and associated pedagogical approach, more recently, others have argued it has political dimensions as it "also aims to develop political actions that guide students in transition processes from subordination to autonomy in order to guide them toward a broader command of their rights as citizens" (Rosa et al., 2016, p. 13). This is a critical positioning of ethnomathematics as it aligns with goals of equity, inclusiveness, and social justice.

Each of these programs of research provides different perspectives on the role of criticality within mathematics education. This research provides insight into: how mathematics is used to format communications to the public; the role of mathematics and statistics education in addressing disadvantage and marginalization; and the connection between culture and mathematical practices. These developments represent increasingly nuanced understandings of what the turn to the critical means within the context of mathematics education—beyond the use of mathematics to evaluate public messaging and informed decision-making alone—representing a maturing facet of theory and practice within the field.

4 Kinds of citizenship

Significant literature related to the notion of numeracy argues for its importance as an educational goal. These arguments tend to align with two justifications of the importance of numeracy: (1) as an underpinning of a mathematically enabled workforce needed to support the STEM innovation required for continuous economic advancement; and (2) as an essential capability for informed and participatory citizenship (e.g., Goos et al., 2019; Tout et al., 2021). Increasingly, there are calls for the second of these justifications to embrace evaluative or *critical* capabilities. Yet, such claims rarely acknowledge that many types of "citizenship" exist internationally and, therefore, what is meant by engaged and responsible citizens varies from nation to nation (e.g., Gal and Geiger, 2022). Further, while there is significant literature that refers to *critical* citizenship, it is not often defined, with an apparent assumption that the meaning is universal. Tudball and Henderson (2014), for example, have argued that the purpose of citizenship education includes the development of the capabilities required for informed citizenship that enable participation in communities at local, national, and global levels. The governments of such communities, however, can be different, for instance, democratic, theocratic, and authoritarian—each of which can have different expectations of what constitutes "good" citizenship. These differences include what are considered inalienable human rights, and the balance between individual freedom and behaviors that ensure the common good. Even if we confine this discussion to democratic societies, there are different perspectives on the ways in which citizens can and should contribute to society in a positive sense.

Westheimer and Kahne (2004), for example, identify three categories of citizenship within democratic societies: personally responsible, participatory, and justice-oriented. Personally responsible citizens contribute to society through their labor (e.g., work, volunteerism), are law abiding, and act responsibly within their communities. Participatory citizens are active in their communities and may take on roles within government or voluntary organizations—working within systems to improve the life circumstances of others. Justice-oriented citizens deliberately take steps to improve the lot of other citizens by, for example, addressing injustice through activism to effect systemic change. Weiland (2017), has drawn on these categories to define what is meant by *critical* citizenship when discussing the role of statistical literacy within society:

> *. . . I am taking the view of a "good" citizen as a blending of participatory and justice-oriented citizenship, which I will refer to as a critical citizenship, where citizens are empowered to participate actively in their community and/or government, and also interrogate the structures at play within their community and government that produce conditions of injustice, and actively work to change those that (re)produce injustices. In today's*

TABLE 2 Frameworks of the Partnership for 21st Century Skills (2002) and the Assessment teaching of 21st century skills project (2009).

Partnership for 21st Century Skills (2002)	Binkley et al. (2012)
Learning and Innovation skills: Critical thinking and problem solving; Communication and Collaboration; Creativity and Innovation	Ways of thinking. Creativity, critical thinking, problem-solving, decision-making and learning
Information, Media and Technology skills: Information Literacy, Media Literacy, ICT (Information, Communications and Technology) Literacy	Ways of working. Communication and collaboration
Life and Career Skills: Flexibility and Adaptability; Initiative and Self-Direction; Social and Cross-Cultural Skills; Productivity and Accountability; Leadership and Responsibility	Tools for working. Information and communications technology (ICT) and information literacy
	Skills for living in the world. Citizenship, life and career, and personal and social responsibility

> *modern societies in a globalized world, there are a plurality of different views, values, and ideas, which citizens must be able to negotiate and navigate in their daily life (p. 25).*

It is this perspective that we will adopt when discussing the notion of critical citizenship—one in which empowerment is central and that addressing injustice is a key responsibility. This connection between citizenship and criticality is a significant development in the turn to the critical—but how is this related to mathematics/numeracy education? In the next section we make this connection through an examination of frameworks and policy documents related to the capabilities needed for participation in society and life-long learning.

5 Recognition of critical capability for participation in society and life-long learning

Rapid societal, economic, and technological change has inspired the development of a range of frameworks that identify and describe the capabilities required to meet the demands of the 21st century. A common theme in such frameworks is the essential role of critical thinking in informed and responsible citizenship. Two examples of such skill sets are presented in Table 2—the Partnership for 21st Century Skills (2002) and Assessment teaching of 21st century skills project (2009). These include reference to problem solving and critical thinking capabilities in addition to creativity, the use of digital tools for communication and accessing information, as well as social and cultural capabilities.

While these frameworks do not make direct reference to mathematics, as general competencies required by all members of society to function in the 21st Century, other frameworks are more specific to mathematics and science. The Key Competencies for Lifelong Learning (European Union, 2019), for example, identifies eight competencies: literacy competence; multilingual competence; mathematical competence and competence in science, technology and engineering; digital competence; personal, social and learning to learn competence; citizenship competence; entrepreneurship competence; and cultural awareness and expression competence. Each of these competencies is defined by essential knowledge, skills, and attitudes. Within *mathematical competence and competence in science, technology, and engineering*, the essential attitudes refer to evaluative or critical capabilities. For example, in mathematics:

> *A positive attitude in mathematics is based on the respect for truth and a willingness to look for reasons and to assess their validity (p. 8).*

While in science:

> *Competence includes an attitude of critical appreciation and curiosity, a concern for ethical issues and support for both safety and environmental sustainability, in particular as regards scientific and technological progress in relation to oneself, family, community, and global issues (p. 9).*

These statements indicate that the notion of criticality is no longer seen as only a general competence within a broad construction of the skills required by citizens for 21st century living and careers, but is positioned as directly connected to the role of mathematics and science in individuals' engagement with their societies. This represents an additional front in which a turn to the critical in mathematics education is evident—as an essential capability required for full participation in society in the 21st century and for life-long learning.

6 The inclusion of the critical perspective in international testing programs

One of the outcomes of increasing globalization is the development of international assessment programs that provide information about individual countries' performance in a range of domains, and as a consequence, comparisons between nations. Two such programs are conducted by the OECD and are related to numeracy: the Programme for International Student Assessment (PISA) (mathematical literacy) and the Programme for the International Assessment of Adult Competencies (PIAAC) (numeracy). The purpose of PISA is to ascertain if 15 year old students can make use of their mathematical knowledge in life-related contexts as a measure of their readiness for their active participation in society. PIAAC takes a different approach by collecting and analyzing data related to the mathematical capabilities of adult populations within different real-world contexts.

Such assessment regimes have been demonstrated to influence educational policy reforms (Stacey et al., 2015). PISA results (including comparative results), for example, have been used to initiate and justify curricular reforms (Lingard, 2017), including mathematics curricula, performance targets, curriculum standards, or assessment practice (e.g., Breakspear, 2012). Such reforms have

TABLE 3 PISA definitions of mathematical literacy.

PISA definitions of mathematical literacy	References
... an individual's capacity to identify, and to understand, and to engage in mathematics and make well founded judgements about the role mathematics plays, as needed for an individual's current and future private life, occupational life, social life with peers and relatives, and life as a constructive, concerned and reflective citizen.	(OECD, 2000, p. 50).
…an individual's capacity to identify and understand the role mathematics plays in the world, to make well-founded judgments, and to use and engage with mathematics in ways that meet the needs of that individual's life as a constructive, concerned and reflective citizen*.	(OECD, 2004, p. 15; OECD, 2009, p. 84)
… an individual's capacity to formulate, employ and interpret mathematics in a variety of contexts. It includes reasoning mathematically and using mathematical concepts, procedures, facts, and tools to describe, explain and predict phenomena. It assists individuals to recognize the role that mathematics plays in the world and to make well-founded judgments and decisions needed by constructive, engaged and reflective citizens*.	(OECD, 2013a, p. 17; OECD, 2016, p. 65)
…an individual's capacity to reason mathematically and to formulate, employ, and interpret mathematics to solve problems in a variety of real-world contexts. It includes concepts, procedures, facts and tools to describe, explain and predict phenomena. It assists individuals to know the role that mathematics plays in the world and to make the well-founded judgments and decisions needed by constructive, engaged and reflective 21st century citizens.	(OECD, 2019, p. 7)

*PISA's definition for mathematical literacy did not change for 2004-2009 and then again 2013-2016.

TABLE 4 PIAAC definitions of numeracy including ALLS.

PIAAC definitions of numeracy including ALLS	References
Numeracy is the knowledge and skills required to effectively manage and respond to the mathematical demands of diverse situations. Numerate behavior is observed when people manage a situation or solve a problem in a real context; it involves responding to information about mathematical ideas that may be represented in a range of ways; it requires the activation of a range of enabling knowledge, factors, and processes.	(Gal et al., 2005, p. 151/152)
Numeracy is defined as the ability to access, use, interpret and communicate mathematical information and ideas in order to engage in and manage the mathematical demands of a range of situations in adult life. To this end, numeracy involves managing a situation or solving a problem in a real context, by responding to mathematical content/information/ideas represented in multiple ways.	(OECD, 2013b, p. 59)
Numeracy is accessing, using and reasoning critically with mathematical content, information and ideas represented in multiple ways in order to engage in and manage the mathematical demands of a range of situations in adult life.	(Tout et al., 2021, p. 93)

included the targeting of specific aspects of curriculum in which student under achievement is identified in PISA results, and adopting PISA like items within national and local assessments (Djordje et al., 2023).

Both PISA and PIAAC items are created according to principles set out in assessment frameworks that are developed before each implementation (or cycle) of these programs. Central to these frameworks are definitions of numeracy (or mathematical literacy in the case of PISA). These definitions act as a type of time-stamp on what was seen as key capabilities/competencies needed for informed and participatory citizenship at a point in history. The definitions for mathematical literacy in PISA are presented in Table 3 with PIAAC definitions of numeracy in Table 4.

PISA's definition of mathematical literacy has always made references to using mathematics to make well founded judgements, a critical capability, but has changed over time in other ways. The 2000 version of the definition of mathematical literacy placed a focus on the use of mathematics in private, occupational, and social life as constructive, concerned, and reflective citizens. This definition was revised in a subtle but important way in the form developed in 2004 and 2009, as mathematics to be *used* and not just *engaged with* in the role of constructive, concerned, and reflective citizen. This more active language underpins the notion that mathematics is a means by which individuals can act in their societies. The refinement included in assessment frameworks published in 2013 and 2016, placed greater emphasis on the capacity to use mathematics in a variety of contexts, and the use of mathematical reasoning, as well as facts and procedures. The use of the word *employ* replaces *use* but maintains the notion that mathematics has an active role in participatory citizenship. This definition also introduces the idea that a mathematically literate person should be able to describe, explain, and predict phenomena as well as make decisions and judgements. These are critical capabilities as understanding and prediction are evidentiary bases for the formation of judgements and prudent decision-making. The most recent definition of mathematical literacy (OECD, 2019) adds the word formulate, which in this context means to transform a real-world situation into a mathematical problem—again attributing an active role of a citizen in the use of mathematics in acting in the world. The progression described here is one in which mathematics is used in an increasingly active way to understand and make predictions about phenomena, informing the judgements and decisions of constructive, engaged, and reflective citizens—attributes of critical citizenship.

A predecessor of PIAAC was the Adult Literacy and Life Skills Survey (ALLS), conducted in 2005. The definition used to frame items in this survey is included here to provide a broader overview, that is, over a greater time span than the 1st and 2nd Cycles of PIAAC alone. In this definition, numeracy (including numerate behavior) is portrayed as the means by which citizens can take a more active role in society, manage the mathematical demands of life, and solve problems in real-work contexts. The definition for the 1st Cycle of PIAAC maintains the broad intent of the ALLS definition but adds two additional capacities—interpreting and communicating mathematical ideas, and working with information represented in multiple ways. These new capacities indicate that becoming numerate is a complex undertaking, requiring the appropriation of sophisticated mathematical capabilities. There is a significant shift in the 2nd Cycle of PIAAC with the inclusion of *reasoning critically* as an activity with which citizens are expected to engage when managing a range of situations encountered in adult life. As with PISA, PIAAC definitions change over time from descriptions which portray an essentially passive use of mathematics when participating in the world to that where mathematics is a key element of the critical reasoning needed to fully engage with society.

There has been a clear turn to the critical in the evolution of definitions for mathematical literacy (PISA) and numeracy (PIAAC). This evolution has deeper significance as both PISA and PIAAC have been demonstrated to influence the national curriculums of participating nations (e.g., Stacey et al., 2015; Lingard, 2017).

7 Criticality, research in mathematics education, and disruption

The COVID-19 pandemic gave rise to a series of research publications—including special issues in a number of journals of high esteem within mathematics education. These publications covered a range of issues that had emerged as impacts of the pandemic as well as discussion of strategies to ensure mathematics and mathematics education learning continued during the crisis (e.g., Drijvers et al., 2021; Vale and Graven, 2023; Villarreal et al., 2023), the role of mathematics teaching/learning in understanding the crisis (Aguilar and Castaneda, 2021; Meyer and Lima, 2023), taking the crisis as an opportunity to introduce rich contexts into mathematics instruction (e.g., da Silva et al., 2021; Siller et al., 2023).

There were also publications that provided critical commentary on how information was disseminated by authorities and the media during the crisis (e.g., restrictions on personal freedoms) and what this meant for being informed and the citizens' agency. Maass et al. (2023), for example, used the COVID-19 pandemic as a context in which the role of mathematical modeling in supporting learning to understand the world critically is highlighted—socio-critical modeling. They argue that citizens need to understand the mathematical (including modeling), ethical, economic, political, and social reasoning behind the restrictive measures imposed during the pandemic if they are to be fully informed when making responsible decisions. To accommodate these considerations, they extended the modeling cycle to include ethical, social, cultural, and economic aspects in decision-making after the validation phase of the modeling cycle.

Geiger (in press a) also argues for the need for critical approaches to mathematical modeling in the time of disruptive events. He describes a study in which the contexts used in recent publications about modeling education were identified. This investigation indicated that most modeling activities for students were set in the context of sport (e.g., fastest, strongest) or commercial activity (e.g., cost-benefit analysis), with very few tasks concerned with ethical, social, or cultural issues. Geiger draws comparision between this situation and the outcome of an analysis of news items published in the mass media, in which modeling is used to report on the progress of the COVID-19 pandemic to provide justification for actions taken by authorities, such as the restriction of personal freedoms. This raises the question of how well education has prepared citizens to interpret and evaluate such justifications or developed their capacities to ask clarifying questions of experts—all critical capabilities. Geiger further argues that the need for these capabilities is not unique to the COVID-19 pandemic as there are other forms of disruption that are now intensifying, for example, global warming and food insecurity.

The capacity for laypersons to critically evaluate experts' use of mathematics is also seen by Kollosche and Meyerhöfer (2021) as a key capability for critical citizenship in democracies. In their view, however, such evaluation is not possible in all circumstances, opening the space for misconceptions and manipulation. This theme was also perused by Gal and Geiger (2022), who examined the mathematical/statistical demands on members of society by investigating the manner in which information was presented to the public during the COVID-19 pandemic. The outcome of this study was a typology of such demands which they described as Statistical and Mathematical Products (StaMPs). StaMPs include nine categories of intertwined mathematical and extra-mathematical demands: (1) descriptive quantitative information, (2) models, predictions, causality and risk, (3) representations and displays, (4) data quality and strength of evidence, (5) demographics and comparative thinking, (6) heterogeneity and contextual factors, (7) literacy and language demands, (8) multiple information sources, and (9) critical demands. They argue that a critical capability—the capacity to evaluate and scrutinize claims in the media rather than just reading and interpreting—is now a key dimension of informed, participatory, and contributing citizenship. This view is consistent with that of O'Sullivan et al. (2021), who maintain that citizens, both young people and adults, need to develop a healthy skepticism, informed by evaluative mathematical capabilities, of what is reported in the media. They see these capabilities as key to a numerate society in which citizens are capable of understanding and engaging with future national and global challenges, for example, the COVID-19 pandemic.

The issue of how information about the COVID-19 pandemic was delivered into public forums is also discussed by Skovsmose (2021). In this study, he provides a critique of the role of mathematics in communicating information about the crisis to the public. He sees this as taking place in three ways: picturing, constituting, and formatting. The first relates to how models can be used to represent reality, the second to the ways in which mathematics is intrinsic to the dynamics of a crisis, and the third relates to ways of acting that can be adequate, counterproductive, or catastrophic. Each of these ways of communicating with mathematics is subject to socio-political influences which can impact on the way in which information is presented to the public. For example, representations of data can be developed from a particular perspective, despite giving the impression of being objective or neutral (e.g., Rubel et al., 2021), raising questions about the ethical use of mathematics in this type of reporting.

The ethics of actions taken in response to the crisis is also explored by Atweh et al. (2023). They report on how two ethical constructs, responsiveness and responsibility, were employed during the decision-making processes used by teacher educators during the crisis when addressing issues of equity. This included ensuring curriculum integrity when shifting to online teaching, adapting teaching practice, and migrating assessment practices to new circumstances. Each of these issues was mitigated by potential inequality concerning internet access.

It is important to recognize that the call for greater criticality is not confined to the COVID-19 pandemic alone, as other disruptions are also the focus of researchers' contributions to the discussion. Barwell (2013), for example, raises the issue of formatting in relation to climate change, in what he describes as post-normal science. In this context, he highlights the important role of citizens in contributing to reflective dialogue that is complex and involves a high degree of uncertainty. Consistent with this position (Coles, 2023) has pointed to the need in mathematics education to bring together socio-political and ecological concerns in what he terms socio-ecological practices. He argues that this constitutes a two-way relationship—what socio-ecological means for mathematics education and what responsibility mathematics education has toward the socio-ecological. This relationship can be considered under themes that include: questioning what gets *centered* in teaching/learning; moving toward a communal mathematics; engaging in a dialogic ethics; working against the epistemological "error" of focusing on the individual as the unit of learning. Practices associated with these themes inform socio-ecological pedagogies that recognize and embody alternative ethical, political, and aesthetic relationships (Valero, 2023).

Taking an aligned but distinct perspective, Geiger and colleagues (e.g., Geiger, in press b; Unshelm et al., under review) are investigating the use of mathematical modeling and big data to critically evaluate claims in the media related to sustainability, drawing on Duncan et al. (2018)'s notion of Gasp of Evidence. A goal of this study is the development of a set of evidentiary practices that inform teaching and learning related to socio-political-ecological—providing the basis for laypeople to form their own opinions when faced with differing claims by both expert and non-expert commentators. In a complementary study, Geiger et al. (2023a) are attempting to establish teaching-learning practices that support students' Critical Mathematical Thinking (CMT). They argue CMT is needed when faced with real-world problems that require consideration of equity, inclusiveness, and social justice in addition to mathematical knowledge and reasoning, evaluative capabilities, and positive dispositions to the use of mathematics to find responses to this form of problem. A framework for the dimensions of CMT is presented in Table 5.

The research literature discussed in this section indicates there are two principal foci when discussing issues related to criticality in the field: (1) how teaching and learning practices need to change in ways that address forms of disruption; and (2) how communication about disruption should be interrogated and evaluated. These have implications for practices in numeracy—those associated with teaching/learning, and informed critical citizenship. These represent a further evolution of what it means to be, and to become, numerate—an evolution that involves a critical turn.

TABLE 5 Critical mathematical thinking (CMT) framework.

Dimensions	Elaborations
Mathematical capability	Power over language, skills, and practices of using and applying mathematics, for example, understanding mathematical concepts and principles, identifying patterns and relationships, manipulating mathematical symbols and expressions
Critical capability	Examining ideas, drawing conclusions, clarifying meaning, identifying, and analyzing arguments, awareness of informal and cultural knowledge (e.g., social, political, environmental) and their influence on conclusions, considering the consequences of decisions and the ethical assessment of their impact.
Evaluating	Assessing claims and arguments (e.g., assessing strength of evidence, quality of data, reliability of sources), generating questions, generating aligned problems (e.g., validity of a solution if the circumstances change).
Reasoning	Logical thinking, inferring, proposing, and checking hypotheses, generalizing, interpreting different information sources, generating evidence-based arguments (including integrated mathematical and extra-mathematical practices).
Disposition	Believing mathematics is relevant to a real-world problem, showing initiative, taking intellectual risks, and displaying perseverance.

8 Discussion and conclusion

In this article, we have argued that there has been an increasing focus on critical aspects of numeracy education and practice—a critical turn. While this critical turn was in development prior to the COVID-19 pandemic, attention to the notion of criticality, as a central aspect of numeracy, intensified during and now immediately after the crisis. The pandemic, however, is not the only disruption having impact on society at large—others, such as global warming, food and energy security, and economic upheaval appear to be intensifying. How we understand and respond to these challenges using mathematics requires a critical perspective—reflecting on potential impacts and ways in which they are addressed in a responsible manner.

A turn to the critical is evident in the historical development of what we understand as numeracy, from the origins of the term in the Crowther Report 15–18 (Ministry of Education, 1959) to more recent frameworks and models, for example, Numeracy for the 21st Century (Goos et al., 2014). This development has typically taken place in response to societal demands, for example, the needs of the workplace in the case of the Mathematics Counts report (Cockcroft, 1982), Steen (1999) response to a "data-drenched" society, or Goos et al. (2014)' development of a Numeracy Model for the 21st Century, which addressed a new curriculum requirement. In the same way, we see a heightened emphasis on criticality, as an aspect of numeracy, as a response to the current times defined by disruption.

In our discussion of the notion of criticality, we have indicated that there are different conceptualisations for what "being critical" means in the context of mathematics education. Despite these differences, at the heart of each conceptualization is a concern for the human condition and the important role of mathematics education in protecting and promoting equity, inclusiveness, and social justice (e.g., Skovsmose, 1990; Gutstein et al., 1997; D'Ambrosio, 1999; Frankenstein, 2009). This means that each conceptualization is also connected to concern for those who are disadvantaged or marginalized. The source of such disadvantage or marginalization is often seen as systemic, for example, from an ethnomathematical perspective, disadvantage, and marginalization can be attributed to the adoption of Eurocentric teaching and learning practices which were not effective in the cultural context of Brazil.

The recognition of critical thinking and critical approaches to decision-making within life-related situations is seen in frameworks that outline the capabilities needed for informed and participatory citizenship (e.g., Assessment teaching of 21st century skills project, 2009). Such capabilities have also been linked to science and mathematics, for example, the Key Competencies for Lifelong Learning (European Union, 2019). Within this framework, mathematics is key to the critical evaluation of claims made in public forums and concern for sustainability in the context of rapid societal, economic, and technological change.

An understanding of the importance of critical capabilities is also evident in international testing programs such as PISA and PIAAC. Changes can be seen in the definitions of numeracy/statistical literacy and the assessment frameworks for each of these programs that reflect increasing recognition of the importance of developing critical capabilities—both as an outcome of schooling and in the day-to-day work and life practices of adults.

There has been significant attention to the critical and evaluative role that mathematics can play in responding to different forms of disruption. This is evident in both the number of publications that have a focus on the COVID-19 pandemic, but also on other aspects of disruption, for example, issues related to sustainability. Studies related to this focus are providing insight into ways in which schools can respond to these different forms of crises—through teaching and learning practices (e.g., Coles, 2023; Valero, 2023), and the development of capabilities that support the interpretation and evaluation of communication about disruption (e.g., Barwell, 2013; Skovsmose, 2021). The former calls for consideration of what gets *centered* (Coles, 2023) in mathematics teaching/learning. In the latter, the notion of formatting is a powerful way to understand how information that is disseminated in public forums is shaped—which can reflect the highest levels of integrity in relation to reporting but can also include misinformation and disinformation. Ongoing projects, such as those focused on using mathematical modeling to investigate issues related to sustainability (e.g., Geiger, in press b; Unshelm et al., under review) are aimed at providing empirical insight into how the integrity of claims in public forums can be established through the deployment of evidentiary practices. A turn to the critical that seeks to go beyond theoretical perspective and polemic.

The turn to the critical within numeracy education, described in this article, has implications for future directions in theory and practice. In terms of theory, further research is required into the nature of criticality within the context of numeracy education. In this article, we have indicated that nuanced understandings of the role of mathematics and statistics in informing the public of the progress of disruptive events (including formatting) and in addressing disadvantage and marginalization—but what other ways can a critically numerate citizen act in the service of society at large? Further, how can the capabilities associated with critical numeracy be brought to bear on other emerging disruptive phenomena, for example, the evolution of artificial intelligence? Or, could effective use of artificial intelligence, for instance, be a new capability required to be critically literate.

The initiatives described above have implications for practice, including the work of policy makers, curriculum writers, and the mathematics education community. What should be *centered* within socio-ecological pedagogies, however, must be informed by an understanding of the kind of citizenship education required to promote a more equitable, inclusive, and just society. In Weiland's (2017) view, this means citizens who are empowered to act in society in ways aimed at changing structures that produce or reinforce injustice. We have argued in this article that this empowerment requires a critically numerate citizenry, and, from the perspective of Atweh et al. (2023), citizens who are responsive and responsible. Geiger et al. (2023b) have suggested, that the choice that needs to be made is between fostering "…individuals who believe it is enough to do no harm or those who take a critical view with the aim of improving the life opportunities of all members of society" (p. 937). If the latter is the goal, numeracy education must embrace the evolving critical turn, that is, to be numerate is to be critical.

Data availability statement

The original contributions presented in the study are included in the article/supplementary material, further inquiries can be directed to the corresponding author.

Author contributions

VG: Conceptualization, Writing – original draft, Writing – review & editing. MS: Writing – review & editing.

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The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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References

Aguilar, M. S., and Castaneda, A. (2021). What mathematical competencies does a citizen need to interpret Mexico's official information about the COVID-19 pandemic? *Educ. Stud. Mathem.* 108, 227–248. doi: 10.1007/s10649-021-10082-9

Assessment and teaching of 21st century skills project (2009). Available online at: https://www.cisco.com/c/dam/en_us/about/citizenship/socio-economic/docs/assessment_teaching_21s_skills.pdf (accessed June 16, 2019).

Atweh, B., Kaur, B., Nivera, G., Abadi, A., and Thinwiangthong, S. (2023). Futures for post-pandemic mathematics teacher education: responsiveness and responsibility in the face of a crisis. *ZDM Mathem. Educ.* 55, 65–77. doi: 10.1007/s11858-022-01394-y

Barwell, R. (2013). The mathematical formatting of climate change: Critical mathematics education and post-normal science. *Res. Mathem. Educ.* 15, 1–16. doi: 10.1080/14794802.2012.756633

Bennison, A. (2016). A sociocultural approach to understanding identity as an embedder-of-numeracy: a case of numeracy and history. *Eur. Educ. Res. J.* 15, 491–502. doi: 10.1177/1474904116643327

Binkley, M., Erstad, O., Herman, J., Raizen, S., Ripley, M., MillerRicci, M., et al. (2012). "Defining twenty-first century skills," in *Assessment and Teaching of 21st Century Skills: Methods and Approach*, eds P. Griffin and E. Care (Dordrecht: Springer), 17–66.

Bolstad, O. H. (2023). Lower secondary students' encounters with mathematical literacy. *Mathem. Educ. Res. J.* 35, 237–253. doi: 10.1007/s13394-021-00386-7

Breakspear, S. (2012). *The policy impact of PISA: An exploration of the normative effects of international benchmarking in school system performance (OECD Education Working Papers, No. 71).* Organisation for Economic Co-operation and Development Publishing.

Cockcroft, W. H. (1982). *Mathematics Counts*. London: HM Stationery Office.

Coffey, P., and Sharpe, R. (2023). An investigation into the teaching of numeracy in subjects other than mathematics across the curriculum. *Int. J. Mathem. Educ. Sci. Technol.* 54, 860–887. doi: 10.1080/0020739X.2021.1978570

Coles, A. (2023). "Teaching in the new climatic regime: Steps to a socio-ecology of mathematics education," in *Proceedings of the 46th Conference of the International Group for the Psychology of Mathematics Education*, eds. M. Ayalon, B. Koichu, R. Leikin, L. Rubel, and M. Tabach (Haifa: PME), 17–34.

da Silva, A. S., Barbosa, M. T. S., de Souza Velasque, L., da Silveira Barroso Alves, D., and Magalhães, M. N. (2021). The COVID-19 epidemic in Brazil: How statistics education may contribute to unravel the reality behind the charts. *Educ. Stud. Mathem.* 108, 269–289. doi: 10.1007/s10649-021-10112-6

D'Ambrosio, U. (1985). Ethnomathematics and its place in the history and pedagogy of mathematics. *Lear. Mathem.* 5, 44–48.

D'Ambrosio, U. (1989). On ethnomathematics. *Philos. Mathem.* 2, 3–14. doi: 10.1093/philmat/s2-4.1.3

D'Ambrosio, U. (1999). Literacy, matheracy, and technoracy: a trivium for today. *Mathem. Think. Learn.* 1, 131–153. doi: 10.1207/s15327833mtl0102_3

D'Ambrosio, U. (2001). General remarks on ethnomathematics. *ZDM Mathem. Educ.* 33, 67–69. doi: 10.1007/BF02655696

D'Ambrosio, U. (2003). "The role of mathematics in building a democratic society," in *Quantitative literacy: Why numeracy matters for schools and colleges*, eds. B. L. Madison, and L. A. Steen (Princeton, NJ: National Council on Education and the Disciplines), 235–238.

D'Ambrosio, U., and D'Ambrosio, B. S. (2013). The role of ethnomathematics in curricular leadership in mathematics education. *J. Mathem. Educ. Teach. Coll.* 4, 19–25. doi: 10.7916/jmetc.v4i1.767

Djordje, K., Stephens, M., Solares-Rojas, A., and Guberman, R. (2023). "Impacts of TIMSS and PISA on mathematics curriculum reforms," in *Mathematics Curriculum Reforms Around the World*, eds. Y. Shimizu, and R. Vithal (Cham: New ICMI Study Series), 359–374. doi: 10.1007/978-3-031-13548-4_22

Drijvers, P., Thurm, D., Vandervieren, E., Klinger, M., Moons, F., van der Ree, H., et al. (2021). Distance mathematics teaching in Flanders, Germany, and the Netherlands during COVID-19 lockdown. *Educ. Stud. Mathem.* 108, 35–64. doi: 10.1007/s10649-021-10094-5

Duncan, R. G., Chinn, C. A., and Barzilai, S. (2018). Grasp of evidence: Problematizing and expanding the next generation science standards' conceptualization of evidence. *J. Res. Sci. Teach.* 55, 907–937. doi: 10.1002/tea.21468

European Union (2019). *Key competences for lifelong learning.* Available online at: https://op.europa.eu/en/publication-detail/-/publication/297a33c8-a1f3-11e9-9d01-01aa75ed71a1/language-en (accessed March 31, 2024).

Frankenstein, M. (1983). Critical mathematics education: an application of Paulo Freire's epistemology. *J. Educ.* 165, 315–339. doi: 10.1177/002205748316500403

Frankenstein, M. (1990). Incorporating race, gender, and class issues into a critical mathematical literacy curriculum. *J. Negro Educ.* 59, 336–347. doi: 10.2307/2295568

Frankenstein, M. (2009). "Developing a criticalmathematical numeracy through real-life word problems," in *Words and worlds. Modelling verbal descriptions of situations*, eds L. Verscheaffel, B. Greer, W. van Dooren, and S. Mukhopadhyay (Rotterdam: Sense Publishers), 111–130. doi: 10.1163/9789087909383_008

Freire, P. (1968). *Pedagogy of the Oppressed* (M. B. Ramos, Trans. 1990 ed.). London: Penguin.

Freire, P., and Macedo, D. (1987). *Literacy: Reading the Word and the World.* Westport, CT: Bergin and Garvey.

Gal, I. (2004). "Statistical literacy: meanings, components, responsibilities," in *The challenge of developing statistical literacy, reasoning and thinking*, eds. J. B. Garfield, and D. Ben-Zvi (Dordrecht: Kluwer), 47–78. doi: 10.1007/1-4020-2278-6_3

Gal, I., and Geiger, V. (2022). Welcome to the era of vague news: Mathematics, statistics, evidence literacy, and the Coronavirus pandemic media. *Educ. Stud. Mathem.* 111, 5–28. doi: 10.1007/s10649-022-10151-7

Gal, I., van Groenestijn, M., Manly, M., Schmitt, M. J., and Tout, D. (2005). "Adult numeracy and its assessment in the ALL survey: a conceptual framework and pilot results" in *Measuring Adult Literacy and Life Skills: New Frameworks for Assessment,* eds. S. T. Murray, Y.Clermont, and M. Binkley (Ottawa: Statistics Canada), 137-—191.

Geiger, V. (2016). "Teachers as designers of effective numeracy tasks," in *Proceedings of the 39th annual conference of the Mathematics Education Research Group of Australasia*, eds. B. White, M. Chinnappan, and S. Trenholm (Adelaide: MERGA), 252–259.

Geiger, V. (2018). "Generating ideas for numeracy tasks across the curriculum," in *Proceedings of the 41st annual conference of the Mathematics Education Research Group of Australasia*, eds. J. Hunter, P. Perger, and L. Darragh (Auckland: MERGA), 314–320.

Geiger, V. (2019). Using mathematics as evidence supporting critical reasoning and enquiry in primary science classrooms. *ZDM Mathem. Educ.* 51. 929–940. doi: 10.1007/s11858-019-01068-2

Geiger, V. (in press a). "The need for critical approaches to mathematical modelling in the time of disruptive events" in *Researching Mathematical Modelling Education in Disruptive/Challenging Times,* eds. H.-S. Siller, G. Kaiser, and V. Geiger (Cham: Springer).

Geiger, V. (in press b). "The role of mathematics education in responding to disruptive times" in the *Proceedings of the 15th International Congress on Mathematics Education (Invited Lecture)* (Sydney: ICMI).

Geiger, V., Beswick, K., Fielding, J., Scheiner, T., Kaiser, G., Goos, M., et al. (2023a). "Investigating critical mathematical thinking when applying mathematics to real-world problems," in *The 13th Congress of the European Society for Research in Mathematics Education*. Budapest, Hungary.

Geiger, V., Gal, I., and Graven, M. (2023b). The connections between mathematics education and citizenship education. *ZDM Mathem. Educ.* 55, 923–940. doi: 10.1007/s11858-023-01521-3

Geiger, V., Goos, M., and Forgasz, H. (2015). A rich interpretation of numeracy for the 21st Century: A survey of the state of the field. *ZDM Mathem. Educ.* 47, 531–548. doi: 10.1007/s11858-015-0708-1

Geiger, V., Mulligan, J., Date-Huxtable, L., Ahlip, R., Jones, D. H., May, E. J., et al. (2018). An interdisciplinary approach to designing online learning: Fostering pre-service mathematics teachers' capabilities in mathematical modelling. *ZDM Mathem. Educ.* 50, 217–232. doi: 10.1007/s11858-018-0920-x

Geiger, V., Yasukawa, K., Bennison, A., Fielding-Wells, J., and Sawatzki, C. (2020). "Facets of numeracy: teaching, learning and practices," in *Research in Mathematics Education in Australasia: 2016-2019*, eds. J. Way, J. Anderson, J. Bobis, H. McMaster, K. Cartwright, and C. Attard (Singapore: Springer), 59–89. doi: 10.1007/978-981-15-4269-5_4

Goos, M., Dole, S., and Geiger, V. (2011). Improving numeracy education in rural schools: a professional development approach. *Mathem. Educ. Res. J.* 23, 129–148. doi: 10.1007/s13394-011-0008-1

Goos, M., Geiger, V., and Dole, S. (2014). "Transforming professional practice in numeracy teaching," in *Transforming Mathematics Instruction: Multiple Approaches and Practices*, eds. Y. Li, E. Silver, and S. Li (New York: Springer), 81–102. doi: 10.1007/978-3-319-04993-9_6

Goos, M., Geiger, V., Dole, S., Forgasz, H., and Bennison, A. (2019). *Numeracy Across the Curriculum: Research-Based Strategies for Enhancing Teaching and Learning.* London: Routledge. doi: 10.4324/9781003116585

Gutstein, E. (2003). Teaching and learning mathematics for social justice in an urban, Latino school. *J. Res. Mathem. Educ.* 34, 37–73. doi: 10.2307/30034699

Gutstein, E. (2005). *Reading and Writing the World With Mathematics: Toward a Pedagogy for Social Justice*. New York: Routledge.

Gutstein, E., Lipman, P., Hernandez, P., and de los Reyes, R. (1997). Culturally relevant mathematics teaching in a Mexican American context. *J. Res. Mathem. Educ.* 28, 709–737. doi: 10.2307/749639

Jablonka, E. (2003). "Mathematical literacy," in *Second International Handbook of Mathematics Education*, eds. A. Bishop, M. A. Clements, C. Keitel, J. Kilpatrick, and F. S. K. Leung (Dordrecht: Kluwer), 75–102. doi: 10.1007/978-94-010-0273-8_4

Jablonka, E. (2020). "Critical thinking in mathematics education," in *Encyclopedia of Mathematics Education*, ed. S. Lerman (Cham: Springer), 159–163. doi: 10.1007/978-3-030-15789-0_35

Kemmis, S., Wilkinson, J., Edwards-Groves, C., Hardy, I., Grootenboer, P., and Bristol, L. (2014). *Changing Practices, Changing Education*. Singapore: Springer. doi: 10.1007/978-981-4560-47-4

Kollosche, D., and Meyerhöfer, W. (2021). COVID-19, mathematics education, and the evaluation of expert knowledge. *Educ. Stud. Mathem.* 108, 401–417. doi: 10.1007/s10649-021-10097-2

Lewandowsky, S., Ecker, U. K. H., and Cook, J. (2017). Beyond misinformation: Understanding and coping with the "post-truth" era. *J. Appl. Res. Memory Cogn.* 6, 353–369. doi: 10.1016/j.jarmac.2017.07.008

Lingard, B. (2017). *The OECD's PISA: Purposes, impact and usage. Paper presented at the International symposium "Educational governance in the era of international assessment"*. Japan Women's University. Available online at: https://www.jeas.jp/jeas/wp-content/uploads/2017/10/Lingard-B.-The-OECD%E2%80%99s-PISA-Purposes-Impact-and-Usages.pdf (accessed March 31, 2024).

Maass, K., Geiger, V., Ariza, M. R., and Goos, M. (2019). The role of mathematics in interdisciplinary STEM education. *ZDM Mathem. Educ.* 51, 869–884. doi: 10.1007/s11858-019-01100-5

Maass, K., Zehetmeier, S., Weihberger, A., and Flößer, K. (2023). Analysing mathematical modelling tasks in light of citizenship education using the COVID-19 pandemic as a case study. *ZDM Mathem. Educ.* 55, 133–145. doi: 10.1007/s11858-022-01440-9

Meyer, J. F. C. A., and Lima, M. (2023). Relevant mathematical modelling efforts for understanding COVID-19 dynamics: an educational challenge. *ZDM Mathem. Educ.* 55, 49–63. doi: 10.1007/s11858-022-01447-2

Ministry of Education (1959). *15 to 18: A Report of the Central Advisory Council for Education*. London: HMSO.

OECD (2000). *Measuring student knowledge and skills: the PISA 2000 assessment of reading, mathematical and scientific literacy*. Available online at: www.oecd.org/education/school/programmeforinternation-alstudentassessmentpisa/33692793.pdf (accessed March 31, 2024).

OECD (2004). *Learning for Tomorrow's World: First Results from PISA 2003*. Paris: OECD. doi: 10.1787/9789264006416-en

OECD (2009). *Assessment Framework: Key Competencies in Reading, Mathematics and Science*. Paris: OECD.

OECD (2013a). *PISA 2012 Assessment and Analytical Framework: Mathematics, Reading, Science, Problem Solving and Financial Literacy*. Paris: OECD Publishing.

OECD (2013b). *OECD Skills Outlook 2013: First Results from the Survey of Adult Skills*. OECD Publishing.

OECD (2016). *PISA 2015 Assessment and Analytical Framework: Science, Reading, Mathematics and Financial Literacy*. Paris: OECD. doi: 10.1787/9789264255425-en

OECD (2019). *PISA 2018 Assessment and Analytical Framework*. Paris: OECD Publishing. doi: 10.1787/b25efab8-en

O'Sullivan, K., O'Meara, N., Goos, M., and Conway, P. (2021). How Covid-19 has reinforced the importance of a numerate society, *Irish Educ. Stud.* 40, 341–347. doi: 10.1080/03323315.2021.1915844

Partnership for 21st Century Skills (2002). *Learning for the 21st century: A report and mile guide for 21st century skills.* Available online at: http://www.21stcenturyskills.org/images/stories/otherdocs/p21up_Report.pdf (accessed June 16, 2019).

Rosa, M., D'Ambosio, U., Orey, D., Shirley, L., Alangui, W., Palhares, P., et al. (2016). *Current and Future Perspectives of Ethnomathematics as a Program.* Cham: Springer Open. doi: 10.1007/978-3-319-30120-4

Rubel, L. H., Nicol, C., and Chronaki, A. (2021). A critical mathematics perspective on reading data visualizations: Reimagining through reformatting, reframing, and renarrating. *Educ. Stud. Mathem.* 108, 249–268. doi: 10.1007/s10649-021-10087-4

Sawatzki, C., and Sullivan, P. A. (2017). Teachers' perceptions of financial literacy and the implications for professional learning. *Austral. J. Teacher Educ.* 42, 51–65. doi: 10.14221/ajte.2017v42n5.4

Siller, H. S., Elschenbroich, H. J., Greefrath, G., and Vorhölter, K. (2023). Mathematical modelling of exponential growth as a rich learning environment for mathematics classrooms. *ZDM Mathem. Educ.* 55, 17–33. doi: 10.1007/s11858-022-01433-8

Skovsmose, O. (1990). Mathematical education and democracy. *Educ. Stud. Mathem.* 21, 109–128. doi: 10.1007/BF00304897

Skovsmose, O. (1994). Towards a critical mathematics education. *Educ. Stud. Mathem.* 27, 35–57. doi: 10.1007/BF01284527

Skovsmose, O. (1998). Linking mathematics education and democracy: Citizenship, mathematical archaeology, mathemacy and deliberate interaction. *ZDM Mathem. Educ.* 30, 195–203. doi: 10.1007/s11858-998-0010-6

Skovsmose, O. (2021). Mathematics and crises. *Educ. Stud. Mathem.* 108, 369–383. doi: 10.1007/s10649-021-10037-0

Stacey, K., Almuna, F., Caraballo, R. M., Chesné, J.-F., Garfunkel, S., Gooya, Z., et al. (2015). "PISA's influence on thought and action in mathematics education," in *Assessing Mathematical Literacy*, eds. K. Stacey and R. Turner (Cham: Springer), 275–306. doi: 10.1007/978-3-319-10121-7_15

Steen, L. A. (1999). Numeracy: the new literacy for a data-drenched society. *Educ. Leader.* 57, 8–13.

Steen, L. A. (2001). "The case for quantitative literacy," in *Mathematics and democracy: The case for quantitative literacy*, ed. L. Steen (Princeton: National Council on Education and the Disciplines), 1–22.

Tout, D. (2020). Evolution of adult numeracy from quantitative literacy to numeracy: Lessons learned from international assessments. *Int. Rev. Educ.* 66, 183–209. doi: 10.1007/s11159-020-09831-4

Tout, D., Dermonty, I., Diez-Palomar, J., Geiger, V., Hoogland, K., and Maguire, T. (2021). "PIAAC cycle 2 assessment framework: numeracy," in *The assessment frameworks for cycle 2 of the Programme for the International Assessment of Adult Competencies*, ed. OECD (Paris: OECD), 64–154. doi: 10.1787/c4221062-en

Tudball, L., and Henderson, D. (2014). Contested notions of civics and citizenship education as national education in the Australian curriculum. *Curric. Teach.* 29, 5–24. doi: 10.7459/ct/29.2.02

United Nations General Assembly (2015). *Transforming our world: The 2030 agenda for Sustainable Development.* UN Doc A/RES/70/1.

Unshelm, N., Digan, S., Geiger, V., Siller, H.-S., and Greubel, A. (under review). "Empowering students to evaluate media claims with big data," in *International Horizons in Mathematics Modelling Education,* eds. T. Ikeda, A. Saeki, V. Geiger, and G. Kaiser (Cham: Springer).

Vacher, H. L. (2017). Ten years, twenty issues, and two hundred papers of numeracy: toward international reach and transdisciplinary utility. *Numeracy* 10:1. doi: 10.5038/1936-4660.10.2.1

Vale, P., and Graven, M. (2023). Strategies implemented by South African teachers to ensure continuing mathematics education during COVID-19. *ZDM Mathem. Educ.* 55, 163–176. doi: 10.1007/s11858-022-01408-9

Valero, P. (2023). "Mathematical subjectivation: death sentence or chances for a terrestrial life?," in *Proceedings of the 46th Conference of the International Group for the Psychology of Mathematics Education*, eds. M. Ayalon, B. Koichu, R. Leikin, L. Rubel, and M. Tabach (Haifa: PME), 53–68.

Villarreal, M. E., Villa-Ochoa, J. A., and Galleguillos, J. (2023). Experiences of preservice mathematics teachers during their education in times of pandemic. *ZDM Mathem. Educ*. 55, 235–248. doi: 10.1007/s11858-022-01461-4

Weiland, T. (2017). Problematizing statistical literacy: an intersection of critical and statistical literacies. *Educ. Stud. Mathem.* 96, 33–47. doi: 10.1007/s10649-017-9764-5

Westheimer, J., and Kahne, J. (2004). What kind of citizen? The politics of educating for democracy. *Am. Educ. Res. J.* 41, 237–269. doi: 10.3102/00028312041002237

Yasukawa, K., Hamilton, M., and Evans, J. (2017). A comparative analysis of national media responses to the OECD Survey of Adult Skills: policy making from the global to the local? *Compare* 47, 271–285. doi: 10.1080/03057925.2016.1158644

Zevenbergen, R. (2004). Technologizing numeracy: Intergenerational differences in working mathematically in new times. *Educ. Stud. Mathem.* 56, 97–117. doi: 10.1023/B:EDUC.0000028399.76056.91

Zhou, Y., and Shen, L. (2022). Confirmation bias and the persistence of misinformation on climate change. *Commun. Res.* 49, 500–523. doi: 10.1177/00936502211028049