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Distance Lesson Study in Mathematics: A Case Study of an Italian High School

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The COVID-19 pandemic put teachers and students in virtual classrooms, necessitating the Lesson Study to be adapted to such a new context. In this paper, we investigate how the roles of teachers, observers, and students are changing, considering the use of Information and Communication Technology (ICT) as the only medium of communication. The research methodology is based on the Documental Approach to Didactics, including Instrumental Genesis. It is a qualitative internal case study. The data was collected through video analysis. The Lesson Study (LS) methods were based on a cyclic model (co-planning, teaching, observation, debriefing, and revision), adapted to Distance Learning (DL). The topic is about Statistics, starting from the COVID-19 diffused data. The experimental work allowed us to observe the inter-connections between LS and DL. We deduced that ICT could be considered as a resource for the documental genesis, which generates processes aimed at social knowledge mediated by the teacher. We supposed that the observing phase was less invasive than during virtual experimentation. Nevertheless, the observers had difficulties understanding some students' and teachers' emotional aspects and analyzing the group activities. The technological resources were an added value for the LS. We present an LS methodology, entirely carried out through DL. The main value consists of showing both affordability and constraints of the DL with respect to the LS and vice versa.

INTRODUCTION

Since March 2020, because of the COVID-19 pandemic and pandemic-induced lockdowns, schools and universities have been suddenly closed in many countries. People have been forced to remain indoors, being deeply impacted on emotional and physical levels.

Teachers, students, and researchers have been facing new educational challenges in learning and educational research. Teachers have become task designers (Rocha et al., 2020). Social distancing has led to redefining some teaching areas, specifically the space-time organization of lessons and educational ethics. From this point of view, changes in teaching and educational design were necessary because of distance learning (DL) (Porter, 1997), and as recommended by UNESCO (UNESCO, 2020). It became necessary to plan *in situ* to consider the context variables, understand, and propose methodologies and educational activities that were more appropriate for students during the emergency. On the emotional, creative, narrative, and communicative levels, the challenge was to organize a learning activity that could generate a situated and socially accepted

educationa practice. Information and communication were the only tasks that were carried out on the Internet. A complete metamorphosis of thinking and acting in education was required to achieve new balances between static and dynamic resources, using and designing teaching resources, and individual and collective work (Pepin et al., 2017). To address these challenges, the Documentational Approach to Didactics (DAD) (Trouche et al., 2020) suggested a change in paradigm by analyzing teachers' work through the lens of resources "in" and "for" teaching, and what the teachers prepare for (supporting) their classrooms.

In this context, the authors experimented with how the Lesson Study (LS) may fit the DL methodology, thus experimenting with a Distance Lesson Study (DLS). The DAD was used as a magnifying glass to analyze the research data. The experimented LS was used as training for the teachers, and all the research analysis was focused on the teachers' perspective.

During the current academic year (2021–2022), a Lesson Study experimentation was started in a blended mode. Five in-service high school teachers and three mathematics education researchers at University of Salerno were converted to a distance mode. This study is an internal case study based on qualitative research, with the empirical data collected through video analysis.

This paper shows a Distance LS, entirely carried out using DL. The activities related to LS were modified, both in the co-planning and in the "virtual classroom" experimentation phases. The teachers were involved in time re-scheduling, content modification, and the production of new documental resources. The resources sharing in a peer-to-peer mode between teachers, through technological instruments, allowed them to feel less isolated in this new educational challenge.

In this emergency period, statistical data regarding the pandemic has been shared daily on mass media. For this reason, we decided that statistics applied to reality as the topic for the LS regarding the understanding and correct use of these interesting statistical data.

As for the conceptual framework used to analyze this case study, we used the DAD framework, the instrumental approach proposed by Rabardel (e.g., Vérillon and Rabardel, 1995) in cognitive ergonomics, and then integrated it into mathematics education (Guin et al., 2005) and the socio-constructivist ideas of Vygotsky (1978).

We investigated how the roles of teachers, observers, and students have adapted due to the logistical restrictions with ICT as the only communications medium (Choppin et al., 2020).

Specifically, we focused on the following key research questions (Research Questions, RQ):

RQ1: What are the main features of this Distance Lesson Study?
RQ2: How does Distance LS support teachers learning from DAD's perspective?
RQ3: Which material, cognitive and social resources favored Distance LS?

GENERAL REVIEW

From the literature on Distance Learning and Lesson Study, we deduced the main features of both these learning/teaching methodologies. Therefore, we planned and experimented with a Distance Lesson Study (DLS), including all the features and preserving the essential elements of the Lesson Study. This research serves as a bridge between two worlds that are pretty distant from each other: Distance Learning (DL) and Lesson Study (LS). To the best of our knowledge, LS in DL mode has not been planned and experimented in Italy, up to now.

Distance Learning

Distance Learning is an educational methodology based on a teacher-learner interaction that is not face-to-face (de Carvalho Borba and Llinares, 2012). With the pandemic, it became the only possible contact between teachers and students since all other possible forms of "closeness" was restricted due to social distancing.

Ever since one of the earliest attempts in 1728—as advertised in the Boston Gazette for "Caleb Philipps, Teacher of the new method of Short Hand", who sought students wanting to learn through weekly mailed lessons (Holmberg, 2005)—distance learning has had significant progress, mainly thanks to the web and proliferation of information and communication technologies. Nowadays, many courses are delivered worldwide through distance learning, and entire degree courses may be attended in this way. Some institutions offer customized self-paced programs, where the time taken to complete the course is determined by the students' own time, skills, and commitment levels.

Courses may be taken in Asynchronous Online Learning (AOL) and Synchronous Online Learning (SOL) modes, depending on the student's needs. AOL modes organize learning activities on web-based environments that provide communication between the class members, and the interaction with the contents in times and places that are flexible, and according to the convenience of each individual. Technologies that enable AOL include email, text messages, blogs, online discussions, wikis, streaming media, or a digital content management system. SOL modes are learning activities in a web-based environment that allow real-time communication between class members and interaction with the contents. Technologies that enable SOL include text, audio, video-based chat, video conferencing, simultaneous presentations from physically distributed sites, and application sharing. In this experimentation, we used the SOL mode. It was preferred because it facilitates better communication between teachers and students, thus promoting learning/teaching in a closer manner.

In the last 2 years, many scholars have analyzed the pandemic situation from many points of view. Among them, many have studied the conditions of students and teachers from a teaching, social and psychological point of view (e.g., Borba, 2021; Cassibba et al., 2021; Yilmaz et al., 2021). Bakker et al. (2021) wondered if the pandemic changed the perspectives on the themes like mathematics education research. According to the

authors, the pandemic worked as a lens on the already important issues of social and educational problems. Some authors (Siregar and Siagian, 2021) evaluated the online learning of mathematics, highlighting the change in its learning method. For example, they designed new teaching approaches and orienting mathematics learning toward solving social problems such as species extinction, climate change, and building a sustainable future. They showed the usefulness of online communication media such as WhatsApp and the lack of interaction between instructors and students. In several countries worldwide, authors investigated the distance learning process of teaching during school closure. Some authors highlighted difficulties due to economic factors or teachers' inability to adapt to a sudden change in their teaching environment or problems due to limited access to the internet, such as the case of Indonesia. Others have highlighted different approaches to organizing distance learning during the pandemic. Few others studied how to use resources and undertake laboratory activities in distance learning. A special issue (Chan et al., 2021) by *Educational Studies in Mathematics*, collated voices from researchers internationally on mathematics education, particularly teaching mathematics during the pandemic. Leveraging data from a mathematics learning software as a substitute assessment, Rutherford et al. (2021) found that students had a lower engagement with the software during the pandemic. Still, students who did engage showed increased performance. Students also experienced changes in motivation with lowered mathematics expectancy and lower emotional cost for mathematics. Results illustrate the potential and pitfalls of using educational technology data instead of traditional assessments and draw attention to access and motivation during at-home schooling. So, there are indeed many authors who have written in this past year on mathematics and distant learning. Ours is intended to be the voice of researchers who, like others, have had to reorganize their teaching by reporting a case study very similar to many other situations in the world. In the authors' previous works, the importance of using information technologies in the educational dialogue to prevent drop out and improving students' engagement, motivation, and participation linked to the Situation Awareness (SA) model has been highlighted (Branchetti et al., 2021; Capone and Lepore, 2021; Capone, 2022a,b). They investigated the possible effects of the long-term use of distance learning as the only means of teaching in an emergency, in addition to other social factors like students' motivation, engagement, and participation.

Lesson Study

The Japanese 授業研究(*Jugyokenkyu*), known to the world as Lesson Study (Yoshida, 1999), originated in Japan around 1870 as an answer to professional development needs for qualified teachers (Isoda et al., 2007).

Working in a small group, teachers collaborate to study content instruction, meet to discuss learning goals, plan an actual classroom lesson (called a "research lesson"), observe how their ideas work in live lessons with students, and then report on the results so that other teachers can benefit from it to improving mathematics teaching and learning.

In **Figure 1**, we show the Lesson Study generic process outline as Buchard and Martin (2017) proposed, including main features (recruit participants, choose a suitable lesson, lesson planning, research lesson, discussion, and sharing).

Following the Third International Mathematics and Science Study (TIMSS) Video Study (Stigler et al., 1999), Lesson Study gained both educators' and researchers' interest worldwide, mainly due to the policies and practices that promote collaboration and cooperation between teachers. Lesson Study was introduced in the United States following Catherine Lewis' research (Lewis, 2000) and Makoto Yoshida's doctoral thesis (Yoshida, 1999), creating an Anglo-American tradition. It has also been introduced in the Netherlands, Italy, Spain, Portugal, Switzerland, Great Britain, Indonesia, Malaysia, South Africa, Australia, Canada, Hong Kong, Sweden, Thailand, Vietnam as well as select countries in South America from where research groups participate in the conferences organized by the World Association of Lesson Studies (WALS). In February 2020, the International Commission on Mathematical Instruction (ICMI) Study congress recently focused on "Teachers of Mathematics working and learning in collaborative groups". The underlying assumption in this ICMI Study is that teachers learn through collaboration; however, it can be challenging to investigate and explain how this learning occurs and gather evidence of what and how teachers learn. In the Lesson Study context, several researchers worldwide have identified changes in teachers' beliefs or disposition toward working and learning, their mathematics knowledge for teaching, and their teaching practices (Xu and Pedder, 2015; Huang and Shimizu, 2016).

In Italy, the Lesson Study has been experimented with in different ways, mainly with students aged 6–12 (Bartolini Bussi et al., 2017; Ribeiro et al., 2019), with prospective secondary school mathematics teachers (Robutti et al., 2016), and with in-service high school teachers (Capone et al., 2020; Capone, 2022a). Specifically, in the Italian experience, one of the core elements of the LS, the Lesson Plan (Bartolini Bussi et al., 2019), where all the activities are scheduled, was modified, including a detailed description of the class details. In Italy, cultural background plays an important role, emphasizing where the teaching-learning processes occur and the teachers work (Mellone et al., 2019), with growing attention to cultural beliefs (Bartolini Bussi et al., 2019).

The Lesson Study experiment described in this paper is in continuity with other Italian studies and the documents presented by the authors at WALS, 2019, the Second International Advisory Committee on Mathematics Education (ACME) Symposium on Mathematics Education, 2020, and ICMI Study 25 (ACME, 2020). To plan our Distance Lesson Study, specifically, organize the Lesson Plan, we started with the research of Bartolini Bussi et al. (2019), Mellone et al. (2019), and Ribeiro et al. (2019). Our experimentation was carried out at a high school and the Lesson Plan was modified to include the distance learning needs and the resources, agreeing with the conceptual framework (DAD), which is described in the following section.
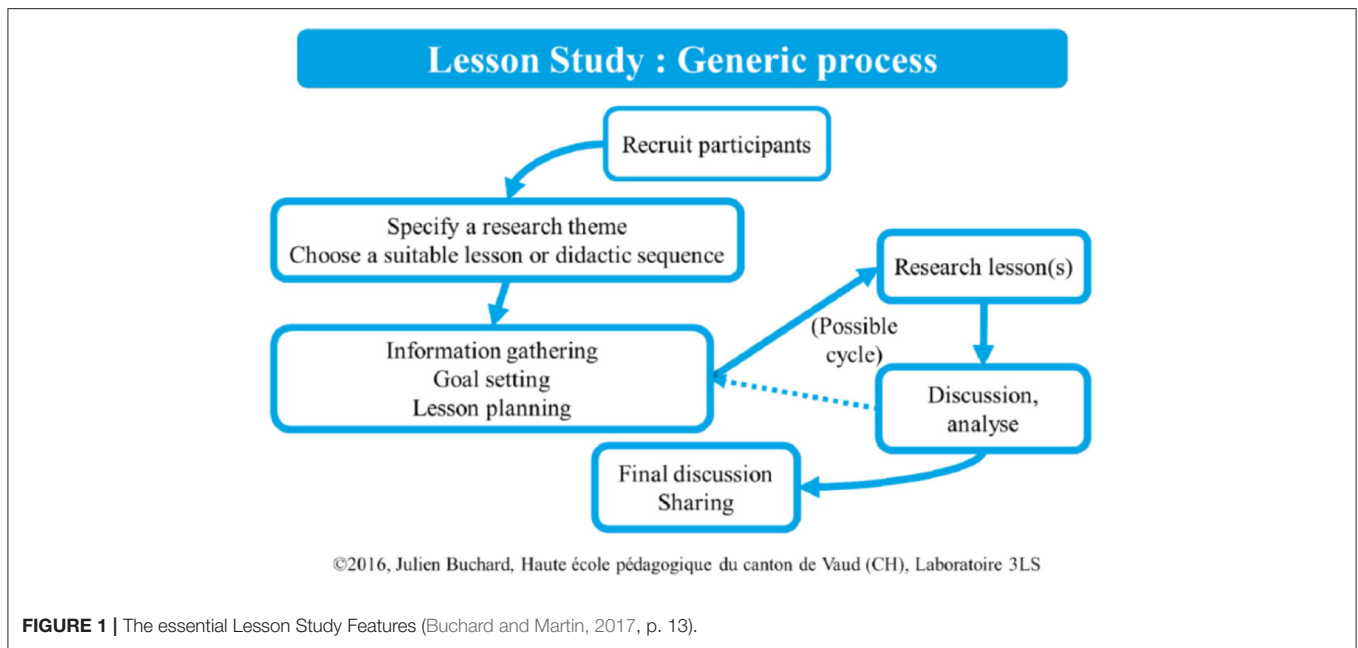
FIGURE 1 | The essential Lesson Study Features (Buchard and Martin, 2017, p. 13).

Many scholars (e.g., Aykan and Yildirim, 2022) investigated the integration of the Lesson Study Model into distance education during the COVID-19 pandemic, focusing on several dimensions: Lesson Study in distance learning, lesson planning processes, challenges of lesson planning, evaluation, and assessment methods, new strategies, methods, and techniques. Huang et al. (2021) in their article, "Digital technologies, online learning and lesson study," show how various technologies could be used to modify and strengthen lesson study. These studies provide evidence of ways in which technology has been used to support teachers and researchers in carrying out LS in virtual and blended environments. Some adopted effective features of traditional LS have been found to be effective in online LS while challenges of using technology are considered. For example, they provided a promising solution using a Swivl robot and multiple iPads to simultaneously video record instructions for the entire classroom and their group activities. The video was then uploaded to Swivl Cloud for the facilitator to annotate with prompt questions, allowing LS members to review and comment before debriefing. Hence, technology allowed teachers to watch the activities of all the groups to learn what happened when the teacher was not in that group in class and repeatedly watch specific clips that the facilitator annotated with questions. According to Huang et al. (2021), conducting online LS effectively to support teachers' learning to teach in such a novel virtual environment is an unprecedented endeavor for educators and researchers.

CONCEPTUAL FRAMEWORK

The used conceptual framework was adapted from the idea of Rabardel on instrumental genesis (Rabardel, 1995) and was then customized for mathematics education (Guin et al.,
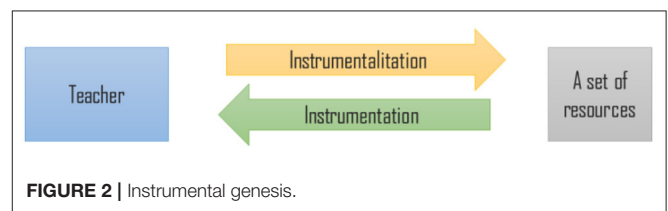
FIGURE 2 | Instrumental genesis.

2005). This process works in both ways: the affordances of the resources influence the teachers' practice instrumentation process, as the teachers' dispositions and knowledge guide the choices and transformation processes between different resources (the instrumentalization process). The teachers develop their particular "schemes" of using these resources, thus obtaining and instrumenting (see **Figure 2**).

In this paper, "scheme" is defined according to Vergnaud's definition: "A *scheme* is a stable organization of the activity of a given subject for a given aim. This stability is reached along various situations corresponding to the same aim: a teacher meeting a new situation (characterized by a new aim) can mobilize and adjust existing schemes or can develop a new scheme." (Vergnaud, 1998).

Starting from the general idea of instrumental genesis, Gueudet and Trouche (2009), together with Birgit Pepin (Gueudet et al., 2012) developed a theoretical framework, the Documentational Approach to Didactics (DAD), where some resources become documents, with renewed schemes of usage:

Resources + schemes of usage = document

In the instrumental genesis, it is possible to distinguish between artifact, which is available for use, and instrument, developed by the teachers/students, thanks to specific schemes of usage.

Instead, in the DAD, the idea of instrumental genesis (Rabardel, 1995) has been used to argue the documental genesis, i.e., a development process of a document (Gueudet and Trouche, 2009). A document developed from a set of resources and provides new resources. Therefore, there is a dialectic nature in the interaction between the teacher and resources in the DAD processes, combining instrumentation and instrumentalization (Vérillon and Rabardel, 1995), thus producing a document.

It relies on Vygotskyan's socio-cultural theory, focusing on mediation in the cognitive processes (Vygotsky, 1978).

Recently (Trouche et al., 2020), it has been updated due to the emerging digitalization of information and communication. Because of the spread of mass media and ICT in learning/teaching processes, there was a further need to design new resources for teaching. Authors agree with Adler (2000) idea of "thinking about a resource like suggests the verb' resource', to the source again or differently". Resources can feed and renew the teacher's activity (Remillard, 2005) and support their commitment to teaching. It is possible to distinguish three different types of resources: material curriculum resources (e.g., textbooks, digital curriculum resources, manipulable objects, and calculators), social resources (e.g., a conversation in a forum), and cognitive resources (e.g., pictures and/or theoretical tools used to work with teachers). In our experimentation, the available resources were organized, taking into account distance learning in SOL mode.

A helpful tool in teaching practice is the Schematic Representation of a Teacher's Resources System (Pepin et al., 2017), as modified by Rocha (2018) with more emphasis on the reflective phase: Reflective Mapping of Teacher Resource System (RMRS). Teachers draw a map of their resources, highlighting the resources they retain more appropriately, including the eventually used databases and their purposes.

Therefore, in our DLS experimentation, Documentational Approach to Didactics (DAD) has been used as the conceptual framework in order to understand teachers' professional development by studying their interactions with the resources they design, organize, use and share in/for their teaching, thus producing effective documents.

MODEL AND PROCESS OF THE DISTANCE LESSON STUDY

Figure 3 shows the methodology we used to design the Distance Lesson Study. It is a cyclic model.

The LS didactic cycle includes the co-planning, teaching, observation, debriefing, and revision phases.

All these phases have been re-designed and customized to distance learning.

In the co-planning phase, the teachers complete and share their Reflective Mapping of Teacher Resource System (RMRS), highlighting the resources they retain more appropriately. By analyzing the RMRS of each teacher, they choose the material, social and cognitive resources to be used during the teaching phase. These resources are included in the Lesson Plan (LP).
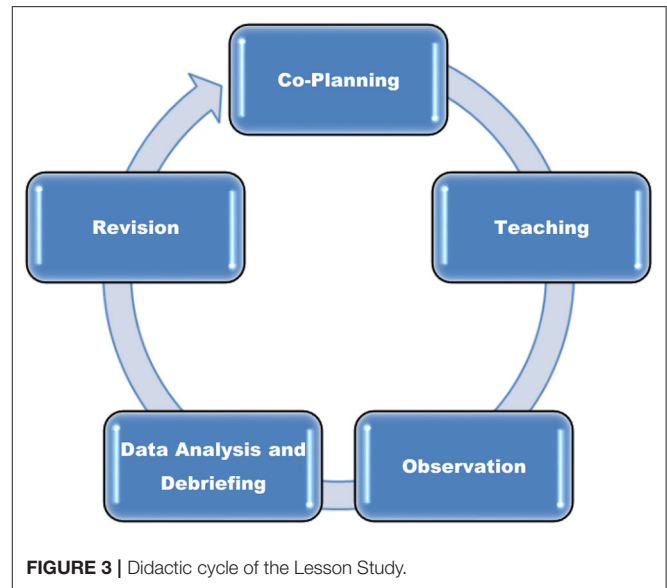
FIGURE 3 | Didactic cycle of the Lesson Study.

The LP is the main document to be organized in the co-planning phase. It includes a detailed description of the classroom, the prerequisite analysis, the motivated choice of the main topic, and how this lesson is linked to the learning unit. Moreover, in the LP, all the activities to be carried out in one hour of the lesson are organized, specifying a brief description, the working mode, the time for each activity, and the resources. The LP also includes observation indicators from which an observational grid is developed by the working group in this phase. This grid is used by the observers (teachers/researchers) who attend the lesson. The same grid is to be filled out by teachers/researchers who are not physically present but share their analysis after viewing the video.

This phase is entirely on the SOL mode, by using software on the cloud to share the documents and the resources.

After the co-planning phase, a teacher designated as the pilot teacher, in the SOL mode too, has to carry out the teaching phase by using video conferencing and sharing resources; the lesson must be video-recorded, and the video has to be shared with the working group.

A group of observers (including both researchers and teachers) must attend the video conferencing, and fill out the observational grid. Further, researchers/teachers who did not attend the teaching phase have to see the video and fill the same observational grid.

From the video analysis and the discussion based on the observational grids, a debriefing phase is carried out, also on the SOL mode. The affordance and constraints of the experimentation are discussed to assist with the revision of the LS. From this revision phase, a new cycle of the LS may begin again with an LP which has to be modified to overcome the constraints of the previous experimentation.

METHODS

Setting and Participants

The Lesson Study was carried out with 21 participants including 15 tenth-grade students, 5 teachers, and 3 researchers. Teachers and researchers were part of the working group. A volunteer teacher was designated as the pilot teacher (*Maria*). She had a Ph.D. in Physics and taught mathematics and physics for 20 years. Moreover, she was already involved in a previous LS experimentation and was good in ICT. Before the LS experimentation, almost all students had achieved the prerequisites identified by the working group. There were no students with special educative needs.

We carried out three LS cycles. However, in this study, we focus on the results of the first LS cycle undertaken during the lockdown (April 2020). The second cycle was during the second lockdown in Italy (March 2021). The third one (November 2021) was carried out in a different external condition. when classes were no longer in the distance mode; nevertheless, all the digital resources were included in the learning process. The same workgroup continued for all three cycles, but there was a different pilot teacher for each cycle.

In the first cycle, the teaching phase was carried out by the pilot teacher (T1), and it was observed by two teachers (T2, T3) and one researcher (R1). Two more teachers (T4, T5) and two researchers (R2, R3) analyzed the video recording and attended the debriefing phase.

The Process of the Project

The co-planning phase was carried out in SOL mode: three meetings, once a week, lasting about an hour (57', 53', 1 h 05', respectively), and the video was recorded by the pilot teacher. The used material resources were Google Meet, Google Classroom, and the Office OneNote software.

During the first meeting, the working group focused on the choice of the LS topic.

During the lockdown period, students were overwhelmed every day by statistical graphs showing data about their reality. The group agreed that the main topic should be Statistics to base the LS on concrete problems.

A document was produced from the discussion about the topic (**Figure 4**), including this subject choice's main needs and educational proposals.

First of all, statistics was chosen to understand and correctly use a large amount of real data about COVID-19 (a real problem, in red, and COVID-19, in purple, in **Figure 4**).

The debate about the role of statistics in the mathematics curriculum is still open. Moreover, it is a curricular topic (mathematics curriculum, in green, in **Figure 4**). As suggested by Batanero and Díaz (2010), reasons for including statistics at the school level were repeatedly highlighted over the last decades of the past century (e.g., Hawkins et al., 2014). For example, the usefulness of statistics for daily life, the important role of statistics in developing critical reasoning, and the instrumental role of statistics in other disciplines and many professions.
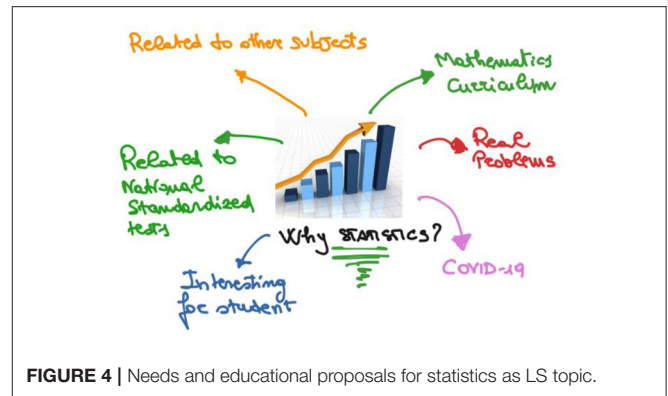
FIGURE 4 | Needs and educational proposals for statistics as LS topic.

Statistics is also included in the national standardization test (Invalsi) and the international OCSE-PISA in the Data and Previsions nucleus, and Uncertainty and Data, respectively.

Finally, all the teachers agreed that it is an exciting topic for the students and effective for analyzing concrete problems. Also, it could be used effectively to plan and experiment with some interdisciplinary learning units (related to other subjects, in orange in **Figure 4**), too (Capone et al., 2021; Capone, 2022a,b).

During the second meeting of the co-planning phase, teachers contributed to designing a learning unit about statistics with their knowledge and cognitive resources. Each teacher was also invited to create a Reflective Mapping of Teacher Resource System map. The first map they produced was without a specific scheme of usage. After comparing the maps followed by a dialectic discussion between teachers and researchers, they decided to organize the RMRS by categorizing the resources into material, social and cognitive resources and, within each category, whether it was a digital or "no-digital" resource. **Figure 5** shows the pilot teacher's RMRS before the discussion and the map she organized by the specific scheme of usage. The produced document was useful for research and experimentation. The resources were included in the Lesson Plan as organized by using this scheme.

Moreover, this document was also used as a resource during the second LS cycle. All the teachers retained their RMRS organized in the first cycle and just modified some resources. **Figure 6** shows the pilot's RMRS produced in the second cycle. It is an example of the "nesting process": a document created in the first LS cycle becomes a resource in the second one.

The teachers then analyzed the material and social resources, which are all digital (Trouche et al., 2020), and categorized them as:

- Applications allowing teachers and students to communicate and share resources: social cloud platform (Padlet), social educational platform (Edmodo), and social networks (WhatsApp, Messenger).
- Institutional applications (Google Suite for Education on the school's web domain) allowing teachers to communicate with students and collect students' and teachers' production, organizing a documents repository.
- Applications allowing students to operate in mathematics and precisely graph some statistical data (Excel, Numbers).
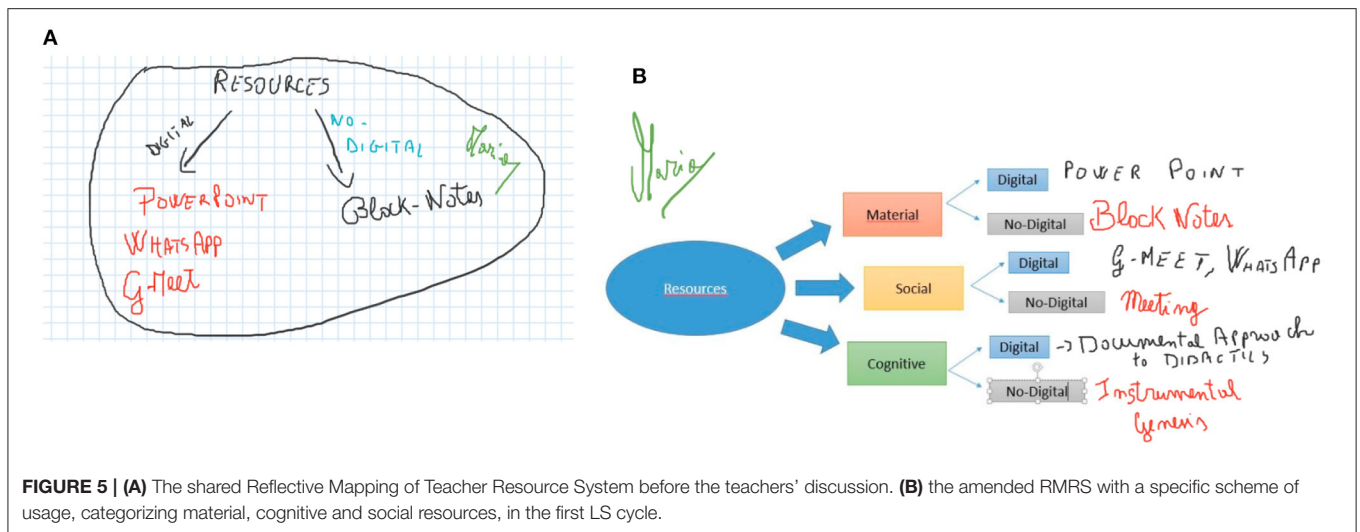
FIGURE 5 | (A) The shared Reflective Mapping of Teacher Resource System before the teachers' discussion. **(B)** the amended RMRS with a specific scheme of usage, categorizing material, cognitive and social resources, in the first LS cycle.
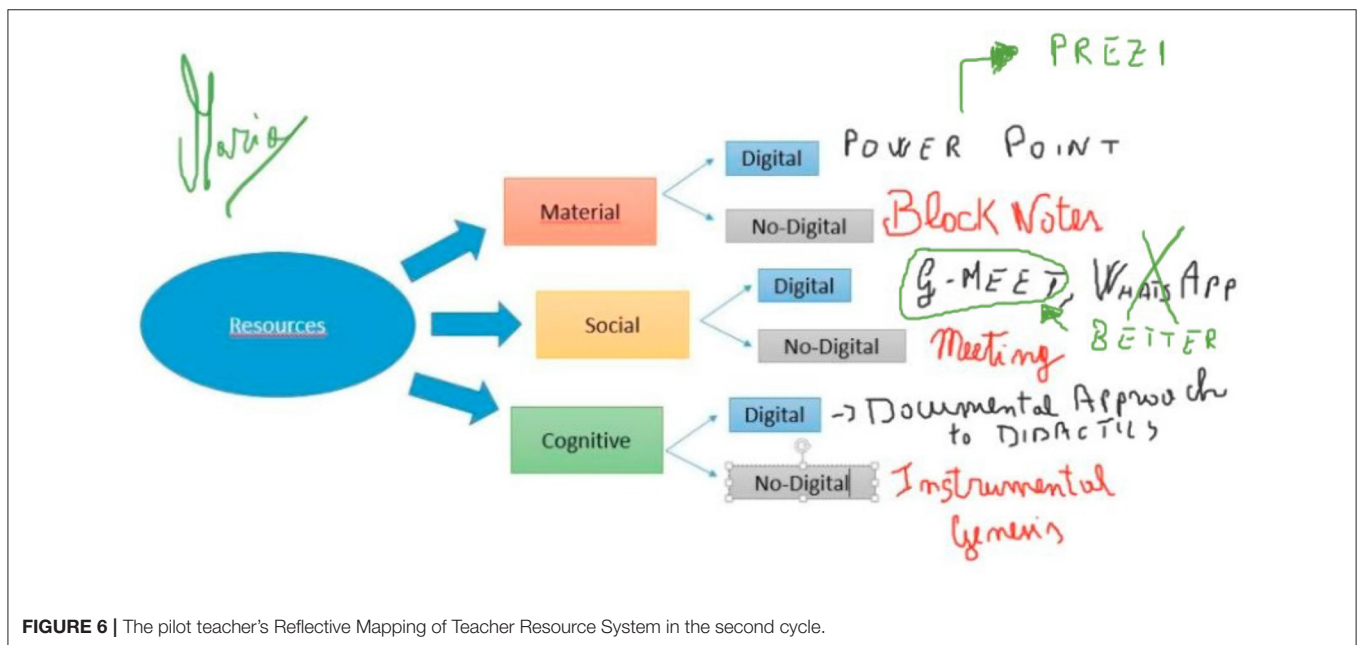
FIGURE 6 | The pilot teacher's Reflective Mapping of Teacher Resource System in the second cycle.

- Web resources to collect data (Youtube, Online Newspapers, TV on demand).
- Software to better organize the communication (Office Suite, Google Suite).

The third meeting of the co-planning phase focused on drafting the Lesson Plan and the observational grid.

The organization of the lesson plan (LP) started with the "Lesson Plan in [BUSSI]" resource. The authors modified it, taking into account the main characteristics of DL and DAD features. The main novelty of this LP was that it was organized in the Lesson Study model, experimented in the presence mode (Capone et al., 2020), and included a "Resources" column: the DAD framework suggests how to include the resources in the LP, specifying the material, social and cognitive ones.

Table 1 shows the first cycle of the LP document.

The lesson plan has more effort in the distance learning mode because a lesson that is not in real-time could create some problems in the virtual classroom's management.

At the end of the Lesson Study cycle, each teacher/researcher suggests further modifications to the Lesson Plan document, e.g., changing the material resources (one software is preferable to others) or even modifying the activity/time management. This modified document becomes a resource for the next Lesson Study cycle, which can be modified by the teachers themselves (and thus transformed into a new document), adapting it to new teaching needs, e.g., to a class with a different profile.

For example, in **Table 2**, a row of the LP as organized in the second cycle is also shown.

By comparing **Table 1** and **Table 2**, we may observe that the teachers, in the second cycle, preferred to add a column: group,

TABLE 1 | Lesson plan of the first Lesson Study cycle.

Activities	A brief description of the activity	Time (min)	Material curriculum resources	Social resources	Cognitive resources	Observation (Who, What, How)
			Resources			
Introducing the lesson and presenting the topic	"Type of graphs which may be used in statistics", multimedia material provided and discussed by the teacher. Attached to Google Classroom	12'	PowerPoint	Google Suite: GMeet and Classroom	Instrumental genesis	**What to look at? About students** - Aware of use of resources and production of documents - Willingness to learn - Reliability and sense of responsibility - Language
Daily Problem formulation	The daily problem is "Pick a COVID-19 statistical survey, look for statistics charts on the web or in newspapers related to the chosen survey and critically analyze them".	2'	Web, Google Suite: Documents or Office Word	Google Suite: GMeet and Classroom	Distance learning	**What behavioral indicators?** - He/she is interested in digital technologies and their use. - He/she is able to use mathematical thought models - He/she is able to develop and apply mathematical thinking and understanding to analyze daily situations. - He/she manages time and information He/she uses the specific discipline language
Daily Problem: presentation and clarifications	In SOL mode, via GMeet, the teacher shows some sample diagrams from online newspapers and/or scientific journals.	3'	Web, online newspapers/ journals	Google Suite GMeet	Documental genesis	
Problem activities	The grouped students work on the daily problem. At the end of the activity, the group will have to produce a document published on Classroom, to be shared with everyone.	20'	Online newspapers TV on demand Youtube Excel • Numbers	Google Suite: GMeet, Classroom WhatsApp Messenger	Documental genesis	**How?** - Observational grid - Video recorded lesson **About Teacher** - Link between teacher and resources - Communication via social (clarity of deliveries) - Scaffolding and Coaching (how it is present in students' activities) - Interaction with the class - Times management
Presentation of the Daily Problem results by students	Three minutes are given for each group to present the problem's solution to their classmates through GMeet. (7 Groups of 3 students)	21'	Office Suite: Word PowerPoint Google Suite: Documents, Presentation	Google Suite: GMeet and Classroom	Debate	
Homework	Starting from the statistical data analyzed within the group or from any document produced by groups during the Lesson Study, create an appropriate graph using the same statistical data	2'	Google Suite: Documents Or Office Suite: Word	Google Suite: GMeet and Classroom	Peer to peer, in the Vygotskijan sense	**What behavioral indicators?** - He/she is aware of the resources selected - He/she provides schemes of usage to students for a conscious use of resources - He/she provides students with all the essential features for delivery. - He/she takes action in students' activities in an appropriate way - He/she takes action in student presentations in a non-invasive way - He/she manages his/her own and students' times **How?** - Observational grid

TABLE 2 | The lesson plan schedule for each activity in the second Lesson Study cycle.

Activities	A brief description of the activity	Groups	Time (min)	Resources			Observation (Who, What, How)
				Material curriculum resources	Social resources	Cognitive resources	
Problem activities	The grouped students work on the daily problem. At the end of the activity, the group will have to produce a document published on Classroom, to be shared with everyone.	Max 3 students	18'	Online newspapers TV on demand Youtube Sitography	Google Suite: GMeet, Classroom	Documental genesis	

where the number of students for each group has to be added. Also, the time, material, and social resources have been modified.

In the observational grid, the focus was on the teacher's behavior (**Table 3**). Specifically, the communication, interaction with the class, and time management were observed in the Lesson Study in the presence mode. In the distance learning mode, a resource section was added to the observational grid.

After the co-planning phase, the pilot teacher carried out the teaching phase. The schedule was maintained; the students took 18 min for the activities on the daily problem and 25 min for the discussions, thus the lesson resulted in 1 h 02 min.

The 1 h of the lesson had been subdivided into some activities, starting with the introduction to the specific topic, continuing with the daily problem, students' activities about it, and the online presentation of students' results, ending with the homework to be shared on the Google Suite Apps, within 1 week. In the introduction to the daily topic, the teacher shared with the students some multimedia resources, organized by herself in agreement with the working group, on the types of graphs used in statistics. Then, she introduced the daily problem: "Pick a COVID-19 statistical survey, look for statistics charts on the web or in newspapers related to the chosen survey and critically analyze them". She surfed the net herself and shared real-time data on the COVID-19 emergency with students to clarify the problem.

According to alphabetical order, seven groups were formed with three students each. The students preferred social networks (WhatsApp) for immediate communication within their group.
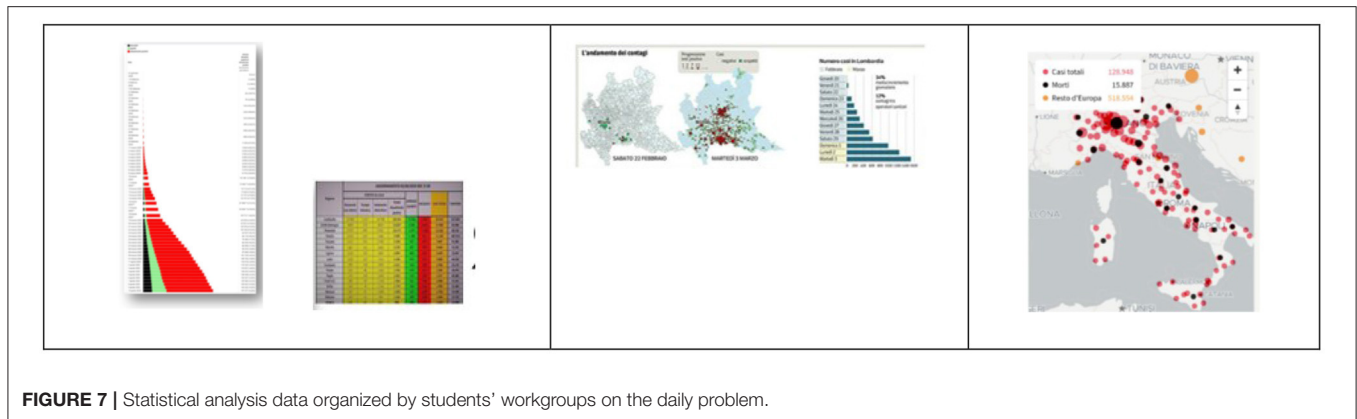
Each group produced a document in the format that they felt most appropriate, showing the data searched for, the data analysis, and their considerations. Some groups also reworked the data, thus producing a graph themselves.

At the end of the activities on the daily problem, all the produced documents were shared on the institutional platform. One member for each group was invited to share their document on the screen, always in SOL mode, and discuss it, emphasizing the web search strategies for the data. The groups focused on different statistical surveys: the percentage of sick/dead/recovered in a specific Italian region/in the entire country/in a comparison between countries; the number of barriers; an economic analysis; a criticism of the virulence of the infection, and so on. They always showed the source of the analyzed data. They properly labeled the type of graph, and, sometimes, they reported the procedures they used to analyze

TABLE 3 | Observational grid.

Categories	Teacher behavioral indicators	Observer answers
Communication	✓ Do they provide students with all the essential elements for delivery? ✓ Do they provide explanations to the students during all the activities?	
Interaction with the class	✓ Do they use gestures to support the delivery's understanding? ✓ Do they use gestures as teaching support?	
Class management	✓ Do they participate in the student's activities? ✓ Do they intervene in students' slideshow?	
Time management	✓ Do they manage the discussion times? ✓ Do they act in delivery times?	
Use of the resources	✓ Did they use all the planned resources? ✓ Material Resources ✓ Social Resources ✓ Cognitive Resources ✓ Did they provide appropriate schemes of usage so that students may produce documents starting from the resources?	

FIGURE 7 | Statistical analysis data organized by students' workgroups on the daily problem.

the data. An example is shown in **Figure 7**, where different statistical analysis data are shown, along with data downloaded from different online newspapers and websites. The first image shows a graph (block chart) and a double-entry table taken from a national newspaper in which the cases of coronavirus in Italy in March and April 2020 are highlighted. The second image shows the evolution of infections in Lombardy in February and March 2020 through a cartogram and a block chart. The third image is a cartogram of aggregated infections by province and deaths by region.

The lesson ended with the individual homework: process the data to produce a statistical graph with the digital resources they considered most appropriate.

An example of the homework is shown in **Figure 8**.

Specifically, the student analyzed the infections in the Lombardy provinces from 25 February to 13 April 2020. The graphs were linear but three-dimensional, thus showing three different variables (data, region, and number of infected). The chart was created by using the Numbers software.

The pilot teacher videotaped the lesson, and the video was shared with the whole working group on Google Classroom.

The observing phase was carried out by three observers (1 researcher and two teachers) attending the Google Meeting in SOL. The rest of the working group analyzed the lesson that was video-recorded in AOL. All members of the working group filled out the observational grid relating to both students and teachers. These forms were shared on Google Classroom as a documental resource.

The video-recording using Google Meet allows the choice of layouts. Here, the teacher chose the grid view to monitor the participants thoroughly. However, this layout could not focus on the teacher. This observation was not as complete as in a face-to-face mode. Therefore, the lack of some observation details, mainly with respect to the analysis of the gestures, impeded the filling of the detailed observational grid. This represented one of the constraints of LS in distance learning mode.

Nevertheless, an explicit advantage of this observation mode is that the students felt comfortable with the virtual presence of the observers. It also enhanced the class and time management. In the debriefing phase (one meeting, 1 h 30') that followed, the working group met, again in SOL mode, to analyze the produced

documents to highlight both the advantages and constraints of this experimentation and to re-design future cycles of LS. In the debriefing phase, a discussion about the teaching phase was also carried out. It was observed that the lesson was on time and minor modifications regarding the time schedule for each activity were suggested to improve it. The attention and the discussion were focused on the importance of the resources used. Both material and social resources were combined in the digital resources, which had a cognitive function, but they facilitated instrumental and documental genesis.

On the topic, and specifically relating to the daily problem, researcher R2, who was absent during the teaching phase but had analyzed the video recording, observed that the use of statistical data immediately caught the students' attention, especially when it was introduced as a tool to understand what was happening around them. Hence, it was a contextualized and well-situated activity.

Data Collection

During the entire process, we collected the following data:

Video recording: 57', 53', 1 h 05' (in the co-planning phase); 1 h 02' (teaching phase); 1 h 30' (debriefing).

Documents: one map on "Needs and educational proposals for the Statistics as LS topic"; RMRS produced by teachers; one PPT for introducing statistical graphs, created by the teachers; two Word documents, to introduce the daily problem and assign homework; seven student working groups documents (two PowerPoints, three Word documents, and two Google Documents), 21 pdf files of homework, and one debriefing document. These documents were published in a free access repository (Google Drive). All of them are available for teachers and researchers, whereas students have access is restricted to students' working group reports and homework.

This collected data formed the elements for a qualitative analysis and helped us answer the research questions through the theoretical frameworks.

DATA ANALYSIS METHODOLOGY

The analysis process commenced from the resources of each teacher. In fact, for each teacher, in agreement with (Trouche,
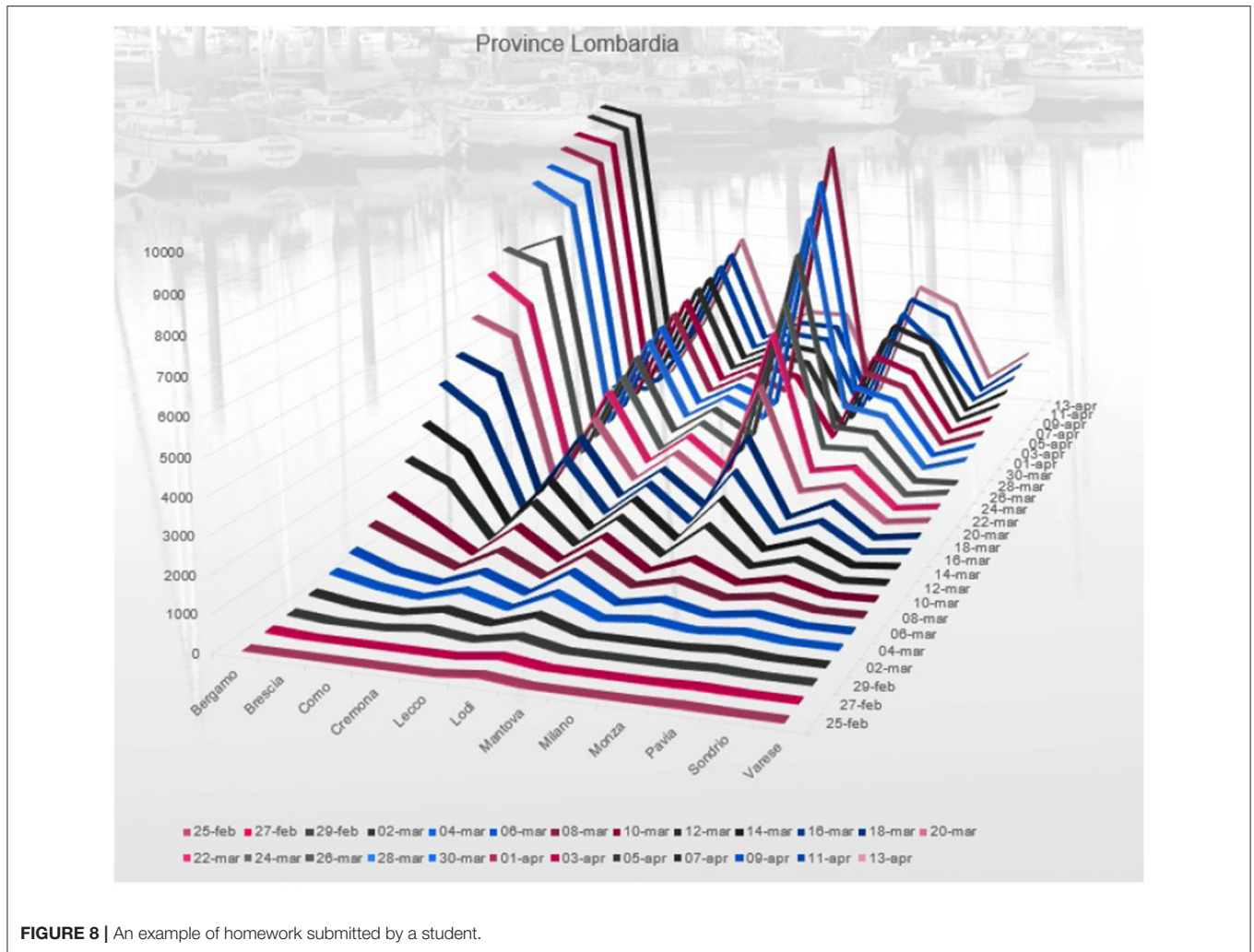
FIGURE 8 | An example of homework submitted by a student.

2016), it was possible to trace the documentational trajectory of the teacher's activity over time. The data was analyzed through DAD and explained below.

The point of view of organizing the resources changes. The pilot teacher managed an RMRS where she collected her resources (**Figure 5A**) and categorized them. We can observe that in the resources as contained by using the scheme of usage, the focus was on the categorization of the resources as material, cognitive and social, and whether these resources were digital or non-digital ones. In the next step, however, from cycle one to cycle two, the document's structure remains the same, so the teacher no longer focuses on "how to classify the resources" but on "which of the resources identified were most effective during the experimentation carried out". When adapting this document to a different circumstance, it is possible to include a different usage scheme, for example, or classify it in a particular way by prioritizing digital resources or not.

Nevertheless, here the DAD has been used mostly in a social sense to analyze the resources/documents of teachers in the networks of practice (Clark-Wilson et al., 2020). Specifically, during the Lesson Study cycle, the teachers shared their resources

and collectively worked to obtain some common documents, which will be new resources for their network. Through the Lesson Study cycle, starting from the shared resources, the group project developed schemes of usage in order to change resources in documents. These documents are the resources for further Lesson Study cycles. This means that a nesting process is generated, where each Lesson Plan document becomes the resource for the following Lesson Study. As previously shown, examples of this nesting process are the RMRS and the LP documents.

FINDINGS

RQ1: What Are the Main Features of the Distance Lesson Study?

In response to RQ1, the experimented DLS had similar architecture to the traditional Japanese ones (Huang et al., 2019), as modified in the High School Italian experimentation (Capone et al., 2020), thus including the following essential features: time scheduling, LP, presence of observers in the teaching phase, and video analysis. The main documents of

the DLS are a shared RMRS, the modified Lesson Plan, the observations grids, and a Debriefing document (see **Table 4**). Nevertheless, the main novelty of the Distance Lesson Study is based on the medium of communication between all actors (i.e., through the Internet) and the use of different resources. The cognitive ones were preserved, whereas both the social and material resources became digital. The use of the digital resources contributed to organizing the "digital documents", which may be available for the whole teachers' community, and emerge as effective instruments for the training/learning of teachers. More particularly, the Lesson Plan document was modified, to include all the new digital resources.

Several differences were evidenced concerning the face-to-face Lesson Study, mainly during the observing phase, which differs from the traditional (in presence) one. A constraint is that the observation was not as multi-prospective as in presence classes, but was performed only through the GMeet channel. Therefore, it was complex to focus on proxemics and kinesics, para-linguistic elements, and group dynamics. Nevertheless, the presence of observers as users was less invasive and intrusive for the students.

Specifically, to answer RQ1, we also analyzed the "observations grids", filled out by everyone in the working group. We focused on the pilot teacher (T1) data from the document analysis.

In the observation grids (**Table 3**), the observers checked if teachers provided students with all the essential elements for delivery. In all the compiled grids, it has been written that T1 shared all the elements for the delivery on Classroom (a Google App); this procedure allowed the students to have the essential elements during the entire duration of the lesson time. They may use these resources whenever they need them. It is an opportunity that was absent in the face-to-face Lesson Study, thus representing an evolution in the documental trajectory of the teacher's activity over time.

T1 confirmed that this aspect also improved the communication channel.

A further behavioral indicator in the observation grid (communication channel) was: "Does he/she provide explanations to the students during all the activities?" R1, an observer in the teaching phase, outlined that just six questions have been asked to the teacher (T1) for explanations during all the activities. R1 observed that in previous Lesson Study experiences, the mean number of questions was 20.

R3 observed: "this difference could be because the students are less involved in e-learning activities and tend to get lost more easily".

Regarding the "interaction with the class", the working group agreed that the Distance Lesson Study distinctly differed from the Lesson Study given in presence. Specifically, the observers had a lot of difficulty observing the gestures, and in general the proxemics, of the teacher.

T1: "I am used to walking between the class desks during the lesson, especially when the students are working in a group, sometimes a glance is enough to interact with the students, so giving them the right directions to continue."

T4: "It's the same for me. It is what I miss about the teaching in presence."

T3: "Nevertheless, from the video, I saw that you used the gestures a lot, although I think that just a few of the students viewed your gestures…."

From the analysis of the completed observation grids, we deduced that no working group member could link the teacher's gestures and the delivery support.

RQ2: How Does This Distance LS Support Teachers Learning From DAD's Perspective?

Here we analyzed the DLS supporting the teachers' learning through the documents they produced during each phase. Each teacher shared their resources (material, cognitive, and social) in the co-planning phase, schematized through an RMRS. With the researchers' contribution, the RMRS of each teacher was modified, and a shared and common RMRS was produced from the intersection of the resources of all the teachers, as shown in **Figure 4**. By analyzing the RMRS of all the teachers, we may deduce the time trajectory of each teacher and the social aspects of the DAD.

In the analysis of the co-planning phase which was video-recorded, the following observations by teachers are noticeable:

T1: "I'm afraid because I don't know if the available resources will be effective for our students. It's the first time I'm in this situation."

T2: "I think the resources will be enough, we already used them in different contexts, and students were always fine about their usage; nevertheless, some problems could arise because of technical details: some connection problems could cause some delays."

T5: "We have to take into account these problems in the time planning of the lesson plan."

By analyzing these observations and some similar ones, researcher R2 deduced that all the teachers, during this phase, shared their material and cognitive resources in order to select better ones to be used for the experimentation, also evidencing constraints and opportunities of each resource. During the teaching phase, the pilot teacher had the chance to verify the effectiveness of the shared RMRS document, thus suggesting (in the debriefing) some modifications for the schemes of usage of the resources themselves. At the end of the debriefing phase, the working group produced a document named "Debriefing document", which summarizes the data in the observation grids, the analysis of the video-recorded lesson, observations of the researchers and teachers, and the data analysis. It thus represents the Lesson Study results document (**Table 4**), showing the interconnections between the Distance Learning Methodology, the Lesson Study main features, and the conceptual framework (Documental Approach to Didactics).

During all the phases, the teachers declared that they had enriched their schemes of use for some resources they already used in their daily teaching. They also developed new schemes in order to produce some documents, which are fundamental in the teachers' professional development. This document's production

Distance learning	Lesson study	Documental approach to didactics
Need to re-design contents	It allows experimenting and testing new tasks, refining them as they are experienced in the different classes.	It helps the teacher in the genesis of documents and the aware use of "Material Resources."
Need to use new methods of teaching	It allows the teachers to discuss and plan the most suitable strategies. Debriefing promotes ex-post comparison.	It guides the teacher to an aware use of "Cognitive Resources."
Need interaction students-students and students-teachers by using alternative resources	It is a collaborative teachers' methodology, thus favoring interaction between teachers	It guides the teacher in the aware use of "Social Resources."

made the LS sustainable and replicable (Capone et al., 2020), constituting a valuable teacher training tool. At the end of the analysis of the videos and the observation grids, the teachers expressed their overall judgment on the activity experienced. A few excerpts are shared below:

T1: "The experience of Distance Lesson Study made me more aware of the need to differentiate teaching. The sudden and indispensable use of technology as the only tool to continue teaching must help us to realize how important it is to be aware of the resources for teaching and to know how to master them."

T2: "It has been a strong motivation to improve my teaching."

T3: "For an effective teaching of mathematics, it is necessary to know how to use more resources: how to select, use, transform existing resources, design and create new materials."

T4: "I hope to be able to continue to design "new things" not only because of the circumstances of the pandemic but also because it must become a practice of the teaching-learning process."

T5: "I think that the cooperative aspect, which is a fundamental feature of the Lesson Study, is one of the main strengths of this experience."

RQ3: Which Material, Cognitive, and Social Resources Favored the Distance LS?

DLS helps the teachers re-plan each content in a very detailed way, structuring it in 1 h of lesson, re-using the available material resources, with a different scheme of usages, through an effort on the cooperative working between teachers. The documentary evidence of this support is the Lesson Plan (see **Table 2**). In the Lesson Plan, the project group summarized all the material, cognitive, and social resources used (or to be used) in the DLS.

As an answer to RQ3, it is possible to outline that the institutional applications for the communication between teachers and students favored the LS on time.

Moreover, technological material resources allowed the students to work on their daily problem and discuss it effectively.

Social resources favored the communications between students-students, students-teachers, teachers-teachers, and teachers-researchers. The teachers and observers deduced that the used social resources allowed for effective class management, starting from the video analysis and direct observations.

Cognitive resources also allowed debating between students and peer-to-peer education. Students could share their material resources with all the students/teachers/observers. Nevertheless, the teacher participated and intervened less in the Lesson Study in the presence mode.

CONCLUSION

In this paper, we experimented with a cycle of Lesson Study, conducted entirely using distance learning, namely the Distance Lesson Study. The use of distance learning became mandatory because of the COVID-19 pandemic. Teachers and students taught/learned and communicated with each other using only digital (both material and social) resources. The focus of this paper was on the teachers. The Lesson Study allowed detailed scheduling and experimentation even from the timing point of view. Therefore, new resources were re-organized to be shared between teachers and it provided effective and useful documents for the teachers and students. The collection, design, and sharing of resources followed a process of documental genesis. The complete cycle of LS helped in the "refining" of the lessons, in the sense that the same lesson, after an appropriate revision, could be reused in a few future cycles, and the documents produced in each cycle became the new resources for the next one. This experimentation deduced that the LS has improved compared to the current LS, mainly in communication and resources/documents processes. Nevertheless, we found that the essential features of the Lesson Study were retained, and they supported distance learning. But the resources, which constitute a fundamental part of DL, were effective for the LS experimentation.

Moreover, the authors observed that some resources might guide the teachers: the material resources to innovate content, the social resources to improve communication between all actors, and the cognitive resources to organize all educational activities better.

Furthermore, an essential element of the Lesson Study is the cooperation between teachers, who see the LS as a teachers' professional development methodology as in the Japanese tradition (Huang et al., 2019). Cooperation has been facilitated by digital (material and social) resources. Specifically, the use of social resources such as G Suite Classroom could be equally effective in the distance learning mode and in face-to-face ones

to document and share the students' activities. Moreover, some of the usage methods of resources adopted in this experiment could be used in face-to-face mode, thus providing the opportunity to include them in daily teaching practices as well.

Finally, we envision this experiment as a milestone for future Distance Lesson Studies in any non-pandemic emergency.

DATA AVAILABILITY STATEMENT

The raw data supporting the conclusions of this article will be made available by the authors, without undue reservation.

ETHICS STATEMENT

The studies involving human participants were reviewed and approved by University of Salerno. Written informed consent to participate in this study was provided by the participants' legal guardian/next of kin.

AUTHOR CONTRIBUTIONS

All authors contributed equally to the testing of the activity and to the writing of the paper. All authors contributed to the article and approved the submitted version.

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