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The Association of Strategy Use and Concrete-Operational Thinking in Primary School

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Concrete-operational thinking depicts an important aspect of cognitive development. A promising approach in promoting these skills is the instruction of strategies. The construction of such instructional programs requires insights into the mental operations involved in problem-solving. In the present paper, we address the question to which extent variations of the effect of isolated and combined mental operations (strategies) on correct solution of concrete-operational concepts can be observed. Therefore, a cross-sectional design was applied. The use of mental operations was measured by thinking-aloud reports from 80 first- and second-graders ($N = 80$) while solving tasks depicting concrete-operational thinking. Concrete-operational thinking was assessed using the subscales conservation of numbers, classification and sequences of the TEKO. The verbal reports were transcribed and coded with regard to the mental operations applied per task. Data analyses focused on tasks level, resulting in the analyses of $N = 240$ tasks per subscale. Differences regarding the contribution of isolated and combined mental operations (strategies) to correct solution were observed. Thereby, the results indicate the necessity of selection and integration of appropriate mental operations as strategies. The results offer insights in involved mental operations while solving concrete-operational tasks and depict a contribution to the construction of instructional programs.

Keywords: strategy use, thinking aloud, cognitive development, process data, primary school

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INTRODUCTION

Concrete-Operational Thinking

The concrete-operational stage depicts an important step in the cognitive development of children (Piaget, 1947). According to Piaget, thinking in this stage is characterized by logical operations, such as conservation, reversibility or classification, allowing logical reasoning. These mental acts cannot be applied in hypothetical situations and are still limited to concrete situations. Therefore, concrete-operational thinking is examined using specific tasks depicting concrete-operational concepts. More recent research indicated that it might not be appropriate to assume a concrete-operational stage, but to examine concrete operations as specific and independent (Berzonsky, 1971; Winkelmann, 1975; Lourenco and Machado, 1996).

Nonetheless, concrete-operational thinking represents an important prerequisite of formal thinking (Inhelder and Piaget, 1958; Powell and Kalina, 2009). It is associated with a range of learning outcomes and academic achievement (Jordan and Brownlee, 1981; Hattie, 2009), such as maths fluency and maths achievement (*conservation ability*; Arlin, 1981; Cooper and Schleser, 2006; Ramos-Christian et al., 2008; Krajewski and Schneider, 2009; Wubbena, 2013; Lambert and Spinath, 2018), as well as reading comprehension and reading achievement (*conservation and classification ability;* Arlin, 1981; Cartwright, 2002; Colé et al., 2014; Cartwright et al., 2017). At the same time, differences in the ability of concrete-operational thinking become clear, as e.g., students with learning disabilities show lower levels of concrete-operational thinking than their peers (Wember, 1986; Riley, 1989; Fakouri, 1991).

The aforementioned differences led to questions concerning the trainability of concrete-operational thinking (Brainerd, 1983). Piaget himself, emphasized the constructivist nature of cognitive development and therefore pays less importance to the influence of third persons (Marchand, 2012). In contrast to this, more recent research highlights the importance of interaction between child and educators for developing cognitive competencies (Vygotskij and Cole, 1981; Fischer and Bidell, 2007). In this sense, when it comes to promoting the ability to solve concrete operational tasks, instructing the use of specific strategies has proven to be a successful means (Brainerd, 1983).

Strategy Use

Strategies are commonly specified as "goal-directed mental operations that are aimed at solving a problem" (Bjorklund, 2012, 265). Furthermore, they can be described as a sequence or a pattern of interdependent mental operations (Pressley and Hilden, 2006). The use of strategies is conscious, controllable and effortful (Pressley and Hilden, 2006; Shaffer and Kipp, 2010). Level and quality of strategy use is associated with a broad range of learning outcomes, e.g., achievement in reading (Hong-Nam et al., 2014; Cromley and Wills, 2016), science (Akyol et al., 2010; Deekens et al., 2017) and mathematics (Torbeyns et al., 2006; Askeland, 2012).

At the same time, similarly to differences in the ability of concrete-operational thinking, differences in the quality and quantity of strategy use arise. In addition to age-related differences (Siegler, 1996), students of the same age differ in their strategy use. In particular, children with learning disabilities show limited and inefficient strategy use in comparison to their peers (Pressley and Levin, 1987; Reid and Lienemann, 2006; Bosson et al., 2010). These interindividual differences are demonstrated in terms of *strategy utilization* and *strategy production deficiencies* (Miller and Seier, 1994; Shaffer and Kipp, 2010; Bjorklund, 2012). *Utilization deficiency* defines an absent benefit of appropriate strategies, whereas *production deficiency* is described as the failure to produce a strategy spontaneously (Miller and Seier, 1994; Schwenck et al., 2007; Bjorklund, 2012). Internal causes of such deficiencies include poor working memory, low metacognitive and declarative knowledge, and low intelligence (Clerc and Miller, 2013). The observed deficiencies can consequently be due to a range of factors, for example not having knowledge about the tasks and relevant strategies, not applying appropriate known strategies, not being able to adapt or integrate multiple known strategies, failing to inhibit earlier strategies, or not being motivated to use strategic approaches (Miller and Seier, 1994; Pressley and Hilden, 2006; Shaffer and Kipp, 2010).

Interventions aimed at promoting strategy use have proven to be successful (Belmont, 1989; Klauer, 2001; Pressley and Hilden, 2006; Reid and Lienemann, 2006; Bosson et al., 2010; Krawec et al., 2013). Strategy instruction proves to be beneficial to students' learning, as students show a higher level of performance, strategic activity and transfer (Fuchs et al., 2003; Bosson et al., 2010; Swanson et al., 2015). However, one important prerequisite for developing strategy instruction are insights into the mental operations involved while solving a specific task. Klauer (2001) argues that the analysis of the mental operations and strategies involved in successful problem solving is a key issue in developing instructional programs.

Strategy Use and Concrete-Operational Thinking

Fakouri (1991) describes difficulties of students with learning disabilities in solving concrete-operational concepts as a consequence of relying on the wrong strategies during problem-solving. Previous research has therefore addressed the question of the trainability of concrete-operational thinking by instructing cognitive strategies (Brainerd, 1983). However, insights in the involved problem-solving processes are a prerequisite for designing such strategy instruction. In addition to the described processes by Piaget (1947), more recent studies focused on describing the involved strategies in variations of concrete-operational thinking (e.g., *conservation*: Bellin, 1965; *seriation:* Fragaszy et al., 2002; *classification:* Freund et al., 1990; *conservation:* Kospentaris et al., 2011 *class inclusion:* Siegler and Svetina, 2006). These studies differ in various ways, such as the age of the participants, the specific tasks that were used and the grain of the identified strategies. Freund et al. (1990) could show, that 3- and 5-year-old children applied different strategies in problem-solving. At the same time, these strategies were unequally associated with the correct solution of classification tasks. Similarly, Fragaszy et al. (2002) describes a less frequent use of strategies during the wrong solution of seriation tasks of 1- and 2-year-old children. Chen et al. (2016) examined processing strategies on matrix completion strategies and could find a more frequent use of processing strategies of high-performing 5- and 6-year-old problem-solvers. These findings indicate that the correct solution of concrete-operational concepts might be explained by qualitative aspects as well as the frequency of the used strategic operations. These differences in strategy use can be expected, based on the described observations of utilization and production deficiencies (Miller and Seier, 1994; Bjorklund, 2012) and need to be considered when designing strategy instructions. So far, an overview about differences in the effectiveness of strategy use of primary-school children are still lacking. Insights in this age group might be of particular interest, as the transition from pre-operational to concrete-operational stage occurs in early primary school (Bjorklund, 2012). Börnert and Wilbert

(2015) explored the strategy use of first and second graders when solving concrete-operational concepts. At the same time, this study did not emphasize differences in the effects of strategy use in the problem-solving processes.

Research Questions

Summarized, the use of specific strategies is of relevance for solving tasks representing concrete-operational thinking. Particularly children with learning disabilities might profit from the instruction of strategies, as they show limited and inefficient strategy use. One prerequisite for instructing strategies involves insight into the individual solving processes of a specific task. So far, information about successful strategies applied with regard to concrete-operational thinking is lacking. As strategies are defined as sets of mental operations, it seems necessary to gain insights into the entire strategy as well as into the components of strategies. The effectiveness of strategy use might be a consequence of knowing appropriate mental operations and combining them in an appropriate manner. Therefore, knowledge of the mental operations involved might contribute to the development of effective instruction. In the present study, we focus on shedding light on the influence of specific mental operations as well as of pattern of these operations (strategies) on the solution of tasks depicting concrete-operational thinking in primary school. We aim to identify successful strategies for approaching the relevant tasks, highlighting the differences in the effects of different strategies. Instead of focusing on differences between participants, we will emphasize differences between problem-solving processes. We assume that differences among the particular single or sets of mental operations applied during problem-solving (i.e., strategies) affecting the probability of successfully solving the tasks do exist (**Figure 1**). In summary, we pose the following three research question:

1. Is there a variation in the contribution of different mental operations to the successful solution of concrete-operational tasks?

It was hypothesized that there are different effects of the observed mental operations on the correct solution in the concrete-operational tasks.

As strategies are defined as a set of mental operations (Pressley and Hilden, 2006); this question can be extended to differing effects of patterns of mental operations on the solution of concrete-operational concepts.

2. Are certain strategies, hereby defined as a set of mental operations, observed more frequently than could be expected based on the individual frequency of the isolated mental operations?

We thereby hypothesized that certain patterns of strategic operations can be observed more frequently than would be expected. The expected frequency is calculated on the frequencies of the use of the individual, isolated mental operations, which depict the components of the respective patterns.

3. Are the identified commonly used *sets of mental operations* (strategies) associated with the correct solution of the tasks?

It was hypothesized that certain patterns of strategic operations are associated with the correct solution of the tasks.

Answering these research questions, might contribute to the identification of patterns of mental operations, being particularly helpful in solving the described tasks. Moreover, the results might offer insights into pitfalls of the use of mental operations, leading to an incorrect solution of the task. In addition to offering insights into these aspects, we want to provide a valid approach for targeting questions concerning the identification of cognitive processes, e.g., strategic activity, in children when developing instructional or diagnostic programs aimed at the promotion of strategic activity.

METHODS

Participants

Eighty first- and second-grade students (76% second grade; 24% first grade) from seven primary schools in Brandenburg (federal state in Germany) participated in this study ($N = 80$). Just over half of all participants were female (54% female, 46% male). The average age of the participants was $M = 7.1$ ($SD = 0.6$) years. Six participants had a migration background (due to information supplied by the teacher). The classroom teachers reported no special educational needs for any participant.

Measures

Concrete-operational concepts were assessed using the subscales *conservation*, *classification*, and *sequences* of the

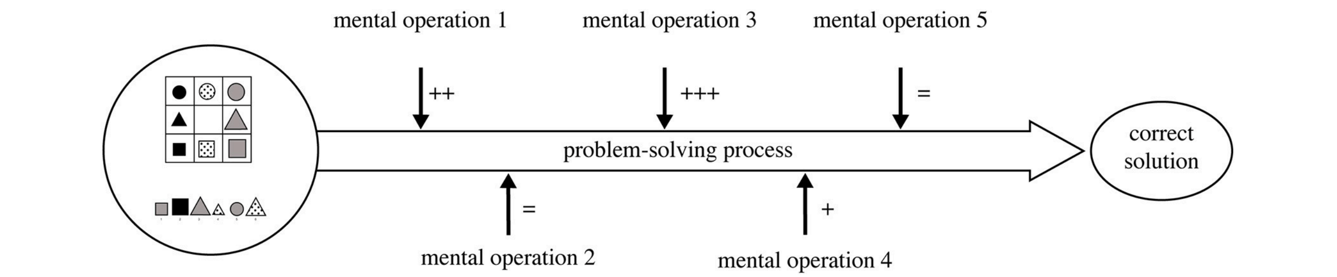
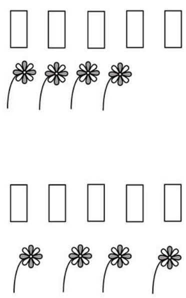
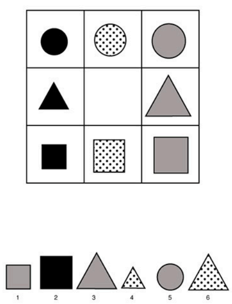
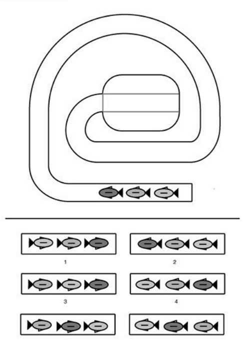


FIGURE 1 | Differing effects of isolated mental operations on correct solution.

TEKO ("Testbatterie zur Erfassung kognitiver Operationen"; Winkelmann, 1975), (see **Figure 2** for task examples). The restest reliability of the implemented subscales ranges between 0.68 < rtt < 0.85. (Winkelmann, 1975). As described by Borst et al. (2012), number conservation tasks consist of two rows with an equal number of objects but which differ in the length of the rows. The tasks used in our study differ in the content of the rows as well as the numbers of objects. Classification tasks are similar to matrix completion tasks and take the form of boxes with 3 × 3 rows containing both a horizontal and a vertical property. One of the fields is empty and the participants are asked to determine the correct answer. These tasks require multiple classification processes. In sequences tasks, a series of three different balls pass through a winding tube and end up in a box. The participants need to identify the correct series of balls. Relational thinking is required to complete the tasks.

Subscale	Mental Operation	used in % of cases
Conservation of numbers		
	O1 Counting of elements	*59 %*
	O2 Matching of elements	*38 %*
	O3 No addition or subtraction of elements	*9 %*
	O4 Recognizing identity (Conservation)	*68 %*
	O5 Reversing Operation	*3 %*
Classification		
	O1 Recognizing pattern	*74 %*
	O2 Checking vertical column	*30 %*
	O3 Checking horizontal row	*32 %*
	O4 Integration of vertical and horizontal property	*15 %*
	O5 Checking answer options	*20 %*
Sequences		
	O1 Recognition of initial series of balls	*52 %*
	O2 Tracking the balls	*41 %*
	O3 Recognition of permanence of central ball	*3 %*
	O4 Recognition of rotation of the balls	*24 %*
	O5 Dividing series into single balls	*56 %*

Due to copyright reasons, example tasks depict similar but not the original tasks. The content of the sequences tasks used is a row of balls instead of a row of fish.

FIGURE 2 | Overview of the subscales and use of related strategic operations in conservation, classification and sequences tasks.

Assessing strategic activity has proven to be a challenge, as it is referring to inner thought processes and can consequently not be observed directly. In research practice, different approaches are used to overcome this difficulty. Among others, collecting verbal data has proven to be promising in assessing inner thought processes (Ericsson and Simon, 1993). Especially on-line methods, summarized under the umbrella term of thinking aloud are discussed as possible methods allowing insights in thinking-processes (e.g., Bannert and Mengelkamp, 2008). Ericsson and Simon (1993) differentiate between different levels of thinking aloud (level 1: talk aloud; level 2: think aloud; level 3: reflect-when-prompted). At the same time, level 3 verbalizations seem to be more appropriate in the context of research with young children, as younger children might have difficulties verbalizing and may profit from prompts reminding them to verbalize. Level 3 verbalizations, however, has an impact on the involved cognitive processes as participants need to integrate the present information (Ericsson and Simon, 1993; Bannert and Mengelkamp, 2008). In contrast to the positive aspects of thinking-aloud methods, major critique arises concerning the completeness of verbal protocols. However, strategy use is mostly conscious, some aspects of problem-solving process might remain non-conscious and can therefore not be verbalized (Kihlstrom, 1987). This problem is supported by observations that some thinking activity is not expressed verbally, indicated by facial expression, non-verbal behavior or mumbling (Schellings et al., 2013). These aspects need to be considered in the transcription and coding process of the recorded verbal data (see Transcription and Coding). Nonetheless, thinking-aloud is a valid approach in assessing strategic activity.

In the light of the aforementioned information, strategy use was assessed using the reflect-when-prompted approach (Ericsson and Simon, 1993). Verbal reports are consequently transcribed and coded (see **Figure 2** for an overview on the coded mental operations).

To control for the influence of intellectual abilities, cognitive abilities were measured using the CFT 20-R (Culture Fair Intelligence Test; Weiß, 2006). An adequate level of retest reliability (0.96) as well as of external validity (0.60–0.75) and construct validity are reported (Weiß, 2006). In addition, classroom teachers were asked to rate the students' current level of performance.

Procedure

Information on the study was sent out to 35 randomly selected schools, resulting in the participation of seven schools. Written informed consent of a parent or legal guardian was required for the children's participation in the study. The decision as to which and how many first- and second-grade classes should participate per school was made by the school representatives. The study was approved by the Federal Ministry of Education of Brandenburg (approval criteria: compliance with data protection regulations and educational relevance of research). Data collection was divided into two stages. In stage one, the intellectual abilities of all students were assessed using the CFT 20-R (Weiß, 2006). In stage two, students were interviewed individually and asked to solve concrete-operational tasks. In addition, students were instructed to verbalize their problem-solving process. The instruction was the following:

> *We are interested in how children solve these puzzles. Since we are not able to see what you are thinking and what is going on in your head, we need your help. Maybe you could say out loud every thought that comes to your mind while solving these tasks. I know that this is very difficult and even I find it hard to do that at times, but maybe you can do it. To support you, I will remind you to tell me what you are thinking. To get used to this, we are going to start with some example tasks.*

As mentioned in the instruction, two tasks were solved in advance to familiarize the participants with the process of verbalizing. These tasks were taken from the *Nobody is as smart as me* program ("Keiner ist so schlau wie ich"; Marx et al., 2009), a German cognitive strategy training that addresses cognitive processes similar to the tasks of interest in this study.

Thereafter, the students solved three tasks from six subscales of the TEKO (Winkelmann, 1975), a German test battery assessing mastery of concrete-operational concepts. However, only the subscales *conservation of numbers*, *classification* and *sequences* were integrated in this study. These subscales are central concepts of the concrete-operational stage (Winkelmann, 1975) and are prerequisites to logical thinking (Woolfolk, 2014). Every task was instructed separately and the students were asked to verbalize their thoughts. If students struggled in the verbalization process, they were prompted to verbalize and were asked more specifically about their cognition.

All interviews were administered in a small room inside the schools during regular classes and recorded on video. Data were collected by the first author and pre-service teachers who were instructed in all procedures and instruments.

Transcription and Coding

In a first step, the recordings of the interviews were transcribed using the software MAXQDA 12 (VERBI Software, 2014). The average length of the videos was 18.01 min. Transcription was administered by three different student assistants after they were instructed in the transcription process and rules. In addition to verbal information, the students were asked to transcribe non-verbal activities of the participants (e.g., finger pointing, counting on fingers, and tilting or shaking the head).

In a second step, the transcripts were coded on the task level. Two external raters were invited for coder training. Following a three-step procedure proposed by Syed and Nelson (2015), a coding manual was introduced, randomly drawn sample data were provided for practice, and the coders' questions were clarified. The coding scheme contains mental operations used in the problem-solving process (see **Figure 2**). Following the definitions of strategies by Pressley and Hilden (2006) as well as Bjorklund (2012), operations are in this context defined as components of a strategy. Consequently, the used strategy is depicted by a pattern of coded operations. In this sense, a strategy, e.g., for solving classification tasks, might consist of multiple operations, such as e.g., firstly checking the columns, secondly checking the rows, thirdly combining both properties and eventually checking the answer options.

The coding scheme was developed inductively and validated by the authors in a pre-study (Börnert and Wilbert, 2015). In the pre-study, utterances of single students were assigned to a pre-existing or new category. During the development process, possible categories depicted either cognitive and task-specific or metacognitive operations (planning, self-control, self-reflection). In the study at hand, however, no metacognitive activity could be observed. The described operations are consequently specific for each subscale and were assigned only once for each task (operation used: yes/no). The category scheme consisted of the category, a description of the respective category and exemplary data.

To assess interrater agreement, 20% of the total recordings were coded by two raters until adequate interrater reliability was documented in the first coding step. Interrater agreement was fair to good for the reliability codings (two raters × transcripts of eight children ≈ 20% of total sample) (Cohen's kappa: $K = 0.45$–0.62; percentage agreement: $P_A = 78$–91). Afterwards, half of the remaining transcripts were assigned to each rater to complete the coding process.

Design

Data analyses focused on the task level instead of the subject level, that is, a data case consisted of one task rather than a person. Therefore, the total sample for data analyses is $N = 240$ (80 children × 3 tasks/subscale). The analyses were performed using the software *R* (R Core Team, 2016) and the packages *lme4* (Bates et al., 2015) and *confreq* (Heine et al., 2015). In a first step, the data were analyzed on a descriptive level. In a second step, the research questions were addressed through different approaches.

Generalized Linear Mixed Model

To address the first research question regarding the individual contributions of single operations to the probability of finding the correct solution, we performed generalized linear mixed models (Cox, 1958) for all three subscales using a maximum-likelihood estimation method and logistically distributed error term.

The *correctness of the problem solution* was the outcome variable (dichotomous) and the *strategic operations* (each dichotomous: yes or no) were the predictor variables. In order to take care of the nested data structure (tasks in persons), multilevel models with random intercepts were applied. Therefore, differences among the participants in problem solving were taken into account. To estimate model validity, we compared the final models with candidate models with gender at the person level and a random intercept as predictors.

Configuration Frequency Analysis (CFA)

To answer the second and third research questions examining specific effects of combinations of mental operations, we chose a two-step approach. In a first step, we performed configuration frequency analysis (CFA) (Lienert, 1969). *Types* identify patterns or configurations of variables which occur more often than expected, whereas *antitypes* identify patterns that occur less often than expected (Stemmler, 2014). In a second step, we performed χ^2-tests to examine whether the identified patterns were associated with correct solution of the task.

RESULTS

Descriptive Statistics

An overview of the predefined mental operations can be found in **Figure 2**. For conservation tasks, the use of individual mental operations ranged between 3% (O5: *reversing operation*) and 68% (O4: *recognizing identity*). Only two mental operations were used in more than half of the verbalized problem-solving processes (O4: 68%; O1: 59%). In contrast, the mental operations O2 (*matching of elements*), O3 (*no addition or subtraction of elements*) and O5 were only used in less than a third of all problem-solving processes (O2: 37%; O3: 11%, O5: 3%). The results are similar for the subscale classification. Only mental operation O1 (*recognizing pattern*) was used in more than half of all relevant tasks (74%), while operations O2 to O5 were only applied in less than a third of all cases. A similar pattern can be found for sequences tasks. Mental operations O1 (*recognition of initial series of balls*: 52%) and O5 (*dividing series into single balls*: 56%) were used in more than half of the tasks, whereas operation O3 (*recognition of permanence of central ball*) was only applied in 3% of all tasks.

The average number of operations used per task in each subscale differs slightly between sequences, classification, and conservation tasks ($1.75 < M < 1.79$). Stronger differences become clear regarding the number of mental operations used per task within the same subscale ($0.63 < SD < 0.90$).

In addition, the rates of correct solution of the tasks vary across all subscales. The highest percentage of correct solutions can be found for the conservation tasks (84%), followed by classification (71%) and sequences tasks (61%). The sum of mental operations used per task is weakly associated with the correct solution ($0.19 < r < 0.29$).

The rates of correct solution of the three different tasks are only weakly associated ($0.07 < r < 0.36$). Similarly, the number of mental operations used per task is only weakly associated between the three tasks ($0.11 < r < 0.23$).

Research Question 1

Generalized mixed linear models were used to answer the first research question addressing the influence of *mental operations* on *correctness of solution* of the task (see **Table 1**). Intellectual abilities significantly predicted *correctness of solution* (*Conservation*: $ß = 0.075$, Std. Error: 0.023, $p \leq 0.01$; *Classification*: $ß = 0.089$, Std. Error: 0.018, $p \leq 0.01$; *Sequences*: $ß = 0.098$, Std. Error: 0.022, $p \leq 0.01$). At the same time, *intellectual abilities* only significantly predicted the *use of mental operations* in classification and sequences tasks (*Classification*: $ß = 0.015$, Std. Error: 0.006, $t = 2.34$, $p \leq 0.05$; *Sequences*: $ß = 0.019$, Std. Error: 0.008, $t = 2.53$ $p \leq 0.05$).

However, as this article focuses on the differences in strategic activity, these main effects do not preponderate. In addition, we controlled for the influence of gender, age, and school on the probability of *correctness of solution*. No significant main effects were identified. Therefore, the described models are reported without including *school*, gender, age or *intellectual ability* as covariates.

All the models described (see **Table 1**) show greater model fit in comparison to the candidate models considering only gender and *subject*, and are consequently more likely to minimize information loss [Model I Conservation: AIC = 203.18 $\Delta_{AIC} = -12.32$, $\chi^2_{(2)} = 16.23$. $p \leq 0.00$; Model II Classification: AIC = 289.91 $\Delta_{AIC} = -4.72$, $\chi^2_{(4)} = 12.71$. $p \leq 0.05$; Model III Sequences: AIC = 308.99 $\Delta_{AIC} = -5.90$, $\chi^2_{(4)} = 13.90$. $p \leq 0.01$]. Differences regarding the isolated contribution of individual *mental operations* to finding the correct solution become clear.

TABLE 1 | Fixed effects of generalized linear mixed models describing the association of correct solution as a criterion and mental operations as predictors.

	OR[a]	ß	Std. Error	*z* value	*p*
I: Conservation[b]					
(Intercept)	1.79	0.58	0.49	1.18	0.23
O1 *Counting the elements*	4.00	1.39	0.51	2.68	<0.01
O2 *Matching*	0.95	−0.04	0.47	−0.09	0.92
O4 *Recognizing identity*	2.94	1.07	0.44	2.42	<0.05
Model fit: $\chi^2_{(2)} = 16.23$. $p \leq 0.00$; AIC = 203.18 $\Delta_{AIC} = -12.32$; ICC = 0.08; random intercept: SD = 0.93, $R^2_{Marginal}$ [c] = 0.14; $R^2_{Conditional}$ [c] = 0.32.					
II: Classification					
(Intercept)	0.75	−0.29	0.39	−0.72	0.47
O1 *Recognizing pattern*	2.74	1.00	0.34	2.88	<0.01
O2 *Checking vertical column*	1.88	0.63	0.38	1.65	0.09
O3 *Checking horizontal row*	1.89	0.63	0.38	1.67	0.09
O4 *Integration of vertical and horizontal property*	1.91	0.64	0.48	1.34	0.17
O5 *Checking answer options*	1.26	0.22	0.39	0.57	0.56
Model fit: $\chi^2_{(4)} = 12.71$. $p \leq 0.05$; AIC = 289.91 $\Delta_{AIC} = -4.72$; ICC = 0.08; random intercept: SD = 0.57; $R^2_{Marginal} = 0.08$; $R^2_{Conditional} = 0.16$.					
III: Sequences					
(Intercept)	0.61	−0.49	0.38	−1.33	0.18
O1 *Recognition of initial series of balls*	1.48	0.39	0.35	1.11	0.26
O2 *Tracking the balls*	2.11	0.74	0.36	2.08	<0.05
O3 *Recognition of permanence of central ball*	0.41	−0.89	0.91	−0.98	0.32
O4 *Recognition of rotation of the balls*	2.42	0.88	0.44	1.97	<0.05
O5 *Dividing series into single balls*	1.96	0.67	0.33	2.02	<0.05
Model fit: $\chi^2_{(4)} = 13.90$. $p \leq 0.01$; AIC = 308.99 $\Delta_{AIC} = -5.90$; ICC = 0.27; random intercept: SD = 0.68; $R^2_{Marginal} = 0.11$; $R^2_{Conditional} = 0.22$.					

[a]*OR, odds ratio.*
[b]*Operations O3 "No addition or subtraction of objects" and O5 "Reversing transformation" were excluded due to zero inflation.*
[c]*$R^2_{Marginal}$ and $R^2_{Conditional}$ were calculated using the R-package "MuMIn" (Bartón, 2016).*

In all described models, mental operations can be identified that have a positive effect on correct solution of the task. In conservation tasks (Model 1), it can be shown that the mental operation of *counting the elements* increases the probability of correct solution (O1; OR: 4.00; $p \leq 0.01$). Likewise, the mental operation *recognizing the identity* (O4) has a positive effect on correct solution (OR: 2.94; $p \leq 0.05$). In contrast, this effect cannot be found for *matching the elements* (O2; OR: 0.95; $p = 0.92$).

Similar results can be described for classification tasks (Model 2). The identified mental operations involved in solving classification tasks differ in their effect on the correct solution of the task as a dependent variable. *Recognizing a pattern* (O1) has a strong effect on correct solution (OR: 2.74; $p \leq 0.01$). At the same time, *Checking the vertical* (M2; OR: 1.88; $p = 0.09$) and *horizontal elements* (O3; OR: 1.89; $p = 0.09$), as well as *integrating both properties into a new element* (O4; OR: 1.91; $p = 0.17$), increase the probability of correct solution; however, these effects are not significant.

In the sequences tasks (Model 3), similar observations can be made. *Tracking the balls* (O2; OR: 2.11, $p \leq 0.05$), *dividing the series of balls into single elements* (O5; OR: 1.96, $p \leq 0.05$), and *recognizing the rotation of the balls* (O4; OR: 2.42, $p \leq 0.05$) each double the probability of a correct solution. In contrast, *recognizing the initial series of balls* has only few and non-significant effects on the probability of a correct solution (O1; OR: 1.48, $p = 0.26$). Using the mental operation of *recognizing the permanence of the central ball* (O3; OR: 0.41, $p = 0.32$) decreases the probability of correct solution.

Research Question 2

As the effects of the isolated operations depend on additional mental operations used, combinations or patterns of mental operations can be explored (see **Table 2**).

In *conservation tasks,* several patterns can be identified as types (i.e., occurring more frequently than expected based on the individual frequency of the isolated mental operations). The most frequent pattern co-occurring with the correct solution of conservation tasks is *Pattern 5* [$\chi^2_{(1)} = 19.86, p \leq 0.01, f_{(o)} = 73$]. This pattern comprises the operations *counting the elements* (O1) and *recognizing the identity* (O4). The frequency of *Pattern 4* is also above chance [$\chi^2_{(1)} = 16.04$, $p \leq 0.01$, $f_{(o)} = 36$]. It comprises *matching the elements* (O2) and, consequently, *recognizing the identity* (O4). However, *Pattern 4* is also identified as a type co-occurring significantly often with the incorrect solution of conservation tasks [*Pattern 1*; $\chi^2_{(1)} = 11.96, p \leq 0.01$, $f_{(o)} = 10$]. In addition, three further models were identified as types; however, the observed frequencies are lower than 10% of the total analyzed patterns and are therefore not explained in detail at this point.

In *classification tasks*, only one pattern of mental operations could be identified as a type leading to correct solution of the tasks. This pattern comprises explicitly the *checking of the horizontal row* (O3) and the *recognition of a pattern* (O1) [*Pattern 1*; $\chi^2_{(1)} = 19.44, p \leq 0.01, f_{(o)} = 34$].

TABLE 2 | Configural Frequency Analysis (CFA) and identified types in patterns of mental operations (strategies).

Task	Pattern (no.)	Task solved[a]	Pattern (O1-O5)[b]	$f^{c}_{(o)}$	$f^{d}_{(e)}$	Type[e]	χ^2	*df*	*p*
Conservation	1	0	0 1 0 1 0	10	3.51	+	11.96	1	<0.01
	2	1	0 0 1 0 1	2	0.05	+	73.33	1	<0.01
	3	1	0 0 1 1 0	10	3.72	+	10.62	1	<0.01
	4	1	0 1 0 1 0	36	18.69	+	16.04	1	<0.01
	5	1	1 0 0 1 0	73	43.58	+	19.86	1	<0.01
Classification	1	1	1 0 1 0 0	34	19.44	+	10.91	1	<0.01
Sequences	1	0	0 0 1 0 1	3	0.33	+	21.57	1	<0.01
	2	0	1 0 0 0 0	20	9.14	+	12.89	1	<0.01
	3	1	0 1 0 1 1	22	3.80	+	87.13	1	<0.01
	4	1	1 0 0 0 0	29	14.45	+	14.65	1	<0.01

[a]*0, wrong solution; 1, correct solution.*
[b]*0, strategic operation not used; 1, strategic operation used.*
[c]*$f_{(o)}$, observed frequencies.*
[d]*$f_{(e)}$, expected frequencies.*
[e]*+, identified as type.*

For the *sequences tasks*, two patterns of mental operations were identified as types co-occurring significantly often with a correct task solution. *Pattern 3* [$\chi^2_{(1)} = 87.13, p \le 0.01, f_{(o)} = 22$] comprises *dividing the series into single balls* (O5), *tracking the balls* (O2), and thereby *recognizing the rotation of the balls* (O4). In contrast, solely the *recognition of series of balls* (O1) is identified as an ambivalent pattern, which significantly co-occurs with correct solutions [*Pattern 4*; $\chi^2_{(1)} = 14.65, p \le 0.01, f_{(o)} = 29$] as well as incorrect solutions [*Pattern 2*; $\chi^2_{(1)} = 12.89$, $p \le 0.01$, $f_{(o)} = 20$] to sequences tasks.

Research Question 3

To examine the third research question as to whether the patterns of strategic operations, identified as types, are significantly associated with the correct solution of the task, additional chi-square tests were performed for each pair of patterns (see **Table 3**). This step was necessary because in configural frequency analyses (CFA) only observed frequencies are compared with the expected frequencies of the same pattern and no assertion can be made with regard to differences between patterns. To gain additional insight into the benefit of a specific pattern, the distribution of the pattern associated with correct solution needs to be compared with the distribution of the same pattern associated with incorrect solution.

In conservation tasks, only one pattern (*Pattern 5*) showed a significantly stronger association with the correct solution than the incorrect one [$\chi^2_{(1)} = 8.22; p \le 0.01$]. In classification tasks, no previously identified type showed a significantly stronger association with the correct solution than the incorrect one. In sequences tasks, only *Pattern 3* was more strongly associated with the correct solution of the task than with the incorrect one [$\chi^2_{(1)} = 8.64; p \le 0.01$].

DISCUSSION

The goal of the study at hand was to carve out differences regarding the influence of isolated and combined mental operations on the correct solution of concrete-operational tasks. Interestingly, only low associations between the different concrete-operational tasks could be described. This indicates, that it might be more appropriate to discuss concrete operations as specific competencies in comparison to associated facets of a concrete-operational stage.

As hypothesized, the use of strategic operations by the participants differs in terms of quantity, but most importantly in terms of the quality of the strategies used. Therefore, the assumption of existing differing effects of the used mental operations can be confirmed. Thereby, differences in the effect of isolated strategic operations become clear. Patterns of strategic operations co-occur frequently with correct solution of the specific concrete-operational tasks. At the same time, only few of these patterns were significantly stronger associated with the correct solution than the incorrect one. These observations can be made for *conservation*, *classification,* and *sequences* tasks.

Specifically, in conservation tasks, the operation *O1 Counting the elements* seems to be more effective than the operation *O2 Matching.* A possible explanation might be that matching is prone to an erroneous allocation of both elements. Use of counting as a strategic approach might result in a more robust performance. In sequences tasks, *O2 tracking the balls* is particular effective. This might be a consequence of the aspect, that the design of the task requires noticing the rotation of the balls. Tracking the balls most likely leads to the perception of the rotation. *O5 Dividing the series into single balls* might contribute to this effect, as this helps the participants to focus on the single elements and therefore decreases cognitive load. In classification tasks, the nature of the differing effects is not as clear, at the same time, differences in the effect are not as strong as in the other tasks. At least, it seems to be of particular importance to realize that a pattern is existing and that this pattern has to be assessed and integrated.

From a more general point of view, one possible explanation for the observed variation in the effect of mental operations

TABLE 3 | Association of identified patterns of mental operations (strategies) and correct solution.

Task	Pattern (no.)	Pattern (O1-O5)[a]	Correct solution[b]	Wrong solution[b]	χ^2	*df*	*p*
Conservation	2	0 0 1 0 1	2/199	0/39	0.37	1	0.542
	3	0 0 1 1 0	10/191	0/39	2.02	1	0.159
	4	0 1 0 1 0	36/165	10/29	1.26	1	0.261
	5	1 0 0 1 0	73/128	5/34	8.22	1	<0.01
Classification	1	1 0 1 0 0	34/136	12/58	0.26	1	0.609
Sequences	3	0 1 0 1 1	22/124	3/91	8.64	1	<0.01
	4	1 0 0 0 0	29/117	20/74	0.07	1	0.79

[a] *0, mental operation not used; 1, mental operation used.*
[b] *Reported indices comprise frequency of pattern/frequency of all other patterns.*

may be based on utilization deficiencies in strategy use (Miller and Seier, 1994; Bjorklund, 2012). Children might produce an appropriate mental operation; however, this mental operation could lack efficiency and show no increase in task performance. Moreover, as the participants are not experienced in the use of certain strategies, it might demand more effort to produce an effective strategy than it would take for older peers (Miller and Seier, 1994). In addition, certain mental operations decrease the probability of finding a correct solution (e.g., *O2 Matching* in conservation tasks; *O3 recognition of permanence of central ball* in sequences tasks). In this context, neo-Piagetian approaches (Houdee and Guichart, 2001; Borst et al., 2012) emphasize processes of inhibiting incorrect mental operations as important aspects of cognitive development. Similarly, Stone et al. (2016) state that utilization deficiencies might be a consequence of individual differences in executive functions (such as inhibition and working memory). Clerc et al (2014) argue that executive functions and metacognition are influencing the transfer of strategy effectiveness.

Focusing on the frequency of the patterns identified, it becomes clear that these patterns include mental operations that are not effective when applied alone. Although not effective when used exclusively, these operations contribute to the successful solution of the task when integrated in a specific combination of mental operations. This goes in hand with the assumptions of Miller (2000) and Waters (2000), who argue that the change from partial to full strategy use is critical for strategy development. In this sense, the results support the assumptions of Miller and Seier (1994), who argue that the lack of integrating strategies might be an important cause of utilization deficiencies. Instructing children in strategy use, therefore, always needs to emphasize the possible integration of several mental operations instead of solely focusing on the promotion of isolated mental operations (e.g., in sequences tasks: *O5 Dividing series into single balls*, *O2 Tracking the balls* and *O4 Recognition of rotation of the balls* or in conservation tasks: *O1 counting the elements* and *O4 recognizing identity*).

At the same time, insights into the frequency of patterns of mental operations are limited, as some patterns co-occur simultaneously with incorrect solution of the task and cannot be identified as specifically associated with the correct solution. This might be due to some strategies being more effective if the tasks' characteristics do not require certain mental operations (e.g., in some sequences tasks, recognizing the rotation of the balls is necessary, while in some it is not). Therefore, patterns of mental operations not including recognition of the rotation might only be effective in the proportion of tasks described above.

Limitations

Although important insights into the mental processes involved in solving the relevant tasks can be described, the present paper shows some major limitations. Firstly, the use of verbalizations for identifying strategy use needs to be discussed with regard to the completeness of information. The ability to verbalize varies among individuals, and some children might not be able to express their thoughts and cognition. Although the authors tried to prevent this situation by carefully introducing the procedure with the help of example tasks, the risk of non-verbalized cognition cannot be ruled out.

Secondly, Pressley and Hilden (2006) argue that children might discover new strategies by performing the tasks. These strategies might lack effectiveness, as they are not practiced. Some identified strategies, therefore, might be more effective when practiced frequently. Consequently, their effect might be underestimated in this study.

Thirdly, the chronological sequence of the mental operations used might also be an important feature of appropriate strategy use. Although we analyzed patterns of mental operations, we did not take the temporal order into account. Future studies should therefore pay respect to this important aspect of strategy use. In addition, problems in strategy use for concrete-operational concepts cannot be generalized to other domains.

CONCLUSIONS

The results do offer some important insights into the strategic processes involved in solving concrete-operational tasks. These insights can be used for the construction of instructional programs aimed at the promotion of concrete-operational thinking. These programs should consider that effective strategy use is a consequence not only of one effective mental operation, but rather of a pattern of well-integrated mental operations. The results offer some hints about which patterns could be particularly helpful or misleading in solving concrete-operational concepts. The necessity of instructing strategy use becomes clear, as a range of participants did not show adequate and successful strategic activity.

At the same time, it can be stated that the chosen approach identifying valid and effective strategies is promising and might be transferred to various domains and concepts. It might serve as an example for an evidence-driven construction of instructional or diagnostic approaches aimed at the systematic promotion of strategic activity, such as dynamic testing (Sternberg and Grigorenko, 2002; Resing et al., 2012).

ETHICS STATEMENT

The study was approved by and carried out in accordance with the recommendations of the Federal Ministry of Education of Brandenburg (Germany) (approval criteria: compliance with data protection regulations and educational relevance of research). Written and informed consent was obtained from the parents/legal guardians of all research participants.

AUTHOR CONTRIBUTIONS

MB-R designed the study, did the acquisition of the sample and the analyses of the data; he wrote the manuscript; JW provided general and scientific support for the conception, method as well as interpretation of this study; he gave important recommendations for data analysis; he assisted with the writing of the manuscript and revised it critically. All authors approved the final version to be published.

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