Risk assessment of goaf instability based on a variable weight model

Linning Guo[1,2], Kepeng Hou[1,2]*, Huafen Sun[1,2] and Yong Yang[1,2]

[1]Department of Resource Development Engineering, Kunming University of Science and Technology, Kunming, China, [2]Yunnan Key Laboratory of Sino-German Blue Mining and Utilization of Special Underground Space, Kunming University of Science and Technology, Kunming, China

Goaf instability poses significant hazards, affecting mine safety and public welfare. This study aims to evaluate the risk of goaf instability to enhance safety measures in mining operations. Thirteen key indicators were identified to construct a comprehensive evaluation index system. By integrating game theory, we combined subjective and objective weights to develop a constant weight model, which was subsequently improved by considering data distribution characteristics to develop a local variable weight model. The variable weight intervals were determined through cumulative frequency analysis of normalized factor indices, and the Monte Carlo method was employed to define weight adjustment parameters. Using the cloud model, we assessed the instability risk of goafs. Our results indicate that the variable weight model provides higher evaluation accuracy compared to the constant weight model, offering clearer and more distinguishable membership degrees for the evaluation outcomes, suggesting its potential for more precise risk assessments in mining operations.

KEYWORDS

goaf, risk assessment, constant weight model, variable weight model, variable weight interval, weight adjustment parameters

1 Introduction

With the escalation of mining operations, numerous unmanaged voids have resulted in overlapping and concentrated void zones. If these voids experience widespread instability, they can precipitate mine air blasts, seismic events, and surface collapse, leading to vegetation loss, soil erosion, and landslides. These geological hazards cause substantial economic damage and have significant societal impacts. Consequently, the instability of mining voids is a severe threat and one of the foremost risks to mine safety and public security. Therefore, it is imperative to assess their risk levels and demarcate hazardous areas for mine safety management (Ma et al., 2012; Zhang et al., 2023; Zhe et al., 2023).

The instability data concerning mining voids exhibit randomness and the classification of instability risk levels is ambiguous, rendering the evaluation process fraught with uncertainty (Guo D. et al., 2024; Ke et al., 2024; Wang et al., 2023). The cloud model effectively addresses the issues of uncertainty, randomness, and ambiguity when assessing the risk of void instability. The core challenge of this method is determining index weights. Commonly used methods for weight determination include the analytic hierarchy process (AHP), gray relational analysis, entropy method, coefficient of variation, and CRiteria Importance Through Intercriteria Correlation (CRITIC) method,

FIGURE 1
Research framework.
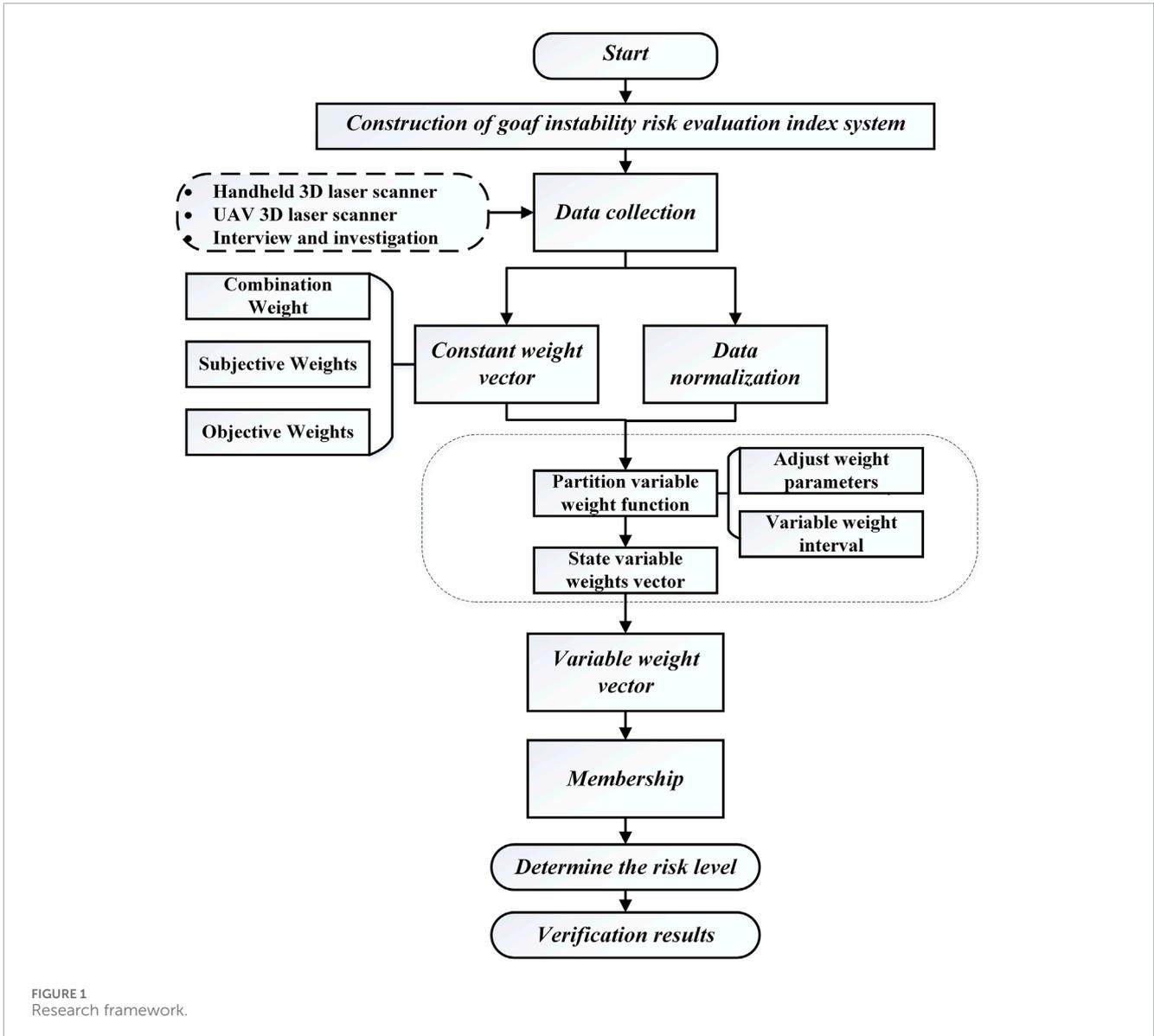
which encompass both subjective and objective approaches. Subjective methods are computationally straightforward and widely applied but are highly subjective, leading to variable results and uncertain accuracy (Liu et al., 2020). Objective methods (gray relational analysis, entropy method, CRITIC) avoid these issues, but are highly data-dependent and susceptible to data fluctuations (Esmaili and Karipour, 2024; Zhang et al., 2024; Zhang and Shang, 2024). Consequently, scholars have employed algorithms such as multiplicative synthesis and linear weighting to combine the weights derived from multiple methods and enhance the accuracy of the evaluation results (Fu et al., 2024; Wang et al., 2024; Zheng et al., 2024).

Although the combination weighting method enhances the accuracy of the index weights, the weights determined by this method remain static for different mining voids. This constant weight model only considers the relative importance of various

factors in the evaluation but overlooks the impact of internal differences in index values on the assessment results, failing to capture the influence of sudden changes in factor indices due to varying evaluation criteria on void instability risks (Guo L. et al., 2024; Yao et al., 2024). Variable weight theory improves upon the traditional constant weight comprehensive evaluation by adjusting the weight of each factor in response to changes in its corresponding state index value, thereby better reflecting the impact of factor state changes in the evaluation decision-making system. This theory has been applied to assess land subsidence and road construction hazards (Guo L. et al., 2024; Han et al., 2023; Jin et al., 2024). Therefore, integrating the variable weight theory into the evaluation of mining void instability risk is beneficial.

This study considered the goaf area of the Baiyang mining area at the Bainiuchang Mine in Mengzi City, Yunnan Province,

FIGURE 2
Regional location map of the Bainiuchang mine.
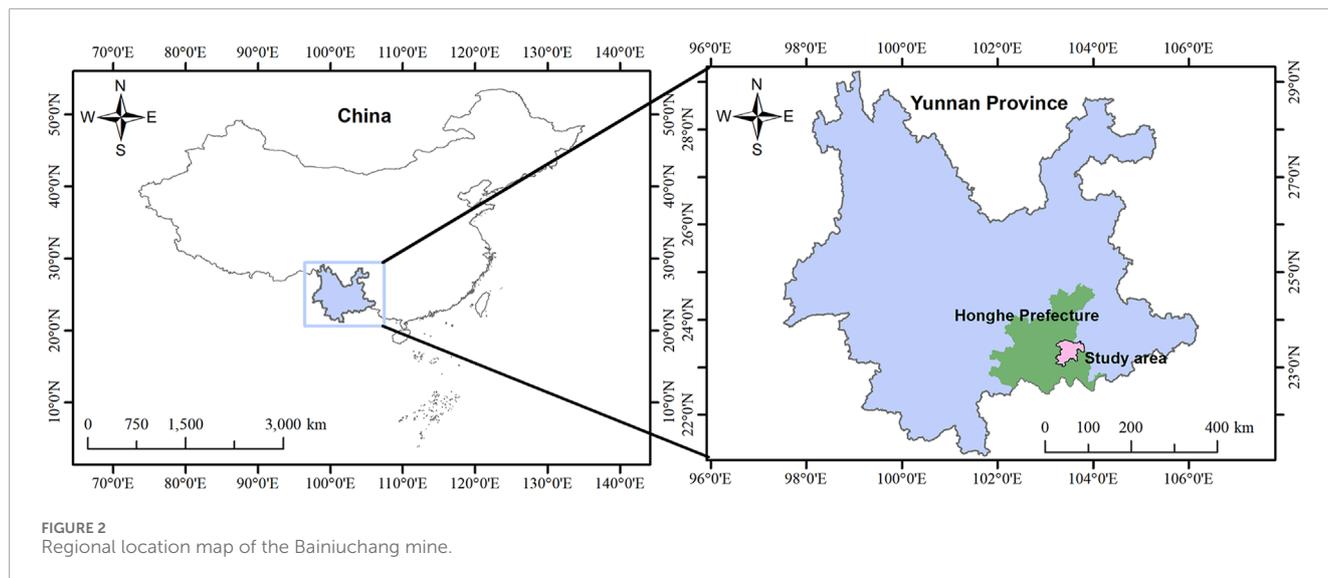
TABLE 1 Survey data of mined-out area.

No	X1	X2	X3/%	X4/°	X5/m³	X6/m²	X7	X8	X9/m	X10/a	X11/m³	X12	X13	Class
1480-1	4	3	68.2	32	1,837	835	22.81	0.11	544	10	84	3	3	IV
1480-2	4	4	73.7	28	4,138	1,881	24.58	0.05	534	10	282	2	4	IV
1480-3	4	4	71.0	23	3,424	1,556	17.80	0.09	526	10	249	2	3	IV
1480-4	4	4	73.5	23	1,019	424	21.63	0.17	520	10	72	2	3	IV
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
1580-1	2	1	62.9	24	5,144	643	12.01	0.30	379	10	0	2	3	IV
1580-2	2	1	68.2	24	4,193	524	5.89	0.46	380	10	0	4	3	IV
1580-3	2	1	65.5	24	2,603	325	14.90	0.57	390	10	0	1	2	IV
1580-4	2	1	67.5	24	4,612	576	7.35	0.40	404	10	0	2	3	IV
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
1640-1	4	3	66.7	25	1,828	366	5.49	0.46	376	13	0	2	3	II
1640-2	2	4	64.6	25	7,827	712	7.56	0.63	396	13	0	1	2	II
1640-3	2	2	61.7	23	4,543	649	9.58	0.28	404	13	0	1	4	II
1640-4	3	4	64.4	25	4,654	517	6.80	0.48	404	13	0	3	3	II
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
1730-10	2	2	63.8	23	1,908	273	12.89	0.44	184	18	0	3	4	IV
1730-11	4	3	66.2	21	5,586	931	11.96	0.24	187	17	0	3	2	IV
1730-12	3	2	64.7	21	3,165	528	16.94	0.21	172	17	0	2	2	IV
1730-13	2	3	64.1	21	6,124	1,021	18.30	0.21	172	17	102	2	3	II

FIGURE 3
Violin plots with different indexes: **(A)** Rock mass quality X3; **(B)** Dip angle of orebody X4; **(C)** Goaf volume X5; **(D)** Maximum exposed area X6.
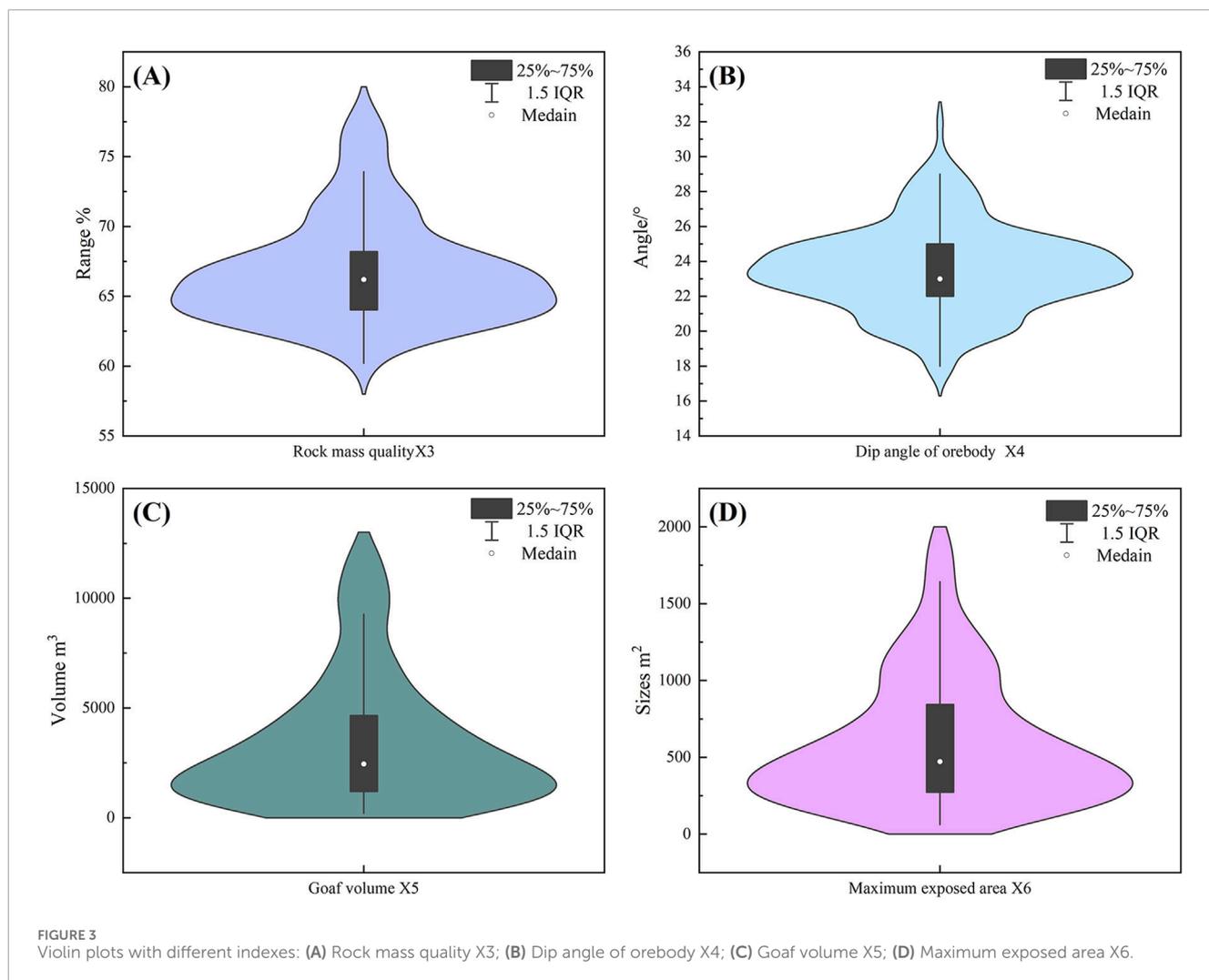
TABLE 2 Mood operator, scale and fuzzy membership.

The tone operator	Same	Slightly	A little	Somewhat	Obviously	Very	Quite	Extremely	Exceedingly	Exceedingly
Scale (v)	0.5	0.55	0.6	0.65	0.7	0.8	0.85	0.9	0.95	1
Fuzzy membership ($\frac{1-v}{v}$)	1	0.818	0.667	0.538	0.429	0.25	0.176	0.111	0.053	0

as an example. The subjective weights were calculated using the importance–binary comparison analysis method, and the objective weights were determined using the Gini coefficient weighting method. The subjective and objective weights were combined based on a game-theory aggregation model. Based on these combined weights, the improved local variable weight theory and cloud model were utilized along with the actual conditions of the goaf area to establish a coupled model for evaluating its instability risk. This can, in turn, guide subsequent reasonable and orderly goaf area management.

2 Methodology

In this study, a comprehensive methodology was employed to assess goaf instability risks. Figure 1 illustrates the research framework. Initially, an evaluation index system tailored to the characteristics of goafs was constructed by selecting 13 key indicators. Data collection involved advanced techniques, such as three-dimensional (3d) laser scanning, unmanned aerial vehicle (UAV) surveys, and field visits. The subjective weights of the evaluation indices were determined using the importance–binary
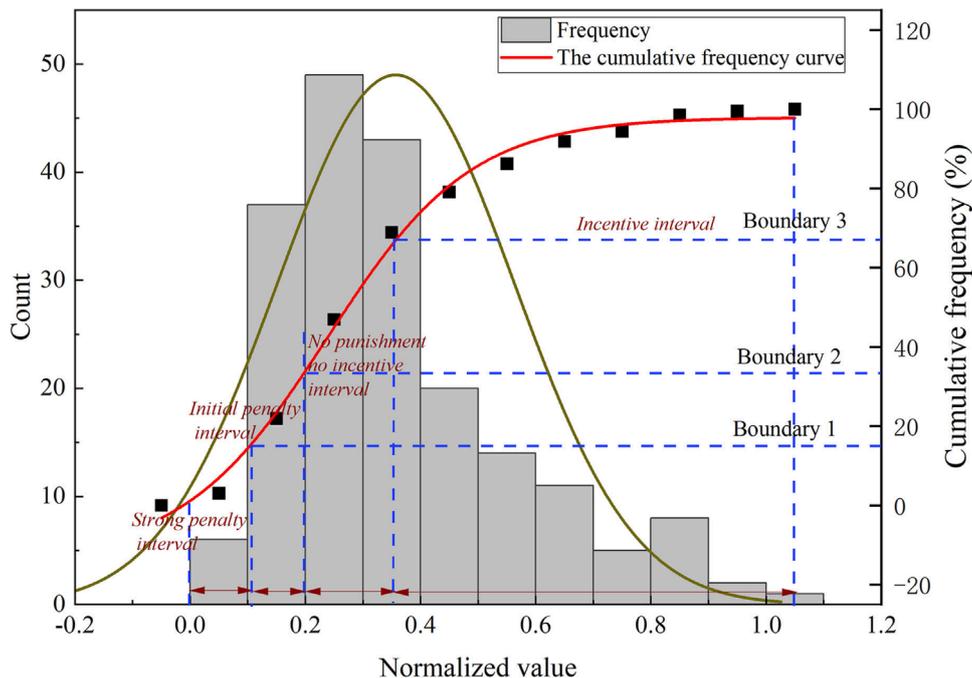
FIGURE 4
Example of a method for determining variable weighting intervals based on cumula-tive frequency.

FIGURE 5
Evaluation index correlation coefficient heat map.

TABLE 3 Importance score of evaluation index.

Evaluation indicators	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11	X12	X13
Standard deviation	0.39	0.34	0.21	0.13	0.14	0.10	0.21	0.17	0.23	0.29	0.17	0.39	0.39
Confliction	12.76	11.78	12.65	11.65	10.39	10.50	12.89	11.78	12.87	13.22	11.97	12.22	11.84
Importance score	5.03	4.03	2.61	1.48	1.45	1.04	2.68	2.06	2.97	3.78	2.07	4.81	4.67
Index ranking	1	4	8	11	12	13	7	10	6	5	9	2	3

TABLE 4 Subjective weight calculation results.

Evaluation indicators	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11	X12	X13
Scale	0.5	0.55	0.65	0.7	0.6	0.6	0.56	0.58	0.68	0.72	0.67	0.52	0.53
Membership	1	0.818	0.538	0.429	0.667	0.667	0.786	0.724	0.471	0.389	0.493	0.923	0.887
Weights	0.114	0.093	0.061	0.049	0.076	0.076	0.089	0.082	0.054	0.044	0.056	0.105	0.101

TABLE 5 The results of objective weight calculation.

Evaluation indicators	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11	X12	X13
Gini coefficient	0.019	0.037	0.006	0.005	0.040	0.050	0.041	0.028	0.066	0.053	0.052	0.002	0.004
Weights	0.047	0.092	0.015	0.013	0.099	0.124	0.101	0.069	0.164	0.132	0.129	0.005	0.010

TABLE 6 Division of evaluation indicators into weighting intervals.

Evaluation indicators	Strong penalty interval	Initial penalty interval	No punishment no incentive interval	Incentive interval
Structural types of rock mass X1	[0, 0.25)	[0.25, 0.5)	[0.5, 0.75)	[0.75, 1]
Geological structure X2	[0, 0.25)	[0.25, 0.5)	[0.5, 0.75)	[0.75, 1]
Rock mass quality X3	[0, 0.11)	[0.11, 0.20)	[0.20, 0.36)	[0.36, 1]
Dip angle of orebody X4	[0, 0.16)	[0.16, 0.27)	[0.27, 0.43)	[0.43, 1]
Goaf volume X5	[0, 0.49)	[0.49, 0.66)	[0.66, 0.83)	[0.83, 1]
Maximum exposed area X6	[0, 0.45)	[0.45, 0.61)	[0.61, 0.8)	[0.8, 1]
Pillar area ratiot X7	[0, 0.04)	[0.05, 0.12)	[0.12, 0.3)	[0.3, 1]
Ratio of rise to span X8	[0, 0.05)	[0.05, 0.11)	[0.11, 0.24)	[0.24, 1]
Embedding depth X9	[0, 0.25)	[0.25, 0.4)	[0.4, 0.63)	[0.63, 1]
Exposure time of goaf X10	[0, 0.24)	[0.24, 0.4)	[0.4, 0.66)	[0.66, 1]
Mined-out area water X11	[0, 0.25)	[0.25, 0.5)	[0.5, 0.75)	[0.75, 1]
Situation of adjacent mined-out areas X12	[0, 0.25)	[0.25, 0.5)	[0.5, 0.75)	[0.75, 1]
Goaf management X13	[0, 0.25)	[0.25, 0.5)	[0.5, 0.75)	[0.75, 1]

TABLE 7 Final variable weight interval classification.

Strong penalty interval	Initial penalty interval	No punishment no incentive interval	Incentive interval
[0, 0.25)	[0.25, 0.42)	[0.42, 0.64)	[0.64, 1]

(1) Problem Statement
- Determine the objective function $f(a1, a2, a3, c)$ and optimize it to find the maximum value of the objective function.

(2) Random Variable Generation
- Generate random values for $a1, a2, a3$ and c within the range [0, 1].

(3) Calculate Sample Values
- For each set of randomly generated $a1, a2, a3$, and c, calculate the corresponding function value $f(a1, a2, a3, c)$.

(4) Statistical Analysis
- Find the set of $a1, a2, a3$, and c that results in the maximum value of $f(a1, a2, a3, c)$ among all the samples.

(5) Result Estimation
- Record and output the values of a1, a2, a3 and c that maximize.

FIGURE 6
The calculation process of weight adjustment parameters.

comparison analysis method, and the objective weights were derived using the Gini coefficient weighting method. These weights were then integrated using game theory to form a constant weight model. To enhance the accuracy, we improved this model by incorporating data distribution characteristics into a local variable weight model. Variable weight intervals were established by analyzing the cumulative frequency of the normalized factor indices, and weight-adjustment parameters were determined using the Monte Carlo method. Finally, a cloud model was employed to evaluate the goaf instability risk, providing a robust and precise assessment.

2.1 Data collection and analysis

2.1.1 Study area

The Bainiuchang Mine is located 42.5 km at a direction of 75° from the downtown area of Mengzi City, Yunnan Province, China (coordinates: 103°45′ to 103°48′, 23°26′30″ to 23°29′30″). The study area (Figure 2), the Baiyang mining area, is located in the northwestern part of the mining area. Mining activities began in 1994, with the peak mining period occurring between 2000 and 2010.

2.1.2 Construction of the evaluation index system for goaf instability risk

Numerous factors contribute to the instability in mined-out areas. Through field surveys and literature review, 13 indicators,

including rock mass structure, geological structure, rock quality, ore body dip angle, goaf volume, maximum exposed area, pillar area ratio, ratio of rise to span, embedding depth, exposure time of goaf, mined-out area water, situation of adjacent mined-out areas, and goaf management (Chen et al., 2024; He et al., 2022; Qin et al., 2019; Ren et al., 2022). An evaluation index system for the instability risk in mined-out areas was constructed. Based on the relevant literature and field surveys, the risk levels were identified as I, II, III, and IV, with the risk level classification detailed in Supplementary Table S1 (Guo et al., 2022; Jia et al., 2016; Lang and Chen, 2015). The corresponding standards for each evaluation indicator are listed in Supplementary Table S2 (Hu et al., 2017; Hu, 2022; Ma, 2015; Ren Honggang, 2020; Wang, 2020).

2.1.3 Data collection

The investigation of the mined-out areas commenced in April 2021 and concluded in October 2022. The team conducted handheld 3D laser scanning surveys of accessible and safe mined-out areas (field surveys and scanning results are shown in Supplementary Figures S1A, C). For areas with high safety risks where direct human entry was not feasible, UAV photography equipment was used for surveying (field survey and scanning results are shown in Supplementary Figures S1B, D). A total of 195 mined-out areas (or groups of mined-out areas) were identified in the Baiyang mining area. Through field surveys and interviews with mining engineering technicians, data on 13 evaluation indicators were collected from 195 mined-out areas in the Baiyang area, as detailed in Table 1.
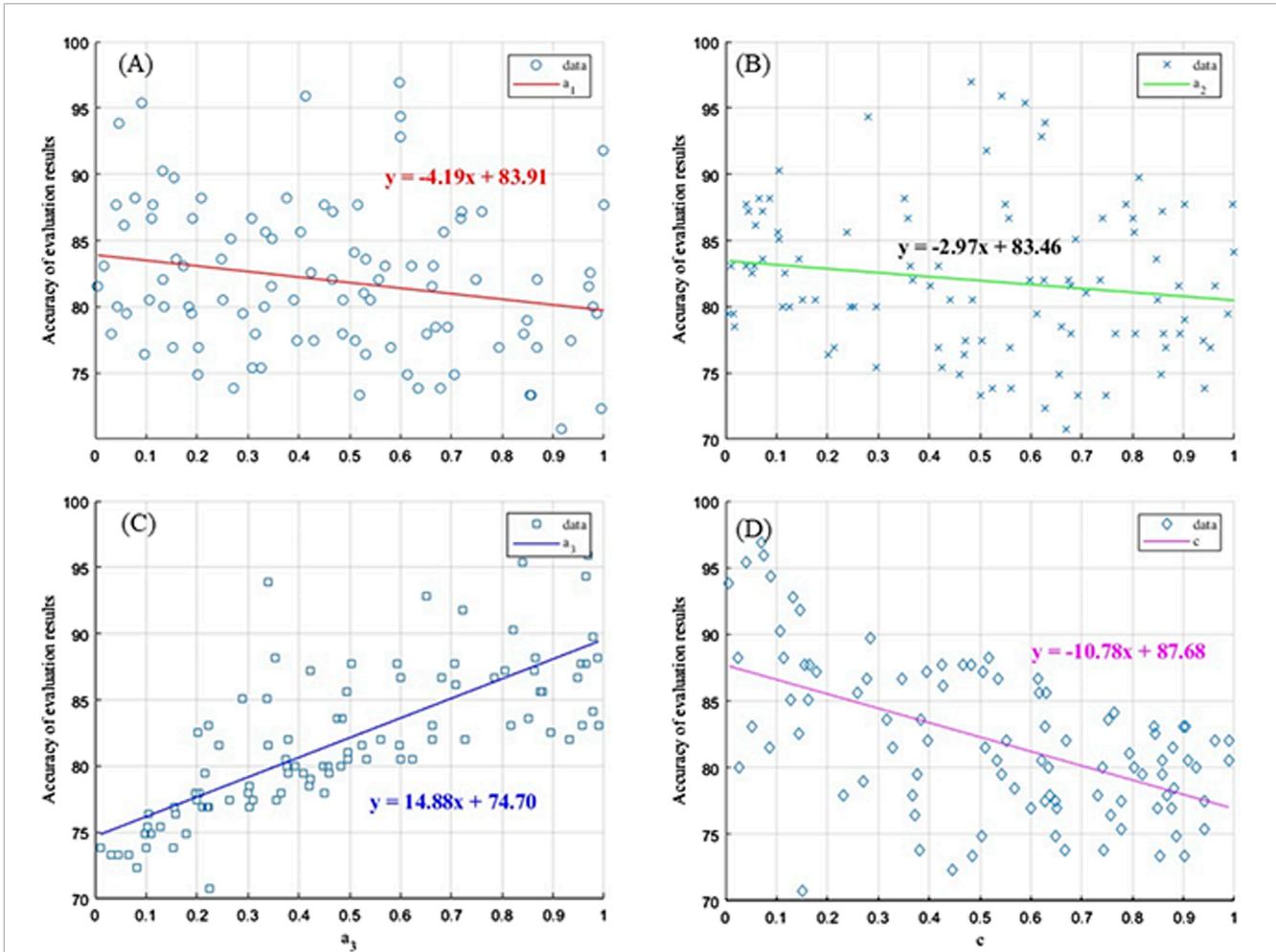
FIGURE 7
The curve of the change of the weight parameter. **(A)** The variation curve of the weight adjustment parameter a1; **(B)** The variation curve of the weight adjustment parameter a2; **(C)** The variation curve of the weight adjustment parameter a3; **(D)** The variation curve of the weight adjustment parameter c.

TABLE 8 The final value of the weight adjustment parameters.

Adjust weight parameters	a1	a2	a3	c
Value	0.29	0.66	0.67	0.01

The integrity and distribution of data in Table 1 were assessed using violin plots. Owing to space constraints, violin plots for only four indicators (X3, X4, X5, and X6) are presented. Figure 3 shows that under natural conditions, the data patterns in Table 1 generally align with a normal distribution.

2.2 Determining constant weights

2.2.1 Importance-subjective weight analysis of binary comparison

To reduce the subjectivity of the computational results and simplify the calculation process, this study employs the CRITIC method to comprehensively measure the contrast strength and conflict of the evaluation indicators. This method objectively quantifies the importance for ranking purposes. A preliminary ranking was established and then adjusted based on expert judgment and the initial ranking. Subsequently, a binary comparison method (Wang et al., 2014; Lu et al., 2014) was used to calculate the subjective weights of the indicators without consistency testing. This approach is relatively straightforward and can replace traditional methods such as the AHP to determine the subjective weights of indicators related to the instability risk of mined-out areas.

2.2.1.1 Calculating the importance of evaluation indicators

The dimensionless processing of evaluation indicator values is performed using Equation 1 (Huan-ling et al., 2023; Maneengam, 2023; Qianjun et al., 2023).

$$MMSx_{ij}^* = \frac{x_{ij} - min(x_j)}{max(x_j) - min(x_j)}$$
$$NMMSx_{ij}^* = \frac{max(x_j) - x_{ij}}{max(x_j) - min(x_j)}$$
(1)
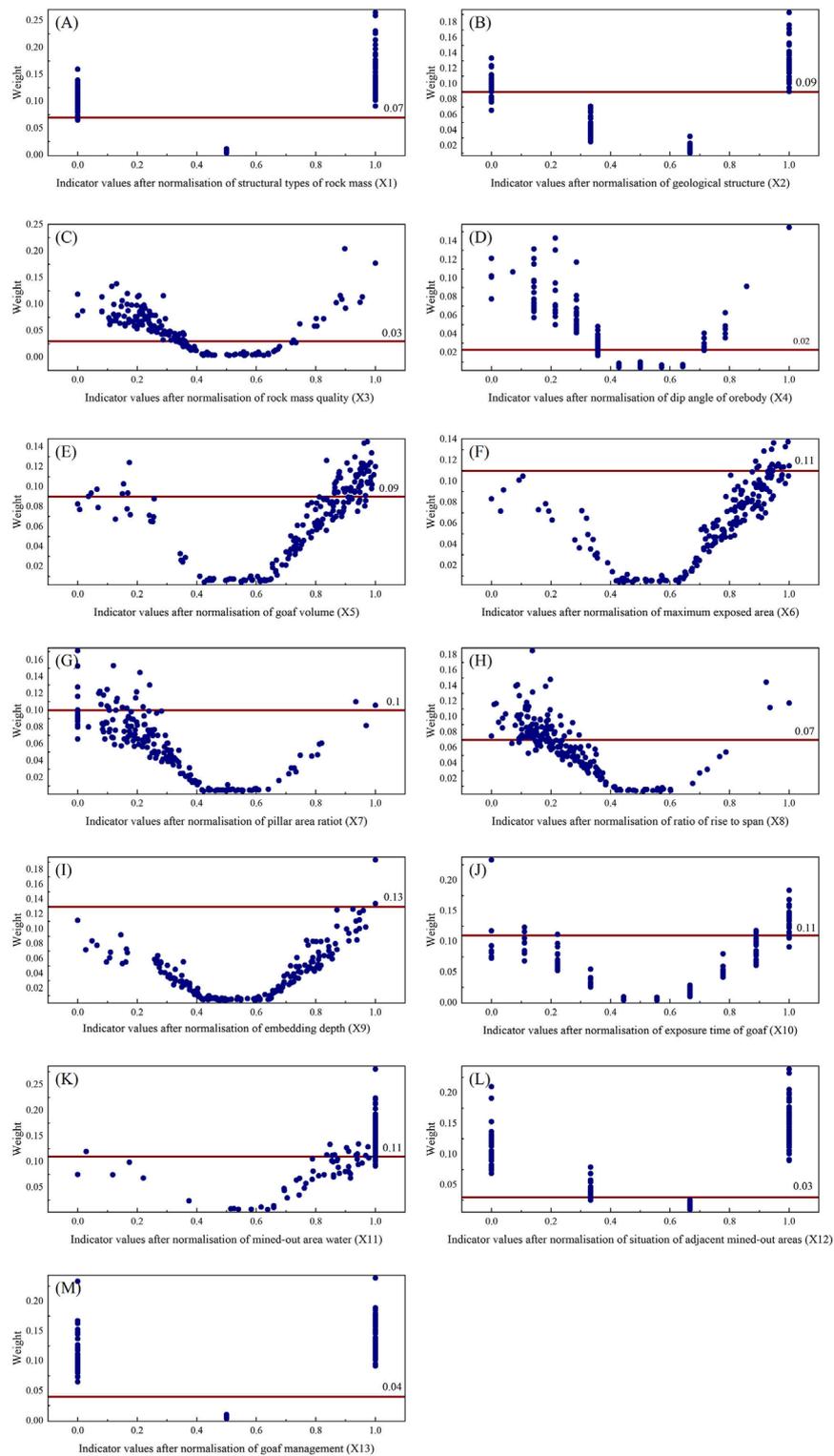
FIGURE 8
Evaluation index constant weight and variable weight comparison chart. **(A)** Rock mass structure X1; **(B)** Geological structure X2; **(C)** Rock quality X3; **(D)** Ore body dip angle X4; **(E)** Goaf volume X5; **(F)** Maximum exposed area X6; **(G)** Pillar area ratio X7; **(H)** Ratio of rise to span X8; **(I)** Embedding depth X9; **(J)** Expo-sure time of goaf X10; **(K)** Mined-out area water X11; **(L)** Situation of adjacent mined-out areas X12; **(M)** Goaf management X13.

TABLE 9 Variable weight analysis table.

Evaluation indicators	Number	Average value	Standard deviation	Maximum	Minimum	Median level
Structural types of rock mass X1	195	0.078	0.064	0.265	0.004	0.093
Geological structure X2	195	0.065	0.047	0.193	0.010	0.041
Rock mass quality X3	195	0.050	0.037	0.204	0.003	0.048
Dip angle of orebody X4	195	0.034	0.034	0.155	0.004	0.024
Goaf volume X5	195	0.068	0.039	0.145	0.004	0.076
Maximum exposed area X6	195	0.060	0.037	0.138	0.004	0.063
Pillar area ratiot X7	195	0.059	0.037	0.171	0.004	0.062
Ratio of rise to span X8	195	0.064	0.035	0.185	0.004	0.067
Embedding depth X9	195	0.038	0.034	0.193	0.004	0.029
Exposure time of goaf X10	195	0.053	0.050	0.233	0.004	0.031
Mined-out area water X11	195	0.125	0.044	0.280	0.007	0.128
Situation of adjacent mined-out areas X12	195	0.086	0.057	0.239	0.010	0.094
Goaf management X13	195	0.077	0.061	0.239	0.004	0.093

The standard deviation (Equation 3) is used to represent the difference fluctuation of the internal values of each index to reflect the contrast strength between the evaluation indices, the correlation coefficient (Equation 4) is used to represent the correlation between the indices, and the conflict between the indices is extracted using Equation 5.

$$\overline{x_j} = \frac{1}{m} \sum_{i=1}^{m} x_{ij} \tag{2}$$

$$S_j = \sqrt{\frac{1}{m-1} \sum_{i=1}^{m} \left(x_{ij} - \overline{x_j} \right)} \tag{3}$$

$$r_{ij} = \frac{\sum_{j,k=1}^{n} \left(x_{ij} - \overline{x_j} \right) \times \left(x_{ik} - \overline{x_k} \right)}{\sqrt{\sum_{j=1}^{n} \left(x_{ij} - \overline{x_j} \right)^2 \times \sum_{k=1}^{n} \left(x_{ik} - \overline{x_k} \right)^2}} \tag{4}$$

$$\eta_j = \sum_{i=1}^{n} \left(1 - r_{ij} \right) \tag{5}$$

Equations 3, 5 are multiplied to obtain the importance score of the evaluation index.

$$C_j = s_j \times \eta_j \tag{6}$$

2.2.1.2 Importance−binary comparison analysis method

By determining the importance ranking of evaluation indicators using Equations 1–6, the importance of the evaluation indicators for the instability risk of mined-out areas was calculated, and a preliminary importance ranking was established (Wang et al., 2014; Lu et al., 2014). Experts were consulted to make

appropriate adjustments to the initial ranking, resulting in a final, practically reflective ranking order.

If the importance of index u_i is not less than that of u_j, it is recorded as $u_i \succ u_j$. After sorting, the influencing factors are $u_1^*, u_2^*, \cdots, u_n^*$, and the relative importance of each index $u_1, u_2, \cdots, u_n$ satisfies the relationship is shown in Equation 7:

$$u_1^* \succ u_2^* \succ \cdots \succ u_n^* \tag{7}$$

2.2.1.3 Selection of scales and calculation of weight coefficients

Based on the ranking of influencing factors, appropriate modal operators and fuzzy membership degrees are selected for each factor from Table 2. The relationship between the scales and membership degrees in Table 2 can be represented by the curve shown in Supplementary Figure S2. When the modal operator falls between two operators in Table 2, interpolation can be used to determine the fuzzy membership degree from Supplementary Figure S2. Finally, a vector matrix is constructed using the fuzzy membership degrees of each factor as weight coefficients, and normalization is applied to obtain the weight vector $B = \left(b_{ij} \right)_{1 \times n}$.

2.2.2 Objective weighting of Gini coefficient weighting method

The Gini coefficient weighting method is an objective weighting method that calculates the Gini coefficient of the evaluation index and normalizes the Gini coefficient of each index to obtain the weight of the evaluation index. The calculation process is as follows (Dai et al., 2024; Zhao et al., 2024; Li, 2015; Li et al., 2014):

TABLE 10 Characteristic parameters of each indicator cloud at different levels.

Evaluation indicators	I Class	II Class	III Class	IV Class
X1	(1, 0.425, 0.01)	(2, 0.425, 0.01)	(3, 0.425, 0.01)	(4, 0.425, 0.01)
X2	(1, 0.425, 0.01)	(2, 0.425, 0.01)	(3, 0.425, 0.01)	(4, 0.425, 0.01)
X3	(20, 16.987, 0.01)	(45, 4.247, 0.01)	(55, 4.247, 0.01)	(80, 16.987, 0.01)
X4	(5, 4.274, 0.01)	(20, 8.493, 0.01)	(40, 8.493, 0.01)	(70, 16.987, 0.01)
X5	(9, 0.807, 0.01)	(5.2, 2.378, 0.01)	(1.52, 0.747, 0.01)	(0.32, 0.272, 0.01)
X6	(3850, 977, 0.01)	(1950, 637, 0.01)	(1000, 170, 0.01)	(400, 340, 0.01)
X7	(2.5, 2.123, 0.01)	(7.5, 2.123, 0.01)	(12.5, 2.123, 0.01)	(22.5, 6.37, 0.01)
X8	(0.015, 0.013, 0.01)	(0.04, 0.008, 0.01)	(0.075, 0.021, 0.01)	(0.8, 0.595, 0.01)
X9	(700, 85, 0.01)	(500, 85, 0.01)	(300, 85, 0.01)	(100, 85, 0.01)
X10	(30, 8.493, 0.01)	(15, 4.247, 0.01)	(8, 1.699, 0.01)	(3, 2.548, 0.01)
X11	(7, 0.849, 0.01)	(4.5, 1.274, 0.01)	(3, 0.849, 0.01)	(0.5, 0.425, 0.01)
X12	(1, 0.425, 0.01)	(2, 0.425, 0.01)	(3, 0.425, 0.01)	(4, 0.425, 0.01)
X13	(1, 0.425, 0.01)	(2, 0.425, 0.01)	(3, 0.425, 0.01)	(4, 0.425, 0.01)

Calculate the Gini coefficient value of the evaluation index:

$$G_k = \begin{cases} \sum_{i=1}^{n}\sum_{j=1}^{n}|X_{ki} - X_{kj}|/2n^2\mu_k & \mu_k \neq 0 \\ \sum_{i=1}^{n}\sum_{j=1}^{n}|X_{ki} - X_{kj}|/(n^2 - n) & \mu_k = 0 \end{cases} \quad (8)$$

where G_k is the Gini coefficient of the k-th index, n is the total number of index data points, and X_{ki} is the i-th value of the k-th evaluation index. μ_k is the expected value of the k-th indicator.

Evaluation index Gini coefficient weight calculation: The Gini coefficient weights of the different indices were obtained by normalizing the Gini coefficient values of each evaluation index.

$$g_k = G_k / \left(\sum_{i=1}^{m} G_i \right) \quad (9)$$

2.2.3 Combination weighting based on game theory

The problem of determining the weight of the index combination is solved using game-theory aggregation. Let the weight vector of m indicators calculated by the importance-binary comparative analysis method be $\vec{W}_1^T$, and the weight vector of m indicators obtained by the Gini coefficient objective weighting method be $\vec{W}_2^T$. Let $\vec{W}^T$ be the linear combination of the weight vectors $\vec{W}_1^T$ and $\vec{W}_2^T$. By calculating, the deviation between $\vec{W}^T$ and $\vec{W}_1^T$ and $\vec{W}_2^T$ is minimized, α_p can be obtained, and then normalized. The combined weight vector is obtained as follows (Dai et al., 2024; Zhao et al., 2024; Li, 2015; Li et al., 2014):

$$\overline{W} = \sum_{P=1}^{2} \alpha_P \overline{W}_P^T \quad (10)$$

$$\min \left\| \sum_{P=1}^{2} \alpha_P \vec{W}_P^T - \overline{W}_l^T \right\| \quad (11)$$

2.3 Determining variable weight

2.3.1 Local variable weight theory

The variable weight theory, introduced by Wang (1985) in the 1980s, posits that the weight of an evaluation indicator dynamically adjusts based on changes in its corresponding state indicator value. Subsequently, Li (1995); Li (1996) provided axiomatic definitions and calculation formulas for three types of variable weights: penalty, incentive, and neutral.

Let the state vector of the evaluation index x_j be $X = (x_1, x_2, \cdots, x_m)$, and let the constant weight vector $W_0 = (w_1^0, w_2^0, \cdots, w_m^0)$ be obtained using the above combination weighting method. Variable Weight Vector $W(x) = (w_1(x), w_2(x), \cdots, w_m(x))$. The variable weight calculation formula (Equation 12), $S(X) = (S_1(X), S_2(X), \cdots, S_m(X))$ is the state-variable weight function.

$$W(X) = \frac{W_0 \bullet S(X)}{\sum_{i=1}^{m} w_i^{(0)} \bullet S_i(X)} = \left(\frac{w_1^{(0)} \bullet S_1(X)}{\sum_{i=1}^{m} w_i^{(0)} \bullet S_i(X)}, \frac{w_2^{(0)} \bullet S_2(X)}{\sum_{i=1}^{m} w_i^{(0)} \bullet S_i(X)}, \cdots, \frac{w_m^{(0)} \bullet S_m(X)}{\sum_{i=1}^{m} w_i^{(0)} \bullet S_i(X)} \right) \quad (12)$$

Duan (2003) proposed a penalty-incentive segmented coordination theorem for state variable weight functions, dividing the penalty phase into initial penalty, strong penalty, and veto stages. Li and Wu (2015); Wu and Li (2016) constructed a four-segment (Equation 13) local variable weight state function based
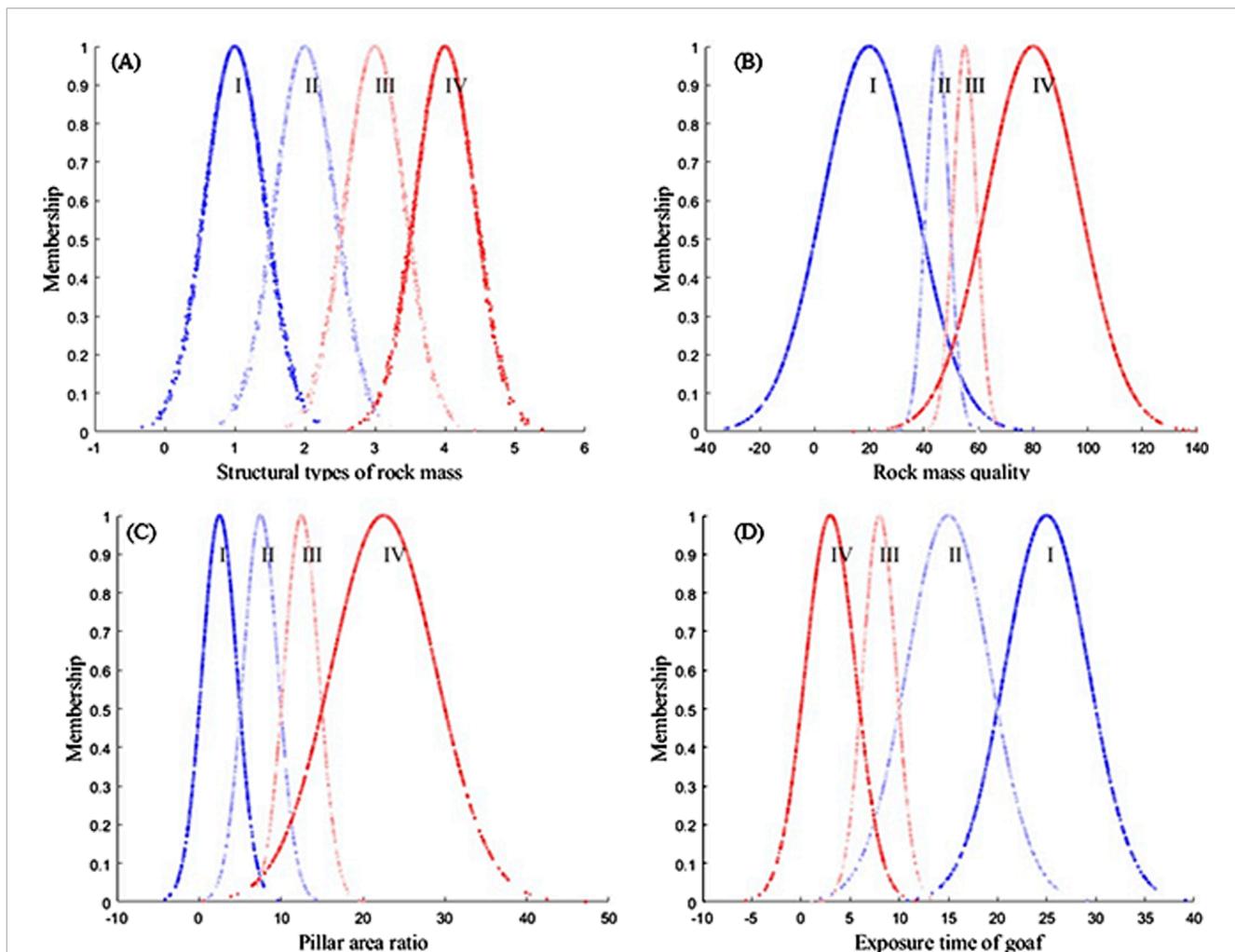
FIGURE 9
Evaluation grade cloud map of different indicators. **(A)** Evaluation grade cloud map of structural types of rock mass; **(B)** Evaluation grade cloud map of rock mass quality; **(C)** Evaluation grade cloud map of pillar area ratio; **(D)** Evaluation grade cloud map of exposure time of goaf.

on the variable weight theorems proposed by Li Hongxing et al., further dividing the incentive interval into initial and strong incentive segments.

$$S_j(x) = \begin{cases} e^{a_1\left(d_{j1}-x\right)} + c - 1 & [0, d_{j1}) \\ c & [d_{j1}, d_{j2}) \\ e^{a_2\left(x-d_{j2}\right)} + c - 1 & [d_{j2}, d_{j3}) \\ e^{a_3\left(x-d_{j3}\right)} + e^{a_3\left(d_{j3}-d_{j2}\right)} + c - 2 & [d_{j3}, 1) \end{cases} \quad (13)$$

Among them, c, a_1, a_2, and a_3 are the weighting adjustment parameters, and d_{j1}, d_{j2}, and d_{j3} are the threshold values for the variable weight intervals of the j-th factor. The variable weight vector is calculated according to Equations 12, 13. Currently, there is no unified method for determining variable weight intervals and variable weight parameters. Li and Wu (2015); Wu and Li (2016) proposed using the K-means method to classify the normalized data of the primary control factors. Then, the threshold values for the variable weight intervals of each factor are determined based on the classification-critical values of various indicators and the calculation

formulas. By sequentially constructing evaluation units that meet the constraints and linking them with the constant weight weights to form equations, four weighting adjustment parameters are solved.

However, this method has the following shortcomings (Li and Wu, 2015; Li et al., 2023; Wu and Li, 2016):

1. Clustering analysis results can only indicate data similarity and aggregation degree but cannot reflect the magnitude relationship of the factor state values. The basis for determining interval thresholds through clustering analysis is insufficient.
2. Using the ideal variable weight back-calculation to adjust the parameters is relatively complex in actual calculations, prone to errors, and can sometimes result in unsolvable situations.
3. The state-variable weight functions for different factors are the same; however, their interval thresholds are different, and the factors are independent of each other during the calculation process.

TABLE 11 Results of the assessment of the theory of variation.

Number	Comprehensive determination degree of instability risk of goaf				Order of evaluation
1480-1	0.0210	0.2185	0.2217	0.4126	IV
1480-2	0.0227	0.2345	0.1142	0.5209	IV
1480-3	0.0145	0.2011	0.1411	0.4334	IV
1480-4	0.0087	0.1499	0.0819	0.5114	IV
⋮	⋮	⋮	⋮	⋮	⋮
1580-11	0.0147	0.3277	0.2439	0.1444	II
1580-12	0.0200	0.3715	0.1412	0.2768	II
1580-13	0.1006	0.1951	0.1831	0.3962	IV
⋮	⋮	⋮	⋮	⋮	⋮
1730-11	0.0219	0.2534	0.1312	0.3752	IV
1730-12	0.0259	0.3089	0.0892	0.3348	IV
1730-13	0.0347	0.3893	0.1274	0.2827	II

2.3.2 Improved variable weighting model

Based on the characteristics of the variable weight model for evaluating the instability risk of mined-out areas, improvements were made to address its shortcomings.

1. For the local state function, the penalty and incentive intervals in Equation 13 were adjusted. The state function is shown in Equation 14 and illustrated in Supplementary Figure S3.

$$S_j(x) = \begin{cases} e^{a_1(d_{j1}-x)} + e^{a_2(d_{j2}-d_{j1})} + c - 2 & [0, d_{j1}) \\ e^{a_2(d_{j2}-x)} + c - 1 & [d_{j1}, d_{j2}) \\ c & [d_{j2}, d_{j3}) \\ e^{a_3(x-d_{j3})} + c - 1 & [d_{j3}, 1) \end{cases} \quad (14)$$

2. For variable weight intervals, the evaluation indices were divided into qualitative and quantitative indices. For the quantitative indices, the variable weight intervals were determined by analyzing the cumulative frequency of the normalized values for each factor (the division process is shown in Figure 4). For the qualitative indices, variable weight intervals were manually designated. The variable weight intervals at each level were determined using the cumulative frequency of the normalized values of the evaluation indices.

3. Regarding the variable weight intervals, according to the determination method described in Step 2, each evaluation indicator is subdivided into four variable weight interval matrices, denoted as V_m. A variable weight matrix I is formed by combining the variable weight intervals of m evaluation indicators. The constant weight vector is multiplied by the variable weight matrix to obtain a comprehensive variable weight interval matrix D.

$$V_m = \left([0, d_{j1}) \quad [d_{j1}, d_{j2}) \quad [d_{j2}, d_{j3}) \quad [d_{j3}, 1] \right)(j = 1, 2, \cdots, m) \quad (15)$$

$$I = \begin{pmatrix} D_1 \\ D_2 \\ \vdots \\ D_m \end{pmatrix} = \begin{pmatrix} [0, d_{11}) & [d_{11}, d_{12}) & [d_{12}, d_{13}) & [d_{13}, 1] \\ [0, d_{21}) & [d_{21}, d_{22}) & [d_{22}, d_{23}) & [d_{23}, 1] \\ \vdots & \vdots & \vdots & \vdots \\ [0, d_{m1}) & [d_{m1}, d_{m2}) & [d_{m2}, d_{m3}) & [d_{m3}, 1] \end{pmatrix} \quad (16)$$

$$V = V_m \bullet W_0 = \begin{pmatrix} D_1 \\ D_2 \\ \vdots \\ D_m \end{pmatrix} \bullet \left(W_0^1, W_0^2, \cdots, W_0^m \right)$$

$$= \begin{pmatrix} [0, d_{11}) & [d_{11}, d_{12}) & [d_{12}, d_{13}) & [d_{13}, 1] \\ [0, d_{21}) & [d_{21}, d_{22}) & [d_{22}, d_{23}) & [d_{23}, 1] \\ \vdots & \vdots & \vdots & \vdots \\ [0, d_{m1}) & [d_{m1}, d_{m2}) & [d_{m2}, d_{m3}) & [d_{m3}, 1] \end{pmatrix}$$

$$\bullet \begin{pmatrix} W_0^1 \\ W_0^2 \\ \vdots \\ W_0^m \end{pmatrix}^T \quad (17)$$

4. Based on the determination of the threshold values for the variable weight intervals, it is also necessary to determine the adjustment parameters a_1, a_2, a_3, and c in the model. These parameters can control and adjust the effect of the weight variable. This study attempts to use the Monte Carlo method to trial calculate the adjustment parameters, stipulating the value ranges for a_1, a_2, a_3, and c. Through multiple trial calculations, the accuracy of the evaluation results is maximized, simplifying the calculation process for back-calculating the adjustment parameters and solving the problem of unsolvable calculations.

2.4 Evaluation of cloud models based on variable weight theory

Using Equations 12, 14, the variable weight vector $W(x_i)$ for each goaf is calculated. The cloud model is then used to calculate the membership degree $\mu(x_{ik})$ of the goaf instability risk. Based on this, the already obtained variable weight vector $W(x_i)$ is multiplied by the membership degrees $\mu(x_{ik})$ of the n evaluation indicators for each goaf, resulting in the comprehensive certainty degree D_k of the instability risk of the goaf (Equation 18). The goaf instability risk level is determined based on the maximum membership degree principle. In this study, a forward cloud generator was used to evaluate the goaf instability risk. The calculation steps are shown in Supplementary Figure S4 (Guo L. et al., 2024; Han et al., 2023).

$$D_k = \sum_{i=1}^{n} \mu(x_{ik}) \bullet W(x_i) \quad (18)$$

FIGURE 10
Comparison of results between the two models. **(A)** Evaluation result accuracy confusion matrix of constant weight model; **(B)** Evaluation result accuracy confusion matrix of variable weight model.
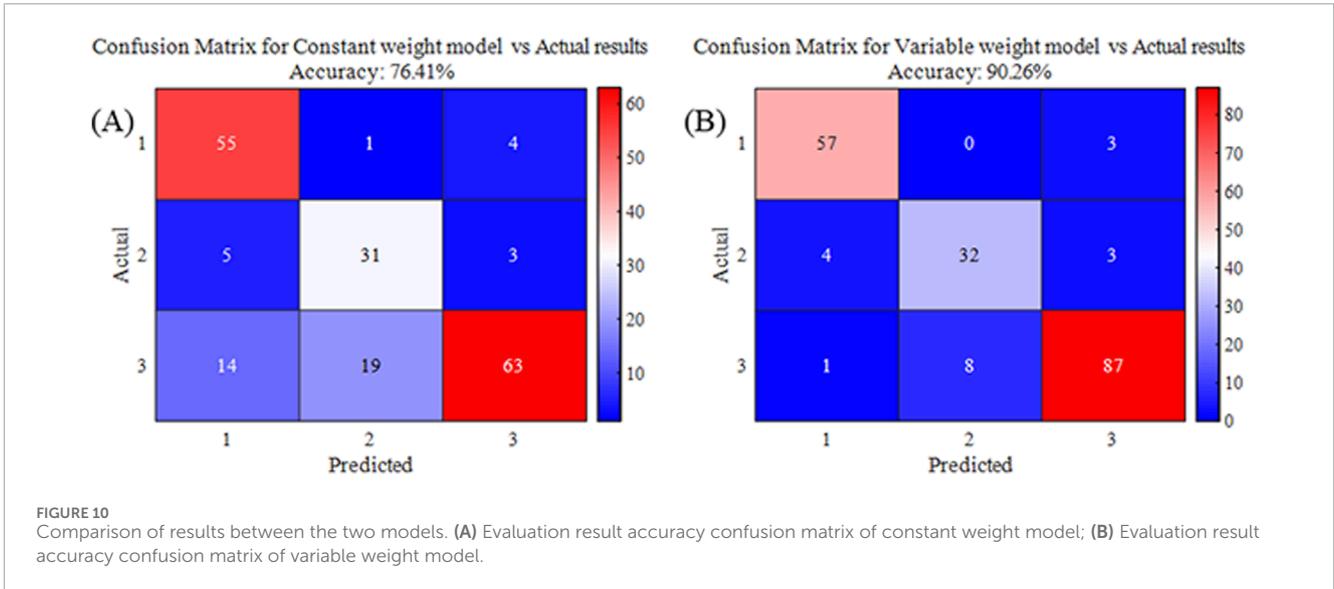
TABLE 12 The difference of comprehensive determination degree of instability risk in goaf between variable weight model and constant weight model.

Number	Variable weight model			Constant weight model		
	Average value	Variance	Coefficient of variation	Average value	Variance	Coefficient of variation
1480-1	0.218	0.138	0.634	0.257	0.205	0.797
1480-2	0.223	0.188	0.841	0.279	0.201	0.721
1480-3	0.198	0.152	0.769	0.261	0.194	0.744
1480-4	0.188	0.193	1.028	0.243	0.240	0.988
⋮	⋮	⋮	⋮	⋮	⋮	⋮
1580-11	0.183	0.117	0.639	0.251	0.174	0.696
1580-12	0.202	0.133	0.659	0.241	0.152	0.628
1580-13	0.219	0.109	0.497	0.261	0.114	0.437
⋮	⋮	⋮	⋮	⋮	⋮	⋮
1730-11	0.195	0.132	0.677	0.267	0.163	0.609
1730-12	0.190	0.134	0.708	0.245	0.165	0.675
1730-13	0.209	0.137	0.656	0.259	0.139	0.536

3 Results

3.1 Determination of constant power weights

3.1.1 Importance-binary comparison method to determine the subjective weight

According to Equation 1, the indicators are processed as dimensionless. Using Equations 2, 3, the standard deviation of the evaluation indicators for the goaf instability risk is calculated.

Using Equations 4, 5, the correlation coefficients of the evaluation indicators are calculated, and a heat map (Figure 5) is drawn. Using Equation 6, the importance scores of the evaluation indicators are calculated and ranked preliminarily. The results are summarized in Table 3.

After consulting relevant experts, the preliminary ranking results of the 13 evaluation indicators were adjusted. The adjusted ranking is: $X1 > X12 > X13 > X2 > X7 > X8 > X5 = X6 > X3 > X11 > X9 > X4 > X10$. The membership degrees of the factors relative to each other were determined based on the ranking results,

FIGURE 11
Histogram of variation coefficient.

FIGURE 12
Box plot of coefficient of variation.
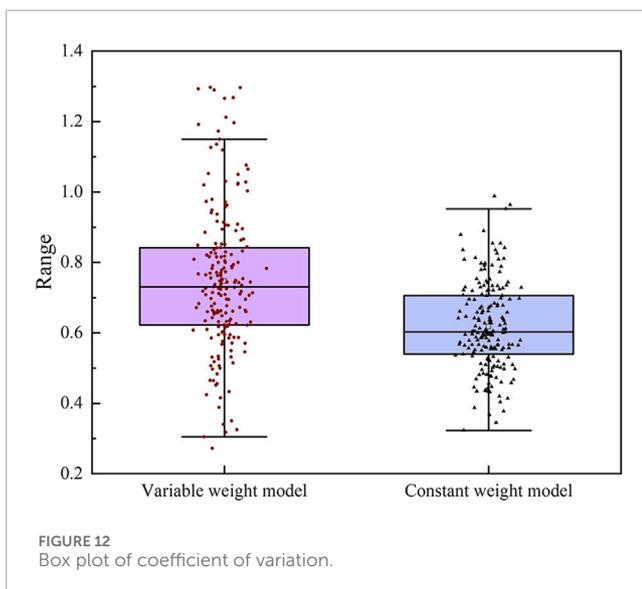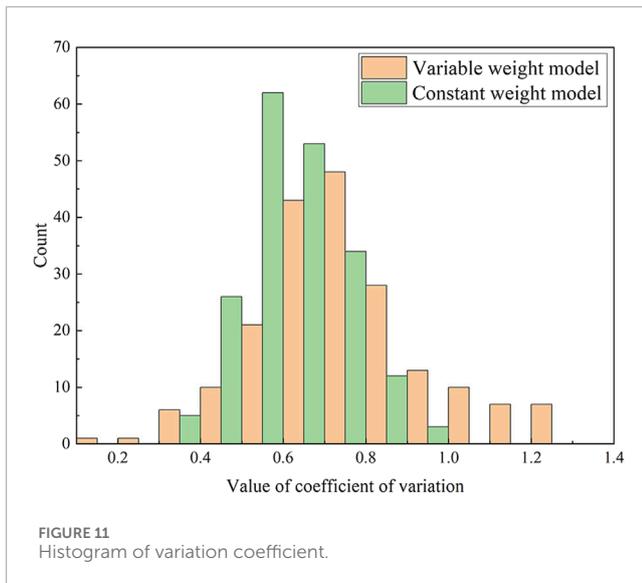
as shown in Table 2 and Supplementary Figure S2. The weights of the evaluation indicators were normalized, and the calculation results are listed in Table 4.

3.1.2 Determine the objective weight

The Gini coefficients of the 13 evaluation indices were calculated using Equation 8 and normalized using Equation 9 to obtain the objective weights of the indices: The calculation results are listed in Table 5.

3.1.3 Combination weight

According to Equations 10, 11, the combination coefficients of the subjective and objective weights were determined as $\alpha_1 = 0.33, \alpha_2 = 0.79$. After normalization, the weight coefficients were reallocated, resulting in $\alpha_1^* = 0.29, \alpha_2^* = 0.71$. The combined weights of the 13 evaluation indices that influence the instability risk of the mined-out areas were calculated using Equation 10: The

results are as follows: W = (0.0666, 0.0920, 0.0286, 0.0233, 0.0919, 0.1099, 0.0977, 0.0731, 0.1315, 0.1064, 0.1078, 0.0345, 0.0367).

3.2 Calculate the variable weight

3.2.1 Determine the variable weight interval

Variable weight intervals were determined using the cumulative frequency of the normalized values of the influencing factors. Owing to the absence of predefined thresholds for categorizing data frequencies as "extreme" or "unique," this study temporarily defined the cumulative frequency thresholds for state values at 15%, 33%, and 67%. Specifically, the normalized value corresponding to a cumulative frequency of 15% was defined as a strong penalty interval, 33% as an initial penalty interval, and 67% as a neutral interval. The remaining interval was considered the incentive interval. The threshold values for the variable weight intervals were determined by plotting the cumulative frequency intervals of the state values of the main controlling factors (Supplementary Figure S5). Thresholds for factors heavily influenced by human intervention can be manually defined. The final interval thresholds are listed in Table 6.

Based on Equations 14–17, the variable weight intervals of the 13 evaluation indices listed in Table 6 were multiplied by constant weights to obtain the comprehensive variable weight interval matrix. The final divisions of the variable weight intervals are listed in Table 7.

3.2.2 Trial calculations of transfer parameters

Assuming that the accuracy rate of the instability risk evaluation results for the goaf has a function f (a1, a2, a3, c) related to the adjustment parameters, the Monte Carlo method is used to determine the combination of a1, a2, a3, and c within their ranges to maximize f (a1, a2, a3, c). The specific steps are illustrated in Figure 6.

With the aid of MATLAB software, 1,000 assignment tests were conducted to obtain the accuracy rate f (a1, a2, a3, c) of the evaluation results as a function of the adjustment parameters a1, a2, a3, c. The resulting variation curve is shown in Figure 7.

a3 adjusts the amplitude of the weight changes in the incentive interval, a1 and a2 adjust the amplitude of the weight changes in the punishment interval, and c controls the weight adjustments of the entire variable weight model. From Figure 7, it can be concluded that the adjustment parameters a1, a2, and c are negatively correlated with the accuracy of the evaluation results, whereas a3 is positively correlated with the evaluation results. In addition, a1 and a2 have weaker adjustment strengths, whereas a3 and ccc have stronger adjustment strengths. The final adjustment parameters of the variable weight model are listed in Table 8.

3.2.3 Variable weight determination

Based on the constructed state variable weight vector formula and the determined variable weight interval and adjustment parameter values, the state variable weight vector Equation 19 for

the risk assessment of goaf instability in the Baiyang mine area was established according to Equation 14.

$$S_j(x) = \begin{cases} e^{0.29*(0.25-x)} + e^{0.66*(0.42-0.25)} + 0.01 - 2 & [0, 0.25) \\ e^{0.66*(0.42-x)} + 0.01 - 1 & [0.25, 0.42) \\ 0.01 & [0.42, 0.64) \\ e^{0.67*(x-0.64)} + 0.01 - 1 & [0.64, 1) \end{cases} \quad (19)$$

According to Equation 12, the variable weight value of each evaluation index is calculated, and the relationship between the constant weight and variable weight was drawn (Figure 8). The variable weight data of the 13 evaluation indices were statistically analyzed using Origin software. The results are shown in Table 9.

From the relationship diagram of the constant and variable weight values of the evaluation indicators (Figure 8), it is evident that the constructed variable weight model effectively adjusts the weights of the evaluation indicators during the process of assessing the goaf instability risk in the Baiyang mining area. Specifically, the variable weight model includes the weights of six indicators: rock mass structure ($X1$), geological structure ($X2$), rock quality ($X3$), ore body dip angle ($X4$), condition of adjacent goafs ($X12$), and management measures ($X13$), which significantly increase their weight values. Conversely, the model has "penalized" the weights of seven indicators: goaf volume ($X5$), maximum exposed area ($X6$), pillar area ratio ($X7$), height-span ratio ($X8$), burial depth ($X9$), exposure time ($X10$), and water accumulation in the goaf ($X11$), reducing their weight values in some goafs.

The average weight of the "neutral" interval (neither penalized nor incentivized) is smaller than that of the other intervals. This is because, under the weight adjustments of the variable weight model, the weights of the factors in the "neutral" interval are compressed. As shown in Table 9, the difference between the average variable and constant weight was not significant. This indicates that the variable weight model not only meets the decision-makers' preferences for the primary control factors under various combinations of states but also considers the relative importance of the indicator weights. Therefore, in terms of the weight adjustment effects, the constructed variable weight model effectively adjusted the weights of the 13 evaluation indicators according to the predetermined variable weight approach, thus satisfying the expected outcomes.

3.3 Evaluation results of the variable weight model

According to the cloud model calculation steps shown in Supplementary Figure S4, the cloud characteristic parameters of the different indicator evaluation standards were calculated, and the results are shown in Table 10. Evaluation grade cloud maps were drawn using MATLAB. Because of space limitations, only a few indicators are shown here in Figure 9.

Using Equation 18, the comprehensive determination degree of the instability risk of 195 goafs in the Baiyang mine area was calculated, and the stability grade of the goafs was determined

according to the principle of maximum membership degree. The results are shown in Table 11.

4 Discussion

4.1 Comparison of evaluation accuracy

Using data from 195 goafs, risk levels were calculated using both variable weight and constant weight models. These results were compared with the actual risk levels to determine the accuracy of the two models. According to Figure 10, the accuracy of risk assessment using the constant weight model was 76.41%, whereas that using the variable weight model was 90.26%. This significant improvement indicates that the variable weight model provides a more accurate assessment of the goaf risk levels than the constant weight model.

4.2 Variation in comprehensive determination of goaf instability risk

According to Equation 18, the comprehensive determination of the risk of goaf instability Di in Baiyang Mine under both constant weight and variable weight conditions was calculated for 195 goafs. Di consists of numerical values of four risk levels, as shown in Equation 20. A difference coefficient ζ was used to measure the discrepancy in the determination of different levels of risk in the goaf (Equation 21). This was compared with the discrepancy in the comprehensive determination of the risk of goaf instability between the constant weight model and the variable weight model (calculated results are shown in Table 12). The bar chart in Figure 11 and the box plot in Figure 12 and illustrate the direct comparison of the distribution of the difference coefficient data for both models.

$$D_k = \begin{bmatrix} I & II & III & IV \\ d_1 & d_2 & d_3 & d_4 \end{bmatrix} \quad (20)$$

$$\zeta = \frac{\frac{1}{4}\left(\sum_{k=1}^{4}\left(d_k - \overline{d_k}\right)\right)}{\overline{d_k}}, \overline{d_k} = \sum_{k=1}^{4} d_k \quad (21)$$

The analysis in Figure 11 shows that the histogram of the variable weight model dataset is higher than that of the constant weight model dataset (approximately 0.75) and the distribution range is wider. The average value of the constant weight model dataset is low and is concentrated near the average value, and the distribution range is narrow. The analysis in Figure 12 shows that the box plot of the variable weight model has a higher median and wider interquartile range (IQR) than the constant weight model. There are more potential outliers in the variable weight dataset, indicating a wider range of values. The results show that the variable weight model has a higher difference coefficient, indicating a wider range of variation in the comprehensive determination. This suggests that the variable weight model better captures the nuances and complexities of the goaf instability risk, providing clearer and more distinguishable membership degrees in the evaluation results.

4.3 Practical significance and future research

These findings enhance the accuracy and reliability of the variable weight model, significantly strengthen mining safety management practices, and ensure safe mining operations. Future research could further refine this assessment framework by incorporating temporal factors and exploring alternative weighting methods. Additionally, extending the application of the variable weight model to other mining environments and geological conditions could provide broader insights into its effectiveness and adaptability.

5 Conclusion

This study employs a systematic approach to construct an index system for evaluating the destabilisation risk of a mining area. The index system is developed based on a comprehensive analysis of the specific characteristics of the mining area in question. In accordance with the tenets of game theory, the subjective and objective weights are integrated and assigned to ascertain the constant weight. Subsequently, in consideration of the characteristics of the indicator values, the local variable weight theory is introduced, and the threshold of the variable weight interval is determined through the normalized cumulative frequency method of the indicators. The Monte Carlo method is employed to determine the adjustment parameters, with the variable weight then applied to each indicator value via the local variable weight state function. The cloud model is subsequently utilised to evaluate the destabilisation risk of the air mining area on this basis. The following main conclusions are drawn in this paper:

1. Although fixed-weight models offer basic assessments, their static weight approaches fail to account for variations and abrupt changes in factor indices. In contrast, the variable weight model dynamically adjusts the weights based on the status values of the influencing factors to overcome this limitation. This flexibility enhances the capability of the model to accurately assess instability risk in mined-out areas, making it a more reliable tool for practical mining safety applications.
2. The variable weight model provided a greater difference in the comprehensive certainty of the goaf instability risk, making the membership degree of the evaluation results clearer and easier to distinguish. This addresses the issue of ambiguity in the results owing to the small differences in membership degrees.
3. In conclusion, the introduction of variable weight theory and the use of the cloud model significantly enhanced the accuracy and reliability of goaf instability risk assessments. Future research can improve the evaluation index system by incorporating time factors and exploring additional weighting methods to refine the model.

Data availability statement

The original contributions presented in the study are included in the article/Supplementary Material, further inquiries can be directed to the corresponding author.

Author contributions

LG: Conceptualization, Data curation, Formal Analysis, Investigation, Methodology, Software, Validation, Visualization, Writing–original draft. KH: Conceptualization, Methodology, Supervision, Validation, Writing–review and editing. HS: Formal Analysis, Software, Writing–review and editing. YY: Investigation, Writing–original draft.

Conflict of interest

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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Supplementary material

The Supplementary Material for this article can be found online at: https://www.frontiersin.org/articles/10.3389/feart.2024.1469834/full#supplementary-material

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