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# Learner-generated drawing as a learning strategy. The effect of teacher-guided intervention program “Learning with Understanding” on composing drawings in math word problems in the primary grades

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The study aimed to examine the possibility of teaching primary school students a learner-generated drawing strategy, among other constructivist learning strategies. The teacher-guided program “Learning with Understanding” began by discussing the broader topics of the learning process, followed by teaching specific strategies, and ended with an overview of all strategies and reflective discussions. During 18 program lessons, primary school teachers taught, practiced, and raised metacognitive awareness of three learning strategies—elaboration of new information with familiar material and daily practice, organization of material into categories and elaboration, and organization of information through drawing. This study examined composing drawings for math word problems before and after the program. The sample consisted of second- and fourth-grade students from eight Estonian schools. The intervention group included 110 students from second grade and 80 students from fourth grade. The control group consisted of 121 second-grade students, and 82 fourth-grade students. Before and after the intervention, students had to solve two math word problems and compose a drawing, if needed. The results showed that before the intervention, neither the control group nor the intervention group students drew almost any drawings. However, after the intervention, both the control group and the intervention group students started to draw more drawings. Also, the intervention group students composed both more drawings and more schematic drawings. The effect of the intervention was visible at both grade levels. Comparing the correctness of answers with the drawing type showed that the fourth grade obtained significantly more correct answers when no drawings were made, while in the second grade,

students had fewer correct answers when they had not compiled a drawing. Thus, we showed that even very young students could learn to compose schematic drawings; however, drawings alone may not be of help to solve the problem.

#### KEYWORDS

composing drawings, learning strategy, math problem solving, intervention, primary grades

## Introduction

The characteristics and developmental peculiarities of children's drawings have been studied for many years (e.g., Barnes, 1892; Cox, 2005). Drawings have been used not only to examine children's general ability level, emotional, and family problems (e.g., Campbell and Bond, 2017; Ivens, 2021), but also to discover children's science conceptions (e.g., Vosniadou and Brewer, 1992; Kikas, 2000), learning (Hsieh and Tsai, 2016), and math (Hatisaru, 2020). Additionally, drawings are also valuable visual learning aids (Mayer, 2017). While drawings have been used to illustrate texts or represent scientific ideas for a long time, the usefulness of constructing drawings as a learning strategy besides learning from drawings has been acknowledged only recently (Van Meter and Garner, 2005; Schmidgall et al., 2019; Ainsworth et al., 2020). Also, the advantages of self-generated drawings have been found only in certain conditions—for schematic drawings, when students have a certain level of knowledge and cognitive skills, and have been taught to compose useful drawings and been supported in their application (Hegarty and Kozhevnikov, 1999; Terwel et al., 2009; Ainsworth et al., 2020).

Although many studies have examined relations between generating drawings and learning, some areas need further investigation. Many interventions use composing drawings in specific subjects like math (Hegarty and Kozhevnikov, 1999; Van Garderen and Scheuermann, 2014) or science (Tippett, 2016). Strategies learnt in one area may be difficult to generalize to other areas. Generating drawings is useful in learning different subjects and it is valuable to use it in different lessons. Moreover, it is only one helpful strategy among several others that students can use and should be taught. Teaching various learning strategies and their application in different lessons allows talking about learning process, reasons why some strategies are useful for learning, what is needed to be successful in applying each strategy etc. It means supporting metacognitive knowledge of learning strategies and skills in their application, which importance has been well documented (Dignath et al., 2008; Fiorella and Zhang, 2018). So far, little attention has been paid to supporting students' metacognitive knowledge and skills during the process of generating drawings. Lastly, teachers tend to

have misconceptions regarding generating drawings and this may inhibit them in supporting students' knowledge and skills of constructive learning strategies (Dignath and Büttner, 2018; Glogger-Frey et al., 2018). Thus, it is important to educate teachers in this process.

Our study aimed to examine the possibility to teach primary school students learner-generated drawing strategy among other constructivist learning strategies. The "Learning with Understanding" intervention program was designed to help primary school teachers teach, practice and raise metacognitive awareness of three learning strategies—elaboration of new information with familiar material and daily practice, organization of material into categories and elaboration and organization of information with drawing. While the effects of the program for supporting elaboration and organization strategies represented fourth grade students (Kikas et al., 2021), the aim of this study was to examine the effects of intervention on second- and fourth-grade students' skills in composing drawings for math word problems.

## Learner-generated drawing as a constructive learning strategy

Drawings are configurations of symbols, images or concrete objects standing for some other entity that constitutes a constructive learning strategy if generated to achieve a learning goal (Van Meter and Garner, 2005; Chi, 2009; Fagnant and Vlassis, 2013; Tippett, 2016; Brod, 2020). For generating drawings, students must first select critical information from the text for processing in their working memory. Next, they mentally organize the verbal elements into a coherent verbal representation that is used to support the construction of the drawing, at the same time integrating given information with the students' existing knowledge from their long-term memory. Finally, learners convert their mental model into a representative drawing on paper that may also include verbal signs (Van Meter and Garner, 2005; Tippett, 2016; Fiorella and Zhang, 2018). Generating is useful for learning drawings and presumes good metacognitive skills as the cognitive processes of selecting, organizing, and integrating occur recursively

and should be guided by self-monitoring and self-regulation (Fiorella and Zhang, 2018).

Drawings and other visual aids are—like words—mediators of knowledge. They are generalizations that help comprehend new material but students also have to learn their meaning (Vygotsky, 1997; Prain and Tytler, 2012). In line with differentiation of everyday and scientific concepts (Vygotsky, 1997), drawings can be divided into pictorial and schematic. Everyday concepts are acquired *via* individual sensory experiences, and similarly, children are used to draw what they see. Everyday concepts are generalizations of personal experiences, and children's pictorial drawings represent the overall situation and emphasize the visual appearance of objects, but also include what they know (Cox, 2005). Scientific concepts do not directly refer to objects, but rather to other concepts; they are generalizations about generalizations, in which perceptual features are recombined into new, supposedly more-informative and abstract structures. Likewise, drawings as a means of a learning strategy—schematic drawings—are generalizations of what has to be learnt, they include abstractions, non-perceptible features and bring out the main idea of the text or the problem (Fiorella and Zhang, 2018).

Acquiring everyday concepts is quite an easy process. Children also compose pictorial drawings at a young age (Cox, 2005). In contrast, learners have difficulties with comprehending scientific concepts, and form misconceptions (Kikas, 2003). Studies have indicated to misconceptions regarding drawings and other visual aids (Stylianou, 2020). For example, students may interpret graphs as iconic, similarly to interpreting picture reading (Leinhardt et al., 1990). These misconceptions may inhibit students' own creation of schematic drawings. Independently, young students and those with lower abilities do not construct or only rarely construct schematic drawings (Hegarty and Kozhevnikov, 1999; Van Garderen and Montague, 2003; Fiorella and Zhang, 2018). Besides misconceptions and drawing skills, starting to compose schematic drawings may increase the cognitive load and result in utilization deficiency and reduced performance (Leutner et al., 2009; Clerc et al., 2014). If the activity of drawing is too demanding, the cognitive capacities are required for composing the drawing and thus are no longer available for information processing. Low metacognitive skills also prevent generating useful drawings (Rellensmann et al., 2016; Fiorella and Zhang, 2018).

Meta-analyses have confirmed advantages of using and generating drawings over passive learning strategies. Learner-generated drawing has been shown to be a more effective learning strategy than verbalizing or guess-and-test strategies (Hembree, 1992) and reading or using text-focused strategies (Fiorella and Zhang, 2018). However, when comparing generated and readymade illustrations, the effects of drawing are mixed and depend on the quality of drawing and amount of teacher support (Fiorella and Zhang, 2018).

## Applying learner-generated drawing in solving math word problems

Math word problems are a subcategory of word problems in which one or more quantitative relationships are described and a numerical answer is required (Van Essen and Hamaker, 1990). Word problems are used in math education starting from first grades as they integrate school math with students' real life experiences (e.g., Van de Weijer-Bergsma and Van der Ven, 2021). However, these are not simple calculation tasks instructed by words, but presume conceptualizing the problem, planning a solution, searching for strategies etc. (Van Garderen and Montague, 2003). Problem solving models have identify two phases: problem representation and problem execution (Krawec, 2014). Problem representation requires a learner to transform linguistic and numerical information into representations that show how the problem information is related. This helps to select appropriate mathematical algorithms and to perform the appropriate calculations (problem execution phase). Composing drawings (also diagrams, representations; see Stylianou, 2020) may be helpful in the problem representation phase (Van Garderen and Scheuermann, 2014). Learner-generated drawing is a tool for analyzing the problem and finding the solution (Van Essen and Hamaker, 1990). Schematic drawings represent the problem's main data and the mathematical relationships between them; these may include both physical features, abstract spatial relationships, and also words (Hegarty and Kozhevnikov, 1999; Fiorella and Zhang, 2018). Good problem solvers compose general schematic drawings of the problem that enhance comprehension (Van Garderen and Montague, 2003; Krawec, 2014).

Studies support the usefulness of constructing drawings by students, but in certain conditions. For instance, it has been shown that the use of schematic representations is positively, whereas the use of pictorial representations is negatively related to success in math problem solving (Hegarty and Kozhevnikov, 1999; Van Garderen and Montague, 2003; Edens and Potter, 2007; Rellensmann et al., 2016). Students' spontaneous diagram use in math word problem solving is also influenced by problem type (Fagnant and Vlassis, 2013) and cultural context (see New Zealand vs. Japan, Uesaka et al., 2007). Studies have indicated that drawing accuracy is related to problem solving performance (Rellensmann et al., 2016), but not always (Van Essen and Hamaker, 1990). Drawings are supportive if students accurately depict the structural relations and processes described in the text or problem.

## Interventions supporting learner-generated drawing strategy

Students need help in learning to use and understand the usefulness of constructive learning strategies, including

generating drawings (Terwel et al., 2009; Tippett, 2016; Fiorella and Zhang, 2018; Ainsworth et al., 2020; Van Meter and Stepanik, 2020). While some researchers have emphasized the importance of teaching students specific schemas for specific tasks (Fagnant and Vlassis, 2013), it is now more widely accepted that students should be taught different types of schemas that may be used for solving various problems (Fiorella and Zhang, 2018). Multiple studies have referred to the importance of metacognitive awareness of learning strategies, including generating drawings (Dignath et al., 2008; Fiorella and Zhang, 2018). Some authors have brought out stages and practices in explicit teaching to use schematic drawings in solving word problems. For instance, Falomir (2018) differentiated three main instructional activities: (1) teaching students how to create and interpret drawings, (2) supporting students in using the drawings and (3) teaching students metacognitive strategies focused on identifying critical components of word problems to ensure mastery of the diagramming process.

Several, but not all, math-specific interventions that provided drawing practice have shown positive effects. For example, Van Essen and Hamaker (1990) found that fifth-graders, but not first- and second-graders, who practiced constructing drawings for arithmetic word problems improved their problem solving performance in comparison with a control group. Students were not explicitly taught how to compose drawings, but experimenters composed drawings together with students. After completing the drawings, students and an experimenter explained their drawings. Csikos et al. (2012) carried out an intervention with third-graders and their teachers, using an experimental-control-group design. In the drawing group, students were asked to generate drawings for each problem and to discuss the role of visual representation in group work and teacher-led discussions. Both the experimental and control groups improved their results from pre- to post-test, but the gain in achievement was notably higher for the drawing group. In both of these interventions, attention was paid to raising students' metacognitive knowledge through explanations and discussion.

Studies have shown that findings depend on drawing and problem types. Fagnant and Vlassis (2013) analyzed the effect of two types of schematic drawings on fourth-graders' solving non-routine arithmetic problems. Half of these were schematic drawings which were close to the informal models that students might construct themselves, and the others were half abstract diagrams derived from the typology of Novick and Hurley (2001). Students were first shown either diagram or schematic drawing accompanied with a math problem. In post-test, students were asked to draw by themselves. Experimental groups did better in post-test and the results slightly favored diagrams rather than schematic drawings similar to self-generated drawings. However, 36% of students did not derive any benefit from learning and the results also depended on problem type and individual student.

De Bock et al. (2003) studied the influence of learner-generated graphical representations on eighth- and tenth- grade students' success to solve non-proportional word problems about area and volume. They found that in the drawing-instruction group, drawings were composed in 94% of the cases, and 83% of these drawings were correct. In contrast, without instruction, only 10% of generated drawings, and only 9% of these drawings were correct. However, although the far majority of students who were instructed, composed correct drawings, it did not help them correctly solve the math problems—students in the drawing-instructed group scored lower in the math test than those in the non-instructed group. The authors highlighted in their findings the lack of attention paid to supporting students' metacognitive knowledge and skills. Also, they noted that the drawing process itself might provide incorrect knowledge when being processed in the working memory.

Learner-generated drawing may also be supported as a part of a wider constructive learning strategy teaching. Studies indicate that in ordinary lessons teachers rarely explicitly teach strategies, and still less support metacognitive knowledge and skills (e.g., Dignath and Büttner, 2018; Coffman et al., 2019). Moreover, teachers' knowledge of learning strategies tends to be limited (Dignath and Büttner, 2018; Glogger-Frey et al., 2018). Various constructive learning strategies have also been supported during self-regulated learning interventions (Dignath et al., 2008). These emphasize the need to include learning about different learning strategies and metacognitive knowledge and skills. It is also important to practice strategies in different contexts, including different subject lessons, which promotes automatization, helps reduce the cognitive load related to monitoring new strategies, and helps to overcome utilization deficiency (Clerc et al., 2014).

Differently from the earlier studies that have not provided separate information on learner-generated drawings, our intervention program "Learning with Understanding" included teaching, practicing, and raising metacognitive awareness of three constructive learning strategies, including self-generated drawings (Kikas et al., 2021). Students learned about strategies first quite generally, then practiced these in three main subjects—mother language, science, and math. Finally, students' metacognitive knowledge and skills in applying the strategies was raised *via* discussions.

## Aims and hypotheses

The aim of this study was to examine what type of drawings primary school students compose for math word problems and what effect the intervention program "Learning with Understanding" has on the frequency and quality of drawings. Second and fourth grade students were selected as participants due to the following reasons. Second grade is the youngest where to start teaching different learning strategies as during

the first grades students study basic academic skills and become acquainted with learning at school. Fourth grade is usually the highest grade in Estonian schools with class teachers teaching all main subjects while starting from fifth grade, subject teachers teach separate subjects. A critical part of the intervention program was practicing learning strategies in different subjects which was easier to accomplish with one teacher. We compared the intervention and control groups, using a pre- and post-test design and carried out analyses separately for second- and fourth-grades. Our research questions and hypotheses were as follows:

First (RQ1), do the intervention and control groups differ in composing drawings in pre- and post-tests? In pre-tests, we expected that only a few students compose drawings and that there are no between-group differences (H1a). In post-tests, we expected more students in the intervention than in the control group to compose drawings for math word problems, at least in the fourth grade (H1b). Earlier, [Van Essen and Hamaker \(1990\)](#) found that only fifth-graders, but not first- and second-graders, composed more drawings after the intervention.

Second (RQ2), how do the intervention and control groups differ in composing schematic drawings in post-test? We expected (H2) that more intervention than control group students would draw schematic drawings. Earlier studies have shown that students start to use more effective learning strategies (creating different models and drawings) only when being taught it ([De Bock et al., 2003](#); [Van Meter and Stepanik, 2020](#)).

Third (RQ3), how are pictorial and schematic drawings related to correctness of solving math word problems? We expected (H3) that schematic, but not pictorial drawings are related to correctly solving math tasks (see [Hegarty and Kozhevnikov, 1999](#); [Van Garderen and Montague, 2003](#); [Edens and Potter, 2007](#); [Rellensmann et al., 2016](#)). Still, not all studies have found these positive relations (e.g., [De Bock et al., 2003](#)).

## Materials and methods

### Sample

The sample included second- and fourth- grade students from eight Estonian schools. Schools were invited to participate through an advertisement. They could choose between two intervention programs—“Learning with understanding” and “We read”—that were implemented concurrently in different schools. The intervention group included children whose schools participated in program “Learning with Understanding.” It was emphasized that participation in the program was voluntary for the teacher and that he/she wanted to integrate the learning strategies into his or her own subjects. The control group was formed from the schools who were in the waiting-list for participating the following year in the program

“We Read.” The control group students completed both pre- and post-tests, but did not participate in any additional practices in classroom.

The intervention group included 110 students from 6 s-grade classes and 80 students from four fourth-grade classes. The control group consisted of 121 s-grade students, from 7 different classes, and 82 fourth-grade students, from 4 classes. There were 193 boys (95 in the second-grade and 98 in the fourth-grade; 88 in the intervention group and 105 in the control group). There were more girls than boys in the intervention groups in the second-grade,  $\chi^2(1) = 9.05, p = 0.003$ . The intervention and control groups did not differ significantly in gender composition in the fourth-grade,  $\chi^2(1) = 3.25, p = 0.07$ .

## Procedure

### Pre-test

Students were tested before the intervention program (end of 2018/beginning of 2019). The students were tested with the agreement of the school management and teachers, and with the informed consent from the parents. Permission was granted both to test the children and to use the test results. All relevant ethical standards were followed when testing the children, and the children had the opportunity to terminate their participation in the testing at any time.

The test included reasoning, reading and math tasks. Solving math problems and composing drawings was a part of the test that was used in this study. Students had to solve math word problems but could decide for themselves whether they would use drawings to solve the task. The guide said “make a drawing if necessary.” The test was on a tablet, was conducted in Estonian and lasted approximately 45 min. The completion took place in the classroom, and the testing was supervised by at least one university team member.

### Intervention

The intervention program was carried out in the first half of 2019 (January-May) and was prepared by the “Learning with Understanding” project team, which included both teachers and researchers (for more details, see [Kikas et al., 2021](#)). The program was created for teachers to support the acquisition of more effective learning strategies in lessons and to deepen students’ awareness and self-regulated learning.

#### *Practices with teachers*

Prior to the intervention program, teachers were given training on learning, memory and understanding, and then the first part of the program was introduced. Teachers were also provided with materials that introduced the idea and principles of the program. In the middle of the program, there was a second meeting to introduce the second part of the program.

It was also an opportunity to discuss issues and difficulties and to exchange experiences.

Throughout the program, teachers received support from researchers. Teachers also completed an online form announcing the tasks they had given to students and providing feedback on the strengths and weaknesses of the intervention. At the end of the program a meeting was held to gather feedback from teachers and to discuss ways to further develop learning strategies.

### *Practices with students*

The program consisted of eight topics, divided into 18-h sections. For students, participation in the program began with addressing the learning process, thinking, memory and memorization, and individual differences. Teachers then introduced three strategies—visualization, making connections (with both personal experience and prior knowledge) and categorization. In the last two lessons, the strengths and weaknesses of the learned strategies were discussed with children and the skills to use one's own strategies were analyzed. Learning strategies were practiced in the mother language (Estonian), math, and science, and subject-specific sample tasks were created for teachers' use.

### *Teaching and practicing drawing*

First, discussions were held on which illustrations are more helpful and supportive for memorizing and understanding important information. Also, discussions on how modeling the tasks helps to solve problems better, and how to create simple, abstract (not necessarily visually attractive) drawings. The practice tasks were selected to be in line with the topics students learnt in ordinary lessons. For example, after reading about the day-night cycle, students were asked to illustrate the movements of the Earth and the Sun in the garden and from a space. The two drawings were compared, and whether the second drawing captured the day-night cycle were discussed.

### **Post-test**

Students were tested again after the intervention program (Spring 2019). The procedure was similar as in pre-test. However, students completed the test on paper. Similarly to pre-test, post-test included reading and math tasks. Solving math problems and composing drawings was a part of the test that was used in this study.

### **Math test**

#### *Math problems*

Math problems were developed by Anu Palu and Eve Kikas. The tasks were picked according to the grade level and that the drawing would have an effect in solving an assignment was taken into consideration. The tasks for both grades were similar and thus more difficult for the second grade than for the fourth grade. The instruction in pre- and post-tests was

the same: "Solve the problems. If necessary, compose a drawing that helps you solve the task better. Write out the solution and the answer." The pre-test included four problems and post-test three, two of which were the same in both tests (numbers and children's names were changed). For comparing the tests before and after the intervention, we chose two of the same problems—classroom and games tasks. The classroom task was: "There were 12 students in the class, five students left during the break and two students came back to the class. How many students are in the classroom now?" The games task was: "Kadri and Lauri have a total of 24 computer games, Kadri has six more games than Lauri. How many computer games does Lauri have?"

### *Drawings and coding*

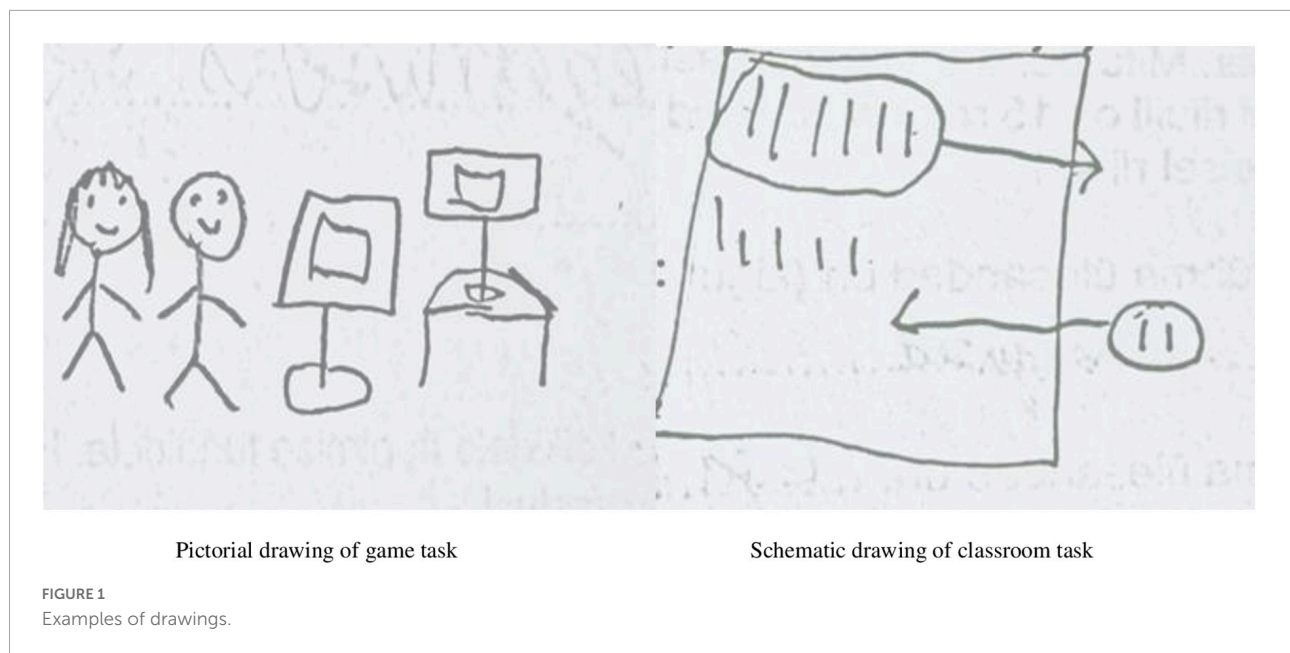
The drawings were coded as no drawing (code 9) if the student did not draw or the drawing could not be interpreted. The rest of the drawings were coded by type, either pictorial (code 1) or schematic (code 2). A pictorial drawing is one that depicts something that may be related to the task, but does not support solving because it does not depict relationships. It might have correct but also superfluous components. A schematic drawing is one that shows relations or numbers (see [Hegarty and Kozhevnikov, 1999](#); [Fiorella and Zhang, 2018](#)). Examples are given in [Figure 1](#).

The drawings were coded by two evaluators. In order to check the reliability of the evaluator's estimates (for the type of drawing) used in the analysis, the reliability between the evaluators was measured with Cohen Kappa. There was a high reliability between evaluators,  $k = 0.92$ , (95% CI 0.87, 0.94),  $p < 0.001$ .

## **Data analysis**

The dependent variable was coded as an ordinal variable of counts (number of drawings made or number of specific type of drawings, either pictorial or schematic). This allowed us to model these counts as explained by the categorical independent variables using an ordinal log-linear approach. More specifically, we used cumulative logit models following [Agresti \(2019\)](#). The analysis was run in R statistical software *via* the statistical software library *multgee* introduced by [Touloumis \(2015\)](#).

Since repeated measurements study design tends to lead to positive correlation between responses, a correction was used that is available in the *multgee* R library as proposed by [Agresti \(2019\)](#) to avoid biased standard error estimators. As suggested by [Hosmer and Lemeshow \(2000\)](#) we also checked that the log-odds do not depend on outcome category or in other words that the "proportionality" condition also known as "parallel logits" condition was met. This was done using the statistical library *vgam* introduced by [Yee \(2010\)](#) and a chi-squared test of difference of model fit between a proportionality assumed



model and proportionality not assumed model. To test RQ1 and RQ2, we started with a model that included the main effects of all categorical independent features and the interactions of all those features.

For RQ1 and RQ3, Configural Frequency Analysis (CFA) was used (von Eye, 1990; Stemmler and Heine, 2016). CFA identifies patterns of answers that occur more (called types) or less (called antitypes) frequently than by chance. The frequencies were tested with a binomial accuracy test because the expected frequencies of some table squares were very low. Bonferroni correction was used to account for multiple testing.

## Results

### RQ1: Differences between the intervention and control groups in composing drawings in pre- and post-tests

The percentage and number of students composing drawings in the intervention and control group in pre- and post-test are shown in Table 1. In both groups and grades, the majority of students did not compose any drawings in pre-test. This indicates that the groups were equal at the beginning of the study and confirms random selection. In contrast, in post-test, more drawings were made in the intervention group, both in the second- (40.0%) and fourth grade (31.2%), although an increase in drawings is also visible in the control group.

We created a model that included the main effects and interactions of all categorical independent variables—test (pre and post), group (intervention and control) and grade (second and fourth). Interactions regarding grade did not improve the model, and so they were removed from the model. We also removed the group's main effect due to following reasons. First, the removal of the group's main effect (Wald = 0.2,  $p = 0.64$ ) did not significantly affect the model at this stage. In the context of the research question, we were interested in the effect of interaction between testing time and group. Second, when checking for the multicollinearity, there was a strong negative correlation between the group and the interaction between the group and the test time ( $r = -0.89$ ). Such correlation creates uncertainty in the model and the standard errors expand enormously. Removing the main effect was reasonable because we were interested in the interaction between the group and the test turn, and it allows a better assessment of the statistical significance to verify the validity of the hypotheses. However, before removing the main effect, we looked at the coefficients and the main effect coefficient showed that in pre-test, the intervention group students were estimated to be 1.21 times more likely to draw than the control group students [Exp (0.20) = 1.22, 95% CI (0.52, 2.85),  $p = 0.64$ ]. This is not statistically significant and thus it can be assumed that the groups were equal in the first test.

We proceeded with a model that included only the main effects of the test turn and grade, and the interaction of the group and the test. The main effect of the test turn was an important variable that improved the model: Wald = 40.49,  $p < 0.001$ . The estimated odds ratio, which expresses the probability of making more or as many drawings as any number of drawings  $k$ , changes five times in the control group during the transition from the

TABLE 1 Percentage and (number) of drawings in intervention and control groups in pre- and post-tests.

Test	Control group			Intervention group		
	Number of tasks with drawing			Number of tasks with drawing		
	0	1	2	0	1	2
<b>Grade 2</b>						
Pre-test	91.7% (111)	5.8% (7)	2.5% (3)	92.7% (102)	5.5% (6)	1.8% (2)
Post-test	73.6% (89)	15.7% (19)	10.7% (13)	60.0% (66)	16.4% (18)	23.6% (26)
<b>Grade 4</b>						
Pre-test	96.3% (79)	2.4% (2)	1.2% (1)	97.5% (78)	1.2% (1)	1.2% (1)
Post-test	78.0% (64)	17.1% (14)	4.9% (4)	68.8% (55)	12.5% (10)	18.8% (15)

TABLE 2 Configural frequency analysis patterns emerging as types.

Pattern	Frequency	Expected frequency	df	z-value	P-value	Type
1120	26	9.24	1	5.38	<0.001	Type
1121	15	6.48	1	3.17	<0.001	Type

Patterns: 1,120, intervention group; post-test, 2 drawings, Grade 2; 1,121, intervention group; post-test, 2 drawings, Grade 4.

first test turn to the second [ $\text{Exp}(-1.64) = 0.19$ , 95% CI (0.12, 0.32)]. Also, grade was an important variable that improved the model: Wald = 4.82,  $p < 0.05$ . The estimated odds ratio, which expresses the probability of making fewer or as many drawings as any number of drawings  $k$ , changes 1.6 times when comparing the fourth grade to the second grade [ $\text{Exp}(0.45) = 1.57$ , 95% CI (1.04, 2.34)].

The interaction between the group and the test turn was statistically significant: Wald = 9.0,  $p < 0.01$ . The effect of the test turn in the control group, which expresses the probability of making more or as many drawings in the control group in the second test turn compared to the first test as in any number of drawings  $k$ , was 1.93 times larger in the intervention group [ $\text{Exp}(-0.66) = 0.52$ , 95% CI (0.34, 0.79)]. This means that while the control group was estimated to have an average of five times more drawings in the second testing, in the intervention group there were about 9.65 times more. This difference is statistically significant.

Neither the inclusion of grade  $\times$  test turn interaction nor the triple interaction of grade  $\times$  test turn  $\times$  group significantly improved the model. However, the frequency table showed some differences between the grades. CFA was conducted to examine if there were patterns that were observed more (types) or less (antitypes) frequent than chance. Based on Bonferroni adjusted  $p$ -value (0.002), two types emerged (see Table 2). These types suggest that significantly more two drawings than expected were made in both grades in post-test in the intervention group. Thus, the CFA analysis suggests that the impact of the program is across grades (by increasing the number of drawings for both tasks).

## RQ2: Differences between the intervention and control groups in composing schematic drawings in post-test

In pre-test, the majority of the control and intervention group students initially made no schematic drawings (see Table 3). In post-test, both in the control and the intervention group, at least one fifth of the students created a schematic drawing at least for one task. The number of students who did not compose schematic drawings at all decreased in both groups, but more in the intervention group. In the second grade, in control group, 9.1% students created a schematic drawing in two tasks while in the intervention group, the percentage was 20.0%. In the fourth grade, the percentages were 4.9 and 13.8%. This suggests that the students in the intervention group began to compose schematic drawings more consistently.

As in the first analysis, we first included in the model all potentially important characteristics: the number of tasks where schematic drawings were used (0, 1 or 2) as a dependent variable, the testing turn (pre- and post-test) and group (control group, intervention group) and the interaction of testing turn and group as independent variables. In this model as the previous one, there is also a strong correlation between group and group-testing time interaction ( $r = -0.94$ ). For the same reasons and following the same logic as explained in research question 1, we removed the main effect for the group. Here the group's main effect coefficient shows the difference between the groups in the first test [ $\text{Exp}(-0.) = 0.81$ , 95% CI (0.11,



TABLE 3 Percentage and (number) of schematic drawings in intervention and control groups in pre- and post-tests.

Test	Control group			Intervention group		
	Number of tasks with schematic drawing			Number of tasks with schematic drawing		
	0	1	2	0	1	2
<b>Grade 2</b>						
Pre-test	97.5% (118)	0.8% (1)	1.7% (2)	93.6% (103)	4.5% (5)	1.8% (2)
Post-test	76.9% (93)	14.0% (17)	9.1% (11)	63.6% (70)	16.4% (18)	20.0% (22)
<b>Grade 4</b>						
Pre-test	98.8% (81)	1.2% (1)	0.0% (0)	100.0% (80)	0.0% (0)	0.0% (0)
Post-test	78.0% (64)	17.1% (14)	4.9% (4)	71.2% (57)	15.0% (12)	13.8% (11)

TABLE 4 Percentage and (number) of correct and incorrect answers by grade and type of drawing.

Type of drawing	Grade 2		Grade 4	
	Correct answer	Incorrect answer	Correct answer	Incorrect answer
No drawing	32.1% (63)	67.9% (133)	58.7% (84)	41.3% (59)
Pictorial drawing	33.3% (2)	66.7% (4)	25.0% (1)	75.0% (3)
Schematic drawing	51.7% (15)	48.3% (14)	73.3% (11)	26.7% (4)

5.88),  $p = 0.84$ ], which means that the random distribution has been successful.

We moved on with a model where the independent variables are the testing turn and the interaction of the testing turn and the group. Characteristic testing turn is also important here and improves the model. Wald = 46.22,  $p < 0.001$ . The estimated odds ratio, which expresses the probability of making more or as many schematic drawings in the control group as at any level  $k$ , varies 10 times between the first testing turn and the second turn [Exp 0.39] = 1.5, 95% CI (0.97, 2.27)].

Here grade was not an important variable and did not improve the model falling slightly above the 0.05 threshold with Wald = 3.25,  $p = 0.07$ . The estimated odds ratio, which expresses the probability of making fewer or as many drawings as any number of drawings  $k$ , changes 1.5 times when comparing the fourth grade to the second grade [Exp (0.4) = 1.5, 95% CI (0.97, 2.27)].

Looking at the interaction, it can be seen that it is statistically significant and improves the model. Wald = 6.97,  $p < 0.001$ . We see that the effect of the testing turn in the control group, which expresses the probability of making more or as many schematic drawings as any number of schematic drawings  $k$ , in the second testing turn compared to the first time, becomes 1.82 times larger in the intervention group [Exp (-0.60) = 0.55, 95% CI (0.35, 0.86)]. This means that while the control group is estimated to make an average of 10 times more schematic drawings on the second test, the intervention group has about 18.2 times more.

### RQ3: Relations between composing schematic and pictorial drawings and solving math word problems

Each student had four scores of correct/incorrect answers and types of drawings—in pre- and post-test and for two tasks. We randomly took one score from each student out of all the tasks he/she solved (both in pre- and post-test) to form a random independent selection of tasks, which allowed us to study the relationship between how the drawing was composed and the correctness of the answer. There were 293 assignments from 293 different students. There were proportionally more correct answers than incorrect ones in the tasks solved by the fourth-grade students, and proportionally more incorrect answers than correct ones in the tasks solved by the second-grade students (see Table 4).

In order to investigate the difference between the drawing types and correctness of answers, we used CFA. The results of the Bonferroni-adjusted binomial accuracy test showed that one antitype and one type emerged (see Table 5). The antitype shows that the second grade students completed the tasks correctly without drawing less frequently than expected under independence. The type suggests that in the fourth-grade, more students than expected solved the problem correctly without making a drawing. While real patterns emerged as far as different drawing types are concerned (schematic vs. pictorial) aside from no drawings, the results seem to suggest that fourth-grade students get more correct answers without drawings than second-grade students without

TABLE 5 Configural frequency analysis of drawing type and grade.

Pattern	Frequency	Expected frequency	df	z-value	P-value	Type
201	63	89.34	1	-3.16	<0.001	Antitype
401	84	62.58	1	2.95	=0.002	Type

Patterns: 201, grade 2; no drawing, correct answer; 401, grade 4, no drawing, correct answer.

drawings so it is possible the drawings benefit second-grade students more.

## Discussion

We examined learner-constructed pictorial and schematic drawings that may be used in solving math word problems. These drawings constitute a learning strategy that should be taught to students (Hegarty and Kozhevnikov, 1999; Van Meter and Garner, 2005; Schmidgall et al., 2019; Ainsworth et al., 2020). The aim of this study was to examine what type—pictorial or schematic—of drawings second- and fourth-grade students compose for math word problems and what effect the intervention program “Learning with Understanding” had on the frequency and quality of drawings. We compared the intervention and control groups’ drawings for two-word problems, similar in pre- and post-test and it appeared that during the pre-test, only some students of both groups composed drawings. However, after implementing the program, more the intervention than the control groups’ students composed drawings, including schematic drawings. The relationship between composing drawings and solving the task correctly differed in the second and fourth grades.

### Composing drawings in pre- and post-test

As expected (RQ1; H1a) and found in previous studies with older students (e.g., Van Essen and Hamaker, 1990; De Bock et al., 2003), initially, neither the second- nor the fourth-grade students composed any kind of drawings. After about 6 months, both the intervention group and the control group composed some kind of drawings (either pictorial or schematic). As expected (H1b), the increase in making any kind of drawings is smaller in the control group, with two tasks performed 10.7% of the time in second grade (vs. 23.6% for the intervention group) and 4.9% for the fourth grade (vs. 18.8% for the intervention group). Different reasons may be offered why the frequency of making drawings in the control group also increased. First, students were tested using tablets during the pre-test—a medium that is not widely used for drawing at this age. Thus, students might compose more drawings in the post-test because it is an easier task on paper. Second, the

increase might occur because of normal development supported by teachers. Control group teachers were in classrooms when students were tested by experimenters and thus were able to see that students were asked to compose drawings for math word problems. It could be that students and teachers, who also taught drawing, discussed the test afterward. Still, the increase in drawings was bigger in the intervention group as well as in the second and fourth grades. Specifically, more students in the intervention group in both grades composed drawings consistently—for both problems.

Similar to previous studies showing that young students rarely compose schematic drawings independently and without specific teaching (Hegarty and Kozhevnikov, 1999; Van Garderen and Montague, 2003; Fiorella and Zhang, 2018), only eight students composed schematic drawings for one problem. As expected (RQ2, H2), in the post-test, the intervention group composed schematic drawings more consistently than students in the control group. Namely, 9.1% of the second grade control group students created a schematic drawing for two problems, while in the intervention group, the percentage was 20%. In the fourth grade, the percentages were, respectively, 4.9 and 13.8%. Earlier studies have similarly shown that students start to use more effective learning strategies (creating different models and drawings) only when being taught (De Bock et al., 2003; Van Meter and Stepanik, 2020). Still, control-group students also drew schematic drawings more in the post-test than in the pre-test. The reasons behind the increase may be similar to those we brought up previously regarding both types of drawings. Moreover, modern Estonian school education emphasizes the so-called language of science (cf. Vygotsky, 1997; Kikas, 2003), and students may learn to find different ways of solving math problems, including with the help of schematic drawings. Math textbooks also include schematic drawings, and teachers tend to interpret using these visual aids as a visualization strategy. Today, the importance of constructing drawings besides learning from drawings is acknowledged (Van Meter and Garner, 2005; Schmidgall et al., 2019; Ainsworth et al., 2020). Constructing schematic drawings is a complex task that presumes good knowledge and cognitive skills—students have to select useful information from the text, organize verbal elements, convert it to visual representation that corresponds to the problem and compose drawings (Van Meter and Garner, 2005; Tippett, 2016; Fiorella and Zhang, 2018). Most students need support and practice in learning to construct helpful schemas

(Hegarty and Kozhevnikov, 1999; Terwel et al., 2009; Ainsworth et al., 2020). Also, the specific value of raising metacognitive awareness that is achieved *via* explicit teaching and discussions is emphasized (Dignath et al., 2008; Falomir, 2018; Fiorella and Zhang, 2018). The intervention lessons included teaching, practicing, and discussing schematic drawings and, thus, more intervention group students constructed schematic drawings than control group students.

The current study results for the second grade differ from the previous findings, as the second-grade students also started making more drawings (20% of post-test drawings were schematic and done for both tasks). This finding differs from Van Essen and Hamaker (1990), who found that only fifth-grade students started to draw more after the intervention, but first- and second-grade students did not. The difference may be related to how students were taught composing drawings as a constructive learning strategy—after first general introduction, more specific tasks were used and students could practice the strategy in different subject lessons. One surprising finding was that more second- than fourth-graders in the intervention group made a drawing either for one or both tasks. However, it should be stressed that students were instructed to compose drawings if needed. As the tasks were the same for both age groups, they were somewhat easier for fourth-graders who might not compose the drawing because they did not need it.

## Relations between the type of drawing and correctness of solving math word problems

Lastly, (RQ3), we aimed to examine relations between drawing type and correctness of solving math word problems. We analyzed the tasks of pre- and post-test completed by the intervention and control groups.

We were surprised that the fourth-grade students completed the tasks correctly without drawing more frequently than expected. In contrast, second grade-students completed the tasks correctly without drawing less frequently than expected. Thus, it seems that second-grade students benefit from drawings more than fourth graders. However, we must be cautious with this interpretation. Namely, as the tasks were the same, they were more difficult for second than fourth graders. Respectively, significantly more correct answers were obtained in the fourth grade than in the second grade. As students were instructed to compose a drawing if needed, fourth-graders might not have seen the need for it and solved the problem without drawing. In addition, the task was on the left, and the place for the drawing was on the right. Since reading is done from left to right in our culture, it is likely that the task was noticed and read before and the need for the drawing after that. The usefulness of drawing in problem-solving also depends on

the task, students' prior knowledge, and cognitive processes (Cromley, 2020).

Altogether, few students composed pictorial drawings. Previously, Van Garderen and Montague (2003) found that sixth-grade gifted students use the schematic type the most, while ordinary sixth-grade students or pupils with learning difficulties use it the least. In addition and contrary to our research both pictorial and schematic drawing types were used to the same extent in all groups. It is possible that since there were fewer tasks in our study and it was not explicitly said that the drawing should be made, a large number of students solved the task without using a drawing. However, if there were more tasks, perhaps more pictorial drawings would have been made.

In both grades, more students obtained correct and fewer students incorrect answers with schematic drawings (Table 4). This difference did not reach statistical significance, probably due to the small sample size. This result confirms earlier findings on the advantages of schematic drawings (Hegarty and Kozhevnikov, 1999; Van Garderen and Montague, 2003; Edens and Potter, 2007; Rellensmann et al., 2016). It has also been argued that schematic drawing represents proportional thinking about the relationships between things, while a pictorial drawing represents things that are redundant in solving a task. It is also possible that students solved the problem before and felt that they did not need a drawing, they would not make one or make a pictorial drawing, and if they used the drawing to check the solution, they would make a schematic one. At the same time, Hegarty and Kozhevnikov (1999) indicated that students might be divided into pictorial and schematic drawers, as some vividly imagine the content of a task while others think spatially about the relationships between things.

## Conclusion and practical implication

As in earlier studies (Hegarty and Kozhevnikov, 1999; Van Garderen and Montague, 2003; Fiorella and Zhang, 2018), we found that only a few young second- and fourth-grade students construct schematic drawings independently. After implementation of the intervention program “Learning with Understanding,” more second- and fourth-grade intervention than control group students composed drawings, including schematic drawings. The finding differs from some earlier intervention studies that showed positive effect mainly in older grades. Different aspects of our intervention might contribute to these effects.

First, as earlier classroom studies have shown that teachers rarely explicitly teach and discuss learning strategies (e.g., Dignath and Büttner, 2018; Coffman et al., 2019), we specifically emphasized the importance of these practices. We educated teachers and provided support throughout the program.

Second, the program for students started with broader topics on the learning process, followed by teaching specific strategies, and ended with an overview of all strategies and reflective discussions (Kikas et al., 2021). Ainsworth et al. (2020) showed that learning is more successful when learners actively develop their understanding of what they are learning. Therefore, it is essential to emphasize the metacognitive level of learning in interventions of teaching learning strategies. Others have also emphasized that raising metacognitive awareness may be a key component of success (e.g., Csikos et al., 2012). Constructing drawings was one learning strategy besides two others.

Third, in line with suggestions by Fiorella and Zhang (2018), in the introductory lesson on visualization strategy, students were taught different types of schemas for various problems (using schemas to learn the text, understanding to build a car with Lego). After that, students were taught to create and use drawings in math (cf. with stages brought out by Falomir, 2018). Cromley (2020) has pointed out that a student may not have a strategy for solving a task that allows them to relate it to other information given in the task (e.g., life situations or previous knowledge). This means that it is important for a student not just to learn what strategies there are but also when to use them and which ones are currently suitable for him/her. Learner-centered interventions focus on solving such situations, and their effectiveness is assessed by how well students are able to apply the strategies themselves to new tasks. This is also what we aimed at with the intervention.

We found even second grade students can be taught to construct and use schematic drawings. However, to overcome the challenges (working memory constraints, low content and learning-related knowledge, and metacognitive knowledge and skills) young students meet when composing schemas (Leutner et al., 2009; Rellensmann et al., 2016; Fiorella and Zhang, 2018), the teaching should be explicit and students should have a possibility to practice drawing-constructing in solving different tasks in different lessons. An important condition is a consistent learning environment, where different aids are used to explain, practice, and discuss learning and the construction of drawings. These discussions specifically raise students' metacognitive knowledge (see Csikos et al., 2012). Practicing different strategies in early grades with simple tasks may build confidence that can be used later in their school career when learning becomes more challenging. The education systems in Estonia and other countries, allow class teachers to teach all main subjects facilitating their possibility to discuss and practice new learning strategies in different lessons.

## Limitations and future directions

Several limitations should also be mentioned. First, students were tested on a tablet in the pre-test but on paper in the post-test. Some internet connection problems occurred during

the first testing that might affect students' outcomes. Also, students have rarely drawn with computers and tablets which might be why so few students created drawings. Although the conditions were similar in the intervention and control groups, in future studies, the medium of the test should be the same throughout the study. Second, students knew that they were participating in the study, so it was not a natural classroom condition. Some children might consider testing unimportant and not take it seriously. As the tasks in both tests were similar, students might be even less motivated in the post-test. However, the motivation of students to participate in the test and complete the tasks greatly influences the results of the research. In future studies, students' motivation and persistence when completing the tests should be assessed and taken into account in the analyses. Third, composing drawings was not obligatory (the instruction was "If necessary, compose a drawing that helps you solve the task better"). It was not possible to distinguish between students who were unable to draw, those who felt they did not have to, or those who didn't have the time or motivation. In future studies, composing drawings could be a separate, obligatory task. Fourth, the effects of the intervention were statistically significant but not high. The reason may be the small sample size and, therefore, further studies should be carried out with larger samples. Fifth, teachers' attitudes, knowledge, and motivation to teach learning strategies play an important role in implementing the program. Each teacher might devote more time to teaching drawing in some lessons than in others (e.g., math compared with science). In the future, teachers' characteristics and preferences should also be examined.

We studied only some aspects of students' self-generated drawing—composing drawings for math word problems. Moreover, a problem-solving task and instruction to draw were given together, and drawing was not obligatory. It might be worth investigating whether the results would be different if the drawing were done first and the solution written afterward. Interviews with children would also give valuable information into the student's perspective and whether they perceive the benefits of drawings and whether they would use it to solve the task or for other reasons. Questions like "Did you draw before solving the problem?" "Did the drawing help you solve the problem?" or "Did the drawing help you check the answer to the task?" could clarify our findings.

Future studies are also needed to examine the different effects of the intervention program. The program chose to look into composing drawings in certain subjects. It would be advisable for future studies to examine students' drawings in other areas (e.g., text comprehension, science problems). Moreover, we compared the differences between the intervention and control groups directly after the program practice, although it is important also to detect the long-term effects of intervention.

## Data availability statement

The raw data supporting the conclusions of this article will be made available by the authors, without undue reservation.

## Ethics statement

Ethical review and approval was not required for the study on human participants in accordance with the local legislation and institutional requirements. Written informed consent to participate in this study was provided by the participants' legal guardian/next of kin.

## Author contributions

KT: writing introduction, methodology, and discussion. SP: data analyses and writing method part. EK: conceptualization of intervention, funding acquisition, writing introduction, and discussion. All authors contributed to the article and approved the submitted version.

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## Conflict of interest

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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