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# The application of gray system-variable fuzzy sets coupling theory on the susceptibility assessment of debris flow hazards

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The occurrence of debris flow hazards has significant destructive effects people's safety and the safety of their property, so the risk assessment of debris flow hazards has great significance. The maximum amount of debris flow at one time ( $X_1$ ), the reserves of loose substance ( $X_2$ ), the watershed area ( $X_3$ ), the length of the ditch ( $X_4$ ), the relative height difference ( $X_5$ ), the gradient of side slope ( $X_6$ ), the longitudinal slope of main ditch ( $X_7$ ), the length ratio of replenishment section ( $X_8$ ), the daily maximum rainfall ( $X_9$ ), and Faults developed in the watershed ( $X_{10}$ ) are selected as the evaluation index at first. Then the gray system-variable fuzzy sets coupling model is introduced; secondly, the relative membership degree matrix about the fuzzy variable sets is established, and the weight coefficients considering the uncertainty of assessment indices are determined based on the entropy weight method; finally, the susceptibility level of debris flow hazards is determined using the mean ranking feature value. The conclusion is drawn that the results obtained based on the coupling technique are consistent with the current specification; its accuracy arrives at 83%, and the method is feasible for the susceptibility level assessment of debris flow hazards. Compared to the traditional variable fuzzy sets model, it avoids the uncertainty of magnitude in the standard analysis, it can ensure the objectivity and accuracy of the calculation result, and its calculation process is simple and efficient. In total, the findings of the proposed model provide an alternate way to assess the susceptibility level of debris flow hazards and improve the evaluation accuracy in the future.

## KEYWORDS

debris flow hazards, variable fuzzy sets, application, level, susceptibility assessment

## 1 Introduction

As the excellent development strategy continues to expand in the west of China, debris flow hazards become a severe problem that hinders economic development and restricts the national quality of life (Gu et al., 2021a). So, risk assessment for debris flow hazards is an indispensable prerequisite to solving the debris flow problem (Gu et al., 2021b; Chen and Zhou, 2019). However, the occurrence and evolution of debris flow is very complex (Wu, 2017). Its influential factors are intricate, and they interact with each other, which is hard to



assess quantitatively (Gu et al., 2021c). So, the susceptibility assessment for debris flow hazards has excellent significance.

The investigations on the susceptibility assessment of debris flow have been performed by researchers in many countries (Gu and Wu, 2019). Their investigation method is composed of three types: the heuristics method (Tan et al., 2018), the physical model method (Tsaparas et al., 2002), and the machine learning method (Liang et al., 2020; Zhang et al., 2020). The heuristics method mainly depends on the index factor of debris flow derived from the subjective experience, and then the debris flow in the investigation area is depicted; the physical model is adopted to try to reappear the respective conditions of the slope when the geological hazards occurred to determine the safe coefficients of ramps (Gu and Wu, 2016), for example, Gu et al. (2022) assess the susceptibility level of debris flow in the Daba River Watershed using the intuitionistic fuzzy sets-TOPSIS model. However, adjusting model parameters requires a lot of time. So, for the model, the prediction of zonal debris flow still has some limitations (Zhou and Gu, 2016; Zhang, 2020). With the development of computer sciences in recent years, the machine learning method is widely applied to assess the susceptibility level of debris flow hazards (Gu et al., 2019; Zhou et al., 2015). Examples include the artificial neural network model (Zhao and Liu, 2005), the decision tree model (Zhou et al., 2017), the logistic regression algorithm (Xiong et al., 2020), the random forest method (Lai et al., 2015), and the support vector machine (Yuan, 2008). These methods can calculate the vulnerability index of debris flow directly, and the interference of artificial subjective factors can be avoided (Zhou et al., 2014).

Although the above method has promoted the development of the assessment theory of debris flow hazards enormously,

improvement is still needed; for example, the calculative process is complex, and the assessment process in many methods is often quantitative or qualitative. Aiming at the shortcomings of the above methods, the entropy weight-variable fuzzy sets are introduced to assess the susceptibility level of debris flow hazards in Horse Neck Field. It has many virtues; for example, the preciseness of the algorithm in practice is very high, and relatively to traditional variable fuzzy sets theory, the suggested method needs no significant data, and its operation is easy. In addition, it can provide a solution scheme for the evaluation indicators, which are difficult to quantify and reduce the influence of human factors (Gu et al., 2022a); the method is an excellent improvement on the traditional fuzzy sets model (Xie et al., 2023).

The paper is organized as follows: in Section 2, the engineering overview is first introduced; in Section 3, theory and methodology based on the entropy-weight variable fuzzy sets model are presented; in Section 4, the assessment model of the debris flow hazards are established, and the assessment results of the proposed model are compared; in Section 5, discussions and comparative analysis are performed; in Section 6, conclusions are drawn.

## 2 Study area

Leibo County lies in the southwest of Sichuan Province in China. Horse Neck Field town is located west of Leibo County, approximately 40 km from Leibo County; its location is plotted in Figure 1. The geographical coordinates of the study area are X: 639468–641324, Y: 3124621–312664, and the annual average temperature is 12–16°C. The winter lasts more than 3 months; it



**FIGURE 2**  
Topography and geomorphology of the study area.



**FIGURE 3**  
The remote sensing image of The slope debris flow.

is a rainy season from May to September, and this period contains 80% of the rainfall for the entire year. The annual average rainfall is 850 mm.

Horse Neck Field Town is on the left bank of the west Suxijiao River. The geomorphologic features are characterized by deep-cutting tectonic erosion. The lowest elevation of the e-shaped valley is approximately 650 m, and its relative height difference with the highest point of the mountain is around 1130 m. The hill runs almost parallel to the gullies, and the overall profile of the slope is steep, with an average gradient of about 500. The topography and geomorphology of the study area are plotted in Figure 2.

There are six branch gullies in the investigation area: Dialou gully, Fatinghou gully, and the slope debris flow in Horse Neck Field Town. The slope debris flow in Horse Neck Field town comprises 1 # gully, 2 # gully, 3 # gully, and Erdingxi gully. A remote sensing image of these four gullies is shown in Figure 3.

### 3 Methodology

#### 3.1 The variable fuzzy set theory

The variable fuzzy theory (Shao et al., 2022) is a further improvement and expansion of the traditional fuzzy set theory based on dialectical materialism and the relativity and dynamic variability of the development of things, it is defined as follows:

It is assumed that  $U$  is the domain of a fuzzy concept,  $F$  and  $F^c$  represent the natures of attraction and exclusion respectively in the field of discourse. For any  $u$ , there exists  $u \in U, \mu_F(u)$  and  $\mu_{F^c}(u)$  are both the relative membership degree function. They represent respectively degree of attraction and rejection of the element  $u$  relative to  $F$  and  $F^c$ . Let:

$$D_F(u) = \mu_F(u) - \mu_{F^c}(u) \tag{1}$$

Where,  $D_F(u)$  is defined as the relative difference function of  $u$  to  $F$ .

Assuming that  $X_0 = [a, b]$  is an attractive interval on the real axis, namely,  $0 < D_F(u) < 1$  interval,  $[a, b]$  and  $[c, d]$  are an interval of repulsive properties, namely,  $-1 < D_F(u) < 0$  interval,  $X = [c, d]$ , and  $X_0 \in X$ , the location relationship is shown in Figure 4.

According to the relevant definition, in Figure 2, the point  $F$  can be interpreted as the point of  $\mu_F(u) = 1$  and  $D_F(u) = 1$ . Then:

When  $x$  is located in the left of  $F$ , the expression is:

$$\begin{cases} \mu_F(u) = 0.5 \left[ 1 + \left( \frac{x-a}{F-a} \right)^\beta \right]; x \in [a, F] \\ \mu_F(u) = 0.5 \left[ 1 - \left( \frac{x-a}{d-a} \right)^\beta \right]; x \in [d, a] \end{cases} \tag{2}$$

When  $x$  is located in the right of  $F$ , the expression is:

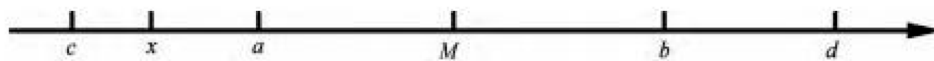
$$\begin{cases} \mu_F(u) = 0.5 \left[ 1 + \left( \frac{x-b}{F-b} \right)^\beta \right]; x \in [F, b] \\ \mu_F(u) = 0.5 \left[ 1 - \left( \frac{x-b}{e-b} \right)^\beta \right]; x \in [b, e] \end{cases} \tag{3}$$

#### 3.2 The gray system theory

Gray system theory (Chen et al., 2022) is a systematic method suggested by Professor Deng Julong to deal with the fuzziness of small samples and lack of information. As the core parameter, the gray correlation degree needs to be calculated by the gray correlation analysis; its basic principle is to determine the degree of correlation among factors by using the similarity of geometric figures of sequential curves. The closer the geometry is, the stronger the correlation is. The basic concepts are as follows:

It is assumed that a sequence of characteristic behaviors of a system is  $X_0 = (x_0(1), x_0(2), x_0(3), L, x_0(n))$ , the sequence of correlation factors can be expressed as follows:

$$\begin{aligned} X_1 &= (x_1(1), x_1(2), x_1(3), L, x_1(n)) \\ &\dots \\ X_m &= (x_m(1), x_m(2), x_m(3), L, x_m(n)) \end{aligned} \tag{4}$$



**FIGURE 4**  
Linear relationship of the point  $x$  in the attractive region and repellent region.

Further information is available (Hang and Wu, 2019):

$$\gamma(X_0, X_i) = \frac{1}{n} \sum_{k=1}^n \gamma(x_0(k), x_i(k)) \quad (5)$$

Then  $\gamma(X_0, X_i)$  represents the relative relationship degree between the sequence  $X_0$  and  $X_i$ .

$$\gamma(x_0(k), x_i(k)) = \frac{\min_i \min_k |x_0(k) - x_i(k)| + \xi \max_i \max_k |x_0(k) - x_i(k)|}{|x_0(k) - x_i(k)| + \xi \max_i \max_k |x_0(k) - x_i(k)|} \quad (6)$$

Where,  $k = 1, 2, 3, \dots, n, i = 1, 2, 3, \dots, m; \gamma_0(x_0(k), x_i(k))$  is called as the association factor;  $\xi$  is called as resolution coefficient;  $\xi \in (0, 1)$ , and  $\xi = 0.5$ .

### 3.3 The coupling model

Their calculation procedure is listed as follows:

- (1) Determination of evaluated objects and index sets. It is assumed that there are  $m$  assessment scheme and  $n$  evaluation indices, its optimal index value is  $A_0 = (a_{01}, a_{02}, a_{03}, \dots, a_{0n})$ , then the combination matrix of the optimal value and the reference value is (Gu et al. (2022b)):

$$B = \begin{bmatrix} a_{01} & a_{02} & a_{03} & \dots & a_{0n} \\ a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ \dots & \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & a_{m3} & \dots & a_{mn} \end{bmatrix}_{(m+1) \times n} \quad (7)$$

- (2) The dimensionless treatment of index. Assuming that  $c_{i1}$  is the minimum magnitude of all schemes at the  $i$ th index and  $c_{i2}$  is its maximum magnitude, then Eq. 8 can be adopted to perform dimensionless calculation,  $x_{ij} \in (0, 1)$ , namely:

$$x_{ij} = \frac{c_{i2} - a_{ij}}{c_{i2} - c_{i1}} \quad (8)$$

Where,  $x_{ij}$  is the dimensionless magnitude of  $a_{ij}$ . So, the dimensionless matrix  $X$  can be obtained as follows:

$$X = \begin{bmatrix} x_{01} & x_{02} & x_{03} & \dots & x_{0n} \\ x_{11} & x_{12} & x_{13} & \dots & x_{1n} \\ \dots & \dots & \dots & \dots & \dots \\ x_{m1} & x_{m2} & x_{m3} & \dots & x_{mn} \end{bmatrix}_{(m+1) \times n} \quad (9)$$

- (3) The calculation of the sequence difference and maximum and minimum difference, are respectively expressed as:

$$\Delta_{0i} = (\Delta_{0i}(1), \Delta_{0i}(2), \Delta_{0i}(3), \dots, \Delta_{0i}(n)) \quad (10)$$

Where,  $\Delta_{0i} = |x_{0j} - x_{ij}|, i = 1, 2, 3, \dots, m; j = 1, 2, 3, \dots, n$

$$\Delta_{\max} = \max_i \max_j \{\Delta_{0i}(j)\} \quad (11)$$

$$\Delta_{\min} = \min_i \min_j \{\Delta_{0i}(j)\} \quad (12)$$

Where,  $i = 1, 2, 3, \dots, m; j = 1, 2, 3, \dots, n$ .

- (4) The determination of gray correlation coefficients

$$\gamma_{ij} = \frac{\Delta_{\min} + \xi \Delta_{\max}}{\Delta_{0i}(j) + \xi \Delta_{\max}} \quad (13)$$

Where,  $i = 1, 2, 3, \dots, m; j = 1, 2, 3, \dots, n, \xi$  is the resolution coefficient, its value

- (5) The calculation of the relative membership degree

$$\mu(u)_{n \times m} = \begin{bmatrix} \mu(u)_{11} & \mu(u)_{12} & \mu(u)_{13} & \dots & \mu(u)_{1m} \\ \mu(u)_{21} & \mu(u)_{22} & \mu(u)_{23} & \dots & \mu(u)_{2m} \\ \dots & \dots & \dots & \dots & \dots \\ \mu(u)_{n1} & \mu(u)_{n2} & \mu(u)_{n3} & \dots & \mu(u)_{nm} \end{bmatrix} \quad (14)$$

- (6) The determination of index weight coefficients

The entropy weighting method is defined as describing the average amount of information using the concept of entropy (Zhou et al., 2021). Its calculative process is listed as follows:

$$r_{ij} = \frac{x_{ij} - x_{i \min}}{x_{i \max} - x_{i \min}} \quad (15)$$

$$r_{ij} = \frac{x_{i \max} - x_{ij}}{x_{i \max} - x_{i \min}} \quad (16)$$

- ① Normalization of different indices

Where,  $x_{ij}$  is the corresponding magnitude of the evaluation indicator in the scheme  $i (i = 1, 2, 3, \dots, m; j = 1, 2, 3, \dots, n)$ .

- ② The determination of index weights

According to the normalized index matrix, the index weights can be depicted:

$$\omega_j = \frac{1 - s_j}{n - \sum_{j=1}^n s_j} \quad (17)$$

Where  $s_j = -k \sum_{i=1}^n b_{ij} \ln(b_{ij})$

$$b_{ij} = \frac{x_{ij}}{\sum_{i=1}^n x_{ij}}$$

- (7) The calculation of the relative membership degree

Its expression is listed as follows:

$$v_F(u)_j = \frac{1}{1 + \left( \frac{\sum_{i=1}^m [\omega_i (1 - \mu_F(u)_{ij})]^g}{\sum_{i=1}^m [\omega_i \mu_F(u)_{ij}]^g} \right)^{\frac{1}{g}}} \tag{18}$$

Where,  $m$  is the identification index value;  $\omega_i$  is the weight;  $F$  is the grade parameter;  $g$  is distance parameter;  $l = 1$  and  $g = 2$  represent heiming distance and Euclidean distance respectively;  $l$  is optimization criteria parameter;  $l$  and  $g$  exist in four combinations, namely,  $l = 1, g = 1, l = 1, g = 2, l = 2, g = 1, l = 2, g = 2$ .

According to Eq. 15, the synthetic membership degree is calculated as:

$$U = (u') \tag{19}$$

Where

$$u' = \frac{v_F(u)_j}{\sum_{j=1}^m v_F(u)_j} \tag{20}$$

(8) The determination of assessment results

Based on Eq. 16, the synthetic membership vector  $U$  can be obtained, their assessment level is listed as follows:

$$H = (1, 2, \dots, c) \bullet U \tag{21}$$

$$\bar{H} = \sum_{i=1}^4 H_i / 4 \tag{22}$$

Finally, according to the magnitudes of  $H$ , if  $n - 0.5 \leq H \leq n + 0.5$ , then the result is level  $n$  ( $n$  is a positive integer).

## 4 The construction of an assessment model

### 4.1 The determination of the evaluation index

The generation mechanism and evolution process of debris flow is very complex, and there are many influencing factors, so the selection of evaluation factors directly affects the final evaluation results. For the assessment model, the evaluation index is often selected based on the actual case in the engineering site. Otherwise, a more significant deviation will occur. According to the actual investigation datum, ten assessment factors in the investigation are selected as the assessment index of debris flow in Horse Neck Field. These indexes are, respectively, the maximum amount of debris flow at one time ( $X_1$ ), the reserves of loose substance ( $X_2$ ), the watershed area ( $X_3$ ), the length of the ditch ( $X_4$ ), the relative height difference ( $X_5$ ), the gradient of side slope ( $X_6$ ), the longitudinal slope of the main ditch ( $X_7$ ), the length ratio of replenishment section ( $X_8$ ), the maximum daily rainfall ( $X_9$ ), and Faults developed in the watershed ( $X_{10}$ ). According to the

TABLE 1 The risk level classification of debris flow hazards.

Level index	I	II	III	IV
$X_1$ ( $10^4 \text{ m}^3$ )	$\leq 1$	1–10	10–100	100–150
$X_2$ ( $10^4 \text{ m}^3/\text{km}^2$ )	$\leq 1$	1–5	5–10	10–15
$X_3$ ( $\text{km}^2$ )	$\leq 0.5$	0.5–10	10–35	35–53
$X_4$ (km)	$\leq 1$	1–5	5–10	10–15
$X_5$ (km)	$\leq 0.2$	0.2–0.5	0.5–1	1–1.5
$X_6$ ( $^\circ$ )	$\leq 15$	15–20	20–25	25–50
$X_7$ ( $\%$ )	$\leq 268$	268–466	466–625	625–900
$X_8$ ( $\%$ )	$\leq 10$	10–30	30–60	60–90
$X_9$ (mm)	$\leq 25$	25–50	50–100	100–150
$X_{10}$ (strip)	$\leq 1$	1–3	3–5	5–10

relevant specifications, the ten evaluation indexes can be classified into four levels in Table 1: susceptibility level I (low), susceptibility level II (medium), susceptibility level III (great), and susceptibility level IV (strong great). The monitoring value of ten assessment indexes about six branch gullies is depicted in Table 2.

### 4.2 The construction of the evaluation frame

A susceptibility assessment for debris flow hazards significantly influences people’s personal safety and that of their property, so assessing the susceptibility level of debris flow hazards is significant.

A new evaluation method of debris flow is provided in the paper, which is based on the variable fuzzy sets theory, as presented in Figure 5. At first, to evaluate the risk level of debris flow hazards, a complete assessment index system is established. Secondly, the weight of each assessment index is determined according to an entropy weight theory. Thirdly, the relative membership degree is determined using the coupling theory. Then, the magnitudes of synthetic certainty degree are determined; finally, the susceptibility level of debris flow hazards is distinguished.

### 4.3 The determination of the risk level of the debris flow hazards

(1) The determination of the weight coefficients

According to the entropy method theory and Table 2, the weight coefficients can be calculated in Table 3.

(2) The determination of the relative membership degree matrix

Based on Table 2, the data in Diaolou Gully is adopted as a sample. The actual magnitude of Diaolou gully in Table 2 usesthe optimal value, and the middle value in the grade interval in Table 1 is regarded as the reference value so that the combination matrix  $B$  can be expressed as:

TABLE 2 The monitoring value.

Name of ditch index	Diaolou gully	1# gully	2# gully	3# gully	Erdingxi gully	Fatinghou gully
$X_1$ ( $10^4$ m <sup>3</sup> )	8.78	0.15	0.09	0.13	0.35	0.23
$X_2$ ( $10^4$ m <sup>3</sup> /km <sup>2</sup> )	12.13	1.9	1.1	2.2	1.5	1.58
$X_3$ (km <sup>2</sup> )	0.85	0.2	0.12	0.18	0.3	0.32
$X_4$ (km)	1.6	0.8	0.73	0.8	0.95	0.97
$X_5$ (km)	0.81	0.6	0.69	0.62	0.72	0.72
$X_6$ (°)	45	37	35	30	22	22
$X_7$ (‰)	504	710	802	706	661	705
$X_8$ (%)	47	45	36	29	63	49
$X_9$ (mm)	130	70	70	70	70	70
$X_{10}$ (strip)	2	4	1	0	5	1

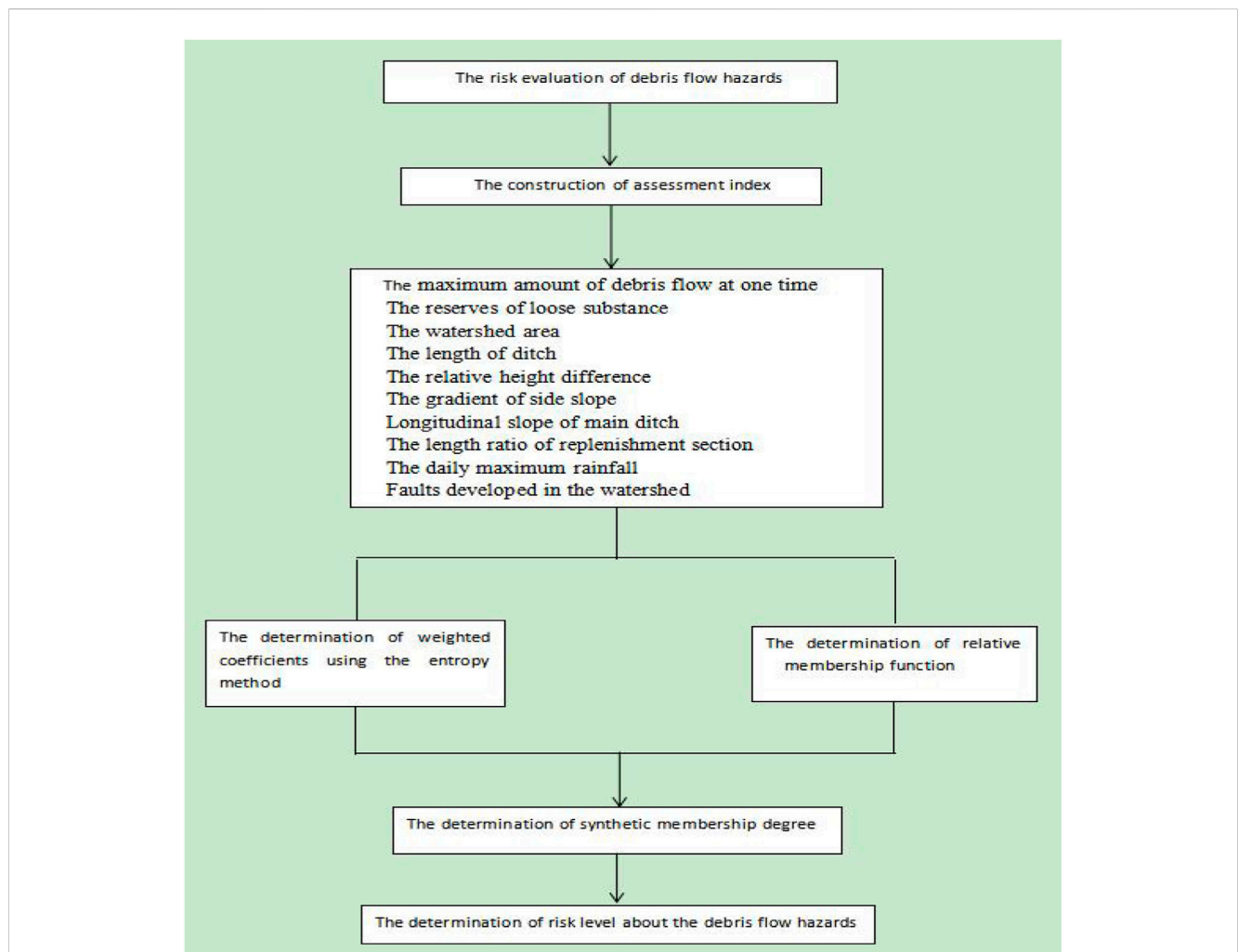


FIGURE 5 The risk evaluation process of the risk level of the debris flow hazards.

**TABLE 3** The weight coefficient matrix.

Index	X <sub>1</sub>	X <sub>2</sub>	X <sub>3</sub>	X <sub>4</sub>	X <sub>5</sub>	X <sub>6</sub>	X <sub>7</sub>	X <sub>8</sub>	X <sub>9</sub>	X <sub>10</sub>
Weight coefficients	0.5188	0.1855	0.0884	0.0157	0.002	0.0132	0.0036	0.0111	0.0134	0.1483

**TABLE 4** The comprehensive relative membership vector.

<i>l</i> & <i>g</i>	$v_F(u)_1$			
<i>l</i> = 1, <i>g</i> = 1	0.8745	0.9099	0.6913	0.7443
<i>l</i> = 1, <i>g</i> = 2	0.8636	0.8908	0.5669	0.6281
<i>l</i> = 2, <i>g</i> = 1	0.9798	0.9903	0.8337	0.8945
<i>l</i> = 2, <i>g</i> = 2	0.9757	0.9852	0.6315	0.7404

**TABLE 5** The normalized comprehensive relative membership degree vector.

<i>l</i> & <i>g</i>	$v'$			
<i>l</i> = 1, <i>g</i> = 1	0.2716	0.2826	0.2147	0.2312
<i>l</i> = 1, <i>g</i> = 2	0.2928	0.302	0.1922	0.213
<i>l</i> = 2, <i>g</i> = 1	0.2649	0.2678	0.2254	0.2419
<i>l</i> = 2, <i>g</i> = 2	0.2927	0.2956	0.1895	0.2222

$$B = \begin{bmatrix} 8.78 & 12.13 & 0.85 & 1.6 & 0.81 & 45 & 504 & 47 & 130 & 2 \\ 0.5 & 0.5 & 0.25 & 0.5 & 0.1 & 7.5 & 134 & 5 & 12.5 & 0.5 \\ 5.5 & 3 & 5.25 & 3 & 0.35 & 17.5 & 367 & 20 & 37.5 & 2 \\ 55 & 7.5 & 22.5 & 7.5 & 0.75 & 22.5 & 545.5 & 45 & 75 & 4 \\ 125 & 12.5 & 44 & 12.5 & 1.25 & 37.5 & 762.5 & 75 & 125 & 7.5 \end{bmatrix}$$

Based on Eqs 8, 9, the dimensionless matrix *X* can be depicted as follows:

$$X = \begin{bmatrix} 0.9842 & 0.9775 & 0.9999 & 0.9984 & 1 & 0.9122 & 0 & 0.9082 & 0.7433 & 0.9976 \\ 0.997 & 0.997 & 0.9989 & 0.997 & 1 & 0.9947 & 0 & 0.9634 & 0.9074 & 0.997 \\ 0.986 & 0.9928 & 0.9866 & 0.9928 & 1 & 0.9532 & 0 & 0.9464 & 0.8987 & 0.9955 \\ 0.9004 & 0.9876 & 0.9601 & 0.9876 & 1 & 0.9601 & 0 & 0.9188 & 0.8637 & 0.994 \\ 0.8374 & 0.9852 & 0.9438 & 0.9852 & 1 & 0.9524 & 0 & 0.9031 & 0.8374 & 0.9918 \end{bmatrix}$$

Then, according to Eqs 10–14, the relative membership degree matrix can be expressed as follows:

$$\mu = \begin{bmatrix} 0.8646 & 0.8079 & 0.9875 & 0.983 & 1 & 0.716 & 1 & 0.5979 & 0.3333 & 0.9925 \\ 0.8812 & 0.9509 & 0.8702 & 0.9509 & 1 & 0.9062 & 1 & 0.8284 & 0.904 & 0.9819 \\ 0.4896 & 0.9408 & 0.7555 & 0.9408 & 1 & 0.923 & 1 & 0.7481 & 0.7012 & 0.9825 \\ 0.5658 & 0.9717 & 0.8349 & 0.9717 & 1 & 0.9143 & 1 & 0.8398 & 0.7576 & 0.9734 \end{bmatrix}$$

(3) Determining the comprehensive relative membership degree vector

According to Eq. 15, and in combination with the matrix of  $\mu$ , the comprehensive relative membership degree matrix can be shown in Table 4.

According to Eqs 16, 17, the normalized comprehensive relative membership degree vector can be obtained in Table 5.

(3) Determination of the susceptibility level of the debris flow hazards

Based on Eqs 18, 19, and in combination with Table 5, the ranking feature value of debris flow in Diaolou gully can be shown in Table 6.

Similarly, the feature value of the six gullies is shown in Table 7.

When validating assessment results, the results in this article are compared with other methods, as listed in Table 8.

The coupling model is applied to assess the debris flow hazards. The whole results are shown in Table 8. It can be found from Table 8 that the susceptibility levels of debris flow hazard from the six different gullies are different. The susceptibility level at the 2 # gully is I; at the 1# and 3 # gullies it is II; and at the remaining gullies it is III. This means that the susceptibility level for debris flow at 2 # gully is low; at 1# and 3# medium; and the susceptibility to debris flow at the rest of the gullies is excellent, so the qualified rate of susceptibility level surrounding debris flow hazards in all valleys arrives at 50%. Because the susceptibility level of debris flow hazards at gullies 1–3# is weak, no measurement need be performed, but a correlated safety procedure should be established. However, the susceptibility levels at the other gullies are significant, so the necessary consolidation measurements should be taken to prevent debris flow hazards; for example, steel mesh should be placed across the relevant slopes in the various gullies.

Based on the comparative results of the assessment model in Table 8, it can be found that the outcomes assessed by the suggested method are consistent with the current specification. Its accuracy rate achieves 83% in the text method, which is higher than the results from the Fuzzy-AHP method (67%) (Li and Sun, 2021). Relative to the Fuzzy-AHP method, the suggested method has higher accuracy, and its result has greater objectivity because the Fuzzy-AHP method is dependent on the expert’s experience. In addition, the suggested method takes into consideration more information from indicators, so its predictable results are more reasonable in comparison with the Fuzzy-AHP model. Thus, when estimating the susceptibility level of debris flow hazards in Horse Neck Field, using the suggested model to reach conclusions is feasible. The method provides accurate results and details about the susceptibility level of debris flow hazards. For example, the reserves of loose substance X<sub>2</sub> at the Diaolou gully is 12.13, which should fall within level IV according to Table 1. In addition, the degree of membership of the other indices obtained by the provided model comes under level III, so the susceptibility level probability at the Diaolou Gully at level III is more extensive than that of levels I, IV, and II. So, the susceptibility level in the Diaolou Valley adheres solely to level III and only barely qualifies as levels I, IV, and II. Furthermore, the susceptibility level of debris flow hazards at 3# gully is more likely to be level II than the level at 1# gully because the mean ranking feature value (1.7311) at 3# gully for class III is higher than that of 1# gully (1.5039). The results obtained using the suggested model accurately demonstrate the susceptibility level of debris flow hazards and further determine the ranking of debris flow hazards at the same level.

**TABLE 6 The feature value of 1# cross-section.**

Name of branch gully	Ranking feature value				Mean value
	$l = 1, g = 1$	$l = 1, g = 2$	$l = 2, g = 1$	$l = 2, g = 2$	
Diaolou gully	2.6054	2.35253	2.6442	2.5411	2.579

**TABLE 7 The assessment values at six cross-sections.**

Name of branch gully	Ranking feature value				Mean value
	$l = 1, g = 1$	$l = 1, g = 2$	$l = 2, g = 1$	$l = 2, g = 2$	
Diaolou gully	2.6054	2.35253	2.6442	2.5411	2.579
1# gully	2.3032	2.1919	2.3482	2.1477	2.2477
2# gully	2.2034	2.092	2.2484	2.048	2.1479
3# gully	2.3018	2.1906	2.3481	2.1475	2.247
Erdingxi gully	2.6094	2.4931	2.6491	2.5078	2.5648
Fatinghou gully	2.6054	2.5021	2.6485	2.4477	2.5509

**TABLE 8 The comparison of results from the different models.**

Name of branch gully	Method in the text	Current specification	Fuzzy-AHP method
Diaolou gully	III	III	III
1# gully	II	II	II
2# gully	II	I	III
3# gully	II	II	II
Erdingxi gully	III	III	IV
Fatinghou gully	III	III	III

## 5 Discussion

- (1) Compared to the traditional variable fuzzy sets model, during the calculative process of relative membership degree, the conventional variable fuzzy set only considers the evaluation interval corresponding to the actual value and adjacent to it, and all other intervals are treated as 0. Traditional methods will lead to the loss of a large number of data related to the indicators; the relative membership degree matrix established by the coupling model considers more information related to the index, and the expression of the membership relationship is more comprehensive and reasonable. In addition, the gray relational analysis has apparent advantages in dealing with the fuzzy calculation of small samples, which are difficult to obtain, such data as debris flow hazards assessment, and the calculation process is simple and efficient.
- (2) In the calculation process, the coupling model uses the gray correlation degree instead of the traditional relative membership degree. Thus, it avoids the uncertainty of value in the standard calculation. Especially when the qualitative problem of non-linear change is conducted, in comparison with the traditional analysis, it is highly dependent on the

- expert experience, and there is no such problem in the coupling model so that it can ensure the objectivity and accuracy of the calculation result as far as possible.
- (3) Although the suggested method has many virtues, it still has some limitations. For example, the great complexity and randomness of different indicators should be given further consideration, and the conversion between qualitative and quantitative characteristics for various indicators can not be realized, so the suggested method still has much room to improve in the future.

## 6 Conclusion

Considering the maximum amount of debris flow at one time ( $X_1$ ), the reserves of loose substance ( $X_2$ ), the watershed area ( $X_3$ ), the length of the ditch ( $X_4$ ), the relative height difference ( $X_5$ ), the gradient of side slope ( $X_6$ ), the longitudinal slope of the main ditch ( $X_7$ ), the length ratio of replenishment section ( $X_8$ ), the maximum daily rainfall ( $X_9$ ), and the Faults developed in the watershed ( $X_{10}$ ), a new assessment method is introduced in this article to assess the susceptibility level of debris flow hazards. The relative membership



degree matrix of the assessment sample is determined at first using the gray system-variable fuzzy sets. Then, the weighting coefficients are calculated by the coupling method. Finally, the susceptibility level of debris flow hazards is determined using the mean ranking feature value.

The proposed method is applied to assess the susceptibility level of debris flow hazards. The result is compared with the current specifications and the Fuzzy-AHP method. The results obtained based on the suggested method are consistent with the current specification, their accuracy reaching 83%. The qualified rate of debris flow in all gullies comes at 50%. In other words, except for 1–3# gullies, for any other valleys, we should adopt necessary measures to prevent the occurrence of debris flow hazards. The results obtained using the suggested model demonstrate the susceptibility level of debris flow hazards accurately and further determine the level ranking of debris flow hazards at the same level.

Compared to the traditional methods, the suggested model has a significant advantage in dealing with the fuzzy calculation of small samples, and its calculation process is simple and efficient; furthermore, it avoids the uncertainty of magnitude in the standard analysis, and it can ensure the objectivity and accuracy of the calculation result as far as possible. For example, the large complexity and randomness of different indicators should given more consideration, and the conversion between qualitative and quantitative characteristics for various indicators cannot be realized, so the suggested method still has space to improve in the future.

In total, the findings of the proposed model provide an alternate way to assess the susceptibility level of debris flow hazards and improve the evaluation accuracy in the future.

## Data availability statement

The raw data supporting the conclusion of this article will be made available by the authors, without undue reservation.

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## Author contributions

X-JZ: Formal Analysis, Investigation, Writing–review and editing. X-BG: Funding acquisition, Methodology, Writing–original draft.

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## Conflict of interest

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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