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EDITED BY
Anlu Zhang,
Huazhong Agricultural University, China

REVIEWED BY
Han Wang,
Guangxi University, China
Xin Yang,
Huazhong Agricultural University, China

*CORRESPONDENCE
Delong Li
ldl@imufe.edu.cn

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Hands-off, artificial construction, or penalty? How to deal with the increasingly polluted coastal wetland ecosystem in China

Yuntao Bai[1], Shuai Ma[1] and Delong Li[2]*

[1]Business School, Shandong Management University, Jinan, China, [2]School of Business Administration, Inner Mongolia University of Finance and Economics, Hohhot, China

As the destruction of coastal wetlands in China becomes more and more serious, the government needs to strengthen the management of the wetland ecosystem. Considering that pollution is an important factor in the destruction of coastal wetlands in China, the government can deal with the problem of wetland pollution through three modes: hands-off, artificial construction, and penalty. In this article, the differential game model is constructed under these three modes. The balanced social benefits of the government and polluting enterprises under the three modes are obtained, and the applicable conditions of various wetland treatment paths are compared. The results show that when the revenue generated by taxation and the indirect income generated by artificial construction are small, the government will choose the laissez-faire mode. However, with the gradual increase of indirect income generated by artificial construction, the government will be inclined to choose the artificial construction mode. When the income from government fines is small, the social benefits of polluters in the hands-off mode are greater than those in the penalty mode. With the increase of fines, the social benefits of polluting enterprises will first decrease and then increase, which will eventually be greater than the social benefits under the hands-off mode.

KEYWORDS

wetland protection, differential game, hands-off, artificial construction, penalty, social benefits

1 Introduction

1.1 Background and research significance

Wetland ecosystems have multiple functions. They provide humankind with a great deal of raw materials, water, and food. At the same time, wetlands can maintain ecological balance and biodiversity and protect the environment (Borgulat et al., 2022). Wetland ecosystems also play an important role in the protection of rare species resources, water conservation, flood storage and drought prevention, and climate regulation, and so on. Wetlands are known as the "kidneys of the earth". However, the destruction of China's coastal wetlands is becoming more and more serious. Soil destruction, environmental destruction, land reclamation from the sea, river diversion, and so on can destroy the local wetland ecological environment. In the process of economic development, people have destroyed most of the Earth's wetland resources.

Wetlands are widely distributed in China, and there are a wide variety of wetland plants. A large number of wetlands are distributed in the coastal area, and the destruction of wetlands in the coastal area is relatively serious. For example, in Fujian Province, China, coastal wetlands disappear at an annual rate of 15.44 km^2/a (Wu et al., 2022). This rate of wetland loss is unprecedented. Among all the destruction factors of coastal wetlands, environmental pollution is the most damaging. This is because China's coastal areas are relatively developed and concentrate a large number of industrial enterprises. The normal operation of these industrial enterprises will produce a large number of pollutants. Industrial pollutants will cause great damage to the wetland ecological environment. It would be difficult to preserve wetlands by shutting down industrial enterprises, which are responsible for much of China's tax revenue. The operation of industrial enterprises is bound to cause the destruction of wetland ecosystems. How to balance environmental protection and economic development effectively is an important issue.

The common methods of wetland management include hands-off, artificial construction, and fines. In the process of wetland management, the hands-off mode saves time and money but is detrimental to the restoration of wetland ecosystems. However, when a wetland ecosystem is seriously damaged, artificial construction and other measures must be taken to ensure the wetland plays its normal role (Takavakoglou et al., 2022). Moreover, the artificial construction of wetlands has increased the financial burden. In the process of wetland management, if the polluters are fined, although it can reduce the financial burden, this is not conducive to the restoration of wetland ecosystems. Each wetland management mode has its own advantages and disadvantages and scope of application. Only by choosing the correct wetland management mode can we better protect the ecological environment of coastal wetlands in China.

The structure of this article is as follows. First, on the basis of setting the corresponding hypothesis and defining the model parameters, this article establishes the differential game model with three modes: hands-off, manual construction, and punishment. Second, the HJB formula is used to solve the differential game model. Thirdly, the optimal governance quantity and social utility of government and social forces are obtained. Fourth, a comparative analysis of social utility is performed through numerical analysis. Finally, the relevant conclusions are discussed. In this article, the factors affecting the amount of wetland management and the applicable scope of different wetland management modes are obtained.

1.2 Literature review

The wetland ecological environment is very important to human production and life. At the same time, wetlands are susceptible to various factors. Some scholars have studied the influence factors of wetlands. This includes both human and natural factors. In terms of human factors, Fernando et al. (2022) analyzed the influence of laws on the protection of the Pantanal wetland. Keshta et al. (2022) studied the impacts of drainage, farmland, and fisheries on the wetland ecological environment. In terms of natural factors, Grinde et al. (2022) analyzed the adverse effects of harmful insects on the wetland ecological environment. Li B et al. (2021) studied the effects of climate, soil properties, and topography on wetlands in the Beijing–Tianjin–Hebei region of China. Matias et al. (2021) studied the impact of the lack of drainage systems on wetlands in northeastern Brazil. These studies mainly introduced the influencing factors of wetland ecosystems from aspects of human factors, such as law and artificial facilities, and also natural factors, such as insects, climate, terrain, and drainage systems.

In facing a damaged wetland ecosystem, it is necessary to evaluate, measure, and identify it. Some scholars have elaborated the evaluation, measurement, and identification methods of wetlands. For example, Tobore et al. (2021) studied the role of geospatial technology in the evaluation of the Baiyangdian wetland. Molecular assays and isotope tracers were used to analyze the spatial and temporal distribution and related activities of microorganisms in coastal wetlands in China (Niu et al., 2022). Zhang et al. (2022) used high-resolution numerical models to simulate wetland fluxes. Yang et al. (2022) proposed an urban wetland identification framework based on an advanced scene-level classification scheme to identify wetlands. These studies mainly evaluated wetland ecosystems from the aspects of geospatial technology, isotope tracer technology, numerical simulation, and so on.

Faced with the destruction of wetlands, some scholars have studied how to control wetland pollution. Some scholars have studied artificial construction to control wetlands. For example, Li et al. (2022) analyzed the effect of the construction of three single-substrate water sources on wetland pollution. Zhong et al. (2022) studied the effect of a sludge treatment wetland (ESTW) reactor on wetland microplastics. Li X et al. (2021) studied the removal of pollutants from constructed wetlands by submerged plants and microbial communities. Ren et al. (2013) analyzed the impact of the construction of water pollution facilities on wetlands. Moreno-Mateos et al. (2010) analyzed the effects of artificial wetland construction on water quality. These studies mainly studied how to manage wetlands from the point of view of specific technology and methods. There are few studies on artificial construction from the perspective of management. Some scholars have studied the employment of penalties to control pollution. For example, Cai et al. (2016) studied the use of penalties to force polluters to produce in socially optimal ways. Bloomer et al. (2009) analyzed the effect of punishment on ozone control. Xue et al. (2020) studied the impact of punishment mechanisms on air quality in China. This environmental punishment-related research is mainly concentrated in the water control, atmospheric control, and other fields. However, there is little research on the use of penalties in the wetlands field.

The above research derives mainly from wetland damage factors, wetland evaluation methods, and how to deal with the problem of wetland damage. However, the study of wetland management measures is often a specific method, instead of effectively weighing different governance approaches. In this study, the methods of wetland management are divided into three modes: hands-off, artificial construction, and penalty. The equilibrium results of each mode are compared and analyzed. Finally, the applicable scope of each wetland management mode is obtained. This provides a reference for China's coastal wetland management.

In terms of management research methods, a large number of achievements have emerged in the use of meta-analysis, system dynamics, econometrics, optimization techniques, and decision systems to study wetland management. For example, Woodward and Wui (2001) used meta-analysis to assess the relative value of different wetland services, the sources of wetland value deviation, and the return to scale displayed by wetland values. Jogo and Hassan (2010) used system dynamics to simulate the effects of different policy regimes on wetland function and economic wellbeing. Ando and Getzner (2006) used econometrics to analyze the role of ownership, ecology, and economics in public wetland conservation decisions. Mirzaei and Zibaei (2021) used an optimization approach to manage water resource conflicts among different water users and usage patterns in a basin. Lin et al. (2006) established a decision support system for wetland restoration. These methods cannot analyze the pros and cons of various decision outcomes of different decision-makers. They do not offer much in the way of ideas for decision-making and cooperation.

However, wetlands have certain resilience, and the wetland ecosystem is constantly changing. These studies do not show that change. In order to solve the shortcomings of the above research, differential game is used in this article. This is a continuous time game. Many scholars have applied it in logistics mode selection (Bai et al., 2022a), environmental governance (Bai et al., 2022b), and supply chain (Zhu et al., 2021), etc. This method can clearly depict the dynamic change process of wetland governance. At the same time, this approach has other advantages of game theory. First, this method can analyze the interaction between wetland managers. Second, it can reveal the benefits and promote the achievement of wetland management cooperation. Third, it can encourage the government to formulate wetland protection policies. However, this approach to wetland management also has some limitations. First, it assumes perfect information and rational decision-makers. Second, the influence of other external circumstances is ignored. Third, it is difficult to deal with overly complex problems.

2 Methodology

2.1 Problem description, hypothesis, and variable definition

2.1.1 Problem description

Wetlands have the reputation of being the "kidney of the earth". The conservation of wetlands plays a vital role in the global ecosystem. In order to describe the whole process of wetland ecosystem governance at all times, this article uses the time-continuous game method of the differential game method. For the sake of convenience, the game is divided into government and polluting enterprises. They are selected as players for the following reasons. Environmental problems directly affect the interests of the general public, and the government, as the organization that protects the rights of the public, plays a leading role in the process of environmental governance. Enterprises can make pollution control decisions because they produce wastewater, waste gas, and other pollutants in their production and business activities, and they are also one of the bearers of environmental responsibility. In accordance with national laws and regulations and environmental protection requirements, enterprises need to control their own pollution in order to achieve the coordinated development of production and environmental protection. As enterprises have a more thorough and comprehensive understanding of the specific situation of their own pollution, therefore, they can make pollution control decisions to better meet the requirements of environmental protection while minimizing the adverse impacts on their business operations. In order to effectively control wetland ecological environments, the following three governance modes are mainly adopted in various countries:

(1) Hands-off mode. A wetland ecosystem has a certain ability of self-regulation, but there is a limit to this ability to regulate. When this limit is exceeded, ecosystems are vulnerable to destruction. However, when the damage to the wetland ecosystem is small, the ecosystem can self-restore its original population, ecosystem, and landscape. At this point, in order to save costs or improve benefits, the hands-off mode is often employed.

(2) Artificial construction mode. When the ecological environment pollution of a wetland is serious, relying only on the self-recovery ability of the wetland cannot achieve the expected effect. At this time, it is necessary to conduct artificial intervention to the wetland ecosystem. For example, wetland ecosystems are affected by the oxygen environment and purification function. In order to improve the oxygen environment of the wetland ecosystem, siphons and air ducts can be added. In order to improve the purification function of the wetland ecosystem, magnetite and other substrates can be arranged in the wetland to achieve this (Lu et al., 2022).

(3) Penalty mode. Using the artificial construction mode cannot fundamentally solve the problem of wetland damage. Fines can act as a deterrent to wetland destruction. In order to protect the ecological environment of its wetlands, China has introduced a relevant environmental protection law. The law clarifies the rights, obligations, and legal responsibilities of all parties. In the process of punishment, the value of the wetland resources and their ecological value are considered. Those who occupy or destroy important national wetlands without permission will be punished accordingly (Liu et al., 2021). This punishment mode plays a very important role in protecting China's important wetland ecosystem.

The relationship between the three different wetland protection modes is shown in Figure 1. In this figure there are two players (government and enterprise) and three wetland control modes. In order to maximize their own interests, each player can choose the appropriate wetland control mode. When one wetland control mode cannot meet the requirements, another wetland control mode will be selected. The arrows in the figure show the player constantly choosing between the different wetland control modes.

2.1.2 Hypothesis

In order to compare and analyze the application scope of the three wetland governance modes, namely, hands-off, artificial construction, and penalty, and to delineate the differential game model of wetland ecological governance, this article establishes the following assumptions based on the causes of wetland destruction, the time state, wetland governance classification, and information mastery degree.

(1) The destruction of coastal wetlands in China is mainly caused by environmental pollution. Many factors can cause the destruction of coastal wetlands. For example, wetland reclamation, excessive use of biological resources, excessive use of wetland water resources, blind urban construction, and other irrational uses lead to wetland ecosystem degradation. This will cause the wetland area to shrink and wetland water quality to decline. Eventually, water resources will be reduced or even exhausted, and the function of wetlands will be reduced or even lost. If all the factors are analyzed, it is difficult to draw conclusions about how pollution affects wetlands. Not only industrial enterprises will pollute wetlands but also agricultural production and domestic sewage, which can destroy wetland ecology. A lot of agricultural production is carried out by individual farmers, who are usually part of small-scale enterprises run by one person or a family. However, individual farmers, like large businesses, produce goods or services for profit and sell them for returns to stay in business. At the same time, the sales of individual farmers' products are often realized through enterprises. For the sake of convenience, individual farmers are also classified as polluters. As for domestic sewage, although this is due to the discharge of residents, it is generally not directly discharged into a wetland. Instead, it is discharged to a sewage treatment plant, then treated with sewage, and then discharged into a river or wetland. Sewage treatment plants are essentially enterprises that pollute wetlands. Therefore, all pollution is considered to be emitted by polluting enterprises in this article.

(2) The government's governance decisions are in a state of continuous dynamic change. China's coastal economy is relatively developed, so there are many polluting enterprises around coastal wetlands. The pollutants emitted by these polluting enterprises are constantly changing. Therefore, the status of coastal wetland ecosystem in China is also in dynamic change. When the destruction of a wetland ecosystem is serious, the Chinese government will increase its wetland management efforts. With the strengthening of the management, the condition of the wetland ecosystem will be improved. When the wetland ecosystem is well protected, the intervention in the ecosystem will be reduced. In the process of wetland control, the amount of pollutants discharged and the amount of management by the government and enterprises can be controlled, making them the control variables.

(3) Wetland management conforms to the principles of economics. The core idea of economics is the effective use of resources, and wetland management should also take this as the goal. As a natural resource, wetlands have a unique ecological environment and important ecological functions, such as water conservation, biodiversity protection, and watershed regulation. Therefore, wetland management should not only consider ecological environment protection but also the improvement of economic benefits. Wetland control needs to comprehensively consider the ecological, social, and economic interests and adopt various

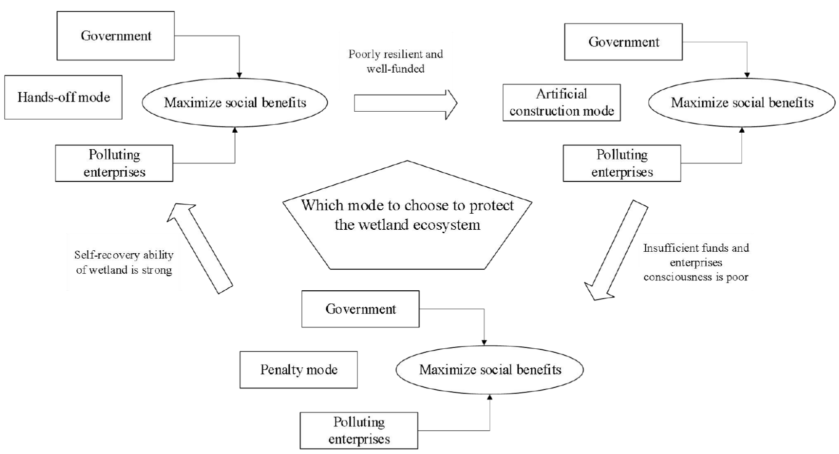

FIGURE 1
Relationship between three different wetland management modes.

means and measures to realize the effective utilization and protection of wetland resources. The costs and benefits in the process of wetland control also conform to the cost–benefit function of economics.

(4) Governments and enterprises are concerned about their reputations. If the government and enterprises pay attention to their own reputation, then the reputation can actually reflect the real state in the game process. State variables are variables used to describe the state of the system, which can reflect various states and changes of the system. It is impossible for enterprises to produce and control pollution all the time, but the reputation of the government and enterprises can exist all the time. Therefore, the production volume of the enterprise, the government's artificial construction, and the government's fine cannot be used as state variables, but reputation can be used as a state variable. It is the existence of reputation that motivates companies and governments to clean up pollution, despite the fact that such actions do not conform to the principle of cost–benefit.

2.1.3 Variable definition

When constructing the differential game model in this article, many parameters and variables are designed. These parameters and variables are defined as shown in Table 1.

2.2 Differential game of different wetland management mode

"Differential game" refers to a time-continuous game played by multiple players in a time-continuous system. It has the goal of optimizing the independence and conflict of each player and can finally obtain the strategy of each player evolving over time and reach the Nash equilibrium. Wetland ecosystems are constantly changing. In addition, the amount of pollution discharged by enterprises, government taxes, government penalties, and the amount of artificial construction is constantly changing. Therefore, this article uses differential game, a time-continuous game method.

In the context of differential game, the strategies of government and enterprises to manage wetlands are time-dependent functions. The wetland governance actions taken by each participant are influenced by the strategies taken by the others, resulting in a continuous evolutionary process of participants searching for the best wetland control actions. This dynamic interaction can be described by differential equations that capture the rate of change in the relationship between variables (Arnone et al., 2022). In the specific context of this study, both the status quo of wetlands and the control decisions of governments and enterprises are constantly changing. Therefore, the differential game theory has strong relevance and applicability in wetland management research.

Compared with other methods, such as differential equation, stochastic strategy, dynamic programming, and so on, differential game has strong applicability. While the differential equation cannot explain the strategic interactions and outcomes between decision-makers, differential game effectively captures the conflicts and cooperation that occur between the government and enterprises in the context of wetland control. On the other hand, stochastic game models focus on probabilistic information about future events in potential wetland control processes, emphasizing randomness rather than temporal continuity (Wu and Zhang, 2022). Therefore, the applicability of the stochastic strategy in this article is limited. Differential games mainly study decision-making problems of multiple parties, in which each player hopes to gain the maximum benefit through their own decision. Dynamic programming is mainly concerned with problems with an optimal substructure, that is, the optimal solution of a large problem is obtained by decomposing the solution of several subproblems and combining the optimal solution of the subproblems. Differential game is more widely used in the fields of economics, engineering, management, and biology, etc., mainly to solve the decision problem of multi-party interaction. Dynamic programming is mainly used in management science, operations research, and computer science, among other fields, to solve complex optimization problems (Howard, 1966). Therefore, this article uses the differential game method.

2.2.1 Hands-off

In the hands-off mode, the social welfare functions of local government and polluting enterprises near coastal wetlands are:

$$J_{N1} = \int_0^{\infty} [bT_N^2(t)\ln(e+I_N) - c_T T_N(t) + lx_{N1}(t)]e^{-\rho t}dt \tag{1}$$

$$J_{N2} = \int_0^{\infty} \left[b_Q Q_N(t) - \frac{c_Q}{2}Q_N^2(t) - aG_N(t) + lx_{N2}(t)\right]e^{-\rho t}dt \tag{2}$$

Among them, $bT_N^2(t)$ represents the direct revenue that taxation brings to the government under the hands-off mode. $bT_N^2(t)\ln(e+I_N) - bT_N^2(t)$ represents the indirect revenue that the government gets from taxes under the hands-off mode. $bT_N^2(t)\ln(e+I_N)$ represents the direct and indirect benefits that taxes bring to the government under the hands-off mode. $c_T T_N(t)$ represents the cost of collecting taxes under the hands-off mode. $lx_{N1}(t)$ expresses the impact of the destruction of coastal wetlands on the reputation of local governments under the hands-off mode. $b_Q Q_N(t)$ represents the benefits of letting polluters produce their products. $\frac{c_Q}{2}Q_N^2(t)$ represents the cost of producing goods for polluters under the hands-off mode. $aG_N(t)$ represents the cost of pollution control of polluters under the hands-off mode. $lx_{N2}(t)$ means the influence of reputation on polluters under the hands-off mode.

The changes in the reputation of local governments and polluting enterprises near coastal wetlands are as follows:

$$\dot{x}_{N1}(t) = -\lambda_1 T_N(t) + \delta x_{N1}(t) \tag{3}$$

$$\dot{x}_{N2}(t) = -\lambda_Q Q_N(t) + \lambda_G G_N^2(t) + \delta x_{N2}(t) \tag{4}$$

Among these formulas, $\lambda_1 T_N(t)$ represents the reduction in the government's reputation caused by taxes under the hands-off mode. $\delta x_{N1}(t)$ represents the decay of a government's reputation over time under the hands-off mode. $\lambda_Q Q_N(t)$ represents the decline in corporate reputation as a result of pollution under the hands-off mode. $\lambda_G G_N^2(t)$ represents the increase in corporate reputation that comes with a

TABLE 1 The main definitions of the variables and parameters in this article.

variables and parameters	specific meaning
Y={N,C,P}	three coastal wetland management modes (hands-off, artificial construction, and penalty)
independent variable	
$T_Y(t)$	government tax revenue under the coastal wetland management mode Y
$Q_Y(t)$	the amount of pollution discharged by polluting enterprises under the coastal wetland management mode Y
$G_Y(t)$	pollution treatment amount of polluting enterprises under the coastal wetland management mode Y
$F_C(t)$	the government's artificial construction input under the coastal wetland management mode Y
$P_P(t)$	punishment of polluting enterprises under the coastal wetland management mode Y
$x_{Y1}(t)$	the reputation of the government under the coastal wetland management mode Y
$x_{Y2}(t)$	the reputation of polluting enterprises under the coastal wetland management mode Y
parameter	
ρ	the discount rate that occurs over time, the discount factor, $0 \le \rho \le 1$
δ	the decay rate of reputation, $\delta > 0$
b	the direct income from unit tax, $b > 0$
I_N	the indirect income from unit tax, $I_N > 0$
I_C	indirect income from artificial construction, $I_C > 0$
I_P	indirect income from pollution penalty, $I_P > 0$
c_T	the cost of collecting taxes, $c_T > 0$
c_Q	the unit cost of producing a product, $c_Q > 0$
l	the positive impact of the unit's reputation, $l > 0$
λ_1	the negative effect of taxes on the government's reputation, $\lambda_1 > 0$
b_Q	revenue generated per unit of pollutant discharge, $b_Q > 0$
b_F	unit cost of artificial coastal wetland construction, $b_F > 0$
b_P	the increase in government revenue resulting from punishment, $b_P > 0$
λ_Q	the negative impact of unit pollution on corporate reputation, $\lambda_Q > 0$
λ_G	the positive impact of unit pollution control quantity on enterprise reputation, $\lambda_G > 0$
λ_F	the positive influence of unit artificial construction degree on government reputation, $\lambda_F > 0$
a	the cost of cleaning up each unit of pollution, $a > 0$
a_P	the increased cost of pollution control caused by punishment, $a_P > 0$
function	
$J_{Y1}(t)$	the government's social welfare function under the coastal wetland management mode Y
$V_{Y1}(t)$	the government's benefit function under the coastal wetland management mode Y
$J_{Y2}(t)$	the social welfare function of enterprises under the coastal wetland management mode Y
$V_{Y2}(t)$	the benefit function of enterprises under the coastal wetland management mode Y

hands-off approach to pollution control. $\delta x_{N2}(t)$ represents the decay of corporate reputation over time under the hands-off mode.

2.2.2 Artificial construction

In the artificial construction mode, the social welfare function of local government and polluting enterprises near coastal wetlands are:

$$J_{C1} = \int_0^{\infty} [bT_C^2(t) \ln(e + I_N + I_C) - c_T T_C(t) - b_F F_C(t) + l x_{C1}(t)] e^{-\rho t} dt \quad (5)$$

$$J_{C2} = \int_0^{\infty} \left[b_Q Q_C(t) - \frac{c_Q}{2} Q_C^2(t) - a G_C(t) + l x_{C2}(t) \right] e^{-\rho t} dt \quad (6)$$

Among them, $bT_C^2(t)$ represents the direct income brought by taxation to the government under the artificial construction mode. $bT_C^2(t)\ln(e+I_N+I_C)-bT_C^2(t)$ represents the indirect revenue that taxes bring to the government under the artificial construction mode. $bT_C^2(t)\ln(e+I_N+I_C)$ represents the direct and indirect revenue that taxes bring to the government under the artificial construction mode. $c_T T_C(t)$ represents the cost of collecting taxes under the artificial construction mode. $b_F F_C(t)$ represents the cost of artificial construction of coastal wetlands under the artificial construction mode. $lx_{C1}(t)$ represents the effect of artificial construction on the government's reputation. $b_Q Q_C(t)$ represents the income of the product produced by the polluting enterprise under the artificial construction mode. $\frac{c_Q}{2}Q_C^2(t)$ represents the cost of producing a product for a polluting enterprise under the artificial construction mode. $aG_C(t)$ represents the cost of pollution control of polluters under the artificial construction mode. $lx_{C2}(t)$ represents the influence of reputation on polluting enterprises under the artificial construction mode.

The changes of local government reputation and the reputation of polluting enterprises near coastal wetlands are as follows:

$$\dot{x}_{C1}(t)=\lambda_F F_C^2(t)-\delta x_{C1}(t) \tag{7}$$

$$\dot{x}_{C2}(t)=-(\lambda_Q+\lambda_F)Q_C(t)+\lambda_G G_C^2(t)+\delta x_{C2}(t) \tag{8}$$

Among them, $\lambda_F F_C^2(t)$ represents the increase in government reputation brought about by artificial construction. $\delta x_{C1}(t)$ represents the decay of the government's reputation over time under artificial construction. $(\lambda_Q+\lambda_F)Q_C(t)$ represents a reduction in the reputation of a company caused by pollution under artificial construction. $\lambda_G G_C^2(t)$ represents the increase in corporate reputation brought about by the artificial construction mode of pollution control. $\delta x_{C2}(t)$ represents the decay of corporate reputation over time under the artificial construction mode.

2.2.3 Penalty

In the pollution penalty mode, the social welfare function of the local government and polluting enterprises near the coastal wetlands are:

$$J_{P1}=\int_0^\infty [bT_P^2(t)\ln(e+I_N+I_P)-c_T T_P(t)+b_P P_P(t)+lx_{P1}(t)]e^{-\rho t}dt \tag{9}$$

$$J_{P2}=\int_0^\infty \left[(b_Q-b_P)Q_P(t)-\frac{c_Q}{2}Q_P^2(t)-(a+a_P)G_P(t)+lx_{P2}(t)\right]e^{-\rho t}dt \tag{10}$$

Among them, $bT_P^2(t)$ represents the direct revenue brought by the tax for the government under the penalty mode. $bT_P^2(t)\ln(e+I_N+I_P)-bT_P^2(t)$ represents the indirect revenue that a tax brings to the government under the penalty mode. $bT_P^2(t)\ln(e+I_N+I_P)$ represents the direct and indirect revenue that taxes bring to the government under the penalty mode. $c_T T_P(t)$ represents the cost of collecting taxes under the penalty mode. $b_P P_P(t)$ represents the government's revenue from fines. $lx_{P1}(t)$ represents the impact on the reputation of the government under the penalty mode. $(b_Q-b_P)Q_P(t)$ represents the income of the product produced by the polluter under the penalty mode. $\frac{c_Q}{2}Q_P^2(t)$ represents the cost of producing products for polluters under the penalty mode. $(a+a_P)G_P(t)$ represents the pollution control cost of polluters under the penalty mode. $lx_{P2}(t)$ represents the impact of reputation on polluters under the penalty mode.

The changes of reputation of local government and polluters near coastal wetlands are as follows:

$$\dot{x}_{P1}(t)=-\lambda_P P_P^2(t)+\delta x_{P1}(t) \tag{11}$$

$$\dot{x}_{P2}(t)=-\lambda_Q Q_P(t)+\lambda_G G_P^2(t)+\delta x_{P2}(t) \tag{12}$$

Among them, $\lambda_P P_P^2(t)$ represents a reduction in the government's reputation as a result of punishing polluters. $\delta x_{P1}(t)$ represents the decay of the government's reputation over time under the penalty mode. $\lambda_Q Q_P(t)$ represents the reduction in corporate reputation caused by pollution under the penalty mode. $\lambda_G G_P^2(t)$ represents the increase in corporate reputation that comes from cleaning up pollution under the penalty mode. $\delta x_{P2}(t)$ represents the decay of corporate reputation over time under the penalty mode.

3 Results

In the differential game, the social welfare of polluting enterprises and governments near wetlands is not only affected by the control variables and parameters but also constantly changes with the change of time, state, and self-recovery ability of the wetlands. In order to better calculate the amount of pollutant discharge, control amount, and social benefit of wetland polluting enterprises, as well as the government artificial construction degree, fines, and social benefit, the HJB formula is adopted. The HJB formula is a partial differential equation, which is the core of the optimal control.

3.1 HJB formula

Wetland has a certain ability of self-recovery; when the damage is minor, there is no need to treat the wetland. When the hands-off mode is adopted for a wetland ecosystem, the HJB equations under this mode are:

$$\rho V_{N1}=\max_{T_N(t)}\left\{\begin{array}{r}[bT_N^2(t)\ln(e+I_N)-c_T T_N(t)+lx_{N1}(t)]\\+\dfrac{\partial V_{N1}}{\partial x_{N1}}[-\lambda_1 T_N(t)+\delta x_{N1}(t)]\end{array}\right\} \tag{13}$$

$$\rho V_{N2}=\max_{Q_N(t),\ G_N(t)}\left\{\left[b_Q Q_N(t)-\frac{c_Q}{2}Q_N^2(t)-aG_N(t)+lx_{N2}(t)\right]+\frac{\partial V_{N2}}{\partial x_{N2}}\left[-\lambda_Q Q_N(t)+\lambda_G G_N^2(t)+\delta x_{N2}(t)\right]\right\} \tag{14}$$

When the wetland ecosystem is artificially constructed, the HJB equations of the social welfare function of the government and polluting enterprises under this mode are:

$$\rho V_{C1} = \max_{T_C(t),F_C(t)} \{[bT_C^2(t)\ln(e+I_N+I_C) - c_T T_C(t) - b_F F_C(t) + lx_{C1}(t)] + \frac{\partial V_{C1}}{\partial x_{C1}}[\lambda_F F_C^2(t) - \delta x_{C1}(t)]\} \tag{15}$$

$$\rho V_{C2} = \max_{Q_C(t),\ G_C(t)} \left\{ \left[b_Q Q_C(t) - \frac{c_Q}{2} Q_C^2(t) - aG_C(t) + lx_{C2}(t) \right] + \frac{\partial V_{C2}}{\partial x_{C2}} \left[-(\lambda_Q + \lambda_F) Q_C(t) + \lambda_G G_C^2(t) + \delta x_{C2}(t) \right] \right\} \tag{16}$$

When fines are imposed on polluting enterprises that destroy wetlands, the HJB equations of the social welfare function of the government and polluting enterprises under this mode are:

$$\rho V_{P1} = \max_{T_P(t),P_P(t)} \left\{ \left[bT_P^2(t)\ln(e+I_N+I_P) - c_T T_P(t) + b_P P_P(t) + lx_{P1}(t) \right] + \frac{\partial V_{P1}}{\partial x_{P1}}[-\lambda_P P_P^2(t) + \delta x_{P1}(t)] \right\} \tag{17}$$

$$\rho V_{P2} = \max_{Q_P(t),G_P(t)} \left\{ \left[(b_Q - b_P) Q_P(t) - \frac{c_Q}{2} Q_P^2(t) - (a + a_P) G_P(t) + lx_{P2}(t) \right] + \frac{\partial V_{P2}}{\partial x_{P2}} \left[-\lambda_Q Q_P(t) + \lambda_G G_P^2(t) + \delta x_{P2}(t) \right] \right\} \tag{18}$$

3.2 Result of equilibrium

Proposition 1: In the hands-off mode, the government tax revenue, amount of pollution discharged by polluting enterprises, pollution treatment amount of polluting enterprises, and social benefits for government and polluters are, respectively (see Appendix 1 for details):

$$T_N^*(t) = \frac{c_T + \lambda_1 \frac{l}{\rho-\delta}}{2b\ln(e+I_N)} \tag{19}$$

$$Q_N^*(t) = \frac{b_Q - \frac{l}{\rho-\delta}\lambda_Q}{c_Q} \tag{20}$$

$$G_N^*(t) = \frac{a}{2\lambda_G}\left(\frac{l}{\rho-\delta}\right)^{-1} \tag{21}$$

$$V_{N1}^* = \frac{l}{\rho-\delta}x_{N1} + \frac{1}{\rho}\left[b\left(\frac{c_T+\lambda_1\frac{l}{\rho-\delta}}{2b\ln(e+I_N)}\right)^2 \ln(e+I_N) - c_T\frac{c_T+\lambda_1\frac{l}{\rho-\delta}}{2b\ln(e+I_N)} \right] - \frac{1}{\rho}\lambda_1 \frac{c_T+\lambda_1\frac{l}{\rho-\delta}}{2b\ln(e+I_N)}\frac{l}{\rho-\delta} \tag{22}$$

$$V_{N2}^* = \frac{l}{\rho-\delta}x_{N2} + \frac{1}{\rho}\left[b_Q\frac{b_Q-\frac{l}{\rho-\delta}\lambda_Q}{c_Q} - \frac{c_Q}{2}\left(\frac{b_Q-\frac{l}{\rho-\delta}\lambda_Q}{c_Q}\right)^2 - a\frac{a}{2\lambda_G}\left(\frac{l}{\rho-\delta}\right)^{-1} \right] + \frac{l}{\rho-\delta}\frac{1}{\rho}\left[-\lambda_Q\frac{b_Q-\frac{l}{\rho-\delta}\lambda_Q}{c_Q} + \lambda_G\left(\frac{a}{2\lambda_G}\right)^2\left(\frac{l}{\rho-\delta}\right)^{-2} \right] \tag{23}$$

Conclusion 1: In the laissez-faire mode, the revenue generated by taxes is inversely proportional to the government's equilibrium tax. It is proportional to the cost of collecting taxes. The amount of pollution discharged by enterprises is directly proportional to the income generated by the unit discharge. The amount of balanced treatment of enterprises is proportional to the cost of pollution control. It is inversely proportional to the positive impact of pollution control on corporate reputation.

Proposition 2: In the artificial construction mode, the government tax revenue, government's artificial construction input, amount of pollution discharged by polluting enterprises, pollution treatment amount of polluting enterprises, and social benefits for government and polluters are, respectively (see Appendix 2 for details):

$$T_C^*(t) = \frac{c_C}{2b\ln(e+I_N+I_C)} \tag{24}$$

$$F_C^*(t) = \frac{b_F}{2\lambda_F}\left(\frac{l}{\rho+\delta}\right)^{-1} \tag{25}$$

$$Q_C^*(t) = \frac{b_Q - \frac{l}{\rho-\delta}(\lambda_Q+\lambda_F)}{c_Q} \tag{26}$$

$$G_C^*(t) = \frac{a}{2\lambda_G}\left(\frac{l}{\rho-\delta}\right)^{-1} \tag{27}$$

$$V_{C1}^* = \frac{1}{\rho}\left[b\left(\frac{c_T}{2b\ln(e+I_N+I_C)}\right)^2 \ln(e+I_N+I_C) - c_T\frac{c_T}{2b\ln(e+I_N+I_C)} - b_F\frac{b_F}{2\lambda_F}\left(\frac{l}{\rho+\delta}\right)^{-1} \right] + \frac{l}{\rho+\delta}x_{C1} + \frac{l}{\rho+\delta}\frac{1}{\rho}\left[\lambda_F\left(\frac{b_F}{2\lambda_F}\right)^2\left(\frac{l}{\rho+\delta}\right)^{-2} \right] \tag{28}$$

$$V_{C2}^* = \frac{1}{\rho}\left[b_Q\frac{b_Q-\frac{l}{\rho-\delta}(\lambda_Q+\lambda_F)}{c_Q} - \frac{c_Q}{2}\left(\frac{b_Q-\frac{l}{\rho-\delta}(\lambda_Q+\lambda_F)}{c_Q}\right)^2 - a\frac{a}{2\lambda_G}\left(\frac{l}{\rho-\delta}\right)^{-1} \right] + \frac{l}{\rho-\delta}x_{C2} + \frac{l}{\rho-\delta}\frac{1}{\rho}\left[-(\lambda_Q+\lambda_F)\frac{b_Q-\frac{l}{\rho-\delta}(\lambda_Q+\lambda_F)}{c_Q} + \lambda_G\left(\frac{a}{2\lambda_G}\right)^2\left(\frac{l}{\rho-\delta}\right)^{-2} \right] \tag{29}$$

Conclusion 2: In the artificial construction mode, the government's tax revenue is inversely proportional to the indirect income generated by artificial construction. The balanced amount of pollution discharged by polluting enterprises is inversely proportional to the positive impact of artificial construction on government reputation. The equilibrium input of government artificial construction is proportional to the cost of artificial construction and inversely proportional to the positive influence of unit artificial construction degree on government reputation.

Proposition 3: In the artificial construction mode, the government tax revenue, punishment of polluting enterprises, amount of pollution discharged by polluting enterprises, pollution treatment amount of polluting enterprises, and social benefits for government and polluters are, respectively (see Appendix 3 for details):

$$T_P^*(t) = \frac{c_T}{2b\ln(e+I_N+I_P)} \tag{30}$$

$$P_P^*(t) = \frac{b_P}{2\lambda_P}\left(\frac{l}{\rho-\delta}\right)^{-1} \tag{31}$$

$$Q_P^*(t) = \frac{b_Q - b_P - \frac{l}{\rho-\delta}\lambda_Q}{c_Q} \tag{32}$$

$$G_P^*(t) = \frac{a + a_P}{2\lambda_G}\left(\frac{l}{\rho-\delta}\right)^{-1} \tag{33}$$

$$V_{P1}^* = \frac{1}{\rho}\left[b\left(\frac{c_T}{2b\ln(e+I_N+I_P)}\right)^2 \ln(e + I_N + I_P) - c_T\frac{c_T}{2b\ln(e+I_N+I_P)} + b_P\frac{b_P}{2\lambda_P}\left(\frac{l}{\rho-\delta}\right)^{-1}\right] + \frac{l}{\rho-\delta}x_{P1} + \frac{l}{\rho-\delta}\frac{1}{\rho}\left[-\lambda_P\left(\frac{b_P}{2\lambda_P}\right)^2\left(\frac{l}{\rho-\delta}\right)^{-2}\right] \tag{34}$$

$$V_{P2}^* = \frac{1}{\rho}\left[(b_Q - b_P)\frac{b_Q-b_P-\frac{l}{\rho-\delta}\lambda_Q}{c_Q} - \frac{c_Q}{2}\left(\frac{b_Q-b_P-\frac{l}{\rho-\delta}\lambda_Q}{c_Q}\right)^2 - (a + a_P)\frac{a+a_P}{2\lambda_G}\left(\frac{l}{\rho-\delta}\right)^{-1}\right] + \frac{l}{\rho-\delta}x_{P2} + \frac{l}{\rho-\delta}\frac{1}{\rho}\left[-\lambda_Q\frac{b_Q-b_P-\frac{l}{\rho-\delta}\lambda_Q}{c_Q} + \lambda_G\left(\frac{a+a_P}{2\lambda_G}\right)^2\left(\frac{l}{\rho-\delta}\right)^{-2}\right] \tag{35}$$

Conclusion 3: In the pollution penalty mode, the government's tax revenue is inversely proportional to the indirect income generated by the penalty. The government's equalization penalty is proportional to the increase in revenue generated by the penalty. The balanced amount of pollution discharged by enterprises is inversely proportional to the increase of government revenue brought by punishment. The amount of pollution control of enterprises is proportional to the increase of the pollution control cost of enterprises caused by punishment.

3.2.1 Numerical analysis

The "discount rate" refers to the conversion of future earnings or expenses to the current value by a certain percentage based on the current time. The discount rate is usually less than 1 because it is used to calculate the present value of future cash flows. Because future cash flows cannot be guaranteed, and there is no way to accurately predict the future, it is necessary to convert future cash flows to the present value at a discount rate, so as to consider the factors of risk and time value. Therefore, this article assumes that the discount factor ρ is 0.9. "Reputational decay" refers to the gradual weakening of one's image and credibility in the public eye over time. This decline can be caused by a range of factors, such as negative press coverage and illegal behavior, etc. It cannot decay completely at once but in a certain proportion every year. Therefore, the degree δ of reputational decay of government and polluting enterprises is 0.1. Although, to a certain extent, taxes do bring a certain economic burden to taxpayers, the revenue generated by taxes often exceeds the cost of taxes. This is because tax revenue can be used to provide public goods and services, thereby bringing greater social benefits. Therefore, this article assumes the direct benefit b from a unit tax is 3, and the cost c_T of tax collection is 2.

Corporate reputation can have a positive impact on supporting corporate brand building, enhancing the connection with consumers, attracting investment and talent, and improving supply chain cooperation. Similarly, the reputation of the government can bring about positive effects such as increasing investment, promoting tourism, enhancing the government's influence on society, and improving the government's management ability. Thus, the positive effect of reputation outweighs the negative effect of tax. The positive influence l brought by unit reputation is 1. The negative effect λ_1 of tax on government reputation is 0.5. The damage caused by wetlands is usually because the polluter's income is greater than the fine, which is often a problem in the current policy and regulatory environment. The revenue $b_Q$generated by unit pollutant discharge is 2. The increase $b_P$in government revenue from the penalty is 1.

The production of products by industrial enterprises will produce certain pollution. If the impact of pollution on reputation is greater than the benefits to the industry, the industrial company will stop producing the product. Therefore, the impact of pollution on reputation is less than the profits achieved by industrial enterprises. Therefore, this article assumes the negative impact $\lambda_Q$of unit pollution on enterprise reputation is 0.5, and the unit cost c_Q of producing the product is 2. Similarly, the income obtained from the production of products is greater than the cost of artificial construction of wetlands; otherwise, there is no need to carry out production activities. Therefore, this article assumes the unit cost b_F of artificial coastal wetland construction is 1. Environmental protection is one of the contents of national governance. The government protects the environment not only to promote economic construction and the improvement of people's lives but also to achieve sustainable development and safeguard the global ecological environment and human health. Environmental protection is a part of corporate social responsibility, and environmental protection can affect their own interests. Therefore, the influence of unit control quantity on government reputation is slightly greater than that on enterprise reputation. Thus, this article assumes the positive influence λ_G of the unit pollution control quantity on enterprise reputation is 0.8, and the positive influence λ_F of the unit labor construction degree on government reputation is 1. This article assumes complete information, that is, the government fully understands the emission status of enterprises. If the cost of cleaning sewage is less than the cost of fines, then companies will clean sewage. In this case, there is no need to treat wetlands. Therefore, the cost of cleaning the sewage is greater than the cost of fines. This article assumes the cost a of cleaning up unit pollution is 1.5, and the increased pollution control cost a_P caused by punishment is 0.5.

It is important to note that these values have no units. This is for two reasons. First, the units of different parameters can be reduced. Second, the main point of research of this article is which mode is better, and the size of the parameters can be used to reach the above conclusions. Therefore, when the indirect income I_N per unit tax is 1, it can be calculated that:

$$V_{N1}^* = 1.42 \tag{36}$$

$$V_{C1}^* = -0.37 \times \frac{1}{\ln(e + 1 + I_C)} + 1.56 \tag{37}$$

$$V_{P1}^{*} = -0.37 \times \frac{1}{\ln(e + 1 + I_P)} + 1.47 \quad (38)$$

Figure 2 can also be produced.

Conclusion 4: When the income generated by taxation and the indirect income generated by artificial construction are small, the social income of the government under the artificial construction mode is smaller than that of the government under the hands-off mode. However, with the gradual increase of the indirect income generated by artificial construction, the social benefit of the government under the artificial construction mode is greater than that under the laissez-faire mode.

When the indirect income I_N per unit tax is 2, it can be calculated that:

$$V_{N1}^{*} = 1.88 \quad (39)$$

$$V_{C1}^{*} = -0.37 \times \frac{1}{\ln(e + 2 + I_C)} + 1.56 \quad (40)$$

$$V_{P1}^{*} = -0.37 \times \frac{1}{\ln(e + 2 + I_P)} + 1.47 \quad (41)$$

Figure 3 can also be produced.

Conclusion 5: When the income generated by taxation is large, no matter how the indirect income generated by artificial construction or fine is increased, the social income of the government under the artificial construction or penalty mode is always smaller than that of the government under the hands-off mode.

It can be calculated that:

$$V_{N2}^{*} = 1.16 \quad (42)$$

$$V_{C2}^{*} = 0.44\lambda_F^2 + 1.84\lambda_F + 1.15 \quad (43)$$

Figure 4 can also be produced.

Conclusion 6: When the positive impact of artificial construction on government reputation is small, the social

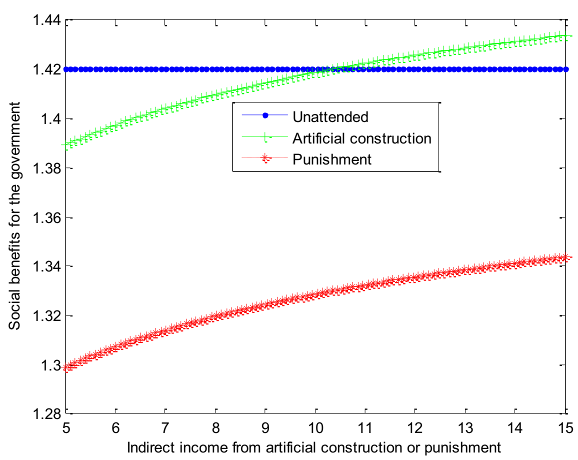

FIGURE 2
The impact of indirect income on the government's social benefits.

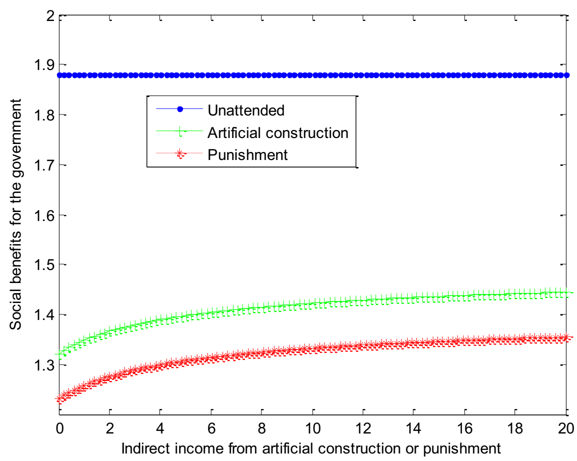

FIGURE 3
The impact of indirect income on the government's social benefits

benefits of polluters under the hands-off mode are greater than those under the artificial construction mode. However, when the positive impact of artificial construction on government reputation is large, the social benefits of polluters under the hands-off mode are smaller than those under the artificial construction mode.

It can be calculated that:

$$V_{N2}^{*} = 1.16 \quad (44)$$

$$V_{P2}^{*} = 0.28b_P^2 - 0.76b_P + 0.25 \quad (45)$$

Figure 5 can also be produced.

Conclusion 7: When the income from government fines is small, the social benefits of polluters under the hands-off mode are greater than those under the penalty mode. With the increase of fines, the social benefits of polluting enterprises will first decrease and then increase, which will eventually be greater than the social benefits under the hands-off mode.

4 Discussion

There are many industrial enterprises near the coastal wetlands in China, and industrial enterprises have seriously damaged these coastal wetlands. China needs to strengthen its coastal wetland pollution control efforts. Although the artificial construction of coastal wetlands in China can effectively protect the local wetland natural landscape and ecological environment, the artificial construction will inevitably be costly. If China imposes fines on wetland destruction, it will reduce the government's financial expenditure but increase the burden on enterprises. Therefore, the application scope of various wetland management modes is an important issue in this article. Since most of the existing research has used traditional methods such as statistical analysis, differential game has not been used to study wetland governance. Moreover, coastal wetland management has not been divided into the artificial construction and punishment perspective, as far as could be found. In this article, the differential game is applied to the field of wetland

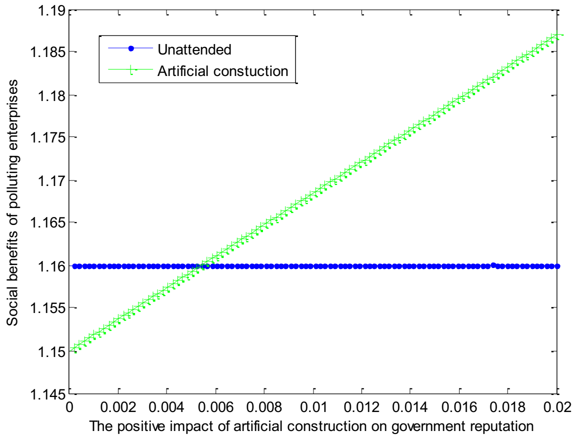

FIGURE 4
The influence of reputation on the social benefits of polluting enterprises

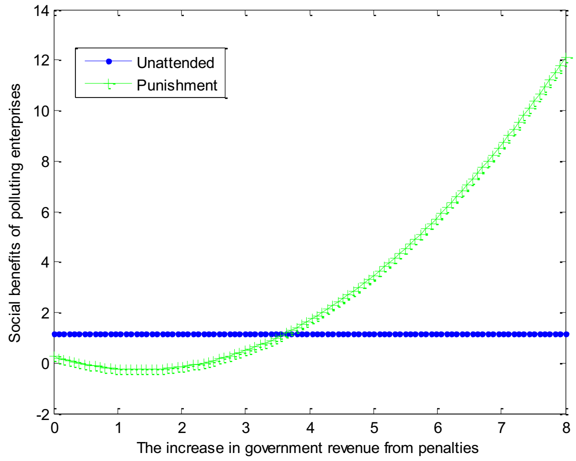

FIGURE 5
The impact of penalties on the social benefits of polluting enterprises

management and how the Chinese government and enterprises protect the coastal wetland ecological environment through their efforts.

The operation of polluting enterprises generates tax revenue. In the face of coastal wetland pollution, many local governments adopt a laissez-faire management mode largely to ensure local tax revenue. When economic development is relatively slow, the more dependent social development is on taxation, the greater the income generated by taxation (Gunter et al., 2021). In order to increase tax revenue and economic development, some local governments are willing to cause environmental pollution, thus destroying the coastal wetland ecosystem (Das and Krishnakumar, 2022). The level of local economic development and pollution discharge can greatly affect the coastal wetland ecosystem. Slower-growing regions are more likely to adopt a hands-off mode because of tax problems. When the cost of controlling the pollution of a coastal wetland ecosystem is high, it indicates that the pollution is serious, and more efforts should be made to control the coastal wetland ecosystem.

Artificial construction of a coastal wetland ecosystem can restore the local natural landscape with environmental value and obtain direct benefits such as tourism income. At the same time, indirect benefits such as purifying water, regulating climate, reducing drought and flood disasters, and protecting wildlife habitats can also be obtained (Zhang et al., 2021). However, most of the government's tax revenue comes from manufacturing, with coastal wetland tourism producing relatively little. It is not easy to observe the indirect benefits generated by artificial construction. When the government relies heavily on tax revenue, it is not conducive to increase the artificial construction of coastal wetlands.

The punishment of polluting enterprises will increase the revenue of the government, which will increase the punishment of coastal wetland polluters. When the government penalizes companies heavily, it discourages them from discharging pollutants. That is because polluters have to weigh the benefits of polluting against the fines. If a fine leads to an increase in the costs of the enterprise, it will cause the enterprise to increase the amount of pollution treatment. For example, Chinese Taipei has learned from the experience of the United States and established a sound legal and regulatory framework, service mode, and market mechanism for its pollution permit system (Li et al., 2017). If companies do not meet emission requirements, they will be penalized. This has played an important role in improving environmental quality.

When a region is relatively undeveloped, it is difficult to transform the ecological benefits of wetland ecosystem into economic benefits. At the same time, a wetland ecosystem has a self-recovery function. At this point, if the artificial construction of the wetland ecosystem is carried out, the cost of input will inevitably be greater than the income obtained. In such cases, governments tend to adopt a hands-off strategy. However, if the economic development of a region is fast and the development of wetlands can bring a lot of tourism and other benefits, the government will increase the construction of wetlands (Sierra et al., 2021).

5 Conclusion

This article assumes that the Chinese government and polluting enterprises can achieve the purpose of coastal wetland management by controlling factors such as the amount of fines, artificial construction investment, pollution discharge, and treatment amount. This article constructs the differential game model under the laissez-faire, artificial construction, and penalty modes and conducts a comparative analysis of them. Studies show that when the revenue generated by taxation is small and the indirect income generated by artificial construction is small, the government will choose the laissez-faire mode. However, with the gradual increase of indirect income generated by artificial construction, the government will be inclined to choose the artificial construction mode. When the income from government fines is small, the social benefits of polluters in the hands-off mode are greater than those in the penalty mode. With the increase of fines, the social benefits of polluting enterprises will first decrease and then increase, which will eventually be greater than the social benefits under the hands-off mode. Therefore, the implementation of the artificial construction mode mainly depends on the income generated by it. Polluters would prefer either impunity or heavy punishment. The contribution of this article to wetland control lies in its ability to take into account the dynamic characteristics of wetland ecosystems and partial information feedback from participants. In this article, the dynamic strategies of wetland management participants can be matched with the dynamic evolution of environmental systems so as to obtain an equilibrium solution.

This article has some shortcomings. For example, it is based on the game theory model to study wetland protection, and there are few specific data related to it. At the same time, while this study does not involve specific wetlands, the specific conditions faced by different wetlands may not be exactly the same. In future research, we can increase the relevant specific data and conduct research on specific wetland protection. The research in this article has certain extensibility. For example, it considers that the destruction of coastal wetlands is caused by pollution, the treatment of coastal wetlands is divided into hands-off, artificial construction, and penalty, and the local government has a comprehensive grasp of the pollution information of enterprises. In future studies, it is possible to consider other factors that may also cause wetland destruction, mixed forms of wetland governance, and the government's grasp of partial pollution information of enterprises, etc., and to study relevant issues. In addition, this research is not only applicable to wetland ecological governance issues, but it also has certain reference significance for creditors' ecological destruction of the Amazon rainforest and the Fukushima nuclear wastewater pollution.

Data availability statement

The original contributions presented in the study are included in the article/supplementary material. Further inquiries can be directed to the corresponding author.

Author contributions

YB: Model building, problem description, main paper writing, result analysis. SM: Graphic drawing, literature collection. DL: Quality control, grammar modification. All authors contributed to the article and approved the submitted version.

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Conflict of interest

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

Publisher's note

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Appendix 1

Taking the derivatives of T_N with respect to (13) and the derivatives of Q_N and G_N separately with respect to (14), and setting them equal to zero, we can get:

$$T_N^{\star}(t) = \frac{c_T + \lambda_1 \frac{\partial V_{N1}}{\partial x_{N1}}}{2b\ln(e + I_N)} \tag{46}$$

$$Q_N^{\star}(t) = \frac{b_Q - \frac{\partial V_{N2}}{\partial x_{N2}}\lambda_Q}{c_Q} \tag{47}$$

$$G_N^{\star}(t) = \frac{a}{2\lambda_G}\left(\frac{\partial V_{N2}}{\partial x_{N2}}\right)^{-1} \tag{48}$$

Substituting (46) into (13) and substituting (47) and (48) into (14), we can get:

$$\begin{aligned}\rho V_{N1} = &\left[b\left(\frac{c_T+\lambda_1\frac{\partial V_{N1}}{\partial x_{N1}}}{2b\ln(e+I_N)}\right)^2 \ln(e+I_N) - c_T\frac{c_T+\lambda_1\frac{\partial V_{N1}}{\partial x_{N1}}}{2b\ln(e+I_N)} + lx_{N1}(t)\right]\\ &+\frac{\partial V_{N1}}{\partial x_{N1}}\left[-\lambda_1\frac{c_T+\lambda_1\frac{\partial V_{N1}}{\partial x_{N1}}}{2b\ln(e+I_N)} + \delta x_{N1}(t)\right]\end{aligned} \tag{49}$$

$$\begin{aligned}\rho V_{N2} = &\left[b_Q\frac{b_Q-\frac{\partial V_{N2}}{\partial x_{N2}}\lambda_Q}{c_Q} - \frac{c_Q}{2}\left(\frac{b_Q-\frac{\partial V_{N2}}{\partial x_{N2}}\lambda_Q}{c_Q}\right)^2 - a\frac{a}{2\lambda_G}\left(\frac{\partial V_{N2}}{\partial x_{N2}}\right)^{-1} + lx_{N2}(t)\right]\\ &+\frac{\partial V_{N2}}{\partial x_{N2}}\left[-\lambda_Q\frac{b_Q-\frac{\partial V_{N2}}{\partial x_{N2}}\lambda_Q}{c_Q} + \lambda_G\left(\frac{a}{2\lambda_G}\right)^2\left(\frac{\partial V_{N2}}{\partial x_{N2}}\right)^{-2} + \delta x_{N2}(t)\right]\end{aligned} \tag{50}$$

Let $V_{N1}^{\star} = k_1 x_{N1} + k_2$, $V_{N2}^{\star} = k_3 x_{N2} + k_4$, wherein, k_1, k_2, k_3, and k_4 are all constants. The parameters of the optimal social welfare function can be obtained by calculation as follows:

$$\begin{cases} k_1 = \frac{l}{\rho-\delta}\\ k_2 = \frac{1}{\rho}\left[b\left(\frac{c_T+\lambda_1\frac{l}{\rho-\delta}}{2b\ln(e+I_N)}\right)^2\ln(e+I_N) - c_T\frac{c_T+\lambda_1\frac{l}{\rho-\delta}}{2b\ln(e+I_N)}\right] - \frac{1}{\rho}\lambda_1\frac{c_T+\lambda_1\frac{l}{\rho-\delta}}{2b\ln(e+I_N)}\frac{l}{\rho-\delta}\end{cases} \tag{51}$$

$$\begin{cases} k_3 = \frac{l}{\rho-\delta}\\ k_4 = \frac{1}{\rho}\left[b_Q\frac{b_Q-\frac{l}{\rho-\delta}\lambda_Q}{c_Q} - \frac{c_Q}{2}\left(\frac{b_Q-\frac{l}{\rho-\delta}\lambda_Q}{c_Q}\right)^2 - a\frac{a}{2\lambda_G}\left(\frac{l}{\rho-\delta}\right)^{-1}\right]\\ \qquad + \frac{l}{\rho-\delta}\frac{1}{\rho}\left[-\lambda_Q\frac{b_Q-\frac{l}{\rho-\delta}\lambda_Q}{c_Q} + \lambda_G\left(\frac{a}{2\lambda_G}\right)^2\left(\frac{l}{\rho-\delta}\right)^{-2}\right]\end{cases} \tag{52}$$

Therefore, it can be concluded that:

$$\begin{aligned}V_{N1}^{\star} = &\frac{l}{\rho-\delta}x_{N1} + \frac{1}{\rho}\left[b\left(\frac{c_T+\lambda_1\frac{l}{\rho-\delta}}{2b\ln(e+I_N)}\right)^2\ln(e+I_N) - c_T\frac{c_T+\lambda_1\frac{l}{\rho-\delta}}{2b\ln(e+I_N)}\right]\\ &-\frac{1}{\rho}\lambda_1\frac{c_T+\lambda_1\frac{l}{\rho-\delta}}{2b\ln(e+I_N)}\frac{l}{\rho-\delta}\end{aligned} \tag{53}$$

$$\begin{aligned}V_{N2}^{\star} = &\frac{l}{\rho-\delta}x_{N2} + \frac{1}{\rho}\left[b_Q\frac{b_Q-\frac{l}{\rho-\delta}\lambda_Q}{c_Q} - \frac{c_Q}{2}\left(\frac{b_Q-\frac{l}{\rho-\delta}\lambda_Q}{c_Q}\right)^2 - a\frac{a}{2\lambda_G}\left(\frac{l}{\rho-\delta}\right)^{-1}\right]\\ &+\frac{l}{\rho-\delta}\frac{1}{\rho}\left[-\lambda_Q\frac{b_Q-\frac{l}{\rho-\delta}\lambda_Q}{c_Q} + \lambda_G\left(\frac{a}{2\lambda_G}\right)^2\left(\frac{l}{\rho-\delta}\right)^{-2}\right]\end{aligned} \tag{54}$$

In this case,

$$T_N^{\star}(t) = \frac{c_T + \lambda_1\frac{l}{\rho-\delta}}{2b\ln(e+I_N)} \tag{55}$$

$$Q_N^{\star}(t) = \frac{b_Q - \frac{l}{\rho-\delta}\lambda_Q}{c_Q} \tag{56}$$

$$G_N^{\star}(t) = \frac{a}{2\lambda_G}\left(\frac{l}{\rho-\delta}\right)^{-1} \tag{57}$$

Appendix 2

Taking the derivatives of T_C and F_C with respect to (15) and the derivatives of Q_C and G_C with respect to (16), and setting them equal to zero, we can get:

$$T_C^{\star}(t) = \frac{c_C}{2b\ln(e + I_N + I_C)} \tag{58}$$

$$F_C^{\star}(t) = \frac{b_F}{2\lambda_F}\left(\frac{\partial V_{C1}}{\partial x_{C1}}\right)^{-1} \tag{59}$$

$$Q_C^{\star}(t) = \frac{b_Q - \frac{\partial V_{C2}}{\partial x_{C2}}(\lambda_Q + \lambda_F)}{c_Q} \tag{60}$$

$$G_C^{\star}(t) = \frac{a}{2\lambda_G}\left(\frac{\partial V_{C2}}{\partial x_{C2}}\right)^{-1} \tag{61}$$

Substituting (58) and (59) into (15) and substituting (60) and (61) into (16), we can get:

$$\begin{aligned}\rho V_{C1} = &\left[b\left(\frac{c_T}{2b\ln(e+I_N+I_C)}\right)^2\ln(e+I_N+I_C) - c_T\frac{c_T}{2b\ln(e+I_N+I_C)} -\right.\\ &\left. b_F\frac{b_F}{2\lambda_F}\left(\frac{\partial V_{C1}}{\partial x_{C1}}\right)^{-1} + lx_{C1}(t)\right] + \frac{\partial V_{C1}}{\partial x_{C1}}\left[\lambda_F\left(\frac{b_F}{2\lambda_F}\right)^2\left(\frac{\partial V_{C1}}{\partial x_{C1}}\right)^{-2} - \delta x_{C1}(t)\right]\end{aligned} \tag{62}$$

$$\begin{aligned}\rho V_{C2} = &\left[b_Q\frac{b_Q-\frac{\partial V_{C2}}{\partial x_{C2}}(\lambda_Q+\lambda_F)}{c_Q} - \frac{c_Q}{2}\left(\frac{b_Q-\frac{\partial V_{C2}}{\partial x_{C2}}(\lambda_Q+\lambda_F)}{c_Q}\right)^2 - a\frac{a}{2\lambda_G}\left(\frac{\partial V_{C2}}{\partial x_{C2}}\right)^{-1} + lx_{C2}(t)\right]\\ &+\frac{\partial V_{C2}}{\partial x_{C2}}\left[-(\lambda_Q+\lambda_F)\frac{b_Q-\frac{\partial V_{C2}}{\partial x_{C2}}(\lambda_Q+\lambda_F)}{c_Q} + \lambda_G\left(\frac{a}{2\lambda_G}\right)^2\left(\frac{\partial V_{C2}}{\partial x_{C2}}\right)^{-2} + \delta x_{C2}(t)\right]\end{aligned} \tag{63}$$

Let $V_{C1}^{\star} = k_5 x_{C1} + k_6$, $V_{C2}^{\star} = k_7 x_{C2} + k_8$, wherein, k_5, k_6, k_7, and k_8 are all constants. The parameters of the optimal social welfare function can be obtained by calculation as follows:

$$\begin{cases} k_5 = \frac{l}{\rho+\delta} \\ k_6 = \frac{1}{\rho}\left[b\left(\frac{c_T}{2b\ln(e+I_N+I_C)}\right)^2 \ln(e+I_N+I_C) - c_T\frac{c_T}{2b\ln(e+I_N+I_C)} - b_F\frac{b_F}{2\lambda_F}\left(\frac{l}{\rho+\delta}\right)^{-1}\right] \\ \qquad + \frac{l}{\rho+\delta}\frac{1}{\rho}\left[\lambda_F\left(\frac{b_F}{2\lambda_F}\right)^2\left(\frac{l}{\rho+\delta}\right)^{-2}\right] \end{cases} \tag{64}$$

$$\begin{cases} k_7 = \frac{l}{\rho-\delta} \\ k_8 = \frac{1}{\rho}\left[b_Q\frac{b_Q-\frac{l}{\rho-\delta}(\lambda_Q+\lambda_F)}{c_Q} - \frac{c_Q}{2}\left(\frac{b_Q-\frac{l}{\rho-\delta}(\lambda_Q+\lambda_F)}{c_Q}\right)^2 - a\frac{a}{2\lambda_G}\left(\frac{l}{\rho-\delta}\right)^{-1}\right] \\ \qquad + \frac{l}{\rho-\delta}\frac{1}{\rho}\left[-(\lambda_Q+\lambda_F)\frac{b_Q-\frac{l}{\rho-\delta}(\lambda_Q+\lambda_F)}{c_Q} + \lambda_G\left(\frac{a}{2\lambda_G}\right)^2\left(\frac{l}{\rho-\delta}\right)^{-2}\right] \end{cases} \tag{65}$$

Therefore, it can be concluded that:

$$V_{C1}^{\star} = \frac{1}{\rho}\left[b\left(\frac{c_T}{2b\ln(e+I_N+I_C)}\right)^2 \ln(e+I_N+I_C) - c_T\frac{c_T}{2b\ln(e+I_N+I_C)} - b_F\frac{b_F}{2\lambda_F}\left(\frac{l}{\rho+\delta}\right)^{-1}\right] + \frac{l}{\rho+\delta}x_{C1} + \frac{l}{\rho+\delta}\frac{1}{\rho}\left[\lambda_F\left(\frac{b_F}{2\lambda_F}\right)^2\left(\frac{l}{\rho+\delta}\right)^{-2}\right] \tag{66}$$

$$V_{C2}^{\star} = \frac{1}{\rho}\left[b_Q\frac{b_Q-\frac{l}{\rho-\delta}(\lambda_Q+\lambda_F)}{c_Q} - \frac{c_Q}{2}\left(\frac{b_Q-\frac{l}{\rho-\delta}(\lambda_Q+\lambda_F)}{c_Q}\right)^2 - a\frac{a}{2\lambda_G}\left(\frac{l}{\rho-\delta}\right)^{-1}\right] + \frac{l}{\rho-\delta}x_{C2} + \frac{l}{\rho-\delta}\frac{1}{\rho}\left[-(\lambda_Q+\lambda_F)\frac{b_Q-\frac{l}{\rho-\delta}(\lambda_Q+\lambda_F)}{c_Q} + \lambda_G\left(\frac{a}{2\lambda_G}\right)^2\left(\frac{l}{\rho-\delta}\right)^{-2}\right] \tag{67}$$

In this case,

$$T_C^{\star}(t) = \frac{c_C}{2b\ln(e+I_N+I_C)} \tag{68}$$

$$F_C^{\star}(t) = \frac{b_F}{2\lambda_F}\left(\frac{l}{\rho+\delta}\right)^{-1} \tag{69}$$

$$Q_C^{\star}(t) = \frac{b_Q-\frac{l}{\rho-\delta}(\lambda_Q+\lambda_F)}{c_Q} \tag{70}$$

$$G_C^{\star}(t) = \frac{a}{2\lambda_G}\left(\frac{l}{\rho-\delta}\right)^{-1} \tag{71}$$

Appendix 3

Taking the derivatives of T_P and P_P with respect to (17) and the derivatives of Q_P and G_P with respect to (18), and setting them equal to zero, we can get:

$$T_P^{\star}(t) = \frac{c_T}{2b\ln(e+I_N+I_P)} \tag{72}$$

$$P_P^{\star}(t) = \frac{b_P}{2\lambda_P}\left(\frac{\partial V_{P1}}{\partial x_{P1}}\right)^{-1} \tag{73}$$

$$Q_P^{\star}(t) = \frac{b_Q - b_P - \frac{\partial V_{P2}}{\partial x_{P2}}\lambda_Q}{c_Q} \tag{74}$$

$$G_P^{\star}(t) = \frac{a+a_P}{2\lambda_G}\left(\frac{\partial V_{P2}}{\partial x_{P2}}\right)^{-1} \tag{75}$$

Substituting (72) and (73) into (17) and substituting (74) and (75) into (18), we can get:

$$\rho V_{P1} = \left[b\left(\frac{c_T}{2b\ln(e+I_N+I_P)}\right)^2 \ln(e+I_N+I_P) - c_T\frac{c_T}{2b\ln(e+I_N+I_P)} + b_P\frac{b_P}{2\lambda_P}\left(\frac{\partial V_{P1}}{\partial x_{P1}}\right)^{-1} + lx_{P1}(t)\right] + \frac{\partial V_{P1}}{\partial x_{P1}}\left[-\lambda_P\left(\frac{b_P}{2\lambda_P}\right)^2\left(\frac{\partial V_{P1}}{\partial x_{P1}}\right)^{-2} + \delta x_{P1}(t)\right] \tag{76}$$

$$\rho V_{P2} = \left[(b_Q-b_P)\frac{b_Q-b_P-\frac{\partial V_{P2}}{\partial x_{P2}}\lambda_Q}{c_Q} - \frac{c_Q}{2}\left(\frac{b_Q-b_P-\frac{\partial V_{P2}}{\partial x_{P2}}\lambda_Q}{c_Q}\right)^2 - (a+a_P)\frac{a+a_P}{2\lambda_G}\left(\frac{\partial V_{P2}}{\partial x_{P2}}\right)^{-1} + lx_{P2}(t)\right] + \frac{\partial V_{P2}}{\partial x_{P2}}\left[-\lambda_Q\frac{b_Q-b_P-\frac{\partial V_{P2}}{\partial x_{P2}}\lambda_Q}{c_Q} + \lambda_G\left(\frac{a+a_P}{2\lambda_G}\right)^2\left(\frac{\partial V_{P2}}{\partial x_{P2}}\right)^{-2} + \delta x_{P2}(t)\right] \tag{77}$$

Let $V_{P1}^{\star} = k_9 x_{P1} + k_{10}$, $V_{P2}^{\star} = k_{11}x_{P2} + k_{12}$, wherein, k_9, k_{10}, k_{11}, and k_{12} are all constants. The parameters of the optimal social welfare function can be obtained by calculation as follows:

$$\begin{cases} k_9 = \frac{l}{\rho-\delta} \\ k_{10} = \frac{1}{\rho}\left[b\left(\frac{c_T}{2b\ln(e+I_N+I_P)}\right)^2 \ln(e+I_N+I_P) - c_T\frac{c_T}{2b\ln(e+I_N+I_P)} \right. \\ \qquad \left. + b_P\frac{b_P}{2\lambda_P}\left(\frac{l}{\rho-\delta}\right)^{-1}\right] + \frac{l}{\rho-\delta}\frac{1}{\rho}\left[-\lambda_P\left(\frac{b_P}{2\lambda_P}\right)^2\left(\frac{l}{\rho-\delta}\right)^{-2}\right] \end{cases} \tag{78}$$

$$\begin{cases} k_{11} = \frac{l}{\rho-\delta} \\ k_{12} = \frac{1}{\rho}\left[(b_Q-b_P)\frac{b_Q-b_P-\frac{l}{\rho-\delta}\lambda_Q}{c_Q} - \frac{c_Q}{2}\left(\frac{b_Q-b_P-\frac{l}{\rho-\delta}\lambda_Q}{c_Q}\right)^2 - (a+a_P)\frac{a+a_P}{2\lambda_G}\left(\frac{l}{\rho-\delta}\right)^{-1}\right] \\ \qquad + \frac{l}{\rho-\delta}\frac{1}{\rho}\left[-\lambda_Q\frac{b_Q-b_P-\frac{l}{\rho-\delta}\lambda_Q}{c_Q} + \lambda_G\left(\frac{a+a_P}{2\lambda_G}\right)^2\left(\frac{l}{\rho-\delta}\right)^{-2}\right] \end{cases} \tag{79}$$

Therefore, it can be concluded that:

$$V_{P1}^{\star} = \frac{1}{\rho}\left[b\left(\frac{c_T}{2b\ln(e+I_N+I_P)}\right)^2 \ln(e+I_N+I_P) - c_T\frac{c_T}{2b\ln(e+I_N+I_P)} + b_P\frac{b_P}{2\lambda_P}\left(\frac{l}{\rho-\delta}\right)^{-1}\right] + \frac{l}{\rho-\delta}x_{P1} + \frac{l}{\rho-\delta}\frac{1}{\rho}\left[-\lambda_P\left(\frac{b_P}{2\lambda_P}\right)^2\left(\frac{l}{\rho-\delta}\right)^{-2}\right] \tag{80}$$

$$V_{P2}^{\star} = \frac{1}{\rho}\left[(b_Q-b_P)\frac{b_Q-b_P-\frac{l}{\rho-\delta}\lambda_Q}{c_Q} - \frac{c_Q}{2}\left(\frac{b_Q-b_P-\frac{l}{\rho-\delta}\lambda_Q}{c_Q}\right)^2 - (a+a_P)\frac{a+a_P}{2\lambda_G}\left(\frac{l}{\rho-\delta}\right)^{-1}\right] + \frac{l}{\rho-\delta}x_{P2} + \frac{l}{\rho-\delta}\frac{1}{\rho}\left[-\lambda_Q\frac{b_Q-b_P-\frac{l}{\rho-\delta}\lambda_Q}{c_Q} + \lambda_G\left(\frac{a+a_P}{2\lambda_G}\right)^2\left(\frac{l}{\rho-\delta}\right)^{-2}\right] \tag{81}$$

In this case,

$$T_P^{\star}(t) = \frac{c_T}{2b\ln(e+I_N+I_P)} \tag{82}$$

$$P_P^{\star}(t) = \frac{b_P}{2\lambda_P}\left(\frac{l}{\rho-\delta}\right)^{-1} \tag{83}$$

$$Q_P^*(t) = \frac{b_Q - b_P - \frac{l}{\rho-\delta}\lambda_Q}{c_Q} \tag{84}$$

$$G_P^*(t) = \frac{a + a_P}{2\lambda_G}\left(\frac{l}{\rho - \delta}\right)^{-1} \tag{85}$$