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# The theoretical framework on humanist ethno-metaphorical mathematics learning model: An impactful insight in learning mathematics

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The educational revolution has posed an immense challenge to the world of education. It demands the development of a generation that can take on the challenges and changes bred by the ever-rapid revolution. It is thus inevitable that education must enable improvements of individual hard skills and soft skills that are required to keep up with such changes, including mathematical hard skills and soft skills. The problem is that not all mathematics learning approaches, particularly in the case of Indonesia, are capable of such improvements and of answering to such demands, challenges, and changes that are posed by the revolution. This research seeks to build a theoretical framework out of a systematic analysis based on various pieces of literature that are relevant and fitting to the theoretical framework under development. In this study, a theoretical framework on humanist ethnometaphorical mathematics learning is developed as a theoretical foundation. This theory is designed for creating a humanist mathematics learning approach based on ethnomathematics and metaphorical thinking to develop students' mathematical hard skills and soft skills and thus enable them to deal with the current and future problems and changes.

#### KEYWORDS

humanist ethno-metaphorical mathematics learning, integrative literature review, metaphorical thinking, mathematical soft skills, mathematical hard skills

## Introduction

The educational revolution in this era has brought with it various significant challenges and changes to human life (Petrillo et al., 2018). Every single aspect moves rapidly, connections form vastly, and novel innovations emerge. This is especially triggered using Internet of Things (IoT)- and

Artificial Intelligence (AI)-based technology in various human needs and occupational fields (Chou, 2018). Such a condition compels humans to make acceleration and enhance their abilities to take on the challenges and changes posed by this era and to leverage them to bring forth changes that are favorable for the world and human civilization (Teck et al., 2019; Maryanti et al., 2020). It is, thus, important to build humans of innovation and creativity who remain in the realm of humanism and nationalism. One way to arrive at this end is to improve hard skills and soft skills and to instill ethics, culture, character, and nationalism in education or school settings (Hendriana et al., 2017b; Anggadwita et al., 2021), as is summarized in Figure 1.

Professional hard skills and soft skills in life and the job world that are technology-based as in today's era are of the utmost importance (Hendriana et al., 2017b; Rohaeti, 2019). In this case, the science of mathematics plays a central role in improving both said hard and soft skills (Hendriana, 2017a). The former include cognitive abilities such as mathematical understanding ability, mathematical reasoning ability, mathematical problem-solving ability, mathematical communication ability, mathematical connection ability, mathematical critical-thinking ability, and mathematical creating thinking ability, to name just a few, while the latter encompasses affective abilities such as mathematical disposition, habits of mind, learning activeness, learning interest, learning motivation, mathematical resilience, self-concept, self-confidence, self-efficacy, self-esteem, and self-regulated learning (Hendriana et al., 2017b). For the innovations that spring out of this era to well-benefit the world, it is of the essence to imprint ethics, culture, character, and nationalism onto every individual (Sasongko, 2019). In the absence of proper imprinting of the aforementioned, it is feared that the power of this era will come to be misused and leave an adverse impact on the world and civilization of mankind (Hendriana, 2017a; Sasongko, 2019). It is, therefore, necessary to make efforts to build hard skills and soft skills and to instill ethics, culture, character, and nationalism hand in hand.

One of the efforts one may make to develop both sets of skills in school instructions is to open up as wide an opportunity as possible for students to express responses, answers, or opinions on mathematical problems they encounter during learning and teaching activities through innovative, creative, humanist, nationalist instructions (Hendriana et al., 2014). Besides, it is recommended to build the habit of free, holistic, comprehensive, innovative, critical, creative, literal thinking ethically to encourage students to build innovativeness, creativity, and criticality in tackling and dealing with the challenges, problems, and changes in life in current era (Liddy, 2012; Hendriana et al., 2017b). This habituation can be commenced in students' environment at school during the learning and teaching process, including in mathematics learning and teaching activities.

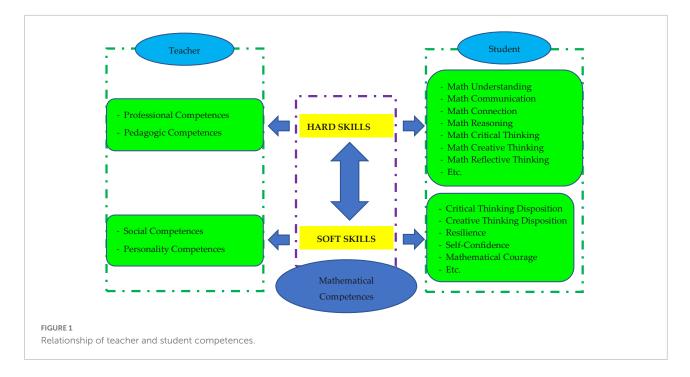
In mathematics instructions, it is no easy task to elicit a positive response out of students' thinking process. However, teachers should still facilitate and appreciate students' thoughts as solution alternatives to the problems presented. Not many a student are able to offer arguments, and it is even extremely few to have the courage to argue, so the solutions presented by students are easily and directly understandable to the teacher and/or their peers. Hence, it is needed of the teacher to dig deeper into students' arguments to extract some information. This is because the arguments conveyed by students at times take the form of expressions that explain their true meanings in indirect manners (Hendriana, 2017a; Boldt, 2021). Such expressions are likening or analogous comparations we often refer to as metaphors (Tim Penyusun Kamus Pusat Bahasa Indonesia, 2005).

Metaphors are indirect expressions in the form of analogous comparisons that carry meanings identical to the original meanings (Tim Penyusun Kamus Pusat Bahasa Indonesia, 2005). This is what is frequently encountered in the way in which students answer and make arguments as a step in the process of solving the problems they are presented with. For this reason, teachers hold a key role in giving students an opportunity to put forward some opinions as arguments out of their thinking processes. Teachers are suggested against shutting up the opportunity for students to expand their thinking. All the answers proposed by students that are accompanied with reasons must be appreciated and declared correct on the grounds that a piece of truth can be believed if it is justified with logical arguments (Hahn, 2018; Shabani et al., 2019).

Teachers must also have or exhibit metaphorical thinking. To some, the use of metaphors in interaction or communication with others is nothing but a rhetoric variation. A speaker would use a certain figurative expression when he/she feels that no literal language would be able to render the same effect as, or compare with, figurative language to deliver the meaning he/she intends to deliver and hence derive the response he/she desires (Ricoeur et al., 1977). Therefore, in instructions, especially mathematics ones, giving students an opportunity to think metaphorically in class is of much criticality. Doing so would stimulate a habit in students' thinking process that is free, holistic, comprehensive, innovative, critical, creative, and literal (Lunenburg, 2011).

Outcomes from mathematics instructions that can improve students' mathematical hard skills and soft skills that are necessary in facing the challenges and changes in educational revolution greatly depend on how students understand a problem they are given and how they solve it innovatively. It is as found by Hendriana (2012) that, during the learning and teaching process, students only imitate and take notes of how to solve problems as their teacher did.

If this goes on and on, students will not solve problems as expected. Students' low mathematical understanding may be resulted from a variety of sources, one of which is the fact that they are rarely given the opportunity to think openly and, as a result, ask questions regarding the problems that they are presented with (As'ari et al., 2019). Attracting questions



from student to teacher or from student to student should be accustomed in every instruction in the classroom through stimulating steps that are taken by the teacher, in which case students may respond.

Teacher's role in facilitating student-centered instructions in the classroom is essential (Estes, 2004; Keiler, 2018; Chen and Tsai, 2021). This role must start with teacher's being able to accommodate every student's opinion, developing understanding and appreciation of student-generated mathematical ideas, and assisting students in developing self-confidence, independence, and curiosity. The fact in the field, however, as found by Hendriana et al. (2018), is that students' level of mathematical courage to ask mathematical questions is still low. This is especially true when students are given the chance to ask non-routine, open-ended questions. This reflects that students' free, holistic, comprehensive, innovative, critical, creative, and literal thinking habit to render thinking results has been sub-optimal and that the opportunity for them to practice this thinking habit has been non-existent. Habituation must be performed routinely and continuously by both the teacher and students so from the internalization process there will form a habit, from which a need and eventually a culture will also rise in the mathematics instructions at school. It is, thus, needed to take a mathematics classroom instructional approach that is fun based on students' culture, way of thinking, and contributions over the course of the instructional process. This approach has double purposes of building students' hard skills, soft skills, and mathematical courage and of giving students to think metaphorically, hence producing humanist, nationalist human beings of innovation and creativity who are able to take on the challenges and changes in this era.

It would be improbable to implement this idea without an underlying theory, so this research emerges to build a theoretical framework for this learning approach in order to promote students' mathematical hard skills and soft skills. This theoretical framework is constructed through systematic review, synthesis, critical analysis, and integration of several pieces of literature on mathematical hard skills and soft skills, humanist mathematics learning approach, ethnomathematics, metaphorical thinking, teacher's professional competences, and problem-posing ability. Results of this research are to contribute as a theoretical foundation for further research during the implementation process in the field.

# **Research methods**

This research uses an integrative literature review as the research method to build a theoretical framework on humanist ethno-metaphorical mathematics learning approach to improve students' mathematical hard skills and soft skills and thus enable them to take on the challenges and changes in this era. The integrative literature review carried out in this research lays a foundation on which a theoretical or conceptual framework is to be built by reviewing, criticizing, and synthesizing literature that is representative of a certain topic in an integrated manner, allowing a new theoretical framework and perspective to be produced (Torraco, 2005). This review is unlike two other kinds of review-systematic literature review and semi-systematic literature review. The two functions as sources for identifying and reviewing critical investigations to ensure the trend and impact of major studies on a certain topic rather than as means for developing or building a theoretical framework on a learning approach (Baumeister and Leary, 1997; Torraco, 2005; Liberati et al., 2009; Wong et al., 2013; Snyder, 2019). Integrative literature review, meanwhile, has a unique contribution to the reconceptualization of an established topic, which in turn can be used to develop a new framework and perspective by providing an overview or description of research trend and effect (Snyder, 2019). Therefore, it was deemed fitting and effective to be used in this research for the purpose of making a theoretical framework on the humanist ethno-metaphorical mathematics learning approach. The role this method assumes is to promote a solution to the need for improved hard skills and soft skills in order to build human beings who are innovative, creative, humanist, and nationalist and who are capable of dealing with the challenges and changes in educational revolution.

Integrative literature review is to be practiced in four stages, namely to design, conduct, analyze, and write a literature review (Torraco, 2005; Snyder, 2019). In the first stage, literature is designed by determining seven key points such as the topics, reasserting the study reasons and objectives, formulating certain research scope and questions, and collecting the literature to be reviewed. The next stage is carried out by conducting an analysis in steps that include determining when the review is to be conducted, examining the reviewing process, and criticizing and synthesizing literature. Literature criticism is performed by a critical analysis which involves close examination of a main idea and its relationship with a problem and by criticizing the existing literature to support our construction framework. Meanwhile, synthesis is conducted by integrating the existing ideas with new ideas to make a new formula for the topic under discussion. In this research, synthesis takes the form of an alternative model or theoretical framework as well as a new way of thinking on the problem under study with an integrative review, which is directly laid down from a previously conducted critical analysis and synthesis (Torraco, 2005).

Our review of publications that addressed the theoretical framework of Humanist Ethno-Metaphorical Mathematics Learning focused on seven topics:

- 1. Mathematical hard skills
- 2. Mathematical soft skills
- 3. The humanist mathematics learning approach
- 4. Ethnomathematics
- 5. Metaphorical thinking
- 6. Teacher's professional competencies
- 7. Problem-posing ability

Using keywords of these seven issues, we searched Google Scholar, ScienceDirect, and SpringerLink, for articles published between January 1, 1973, and October 30, 2021.

We used journal's articles, books, theses, dissertation, and proceeding's articles to cite the sources. We also conducted a manual search of specialist publications and citations from the papers found during the initial investigation. After we had all the documents, we went over them to ensure we could use them to accomplish the research goals. The papers that have been chosen are those that address at least one of the seven research topics.

The third stage is review analysis. In an integrative literature review, data analysis is replaced by a clear logic and conceptual reasoning as a foundation for argumentation and explanation. Both are major features that are used to develop a framework or model proposed and to enable readers to see a link between the research problem, literature criticism, and theoretical findings in a theoretical framework. The last stage is to write a review in a complete, structured manner. The motivation and need of the study are to be delivered. Besides, the reviewing process is explained transparently in aspects such as how literature is identified, analyzed, synthesized, and reported by the researcher. Review results in an integrative literature review are not measured and evaluated as strictly as in empirical studies. Nonetheless, the quality of such research is seen from the depth, accuracy, and substantial contributions that are truly valuable and new in a certain field or topic (Snyder, 2019).

The literature of hard and soft mathematical skills, and problem-posing ability focusing on the indicators and their types of skills are studied in detail and comprehensively to construct the characteristics of the learning targets to be achieved. Furthermore, ethnomathematics and metaphorical thinking literature are explored to design the framework of this learning model. Lastly, teachers' professional competencies are discussed to determine what competencies and characteristics are needed by teachers who will implement this learning model.

## Results

Humanist ethno-metaphorical mathematics learning is a theoretical framework on humanist mathematics learning with metaphorical thinking culture that improves students' mathematical hard skills and soft skills that are necessary to face the challenges in this era. Its urgency in mathematics learning has been growing, given that, over the time, mathematics has been perceived as daunting to students, in which case learning at school that follows a drilling and memorization system oftentimes gives students only the knowledge of formulas rather than the understanding of the meaning and benefit of learning mathematics and mathematics' relationship with their daily life.

Metaphor itself is here meant to refer to another way of expressing a supposed or actual meaning based on students' understanding and experience. In humanist ethno-metaphorical mathematics learning, metaphors are used to express the meaning of a mathematical concept or idea ethnomathematical or mathematically that exists in, or possibly is practiced in, students' culture and daily life, allowing students greater ease to understand the mathematical concept and to find its meaning and understand its benefit in daily life. Using metaphors will also connect mathematical concepts to ethnomathematical ones that have been known in students' everyday life.

In humanist ethno-metaphorical mathematics learning, metaphorical thinking is used to conceptualize abstract mathematical concepts into concrete ones, while ethnomathematics is used to set up a learning context that makes the conceptualization process from abstract to concrete easier. Undertaking mathematical idea construction will involves three forms of conceptual metaphors: first, grounding metaphors, which lay the basis for understanding mathematical ideas that are connected to daily experience; second, linking metaphors, that are present in the process of constructing a link between two things by selecting, asserting, giving flexibility to, and organizing the characteristics of the main topic, with support from a subsidiary topic, in the form of metaphorical questions; and third, redefinition metaphors, that arise when redefining the metaphors and selecting the most suitable one among them with the topic to be taught.

Several metaphorical thinking strategies can be used in mathematics learning, and these involve the following: first, comparing the meanings of metaphors in illustrating mathematics; second, giving students a chance to convey their own metaphors; and third, comparing the meanings of the metaphors across various cultures. The use of cultural contexts and metaphorical thinking will take students' learning beyond just formulae memorization.

With ethnometaphors, students will be able to connect mathematics to their daily life and the cultures that exist around them. They will also be able to take meanings from mathematics learning and understand and communicate them with metaphors or expressions that are closely related to their everyday life and culture, making it easier for them to understand and communicate mathematics. However, enabling students to extract meanings is not the only thing that ethnometaphors can do. They also enable students to apply mathematics in their daily life because ethnomathematics essentially departs from the notion and concept that mathematics is a culture-based human activity that originates in the way in which society responds to mathematics-related phenomena.

Using ethnometaphors in tandem with students' cultural contexts and daily life in learning allows students to learn not only mathematics, but also ethic, moral, social, and nationalist values that come within the culture and daily life that are used as the learning context, enabling the shaping of students' character as humanist individuals. These things are important for students to deal with the changes that are rendered by educational revolution, which is rife with IoT, and AI uses that may both be advantageous and disadvantageous to the civilization and the world. It is thus vital to build character and imprint ethical, moral, social, cultural, and nationalist values on students to direct the changes in this revolution toward a greater benefit for the civilization and the world.

Some things that characterize humanist ethno-metaphorical mathematics learning lay a foundation for improving

students' mathematical hard skills. These skills include mathematical understanding ability, mathematical reasoning ability, mathematical problem-solving ability, mathematical communication ability, mathematical connection ability, mathematical logical thinking ability, mathematical critical thinking ability, and creative mathematical thinking. Furthermore, humanist ethno-metaphorical mathematics learning also improves students' mathematical soft skills. They include habits of mind, learning activeness, learning interest, motivation, mathematical resilience, self-concept, self-confidence, self-ability, and appreciation of science.

Further research should specifically improve each mathematical hard skill and soft skill using humanist ethnometaphorical mathematics learning based on the theoretical framework of this study's humanist ethno-metaphorical mathematics learning. In implementing this learning approach, teachers should assume the role of facilitators who can design humanist learning innovatively. They should have soft skills that are of two categories: personality competencies and social competencies. Falling to the first category are as follows: maturity, virtuous character, wisdom, charisma, and fitness to be a role model for students; understanding of the professional rights and obligations of a mathematics teacher; understanding of the professional tasks and functions of a mathematics teacher; and understanding of the impacts of educational innovations on the development of the nation's character. Meanwhile, the second category includes the following: upholding of norms, values, morality, religion, ethics, and professional responsibilities; practical communication ability; understanding of problems in mathematics education and contemporary mathematics; teamwork and rapid adaptation to the skills needed in a work environment; mastery of educational interactions between teacher, student, parent, and society; understanding of consultative communication between mathematics education, regulator, and societal needs; and ability to use and apply information and communications technologies in mathematics learning.

Teachers must have the innovativeness to design mathematics learning that is interesting, fun, and innovative based on students' ways of thinking and contributions during the learning and teaching activities. This learning design must be set off from the students' needs, conditions, and innovative ideas, which are the product of their learning methods to achieve better learning outcomes. Besides, they must also make use of appropriate learning aids and strategies to motivate students, including the environment, as a learning source to create enjoyable, fun learning.

In humanist ethno-metaphorical mathematics learning, it is imperative for teachers to be able to encourage the students to find their own ways to solve the problems presented, express their own opinions both orally and in writing, and participate in creating and shaping a comfortable learning environment for themselves. In this way, student-centered learning will be created, in which case students will be able to actively develop their own knowledge through the innovative learning that has been designed by the teachers. Hence, strong will be the students' conviction that what they master is not only the materials, but also how they learn those materials meaningfully.

The characteristics of humanist ethno-metaphorical mathematics learning are identical to those of humanist learning in general, as follow:

- 1. It places students as inquirers rather than just recipients of facts and procedures.
- 2. It gives students an opportunity to interact to understand and solve problems in depth.
- 3. It accustoms students to learn problem-solving in various ways.
- 4. It presents open-ended problems that are interesting and challenging.
- 5. It uses evaluation or assessment techniques that are not based solely on procedures.
- 6. It develops understanding and appreciation of studentgenerated mathematical ideas.
- 7. It helps students develop self-confidence, independence, and curiosity.
- 8. It presents learning by drawing a link to daily life.

Humanist ethno-metaphorical mathematics learning has a role in promoting students' mathematical hard skills and soft skills with which students will find it easier to deal with the challenges and changes in this era. However, one does not simply implement this approach, and by doing so improve students' mathematical hard skills and soft skills. Further research that is greater in specificity and implementativeness based on the theoretical framework that has been established in this study will be necessary. In other words, the empirical contribution of the theoretical framework developed here sets up a foundation for developing the humanist ethno-metaphorical mathematics learning approach and its implementation in the learning and teaching activities in the classroom.

## Discussion

## Mathematical hard skills

Mathematical hard skills refer to knowledge, technology, and technical skills mastery related to a field in the science of mathematics (Hendriana, 2017a). They are derived and explained from the existing core competences and basic competences for each grade. These skills are divided into several categories, namely mathematical understanding ability, mathematical reasoning ability, mathematical problem-solving ability, mathematical communication ability, mathematical connection ability, mathematical logical thinking ability, mathematical critical thinking ability, and mathematical creative thinking ability. The indicators of each category are as follows:

1. Mathematical understanding ability

Mathematical understanding ability is a basic competence in mathematics learning that includes the ability to absorb a material, remember and apply a mathematical formula and concept, estimate the truth of a statement, and apply a formula and theorem in solving a problem (Hendriana, 2017a; Louie, 2017). NCTM (1988) puts forth several indicators of mathematical understanding, namely being able to identify a concept verbally and in writing, being able to identify and make examples and other things, being able to use a model, diagram, and symbol to represent a concept, being able to turn one form of representation to another form of representation, being able to recognize various conceptual meanings and interpretations, being able to identify the properties of a concept and recognize the prerequisites to determine a concept, and being able to compare and contrast various concepts.

2. Mathematical reasoning ability

Mathematical reasoning ability is defined as a thinking process that seeks to link known facts to draw a conclusion (Saleh et al., 2018; Hawes and Ansari, 2020). Mathematical reasoning is classified into two categories, namely mathematical inductive reasoning, and mathematical deductive reasoning (Siswono et al., 2020; Kaplan et al., 2021). Mathematical inductive reasoning is based on some conclusion-drawing characteristics: transudative reasoning, or the ability to draw a conclusion from one case with another case; analogical reasoning, or the ability to draw a conclusion based on similarities of processes or data; generalization reasoning, or the ability to draw a general conclusion based on some limited data under scrutiny; the ability to estimate answers of solution and trend, interpolation, and extrapolation; the ability to provide explanation on an existing model, fact, property, relationship, or pattern; and the ability to use a relationship pattern to analyze a situation and establish a conjecture (Hendriana, 2017a). Indicators of mathematical deductive reasoning, meanwhile, include the following: the ability to perform calculation based on a certain rule or formula; the ability to draw a logical conclusion based on rules of inference, appropriate proportion, probability, correlation between two variables, and combination of multiple variables; and the ability to formulate direct evidence, indirect evidence, and evidence with mathematical induction (Hendriana et al., 2014, 2017b; Bruckmaier et al., 2021; Shodikin et al., 2021).

3. Mathematical problem-solving ability

Problem-solving ability is defined as the ability to formulate a new answer or solution beyond simple application of previously learnt rules to achieve a goal. Hendriana (2017a) explains that the indicators of mathematical problem-solving ability include the following: being able to present a problem in a clearer form; being able to state a problem in an operational form; being able to formulate a hypothesis and work procedure to solve a problem; being able to test a hypothesis (by collecting data, processing data, and drawing inference from data); and being able to re-examine the results obtained. There are several strategies that can be used to solve a problem: trials and errors; making a diagram; experimenting with a simpler problem; making a table; finding a pattern; breaking down a goal (detailing a general goal into actual goals); performing calculation; thinking logically (using reasoning and conclusion drawing); analyzing how to achieve a desired goal; and ruling out improbable (Pólya and Conway, 1973).

## 4. Mathematical communication ability

Mathematical communication ability is defined as the ability to deliver a mathematical idea both in speaking and in writing (Hendriana, 2017a; Hendriana et al., 2017b). The indicators of this mathematical ability include being able to reflect and clarify thoughts on mathematical ideas, being able to connect everyday language and mathematical language using symbols, being able to use the skills to read, listen to, evaluate, and interpret mathematical ideas, and being able to use mathematical ideas to form an assumption and devise a convincing argument (Hendriana et al., 2014; Hendriana, 2017a).

#### 5. Mathematical connection ability

Mathematical connection ability is defined as the way to solve interrelated mathematical problems, situations, and ideas into a mathematical model and to apply the knowledge gained from solving a problem to solve another problem (Lappan et al., 1996). This ability is critical to the mastery of conceptual understanding and to problem-solving. Additionally, this ability allows an individual to connect some mathematical ideas to have a deeper mathematical understanding and to see associations of mathematical topics to contexts outside mathematics and to daily experiences. The indicators of mathematical connection ability include being able to draw on connections between mathematical topics and between a mathematical topic and other topics and being able to use mathematics in other subject matters and/or in daily life (Hendriana, 2017a).

6. Mathematical logical thinking ability

Mathematical logical thinking ability is the ability to think inductively and deductively according to the laws of logic and the ability to understand and analyze numerical patterns and solve problems using the thinking ability (Hendriana, 2017a; Demetriou, 2020). The indicators of mathematical logical thinking ability are as follows: being able to draw a conclusion based on analogies; being able to examine or test the validity of an argument; being able to connect facts as problems involving logical thinking; and being able to be building and fixing an assumption (Hendriana et al., 2014, 2017b; Hendriana, 2017a).

#### 7. Mathematical critical thinking ability

Mathematical critical thinking ability is the ability to use the thinking ability actively and rationally with full awareness and to consider and evaluate information (Hendriana, 2017a; Han et al., 2021; Van Peppen et al., 2021). The indicators of this ability include the following: the ability to identify and justify concepts or the ability to provide rationale for conceptual mastery; the ability to generalize or the ability to complete supporting data or information; and the ability to analyze an algorithm or the ability to evaluate or examine an algorithm.

8. Mathematical creative thinking ability

Mathematical creative thinking ability is the ability to find a variety of unprecedented solutions, which are acceptable in their correctness, to open-ended mathematical problems with ease and flexibility (Hendriana, 2017a; Apiola and Sutinen, 2021). Creative thinking ability is inclusive of fluency, flexibility, authenticity, and elaboration. The indicators are as follows: fluency, by which to generate a lot of relevant ideas or answers and have a smooth flow of thinking; flexibility, by which to generate a great variety of ideas, alternate ways or approaches, and have diverse thinking directions; authenticity, by which to offer answers unlike others' or rarely presented by many; and elaboration, by which to develop, improve, and enrich an idea, go into greater details, and expand thoughts (Hendriana, 2017a; Dunstan and Cole, 2021).

## Mathematical soft skills

Mathematical soft skills refer to habits of mind, learning activeness, learning interest, motivation, mathematical resilience, self-concept, self-confidence, self-ability, and appreciation of science (Hendriana, 2017a; Hendriana et al., 2017b). They have a vital role in building an individual's character. In education, learning strategies should optimize the interactions of teacher to student as well as the interactions of student to teacher or student to student. This is necessary to create a healthy, conducive, productive environment and to propagate multidirectional interactions.

The indicators of several mathematical soft skills are outlined below:

1. Habits of mind

The indicators of habits of mind include having the habits to be persistent and insistent, managing what the heart says, empathizing with others' feelings, thinking flexibly, being reflective, confident, and open-minded, practicing metacognitive thinking, working carefully and properly, achieving high standards, posing questions, presenting problems effectively along with some supporting data, drawing on former experiences, making analogies, thinking and communicating clearly and correctly, using the senses keenly, thinking intuitively, estimating solutions, creating, fantasizing, and innovating, showing passion when making responses, having the courage to assume responsibility and take risk, showing some sense of humor and thinking interdependently, and learning continuously (Costa and Kallick, 2005; Jacobbe and Millman, 2009; Hendriana, 2017a).

#### 2. Learning activeness

The indicators of learning activeness include being able to pay attention to the teacher's explanation, being able to understand the problems presented by the teacher, being active in posing and answering questions, being able to cooperate in groups, being able to express opinions, being able to give group mates an opportunity to express opinions and being able to present the work of the group.

#### 3. Learning interest

The indicators of learning interest include taking some joy in learning, showing interest in learning, being engaged in learning, learning diligently, completing mathematics assignments, being committed, and disciplined, and having a studying schedule (Hendriana, 2017a).

### 4. Motivation in mathematics

The indicators of motivation include showing selfconfidence in using mathematics, being flexible in doing mathematical work, showing willingness in setting aside other obligations and tasks, being committed to doing mathematics, being able to defend opinions, and showing insistence and perseverance in completing mathematics assignments.

#### 5. Resilience in mathematics

The indicators of resilience in mathematics include industriousness, self-confidence, hard work, persistence in the face of problems, failures, and uncertainties, desire to socialize, helpfulness, discussion with peers and adaptation to the environment, curiosity, reflection, researching and using various resources, language proficiency, and self-control and awareness of feelings of one's own (Johnston-Wilder and Lee, 2010; Hendriana, 2017a).

#### 6. Self-concept

The indicators of self-concept include showing willingness, courage, insistence, earnestness, seriousness, and interest in learning mathematics, believing in one's ability and success, recognizing one's strengths and weaknesses in mathematics, showing cooperativeness and tolerance to others, appreciating the opinions of others and of one's own, forgiving of others' and one's own mistakes, showing communication ability and knowing one's place, and having perspective on, deriving benefits from, and taking a liking in the subject matter and the learning of mathematics (Hendriana, 2017a; Hendriana et al., 2017b).

#### 7. Self-confidence

The indicators of self-confidence include the following: believing in one's own abilities; showing no anxiety, feeling free, and taking responsibility for one's own deeds; being independent in decision-making; having the courage to express opinions and feeling driven to make achievements; and recognizing one's own strengths and weaknesses (Hendriana et al., 2014, 2017b; Hendriana, 2017a).

### 8. Self-ability

The indicators of self-ability include being able to deal with the problems that arise, being convinced in one's success, having the courage to take on challenges, having the courage to take risks, being aware of one's strengths and weaknesses, being able to interact with others, and being resilient and persistent (Hendriana et al., 2017b).

#### 9. Self-appreciation

The indicators of self-appreciation include believing in one's abilities, being confident in oneself during communication, being confident in one's strengths and weaknesses, taking pride in the results that are achieved, and being confident that others are in need of one's self (Hendriana, 2017a).

## Ethnomathematics

Ethnomathematics etymologically uses three Greek roots, namely ethno for a natural or sociocultural group, mathema for explaining and learning, and tic for ways, arts, and techniques (D'Ambrosio, 2016). Thus, ethnomathematics is defined as a program that learns and combines the mathematical ideas, ways, and techniques practiced and developed by diverse sociocultural or members of cultural groups. It emphasizes exploring the science developed by a cultural group on ideas, ways, and techniques to develop a knowledge system. It acknowledges that there are a variety of ways of doing mathematics that are developed by numerous cultures (D'Ambrosio, 2001). The main foundation that underlies ethnomathematics is the awareness of the vast range of ways to have knowledge of and to do mathematics in relation to values, ideas, notions, ways, techniques, and practices in a diversity of cultures (D'Ambrosio, 2001; D'Ambrosio, 2007). Ethnomathematics represents the ways in which a multitude of cultures make their own mathematical realities through mathematical ideas, notions, ways, techniques, and practices that are used in daily life.

Ethnomathematics is a program to create new knowledge, so it is a must for mathematicians to regard society as a whole, including in perceiving its constituent cultural dimensions (D'Ambrosio, 2016). This is to take into consideration the expectations of each member of society and the traditions that contain high moral values such as appreciation, tolerance, acceptance, awareness, dignity, integrity, and peace, among others. As a result, it allows to keep the ethics of the mathematics users in check, make them human, and prevent the use of mathematics as a basic instrument for conquest, colonization, subordination, absorbance, and even elimination of other civilizations.

Ethnomathematics is associated with the motif by which a certain culture (*ethno*) in its history develops steps to calculate, infer, compare, and classify techniques and ideas (tics) that enable the society within to model the natural and social environment and context to explain and understand mathematical phenomena (*mathema*) (Rosa and Orey, 2016). It is also easier to accept to indigenous peoples of certain populations as the contexts used are easier to access and reach, particularly to those living in rural and coastal areas. Mathematics in traditional practices become more interesting than formal mathematics, given that the latter is conducted in a rigid, cold fashion.

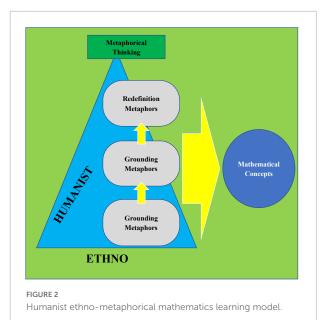
Ethnomathematics is dynamic, holistic, transdisciplinary, and transcultural (d'Ambrosio, 1985; D'Ambrosio, 2016). Its evolution will benefit academic mathematics, especially since in ethnomathematics the way in which mathematics is done is closer to real contexts and the mathematical agents are immersed in reality. Thus, it is deemed necessary to modernize resources that are rich in cultural heritage so that both ethnomathematics and academic mathematics can be well-positioned in the world today.

## Metaphorical thinking

The studies on metaphorical thinking in mathematics learning in Indonesia were pioneered by Hendriana's work (Hendriana, 2002) that concerned problem-posing in reciprocal teaching, which was preceded by an inquiry into peer-learning at the remediation stage. Globally, they were first introduced by Carreira (2001) through his work that was entitled "Where there's a model, there's a metaphor: Metaphorical thinking in students' understanding of a mathematical model", which was subsequently presented in Indonesia by Hendriana (2009). Back then, there was no single work that used metaphors in answering mathematical problems in the form of nonstandard answers as a treatment that could promote students' mathematical hard skills and soft skills. Thus, a learning design was established using several elements known as metaphorical thinking (Carreira, 2001) to improve hard skills (mathematical understanding and mathematical communication abilities) and soft skills (self-confidence) in students (Hendriana, 2009).

Metaphors are classified as connotations in the traditional rhetoric, that is, as a depiction that classifies the presence of a variety of meanings in word use and more precisely in the denotation process (Carreira, 2001). A metaphor is accounted for as the application of a name that belongs to something else proportionately by means of comparison as marked explicitly by a comparative theme. It then gives a unique character to the wholeness of the relationship between the explicit and implicit meanings of a concept. Simply put, a metaphor is said to be a comparison that bridges the relationship between literal meaning and figurative meaning (Ricoeur, 2002).

The fact of a part of a human thinking process and its understanding system is metaphorical (Ricoeur et al., 1977). Metaphors lie in their central role in understanding the



relationship between the language of human knowledge and the world of desire (Carreira, 2001). It is also fair to say that they are linguistic expressions whose meanings are out of the direct reach of the existing symbols as the intended meanings predicate the linguistic expressions (Ricoeur, 2002). Cognitive paradigms perceive metaphors as tools to conceptualize abstract concepts into more concrete concepts (Hendriana et al., 2018). Thus, metaphors are another way of expressing meanings that supposedly or actually are based on students' understanding and experience, as is illustrated in Figure 2.

At the other end of the spectrum, we can also see that a metaphor is a direct comparison of two things with different meanings, whether they are related or unrelated (van Poppel, 2020; Steines et al., 2021). Metaphors can open up a new horizon for one's concrete understanding, thereby able to promote students' communication ability in shedding light on difficult concepts through a combination of some concepts they are already familiar with.

The thinking process that makes use of metaphors in gaining an understanding of a concept can also be referred to as metaphorical thinking (Ricoeur et al., 1977). A metaphor starts off from a concept student have recognized all the way to another concept they have no knowledge of, or they are presently learning. It relies on some properties of the concept and object metaphorized. Metaphorical thinking in mathematics is used to make clear of one's thinking way that is related to his/her mathematical activity (Carreira, 2001; Hendriana et al., 2018). Abstract concepts that are organized through metaphorical thinking are stated in concrete things based on structure and reasoning way on the foundation of a sensorial-motor system called conceptual metaphor (Carreira, 2001).

Some of the metaphorical conceptual forms employed to construct a mathematical idea are as provided below (Ricoeur et al., 1977; Carreira, 2001; Hendriana, 2009):

- 1. *Grounding metaphors*, which are the basis for understanding mathematical ideas that are linked to everyday experience.
- 2. *Linking metaphors*, which establish a relationship between two matters by selecting, asserting, giving flexibility to, and organizing the characteristics of a primary topic with the support of a subsidiary topic in the form of metaphorical statements.
- 3. *Redefinition metaphors*, which redefine metaphors and select the most suiting one to the topic to be taught.

Metaphorical thinking in mathematics starts with modeling a situation mathematically, followed by giving meaning to the models through an approach from a semantic point of view (Ricoeur et al., 1977; Carreira, 2001; Hendriana, 2009). In mathematics learning, metaphor use by students are one way to link mathematical concepts to the concepts they are familiar with in daily life, in which case they express mathematical concepts in their own language to demonstrate their understanding of such concepts (Hendriana, 2009).

Carreira (2001) explains metaphorical thinking in a diagram form as is seen in **Figure 3**.

Some strategies in metaphorical thinking that may be helpful to students in understanding a mathematical topic are as provided below (Carreira, 2001; Hendriana, 2009):

- Use metaphors to illustrate a concept in steps including identifying first the main concepts to be taught, thinking of possible metaphors to illustrate the concepts, choosing one among several possible metaphors that is most suitable, and planning ways for discussing that metaphor/analogy to ward off confusion from the students. In this case, we must believe that the students have adequate knowledge and experience to think metaphorically.
- 2. Give students a chance to express their own metaphors. This is important to prevent misunderstandings that arise from the diversity in cultures and customs, which may lead also to differences in the means and foundations of student understanding in making an analogy of a topic. Then, give the students an opportunity to exchange analogies so they have a chance to discuss with one another.
- 3. Discuss the understanding foundation in metaphorical thinking by analyzing the reasons for which analogies/metaphors are chosen.
- 4. Compare the significances of the metaphors across various cultures.

Thus, metaphorical thinking can be defined as a thinking process to discern and communicate abstract concepts in

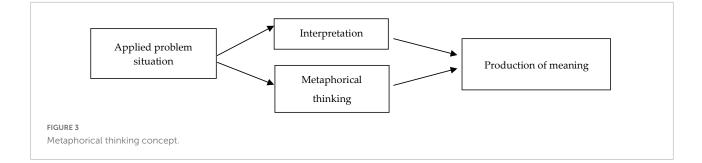
mathematics into more concrete concepts by comparing two things with different meanings.

Metaphorical thinking in mathematics is unlike metaphorical thinking in traditional contexts, with the differences lying in the conceptual understanding and in the application to solve the problems encountered (Hendriana, 2009). In the case of ordinary metaphorical thinking, students are required to describe a concept using the concepts they have formerly recognized without having to completing it in detail.

## Teacher's professional competences

The professional competences of a mathematics teacher cover the following: understanding of mathematics theories and mathematics learning theories; mastery of the implementation of mathematics theories and mathematics learning theories; mastery of the selection, formulation, designing, and delivery of mathematics theories and mathematics learning theories; and mastery of the principles, rules, and concepts of mathematics theories and mathematics learning theories in learning environment contexts (Hendriana, 2017b). A mathematics teacher must also possess some mathematics pedagogic competences, which include the following: understanding of the curriculum and development of mathematics learning plans; understanding of the approaches, methodologies, and techniques in mathematics learning and their implementation in the classroom; understanding of the implementation of mathematics learning and mathematics learning evaluation systems in the classroom; mastery of the skills to assess students' level of understanding and potential in mathematics learning; and mastery of the skills to conduct research in mathematics learning.

A teacher is expected to have some soft skills, which consist of personality and social competences (Caggiano et al., 2020). Personality competences here are comprised of 4 aspects: maturity, virtuous character, wisdom, charisma, and fitness to be a role model for students; understanding of the professional rights and obligations of a mathematics teacher; understanding of the professional tasks and functions of a mathematics teacher; and understanding of the impacts of educational innovations on the development of the nation's character. Meanwhile, the social competences a teacher must have include the following: upholding of norms, values, morality, religion, ethics, and professional responsibilities; effective communication ability; understanding of problems in mathematics education and contemporary mathematics; teamwork and rapid adaptation to the abilities needed in a work environment; mastery of educational interactions between teacher, student, parent, and society; understanding of consultative communication between mathematics education, regulator, and societal needs; and ability to use and apply information and communications technologies in mathematics learning.



To coordinate all the above mentioned, a teacher is demanded to have the ability to innovate through interesting, fun, and innovative mathematics learning activity designs based on students' ways of thinking and contributions over the course of the learning and teaching activities (Hendriana et al., 2019). Innovative learning is learning which is designed based on the needs, conditions, and innovative ideas of the students and is the product of their ways of learning to have better learning outcomes (Hendriana and Rohaeti, 2017). In innovative learning activities, it is expected that students be able to participate actively in various activities that are aimed to develop their skills and understanding with a focus on learning by doing (Hošková, 2010). The teacher is expected to be able to use a string of relevant learning aids and methods to motivate students, including the environment, as a source of learning to create interesting, joyful learning. On the other side, it is also demanded of the teacher to be able to encourage students to find their own ways to solve the problems presented, express their ideas both orally and in writing, and participate in creating and shaping a comfortable learning environment for themselves (Hendriana, 2012). As a result, students who serve as the learning subjects are able to actively develop their own knowledge through innovative learning that has been designed by the teacher. During the learning process, the teacher solely acts as facilitator.

There are several types of innovative learning that are capable of promoting students' hard skills, including the following: indirect learning that can positively contribute to students' creative and critical thinking abilities; learning using games that are based on flash macromedia software that can improve students' mathematical reasoning and communication skills; problem-based learning that can stimulate students' mathematical reasoning, understanding, and communication skills; and discovery learning that can exert a positive effect on students' mathematical communication ability and achieved understanding (Hendriana, 2012, 2017a). Innovative learning of these types can also inspire several soft skills in students, including mathematical disposition and learning independence.

There is another sort of innovative learning that puts a stress not on mechanistic learning, but on humanist learning, in which case mathematics learning has formerly been productive merely in the cognitive aspect but now its productivity has expanded into the affective aspect as well. This learning type is believed to be capable of tackling the hard skill and soft skill problems frequently encountered by students. Learning of this type is also known as humanist mathematics learning (Hendriana, 2012, 2017a).

Humanist mathematics learning is a preliminary essential in giving stimulus to students, hence reducing negative responses to mathematics (Hendriana, 2002, 2009, 2012, 2017a; Hendriana and Rohaeti, 2017). Through pleasure in mathematics, it is hoped that the habit of thinking mathematically can be welltrained. Therefore, students will learn to appreciate and have fondness for mathematics as they have developed a belief on how to formulate and use mathematical means should a need arise.

Such a learning system will be oriented toward the students rather than the learning materials or the learning process and outcomes. Hence, strong will be the students' conviction that what they master is not only the materials, but also how they learn those materials meaningfully. There are several things that can be performed in humanist mathematics learning to give impacts and meanings to students, including the following (Hendriana, 2009):

- 1. Teacher's creativity in devising strategies around the applied curriculum. Not only can he/she teach according to the curriculum implementation direction and technical direction, but the teacher can also develop some strategies around the curriculum to sort and select the materials important to the students and deliver the materials sustainably, even also discard materials of no import.
- 2. Teachers' innovations in learning. The variety of learning methods hold a pivotal role in attracting students' interest in mathematics learning. Innovations of various learning methods according to the teaching materials will keep students from boredom when participating in learning.
- 3. Relating the teaching materials to events or happenings in daily real life. Demonstrating the link between mathematics to realities in life will make mathematics learning more meaningful to students. They may apply the concepts or theories they have learned to solve real problems that they encounter in their daily life, making mathematics more humanist in nature.

Conducting mathematics learning in a humanist manner will surely allow students to take pleasure and interest in learning mathematics. They will put some efforts to let the joy they take in mathematics to grow, and it is expected that such efforts would lead to excellent achievements. Although it is impossible to force all the students to take a liking in mathematics, it is still deemed necessary to motivate them for them to be able to master the concepts in mathematics. Humanist mathematics learning in itself is characterized by the following (Hendriana, 2009):

- it places students as inquirers as opposed to just recipients of facts and procedures;
- 2. it allows students to interact with each other in order to understand and solve problems in depth;
- it accustoms students to learn to solve problems in a variety of ways;
- 4. it presents open-ended problems that are interesting and challenging;
- 5. it uses evaluation or assessment techniques that are not based solely on procedures;
- 6. it develops understanding and appreciation of studentgenerated mathematical ideas;
- 7. it helps students develop self-confidence, independence, and curiosity; and
- 8. it presents learning by drawing a link to daily life.

## Problem-posing ability

In the practice in the field, teachers typically face difficulties in eliciting expected responses despite the various stimuli that have been created. Of the questions posed by teachers regarding the materials delivered, Widodo and Pujiastuti (2006) states, approximately half are close-ended questions that require only short answers out of memory rather than understanding. In relation to this statement, Japa (2014) recommends that teachers should pose problems that are more of the open-ended type in the learning process. This is necessary to allow students to express their opinions in the form of both questions and statements (as a stimulus form), which is expected to inspire responses out of other students that may be used as alternative solutions.

With, one of the weaknesses that students have in presenting problem solutions is the lack of questioning ability on the part of the students themselves. The questioning ability in the mathematics subject will be considered excellent when it covers aspects of quality, relevance, language, and frequency. Widodo and Pujiastuti (2006) opines that, in an analysis, questions will be classified based on certain considerations. The first consideration concerns academic and non-academic questions. While academic questions are related to the subject's materials, whether discussions on them have been complete or still underway, non-academic ones are questions that are related to social, organizatory, and material aspects, all of which are nonacademic. The second consideration is about closed- and openended questions. The former are questions that elicit limited responses and typically are direct toward a single conclusion, whereas the latter are questions that invite multiple answers. The last consideration is associated with cognitive processes that fall into Bloom's taxonomy, namely, to remember, to understand, to apply, to analyze, to evaluate, and to create.

Regarding mathematical questioning ability, Hendriana et al. (2017a) define it as the ability to ask mathematical questions, or one's ability to pose problems, out of a statement considering the relationship between the questions to the questions' contexts, in which case the questions posed are either routine or non-routine, close-ended or open-ended. The categorization of mathematical questioning ability is broken down as follows:

- 1. The mathematical questioning ability is categorized as very low if the questions one raises are incompatible with the contexts of the questions and if the questions are considered routine, close-ended questions.
- 2. The mathematical questioning ability is categorized as low if the questions one raises are incompatible with the contexts of the questions and if the questions are considered routine, open-ended questions.
- 3. The mathematical questioning ability is categorized as moderate if the questions one raises are compatible with the contexts of the questions, but the questions are routine and close-ended.
- 4. The mathematical questioning ability is considered as high if the questions one raises are compatible with the contexts of the questions, but the questions are nonroutine and close-ended.
- 5. The mathematical questioning ability is considered as very high if the questions one raises are compatible with the contexts of the questions and if the questions are nonroutine and open-ended.

Questioning ability development in mathematics is encouraged for students to have the courage to express opinions. Affectively speaking in mathematics learning at school today, students tend to have little courage to say their opinions in their own language. On this, Hendriana et al. (2018) state that the mastery of the mathematical questioning ability has not reached a favored mark in the indicator of non-routine, open-ended problem-posing when seen from the mathematical courage perspective.

This review attempts to combine the said ability with the metaphorical thinking approach. The aim is to enable all students to convey their opinions throughout the learning process, irrespective of the questions given, whether the opinions have been expressed faithfully in appropriate mathematical terms or just through metaphors that describe them. Students would initially find it difficult to express their opinions due to a lack of self-confidence, mathematical anxiety, and other negativities, but a little bit of "cognitive conflict" in relation to their activeness in saying their opinions during the learning process will then be introduced. Afterward, a "reward" is given to those who are able to let their opinions heard during the learning process, allowing the learning process to run joyfully. Such is supported by a learning atmosphere that is built to appreciate every opinion that comes out of the students, making them feel appreciated while they are learning.

Conducting learning in such a way would make students who initially are shy, anxious, and lacking in self-confidence during the learning process find more joy. After taking some comfort, students will gradually become more accustomed to such a way of learning. A few lecturers in an educational institution in West Java, for instance, have conducted, and made it a culture to conduct, learning activities in this manner. For a positive result to be obtained, one may first face some difficulties. Some force and rewarding may be required for a habit to be established. Once a good habit is formed, those around us my feel some positive impacts. It is not an impossibility for the habit to evolve into a culture to which we belong. This will lead us to what we know as humanist, ethno-metaphorical mathematics learning, a pleasurable approach to mathematics learning to students that is based on the culture of thinking metaphorically and appreciation of every student-made contribution during the learning process in the classroom.

## Conclusion

Humanist ethno-metaphorical mathematics learning that has a basis on ethnomathematics and metaphorical thinking is one of the learning approaches that offer a promise in mathematics learning to improve students' hard skills and soft skills, thereby helpful for students to face the challenges and changes in this era. Learning is conducted in the form of conceptual metaphors following humanist mathematics learning strategies. In learning of this sort, the teacher takes an active role as facilitator who must have soft skills that consist of personality competences and social competences. He/she must also could design learning in an innovative, studentcentered fashion for the students to be able to take meanings of the mathematics learning and find their own ways to solve the mathematical problems they are presented with as well as to express their own ideas. Therefore, this situation may help students improve their hard and soft skills during the learning process.

Numerous constraints must be considered when interpreting the findings of this integrative literature review study. This study reviews a variety of sources to develop a theoretical framework for a promising learning model in mathematics education, emphasizing ethnomathematics and metaphorical thinking. This promotion model is based on metaphorical thinking, indicators of soft and hard skills, and ethnomathematics learning syntax. As a result, literature that does not discuss the other literature must be excluded from this study and its model and its implementation in the learning process. However, this study thoroughly examined the literature that supports the constructed learning model.

Finally, we have seen an increase in alternative learning approaches since the pandemics began 2 years ago. Most of them operate optimally in areas with robust internet infrastructure, enabling proper online course-based blended learning. There are—however, few studies focusing on rural areas with limited internet connectivity but a rich cultural heritage. As a result, the findings of this study can be used to implement humanist ethnometaphorical mathematics learning as a promising alternative learning approach for improving mathematical skills in a city of culture through direct teaching and learning. The future researcher can conduct additional research through the collection of empirical data during the learning process. This is to demonstrate the model's efficacy in mathematics teaching and learning activities.

## Data availability statement

The original contributions presented in this study are included in the article/supplementary material, further inquiries can be directed to the corresponding author/s.

## Author contributions

HH: conceptualization. HH and RP: methodology and writing—original draft preparation. MR and ER: validation and data curation. WH and RP: formal analysis. HH, WH, and RP: investigation and writing—review and editing. MR, ER, and WH: visualization. All authors have read and agreed to the published version of the manuscript.

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# **Conflict of interest**

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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