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Risk sharing in quasi-public-welfare water conservancy (PPP) public-private participation projects: an integrated application of Shapley value and utility theory

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Introduction: Quasi-public water conservancy PPP (Public-private participation) projects are closely related to people's livelihoods and involve multiple participating stakeholders. Previous research on risk sharing in such projects has primarily focused on qualitative analysis of risk factors. Due to self-interest considerations, the collaborating parties tend to deflect and transfer risks to each other as much as possible in the risk sharing process. Additionally, some quantitative analysis methods have been predominantly based on a unilateral perspective.

Methods: Therefore, the present study proposed a new model, which is based on the Shapley Value and the Utility Theory, encompassing a comprehensive analysis of multiple factors such as the proportion of capital contribution, bargaining position, risk management capabilities, and risk-taking willingness of heterogeneous subjects. Firstly, the relationship between the risk losses of different stakeholders and their corresponding value scales and utility attributes is comprehensively analysed, and the transformation characteristics and links of their risk preferences on spatial and temporal scales are summarised. Secondly, The utility values of heterogeneous subjects are employed as quantitative indicators to evaluate utility, leading to the construction of a comprehensive utility objective function for these subjects. Finally, The Shapley Value is then applied to modify the risk-sharing ratio based on the Utility Theory.

Results: The research results show that the risk sharing ratio obtained by single use of shapley value theory or utility theory will be biased toward one side, while the result calculated by using the combination of the two methods is 57.35% for the government to share the risk ratio, and 42.65% for the social capital side, which is a more balanced result.

Discussion: The proposed model enriches the risk management method and theory of quasi public welfare water conservancy PPP projects.

KEYWORDS

quasi-public water conservancy project, PPP, risk sharing, Shapley value, utility theory

1 Introduction

Water conservancy is closely related to people’s livelihoods and function as a key factor in promoting regional economic development, ensuring industrial production, and improving living standards. While water conservancy projects hold a pivotal position in the realm of infrastructure development, they also encounter numerous risks and challenges throughout their preparatory, construction, and operational stages (Wu et al., 2021; Ge et al., 2022). In recent years, the adoption of the Public-Private Partnership (PPP) model has gained significant traction in the realm of water conservancy projects (Zhang, 2014; Li, 2017; Qiu et al., 2018). As of the end of September 2018, according to the data from the project database of the China Public Private Partnerships Center, there are 538 water

conservancy engineering PPP (Public-private participation) projects in China, with an investment amount of up to more than 400 billion yuan. Among them, water conservancy construction PPP projects account for 60.78% of all projects, with a total of 327 projects and a total project investment of more than 260 billion yuan. Quasi-public-welfare water conservancy projects play an indispensable role in the field of water supply and other social livelihoods and public services. Since quasi-public-welfare water projects are not intended to make a profit, the pricing of products and services set under the guidance of the government cannot affect the purchase and use of the public for over-high prices. Therefore, the project fees are difficult to cover the cost, resulting in low economic benefits, lack of attractiveness to investors, and difficulty in financing. At the same time, quasi-public-welfare water conservancy projects are

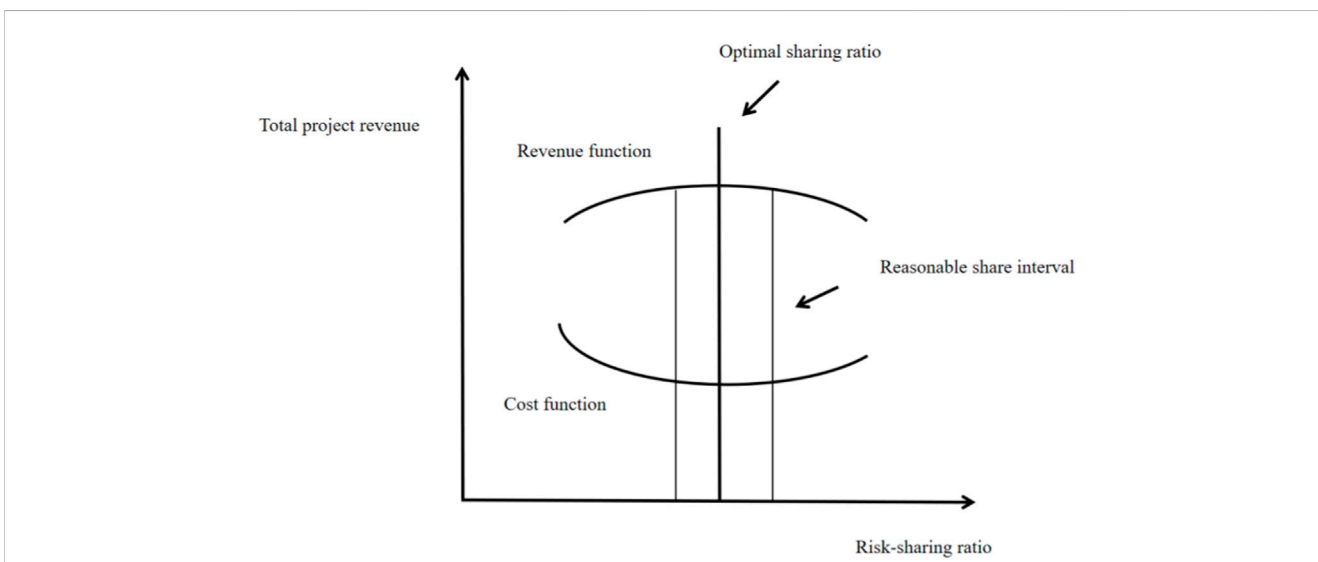


FIGURE 1 Relationship between project benefits and risk-control costs.

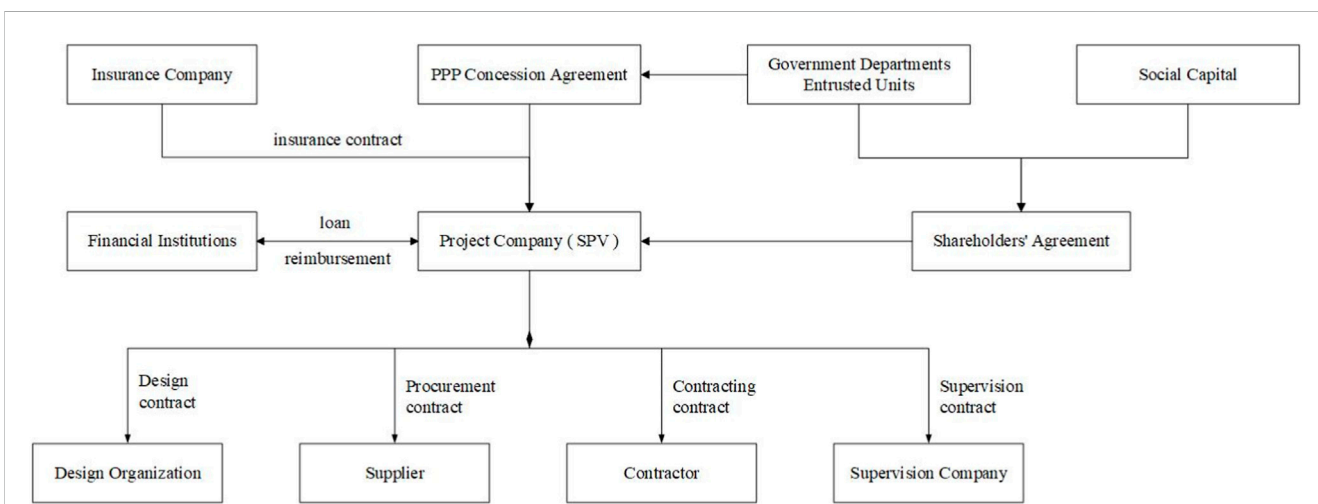


FIGURE 2 PPP project participant-relationship map.

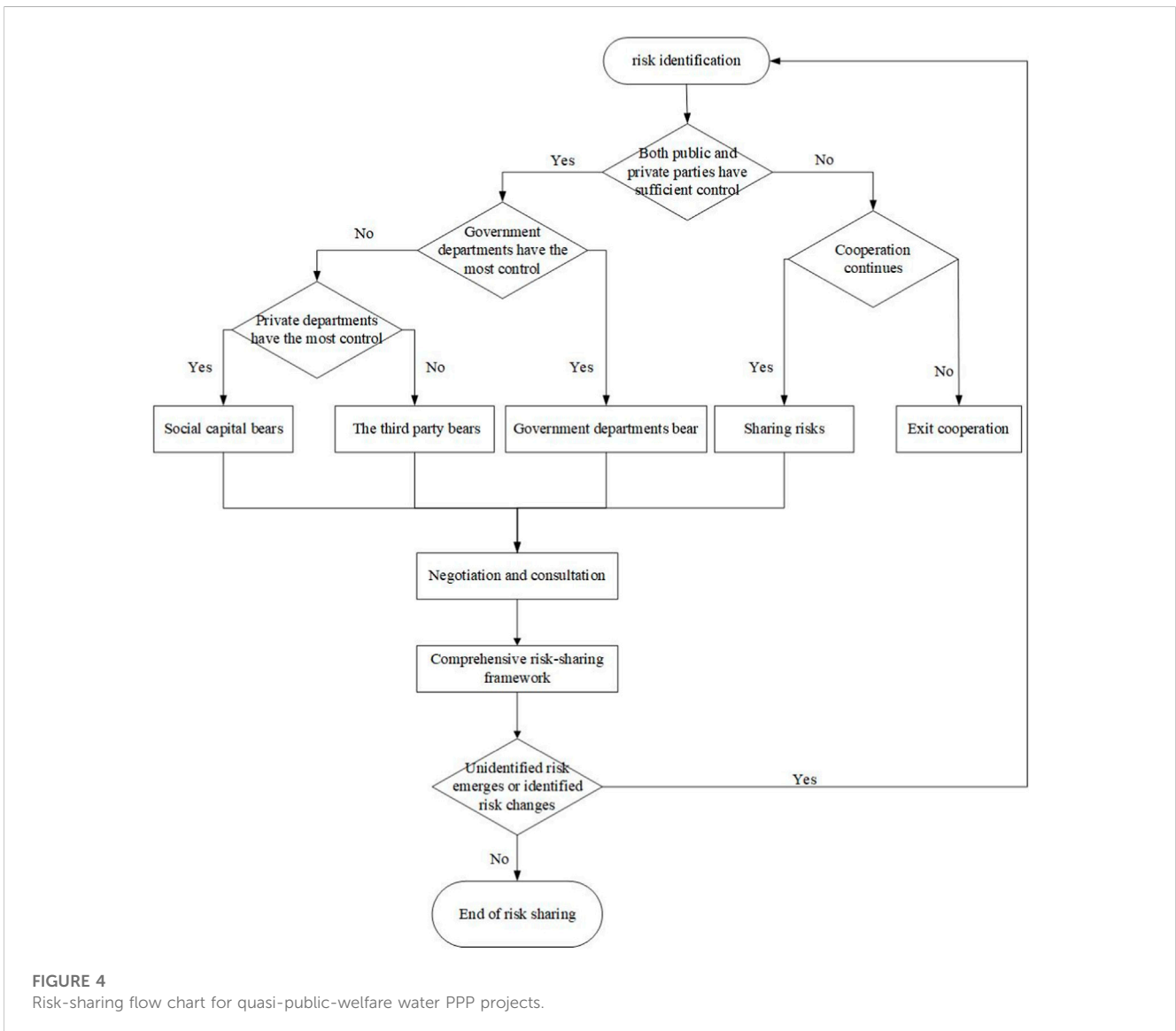
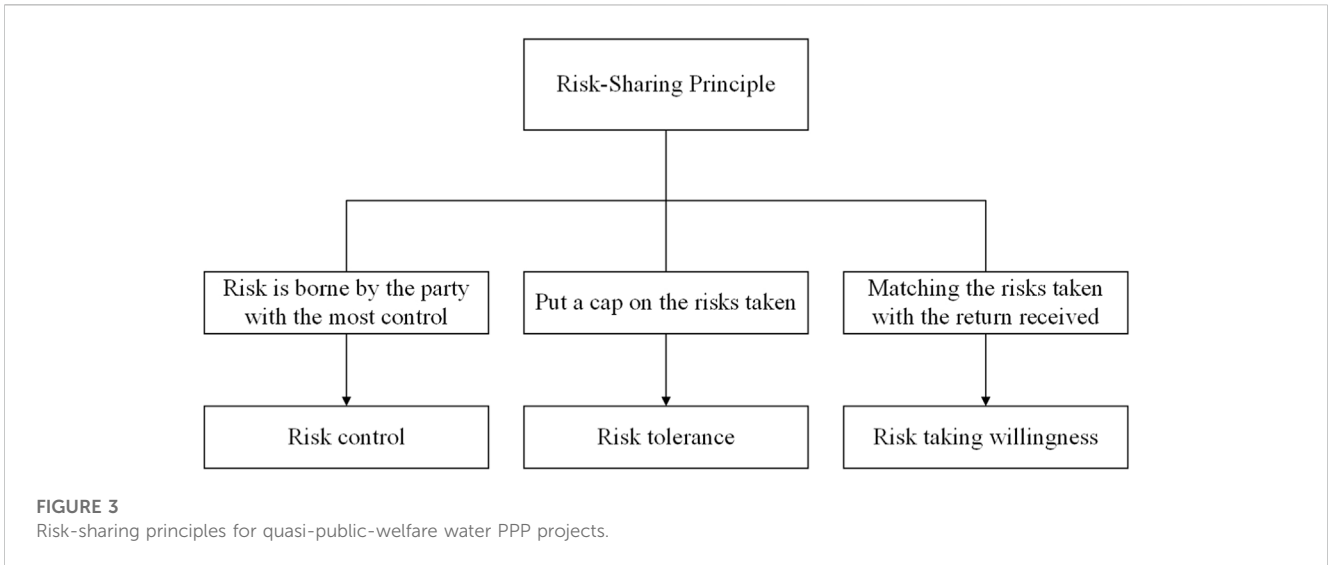


TABLE 1 Risk sharing program.

Risk factor	The most risk-control force		Risk sharing method		
	Government department	Social capital	Government department	Social capital	Shared
Construction cost overrun		√		√	
Financing		√		√	
Engineering change					√
Operation cost overrun		√		√	
Operating revenue		√		√	
Residual value					√
Government officials corruption	√		√		
Charge change		√		√	
Cost payment	√		√		
Interest rate	√		√		
Exchange	√		√		
Inflation	√		√		
Operating cost overruns		√		√	
Tax adjustment	√		√		
Changes in social capital investors		√		√	
Project measurement method subjective		√		√	
Insufficient financial supervision of the project		√		√	
Duration					√
Construction safety					√
Completion					√
Lack of practical experience		√		√	
Unclear definition of responsibility					√
Construction site pollution					√
Waste pollution					√
Climate/geological conditions					√

difficult to implement due to high technical requirements and strict project management requirements, and require coordination of multiple units to collaborate in the meantime during the actual construction process. Given China lacks experience in managing quasi-public-welfare water conservancy PPP projects, many problems occur in the practical application (Ding, 2005; Li et al., 2013; Bi et al., 2016; Gao, 2018).

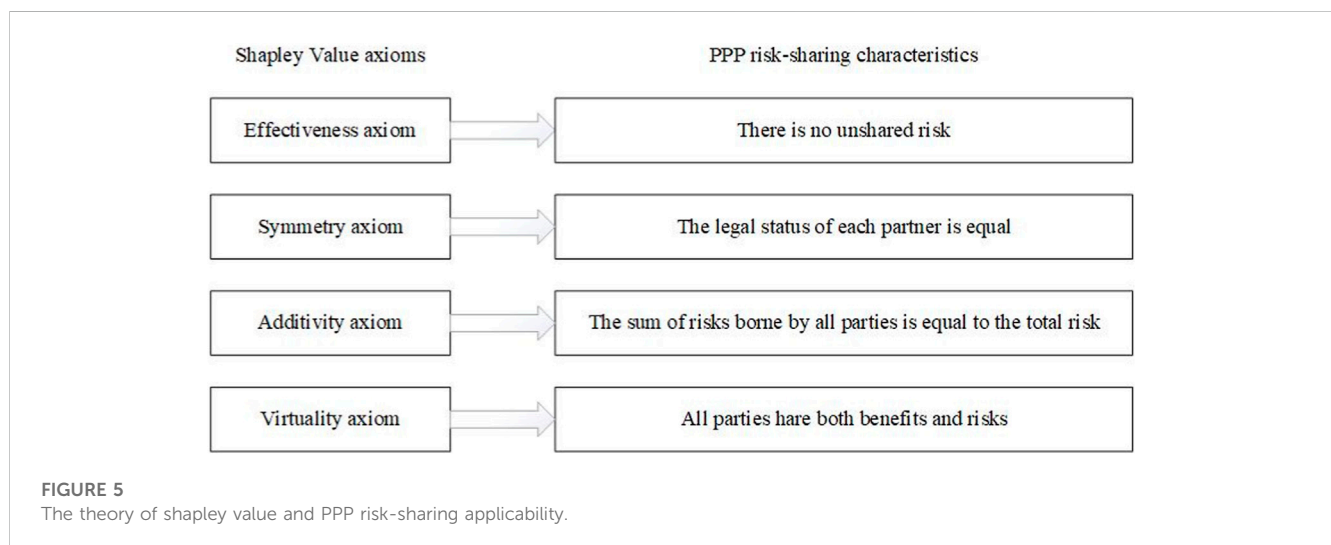
Quasi-public-welfare water PPP projects are characterized by their large scale, involvement of numerous stakeholders, and long cooperation periods that can span several years or even decades. These projects are susceptible to various uncertainties throughout their life cycle, such as changes in the economic environment, which may result in complex and diverse project risks. In order to safeguard their own interests, both partners in PPP projects aim to transfer as many risks as possible to the other party, which often

leads to prolonged negotiation time and increased agreement costs. Furthermore, even after a cooperation agreement is successfully signed, instances of mutual shirking of responsibilities may arise out of project implementation, further exacerbating overall project risks and costs. Therefore, risk allocation is one of the key considerations in research on risk management measures.

Risk sharing, as an important means to control economic risks, is aimed at determining a reasonable allocation of risks that can incentivize the participants in PPP projects, leverage their respective strengths to enhance operational efficiency, reduce the likelihood of risk occurrence or loss, and maximize the overall project benefits. Although the theoretical model of risk sharing is relatively mature, the relevant research mainly focuses on the risk sharing of simple projects. Such projects are often dominated by one party and from a unilateral viewpoint. This traditional risk sharing method is not

TABLE 2 Common risk-sharing model approaches.

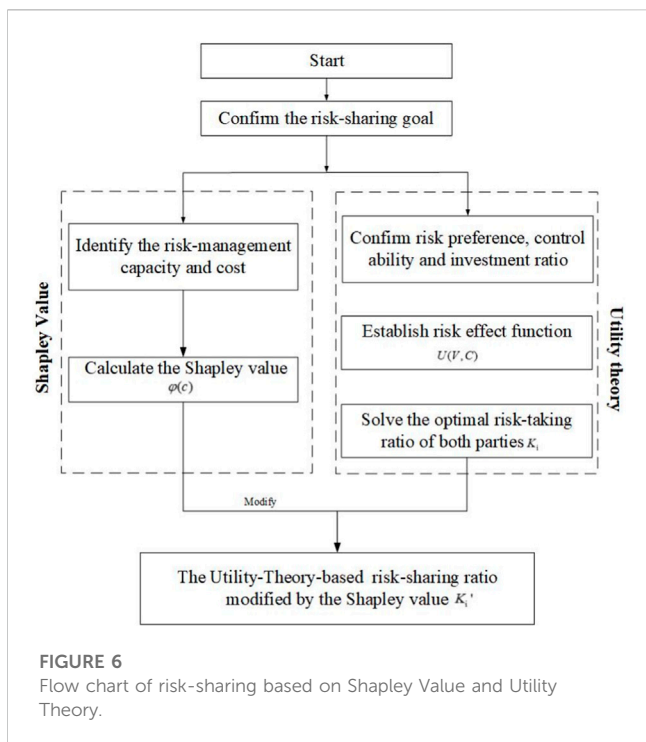
Risk sharing model	Feature	Application cases
Principal-agent model	Assuming that both the principal and the agent are strictly risk-averse, the optimal risk-sharing scheme is that each party bears a certain amount of risk	
Game model	The assumptions of the game model are numerous and the parameter settings that determine the calculation results are mainly based on experience and approximate estimates	The Comprehensive Improvement PPP Project for the Luoyang River reach at Luojiang District
Utility Theory model	High efficiency of risk sharing is made possible by taking into account the ratio of proportion of capital contribution, bargaining position, willingness to take risks, and other factors	The first phase of eastern and middle routes of the South-North Water Diversion Project
Gray relational analysis method	Based on the similarity of development trends among factors. The magnitude of the correlation coefficient can be used to measure the proximity of the evaluated party to the ideal risk taker	A sewage treatment plant PPP project, an Olympic sports center PPP project
Bargaining theory	In response to the respective proposed allocation schemes, the other party will again be caught in a long bargaining process, where the government department and the social capital party share the risk loss	A highway PPP project
Monte carlo simulation	It overcomes the shortcomings of traditional risk-analysis methods such as qualitative estimation of a single risk, and enables a more comprehensive and systematic analysis of project risk sharing	
The theory of Shapley Value	The Shapley method reflects the 'average' contribution to the distribution of benefits for each participant. It also takes into account the degree of contribution of each participant to the total project revenue and its ability to manage risk	Heishanxia river reach water conservancy PPP project



suitable for PPP projects with multi-party participation and equal status. Because of the complexity of the project structure and the specific nature of the water conservancy industry, the research threshold of quasi-public welfare water conservancy PPP projects has increased, coupled with the fact that there are fewer completed projects of this type, the risk sharing research of quasi-public welfare water conservancy PPP projects both is lagging behind the social demand. Therefore, it is necessary to carry out research on quantitative risk-sharing methods for quasi-public welfare PPP projects based on multi-party perspectives to meet the interests of all parties and reduce the overall risk of the project, so as to improve the enthusiasm of social capital invested in quasi-public welfare water projects and create more water projects that benefit society.

2 Research status quo

In the field of project management, the decision-making process regarding risk sharing is of great importance. As a basis for risk sharing decision, Xu (Xu et al., 2022) constructed the risk evaluation index system and the DEMATEL-ANP-FUZZY risk evaluation model of the water conservancy PPP project. Jin, (2010) conducted a study utilizing dealing cost economics and resource organization ability to identify five key characteristics that influence risk sharing in projects. Jin, (2011) further developed a predictive model by combining fuzzy logic and artificial neural networks to accurately forecast effective risk-sharing strategies in dynamic business environments. The proposed Fuzzy TOPSIS decision model aids organizations in determining the optimal risk taker



throughout the risk-sharing process (Khazaeni et al., 2012). Zhao (Zhao et al., 2020) determines the risk factors to be shared among participants through the method of combining GCA method and TOPSIS method. And build a model based on the utility theory to determine the proportion of risks shared among participants. In the context of contract-based projects, Xu et al. (2018) explained how risk-sharing mechanisms in contracts incentivize contractors to collaborate with project owners through a parallel mediating mechanism. Tembo-Silungwe and Khatleli, (2018) introduced the structuration theory to identify the constraints and facilitators of risk sharing. They conducted interviews with practitioners in the Zambian construction industry for finding ways to optimize risk-sharing practices. Li (Li and Wang, 2023) studied the risk sharing problem in the public-private partnership model and constructed a game model of PPP risk sharing dynamic government regulation. Shi et al. (2021) argued against the effect of the Principal-Agent Theory in designing risk-sharing rules for construction projects involving risk-neutral contractors. They highlighted the potential bias in incentive structures and proposed the analysis of risk sharing between owners and risk-neutral contractors to address this issue.

Studies on risk sharing in PPP projects can be categorized into three main groups. The first category involves questionnaire-based

studies that aim to determine the allocation of risks between the public and private sectors. Zhao et al. (2020) employed the Delphi method to identify key risk factors. Meanwhile, in order to achieve rational risk sharing, he determined the risk factors that participants were willing to assume and subsequently calculated the risk sharing ratio by combining the GCA method and TOPSIS method. Shrestha et al. (2018), based on the expertise of 32 practitioners, investigated the risk allocation in PPP water projects in China. They found that poor risk management was often brought about by the misallocation of risk between the two primary actors in the PPP. The second category comprises empirical analyses. Huang et al. (2022) conducted an empirical analysis with the Risk Mechanism Information Integration System (RMIS) and took the Chengdu Metro Rail Transit Line 17 Phase I PPP project as a case study. They established a risk-sharing decision-making model to evaluate different risk allocation options, and determined the optimal decision for each risk factor between social capital and the government. Nguyen et al. (2018) studied 31 risk allocation schemes for 21 road PPP contracts in the U.S. and discovered that the majority of risks were transferred to the private sector. Heravi and Hajihosseini, (2012) analyzed the contract organization of the Tehran-Chalus toll road PPP project and proposed suggestions on the improvements of the risk-sharing schemes for better project performance. The third category combines different methodologies to build models for risk allocation. Wang et al. (Wang, Zhou, Jin, Zhao, and Sun, 2022) developed risk assessment models, including the optimal-worst multi-criteria decision-making method and fuzzy comprehensive evaluation method, to achieve effective risk sharing and benefit allocation among different stakeholders. Their objective was to promote win-win outcomes. Mazher et al. (2019) proposed a multi-attribute risk allocation decision-making method based on non-additive fuzzy integration. In this method, the risk management capabilities of each stakeholder could be effectively aggregated and the acceptable principles for risk allocation could be evaluated. Some scholars have conducted quantitative research on the proportion of risk sharing based on a unilateral perspective: Zhao et al. (2020) based on the model of utility theory to determine the proportion of risk shared among participants from the government’s perspective, and validated it with the PPP project of sponge city in Qingshan District, Wuhan City. Wang et al. (2022) combined the capital investment, contribution, and project participation to revise the Shapley value income distribution model to obtain a more fair and reasonable income distribution and risk sharing system.

Through the analysis of the existing literature, the previous research on risk sharing by scholars at home and abroad focuses on the qualitative analysis of risk factors, i.e., to determine which risks

TABLE 3 Risk-sharing ratio.

Sharing ratio calculation method	Risk-sharing ratio	
	Government department (%)	Social capital (%)
Utility Theory	34.62	65.38
Shapley Value	69.57	30.43
Utility-Theory and Shapley Value	57.35	42.65

are shared by both public and private parties, which risks are borne by each party, and lacks quantitative judgment on the proportion of shared risks; a small number of scholars have made a quantitative study on the proportion of risk sharing in PPP projects, but the majority of them are concentrated in the operational PPP projects with strong profitability, and they focus on considering some factors such as the risk disposal capacity or risk appetite, or they are based on a unilateral viewpoint of the governmental departments or the social capitalists, which is one-sided, and the assumption of conditions are many, and the computation process is complex.

In view of this, in order to ensure that the risk sharing scheme is scientific, fair and effective, combined with the characteristics of quasi-public welfare water conservancy PPP projects, this study taking into account the proportion of capital contribution, bargaining position, and the differences in the ability of both parties to deal with risks. The Shapley value is coupled with the utility theory, the quasi public welfare water conservancy PPP project in the government sector and the social capital party risk sharing ratio to carry out research, and combined with the Chongqing Fengsheng Reservoir Project examples to carry out The risk sharing ratio is calculated with the example of Chongqing Fengsheng Reservoir Project.

3 Principles and process of PPP project risk management

In a PPP project, risk sharing plays a crucial role in distributing various risk elements among the participants. It is a strategy that can be employed to manage the relationship between stakeholders in risk management. Given the complexity of risks, the multiplicity of participants, and the severity of implications brought by risks, risk sharing has always been a challenging and pivotal aspect of project risk management (Xu et al., 2010; Tian, 2014).

Achieving the most reasonable balance of benefit sharing and risk sharing between social capitalists and government departments is essential for reducing project costs, enhancing operational efficiency, and maximizing overall benefits (Xu et al., 2010; Ranjith et al., 2016). Insufficient risks shared with the private sector may lead to decreased efficiency and increased project costs, while excessive risks shared may also diminish the private sector's enthusiasm and inflate the overall project cost likewise. Therefore, identifying the optimal risk-sharing ratio is crucial to maximizing project revenue and minimizing the total cost of risk control, as depicted in Figure 1.

A clear definition of risk-sharing subjects is essential in PPP projects, considering the multitude of participating parties, including government departments, social capital parties, project companies, financiers, contractors and operators, and the rights and obligations of each subject are different (Li, 2017; Wu, 2017; Li, 2018). The scientificity and effectiveness of the risk-sharing scheme can be ensured only when the risk-sharing subjects are precisely identified (Zhao, 2018). Throughout the entire life cycle of PPP projects, government departments, and social capitalists serve as the primary risk-bearing parties. Together, they establish the project company as the project's operating entity, responsible for financing, design, construction, and operation, among other specific tasks. The relationships are presented in Figure 2.

The risk-sharing scheme should be developed by initially identifying and analyzing potential risks at each stage of the project. Subsequently, the risk preferences, risk tolerance, and risk-control abilities of each participant are further examined to formulate an appropriate risk-sharing plan (Jin et al., 2011; Lang et al., 2017).

The fundamental principle of the PPP model lies in the sharing of benefits and risks, with the primary risk bearers being government departments and private investors. As the core participants, they play indispensable roles in quasi-public-welfare PPP projects. This chapter focuses on social capitalists and government departments as the key stakeholders and places emphasis on investigating the risk-sharing mechanisms between them.

In the realm of risk sharing, numerous scholars such as Lam, Li, and others have conducted research on the subject. They have put forth a risk-sharing principle as depicted in Figure 3 (Arndt, 1998; Wang et al., 2022; Li et al., 2005; Ng and Loosemore, 2007; Lam et al., 2008).

(1) Risk is borne by the party with the most control

When a participant in a project has the highest level of control, they possess an advantage in terms of reducing the likelihood of risk occurrence and mitigating risk losses.

(2) Put a cap on the risks taken

In situations where a project incurs substantial risk losses that surpass the capacity of the responsible party, it becomes necessary to prevent project insolvency by stipulating that the risk cannot be solely borne by one party or continued to be shared according to the previously agreed risk-sharing ratio.

(3) Matching the risks taken with the return received

Effective risk sharing should be grounded in comprehensive risk identification and analysis, which entails taking factors such as the probability of risk occurrence, potential losses, costs associated with risk retention, and the expenses associated with risk transfer into consideration. By comparing these elements with the project returns, a well-aligned risk-sharing arrangement can be established.

Within the risk management process of PPP projects, a sound design of procedures serves as a crucial assurance for the implementation of risk sharing (Jin, 2011; Wang, 2017). Following the completion of effective risk identification for the project, the risk-sharing procedures for quasi-public-welfare water PPP projects can be established under the aforementioned risk-sharing principles, as depicted in Figure 4.

In the development of quasi-public-welfare water PPP projects, the risk-allocation process is dynamic and complex. During the project's decision-making phase, a comprehensive analysis is conducted to examine the unique advantages, status disparities, investment and revenue objectives, as well as the risk appetite and control capabilities of both government departments and social capitalists involved in the PPP project. This analysis provides insights into the specific characteristics and circumstances of each party. Following this analysis, suitable risk identification methods are employed to thoroughly explore and analyze

potential sources of risk throughout the project's lifecycle, so as to assess whether these risks fall within controllable limits. The subsequent stage comes to the actual risk sharing when, government departments and social capitalists collaborate to determine the allocation of risks, specifying which risks will be borne by the government, which borne by the social capitalists, and which shared jointly. Once the shared risks are identified, it becomes necessary to establish the risk-sharing ratio for each party involved in order to determine their respective responsibilities and obligations for the shared risks.

The scheme on risk identification and preliminary risk sharing for quasi-public-welfare water conservancy PPP projects, as demonstrated in [Table 1](#), is developed based on the risk-sharing principles put forth by Liu Xinping and others ([Liu and Wang, 2006](#); [Wang, 2017](#)). Additionally, the research conducted by Li Yang et al. on risk-sharing schemes for relevant PPP engineering cases ([Li, 2018](#); [Xu et al., 2022](#)) has been referred to as well.

4 Risk-sharing model

The success of PPP projects hinges on the effective sharing of project risks among the participants. The most notable distinction between PPP and other models lies in the establishment of a reasonable sharing ratio that aims to stimulate the enthusiasm of all parties involved and leverage their respective advantages. If government departments assume an excessive amount of risk, it may lead to financial strain on the government and undermine the original intent of introducing social capitalists through PPP projects. On the other hand, if social capitalists shoulder an excessive amount of risk, it may deter their participation in PPP projects, ultimately impeding their contributions in terms of capital and expertise. Thus, risk sharing in PPP projects must adhere to the principle of achieving mutual benefits and a win-win situation for all parties involved.

The current research on risk sharing in PPP projects can be broadly classified into two categories. The first category involves using questionnaires to gather information and derive a project risk-sharing matrix, which provides decision support for risk sharing. This approach aims to capture the perspectives and insights of project participants to determine an appropriate risk-sharing arrangement. The second category of research focuses on quantitative methods for risk sharing in PPP projects. These methods employ techniques such as Linear Programming and Game Theory to analyze and optimize the outcomes of risk sharing. By means of mathematical models and computational approaches, these methods aim to achieve an objective and efficient allocation of risks among project participants.

[Table 2](#) displays some common quantitative research methods employed in studying risk sharing in PPP projects ([Wang and Yang, 2013](#); [Wang, 2017](#); [Zhang, 2017](#)). These methods offer valuable tools for analyzing and evaluating risk-sharing strategies in PPP projects.

The theoretical model of risk sharing has matured, with the relevant research primarily concentrated on risk sharing in simple projects. However, limited attention has been given to the risk-sharing aspects of quasi-public-welfare water conservancy PPP projects. Consequently, this paper aims to address this gap by focusing on risk sharing in quasi-public water PPP projects. It

takes into consideration the unique characteristics of such projects and seeks to implement practical risk-sharing principles and methods.

The theory of Shapley Value mainly focuses on the disparity in risk-management capabilities between the two parties involved to determine the allocation of risk-sharing ratios. The results from applying the theory primarily reflect the risk-management abilities of both parties and highlight the differences between them. Hence, this paper seeks to combine the Shapley Value approach with the Utility Theory with the latter used to calculate the risk-sharing ratio, and the former employed to adjust the risk-sharing ratio derived from Utility Theory. The risk-sharing model established based on the two theories provides a more comprehensive and rational approach to determining the risk-sharing ratio and proves to be more reasonable than those solely relying on the Utility Theory or the Shapley Value Theory for calculating the risk-sharing ratio.

4.1 The theory of shapley value

In economic activities, when multiple actors join forces through alliances or partnerships, they have the potential to achieve greater gains or minimize losses compared to working individually, thereby creating a win-win or multi-win situation. The theory of Shapley Value focuses on the distribution of benefits or sharing of risks among the members of an alliance. It takes into account the level of contribution made by each member towards the overall goals of the alliance and reflects the interactive process of cooperation among alliance members. The theory of Shapley Value is primarily applied to address issues related to the distribution of cooperation costs and benefits among alliance members ([Yu et al., 2014](#); [Xu et al., 2016](#)).

In recent years, the application of the Shapley Value Theory has extended to various areas such as investment analysis, compensation allocation, risk allocation, and burden apportionment. In the water sector, the Shapley Value has emerged as a prominent method for solving cooperative game problems. It provides a quantitative allocation ratio that is practical and straightforward to apply ([Yang, 2016](#); [Wang et al., 2018](#)).

The calculation of the Shapley value is made in view of the costs and capabilities of both parties involved in managing a particular risk. Its results can reflect the disparity in risk management abilities between the parties. However, some factors such as the ratio of proportion of capital contributions and bargaining positions of the parties are not taken into account. Given the theoretical limitations of the Shapley Value and the unique characteristics of PPP projects, it is necessary to revise the index system of the Shapley Value method. This revision allows for a more comprehensive analysis, considering factors specific to PPP projects ([Fan, 2014](#); [Sheng and An, 2019](#)).

Pinpointing the optimal allocation scheme or "solution" is indeed the most challenging aspect of the Cooperative Game Theory. The Shapley Value method, as a type of cooperative game, presents a straightforward calculation process and allocates benefits based on the partners' marginal benefit to the alliance, which aligns with Adams' equity theory. The latter theory suggests that distributing benefits based on the contribution rate can enhance team cooperation and output, in line with the principles of benefit

sharing and risk sharing in PPP projects. It can also help emphasize each partner's importance within the alliance.

Let N be the set of alliances consisting of n cooperating parties, and $N = 1, 2, \dots, n$. The cooperating parties within the alliance are non-antagonistic and the overall cooperation gains increase in step with participating parties. Let S be a subset of N , representing a cooperative alliance formed among the parties, such that $S \subseteq N$. $v(S)$ denotes the profit of the cooperative alliance S , and $[N, V]$ represents an n -person cooperative game with the characteristic function v . Specifically, $v(\varphi) = 0$, $v(S_1 \cup S_2) \geq v(S_1) + v(S_2)$, where $S_1 \cap S_2 = \varphi$ and φ represents the empty set. By applying the Shapley Value method to evaluate the marginal contributions of each party to the overall cooperation, a fair and reasonable allocation of interests for each party can be obtained.

The total benefits obtained by the cooperative alliance surpass the sum of the individual benefits obtained by each party working alone, i.e.,

$$v(S_1) > \sum_{i \in S} v(i) \tag{1}$$

Here, $v(i)$ represents the profits obtained by participant i when working alone.

Let X_i represent the distribution of benefits obtained by member i from the cooperative alliance. $X = (X_1, X_2, \dots, X_n)$ indicates that the cooperative participants' benefits in the alliance are no less than the benefits obtained by independently completing the project, where $i = 1, 2, \dots, n$.

$$X_i \geq v(i) \tag{2}$$

$$\sum_{i \in S} X_i \geq v(S) \tag{3}$$

Assuming that the members of the cooperative alliance join the alliance N in a random order, there are a total of $n!$ possible orderings. Each ordering has an equal probability of occurrence, which is $\frac{1}{n!}$. When party i forms an alliance S with the preceding $(s - 1)$ individuals, the marginal contribution of party i to the alliance $S \in N/i$ is $v(S) - v(S \setminus i)$. Since there are $(|S| - 1)!$ permutations of the preceding $(s - 1)$ parties and $(n - s)!$ permutations of the remaining $(n - s)$ parties, the total number of permutations for the alliance S formed is $(n - 1)! (|S| - 1)!$. The probability of forming the alliance S is $\frac{(n - |S|)! (|S| - 1)!}{n!}$. Therefore, the expected value of the marginal contribution of participant i in alliance S is precisely the Shapley value.

The calculation formula for the Shapley value $\varphi(v) = (\varphi_1(v), \varphi_2(v), \dots, \varphi_n(v))$ is as follows:

$$\varphi_i(v) = \sum_{S \in S_i} W(|S|) [v(S) - v(S \setminus i)] \tag{4}$$

Substituting $W(|S|) = \frac{(n - |S|)! (|S| - 1)!}{n!}$, we have:

$$\varphi_i(v) = \frac{(n - |S|)! (|S| - 1)!}{n!} (v(S) - v(S \setminus i)) \tag{5}$$

Here, $i = 1, 2, \dots, n$, $|S|$ represents the number of cooperating parties in the alliance; $S \setminus i$ represents all subsets of set N that contain i ; $W(|S|)$ is the weighting factor; $v(S \setminus i)$ represents the overall benefit obtained by the alliance S when party i is excluded; and $v(S) - v(S \setminus i)$ represents the marginal contribution of party i to the alliance S .

The research in this paper focuses on the risk sharing of quasi-public-welfare water conservancy PPP projects, which involves the

allocation of costs within a cooperative alliance. To derive cost sharing, the term "revenue" in the previous formulas needs to be replaced with "cost". The specific calculation formulas are as follows:

$$\varphi_i(c) = \sum_{S \in S_i} W(|S|) [c(S) - c(S \setminus i)] \tag{6}$$

$$\varphi_i(c) = \frac{(n - |S|)! (|S| - 1)!}{n!} (c(S) - c(S \setminus i)) \tag{7}$$

But before that we need to explore the four axioms of the Shapley Value to determine its suitability for risk-sharing studies in PPP projects. The study of risk sharing in PPP projects is grounded on the principle that benefit sharing is premised on risk sharing, and both parties voluntarily cooperate for mutual benefit. Figure 5 demonstrates the analysis of risk sharing in PPP projects based on the four axioms of the Shapley Value:

First, all partners share all the risks of the project, without any risk being left unshared, which aligns with the effectiveness axiom of the Shapley Value. Secondly, although the government department is the initiator of a PPP project and the social capital party plays a dominant role in the PPP project company, the risk-sharing results only take into account factors such as risk-control ability, risk-bearing ability, and risk willingness of each party and the legal equality of each partner is not considered during risk sharing. This satisfies the axiom of symmetry. Thirdly, sum of the risks respectively borne by each party is amount to the total risks of the PPP project, in line with the axiom of additivity. Finally, there is a clear division of work and responsibilities among the participants with each being required to share both project benefits and risks. As a result, no partner solely bears risks without enjoying benefits or solely enjoys benefits without bearing risks, aligning with the axiom of virtuality.

In conclusion, PPP projects, including quasi-public-welfare water PPP projects, align with the four axioms of the Shapley Value method. Consequently, the Shapley Value Theory can be effectively applied to analyze and study the risk-sharing dynamics in quasi-public-welfare water PPP projects. This approach provides a solid foundation for understanding the fair distribution of risks and benefits among the project stakeholders, promoting the overall effectiveness and success of such projects.

4.2 Utility theory

The Utility Theory, originally proposed by mathematician D. Bernoulli (1954), is a decision-making theory widely used in economics. It provides a framework for decision-makers to make choices by considering their individual utility or satisfaction levels. Since different decision-makers may have different utility preferences, even when presented with the same information, they still have diverse perceptions and evaluations.

In the context of risk-sharing decisions, the Utility Theory employs utility values as quantitative indicators. Higher utility values represent higher satisfaction with the expected returns, while lower utility values indicate lower satisfaction. The Utility Theory takes into account various factors such as the proportion of capital contributions of the parties involved and their bargaining positions. It enables the measurement of evaluations, preferences, and attitudes of different individuals toward risks and other factors.

Through the risk-sharing game that considers expected benefits and costs, the utility theory model enables the determination of a reasonable risk-sharing ratio., which provides valuable insights and references for decision-makers involved in the project. By utilizing this model, decision-makers can make more well-grounded and strategic choices, leading to optimized risk management and maximized project revenue.

In quasi-public-welfare water conservancy PPP projects, the utility functions U of the government department and the social capital party can be represented as functions of project benefits V and the cost of project risk control C , i.e., $U = U(V, C)$. Considering that PPP projects usually involve risk sharing and co-operation between multiple participants, each participant has a different sensitivity and preference for risk. With a fixed total investment, each party's perception of risk taking has a non-linear characteristic, i.e., a higher risk control cost leads to a greater loss of utility, while a lower risk control cost has a lesser impact on the increase of utility, and as a quasi-public welfare projects the contribution of economic returns to utility is linear. Therefore, this paper adopts a quadratic utility function for quasi-public welfare PPP projects can comparatively describe this nonlinear relationship, the formula is:

$$U = -\frac{1}{a}C^2 + V + 1 \tag{8}$$

The parameter $a > 0$ is estimated or adjusted according to individual preferences and risk attitudes. Larger values of a imply that individuals are more resistant to the cost of taking risks, and government departments are more sensitive to costs than social capitalists in Quasi-Public-Welfare Water Conservancy PPP Projects.

The optimization model for minimizing project risk-control costs is represented as follows:

$$Max[U_1(V_1, C_1) + U_2(V_2, C_2)] \tag{9}$$

$$MinCr(C_1, C_2) \tag{10}$$

Here, U_1, V_1 , and C_1 represent the utility function, benefits, and risk-bearing cost of the government department, respectively. U_2, V_2 , and C_2 represent the utility function, benefits, and risk-bearing cost of the social capital party.

Assuming the expected total risk cost is C_e , where K_e represents the proportion of risk shared by the government department (with $0 \leq K \leq 1$) and the proportion of risk shared by the social capital party is $1 - K$. The actual cost required for risk control is represented as C_r .

$$C_e = C_{eg} + C_{es} \tag{11}$$

$$C_r = C_{rg} + C_{rs} \tag{12}$$

$$C_{eg} = KC_e, C_{es} = (1 - K)C_e \tag{13}$$

$$C_{rg} = KC_r, C_{rs} = (1 - K)C_r \tag{14}$$

Here, C_{eg} and C_{rg} represent the expected risk cost borne by the government department and the actual risk cost, respectively. C_{es} and C_{rs} represent the expected risk cost borne by the social capital party and the actual risk cost, respectively.

The actual cost of risk is a function of the effectiveness of risk control and management, as well as the shared estimated risk cost. The actual risk cost is determined by the effectiveness function of risk control and the function of shared expected risk costs, represented as:

$$C_{rg} = F(K, C_{eg}) \tag{15}$$

$$C_{rs} = G(1 - K, C_{es}) \tag{16}$$

Here, F and G are the effectiveness functions of risk control for both the public and private sectors. The spillover benefits respectively obtained by the public and private sectors from effective risk control are as follows:

$$V_1 = C_{eg}C_{rg} = K(C_eC_r) \tag{17}$$

$$V_2 = C_{es}C_{rs} = (1 - K)(C_eC_r) \tag{18}$$

Furthermore, the maximum utility function can be derived as:

$$max[U_1(K(C_eC_r), KC_r) + U_2(1 - K)(C_eC_r), (1 - K)C_r] \tag{19}$$

$$min C_r \tag{20}$$

To maximize the overall efficiency of the quasi-public-welfare water conservancy PPP project, a comprehensive utility objective function is constructed for the government and the social capital:

$$f(U_1, U_2) = a_1(U_1 \cdot (U_1^0)) + a_2(U_2 \cdot (U_2^0)) \tag{21}$$

Here, a_1 and a_2 are the weighting coefficients of the negotiation between the government and the social capital, and $a_1 + a_2 = 1$. Generally, a is inversely proportional to the risk-sharing ratio. U_1^0 and U_2^0 are the initial utility values for the government and the social capital without risk impact, and $U_1 \cdot (U_1^0)$ and $U_2 \cdot (U_2^0)$ represent the spillover utilities obtained by the government and the social capital due to risk assumption. Since the overall objective is to maximize the comprehensive benefits, the spillover utilities are positive values, i.e., $U_1 \cdot (U_1^0) > 0$ and $U_2 \cdot (U_2^0) > 0$.

Due to the long cooperation period and complex risks involved in the quasi-public-welfare water conservancy PPP project, it is challenging to determine the total risk cost. The minimum risk cost represents the joint expected cost of risk for both the government and the social capital, which can be expressed as:

$$C_r = \overline{EC} \tag{22}$$

The optimization model for risk-sharing is as follows:

$$max f(U_1, U_2) = max \{a_1(U_1(K(C_e - C_r), K \cdot C_r) - U_1^0) + a_2(U_2(1 - K) \cdot (C_e - C_r), (1 - K) \cdot C_r) - U_2^0\} \tag{23}$$

$$s.t. C_r = \overline{EC}$$

In this model, the objective is to maximize the function $f(U_1, U_2)$, which represents the comprehensive utility of the government and the social capital. The constraint ensures that the actual risk cost C_r is equal to the expected cost \overline{EC} , which represents the desired level of risk control.

To solve for the optimal risk cost-sharing ratio for the government, we can take the partial derivative of $f(U_1, U_2)$ with respect to K . Given that C_r is a function of K and C_e is constant, we obtain:

$$\frac{\partial f}{\partial K} = a_1 \left[(C_e - C_r) \frac{\partial U_1}{\partial V_1} - K \frac{\partial U_1}{\partial V_1} \frac{\partial C_r}{\partial K} + C_r \frac{\partial U_1}{\partial C_1} + K \frac{\partial U_1}{\partial C_1} \frac{\partial C_r}{\partial K} \right] + a_2 \left[(C_e - C_r) \frac{\partial U_2}{\partial V_2} - K \frac{\partial U_2}{\partial V_2} \frac{\partial C_r}{\partial K} + C_r \frac{\partial U_2}{\partial C_2} + K \frac{\partial U_2}{\partial C_2} \frac{\partial C_r}{\partial K} \right] \tag{24}$$

By applying the Lagrange multiplier method to find the optimal solution, we let $a_1 = a$ and denote the optimal risk cost-sharing ratio as $K_0 = u \cdot a$. Thus, we have:

$$\begin{aligned}
 L &= f(U_1, U_2) + \lambda(\overline{EC} - C_r) \\
 &= a[U_1(K_0(C_e - C_r), K_0 \cdot C_r) - U_1^0] + \\
 &\quad (1 - a)[U_2((1 - K_0)(C_e - C_r), (1 - K_0) \cdot C_r) - U_2^0] \\
 &\quad + \lambda(\overline{EC} - C_r) \tag{25}
 \end{aligned}$$

Taking $\frac{\partial L}{\partial a} = 0$ and $\frac{\partial L}{\partial \lambda} = 0$, we obtain:

$$\begin{aligned}
 \frac{\partial L}{\partial a} &= a \left[(C_e - C_r) \frac{\partial U_1}{\partial V_1} u'(a) + C_r \frac{\partial U_1}{\partial C_1} u'(a) \right] \\
 &\quad + [U_1(K_0(C_e - C_r), K_0 \cdot C_r) - U_1^0] - (1 - a) \\
 &\quad \left[(C_e - C_r) \frac{\partial U_2}{\partial V_2} u'(a) + C_r \frac{\partial U_2}{\partial C_2} u'(a) \right] \\
 &\quad - [U_2((1 - K_0)(C_e - C_r), (1 - K_0) \cdot C_r) - U_2^0] \\
 &= 0 \tag{26}
 \end{aligned}$$

$$\frac{\partial L}{\partial \lambda} = \overline{EC} - C_r = 0 \tag{27}$$

By Eq. 27, can be determined. Substituting C_r into Eq. 25 yields the value of a , which allows the determination of K . Consequently, the risk allocation between the parties involved will be established. The determined risk allocation not only effectively motivates the cooperation of all participants, enabling risk sharing, but also reduces the overall project risk cost.

The application of the Utility Theory for quasi-public-welfare water PPP projects assumes that the following conditions are all satisfied.

- (1) Cost minimization and benefit maximization as key performance indicators for both partners;
- (2) Independent existence of each risk;
- (3) Transparent communication and symmetric information about risks among partners;
- (4) The cost of risk can be estimated and the shared risk can be identified;
- (5) Both government departments and private institutions are groups with bounded rationality to pursue their own interests.

4.3 Risk sharing calculation

Though taking into account factors such as the proportion of capital contributions and bargaining position of both parties, the Utility Theory neglects the disparity in the parties' risk-management abilities. On the other hand, the Shapley values calculation is based on the cost and ability of the parties to handle a specific risk, providing insights into their differing risk-management capabilities. To address these considerations comprehensively, a combined approach of the Shapley Value and the Utility Theory is employed. The Shapley value is calculated for different sharing agents, reflecting their respective abilities in risk handling. Meanwhile, the Utility Theory is used to calculate the risk-sharing ratio based on the preferences and evaluations of the sharing parties. The Shapley value is then adopted to modify the

risk-sharing ratio. By integrating these two approaches, a risk-sharing model based on the Shapley Value and the Utility Theory is established. Firstly, the relationship between the risk loss of heterogeneous subjects and the corresponding value scale and utility attribute is calibrated by comprehensively analyzing the factors such as the proportion of capital contribution, negotiation status and risk disposal ability of heterogeneous subjects, and the transformation characteristics and relationship of risk preference of heterogeneous subjects on time and space scale are summarized. Secondly, the utility value of the heterogeneous subject to the evaluation utility is used as the quantitative index to construct the comprehensive utility objective function of the heterogeneous subject. Finally, The Shapley Value is then applied to modify the risk-sharing ratio based on the Utility Theory. The risk-sharing process based on the two theories is depicted in Figure 6, illustrating the comprehensive and balanced consideration of these factors.

The steps are as follows.

The first step: Determining risk-sharing parties.

During the risk management process, it is essential to establish the allocation of risks between government departments and the social capital party provided that those mutually shared risks have also been identified. Since the objective of risk-sharing is to address the risks that both parties are responsible for, before initiating the sharing process, it is crucial to establish a clear understanding of which risks fall under the shared responsibility of both parties.

The second step: Calculating the share of risk based on Utility Theory.

Provided that the risk-sharing objectives are determined, based on the risk preference, control ability, and proportion of capital contribution of the government and social capital towards the risk, the expected loss brought by risk disposal and negotiation weighting coefficients are determined. The utility functions of the government and social capital in quasi-public-welfare water conservancy PPP projects can be expressed as a function of project returns V and the cost of project risk control C , that is, $U = U(V, C)$.

Based on Eqs 8–20, we construct the comprehensive utility objective function for the government and social capital as follows:

$$f(U_1, U_2) = a_1(U_1 U_1^0) + a_2(U_2 U_2^0) \tag{28}$$

where a_1 and a_2 represent the weighting coefficients of the negotiation between the government and social capital, respectively, and $a_1 + a_2 = 1$. In general, a is inversely proportional to the risk-sharing ratio. U_1^0 and U_2^0 denote the initial utility values of the government and social capital respectively, in the absence of risk influence.

The optimization model for risk-sharing can be formulated as follows:

$$\begin{aligned}
 \max f(U_1, U_2) &= \max \{a_1(U_1(K(C_e C_r), K \cdot C_r)U_1^0) + \\
 &\quad a_2(U_2((1 - K) \cdot (C_e C_r), (1 - K) \cdot C_r)U_2^0)\} \tag{29} \\
 \text{s.t. } C_r &= \overline{EC}
 \end{aligned}$$

Taking partial derivatives of K with respect to $f(U_1, U_2)$, where C_r is a function of K and C_e is a constant, we can derive the optimal risk-cost sharing ratio K_1 for the government sector and the optimal risk-cost sharing ratio $K_2 = 1 - K_1$ for the private capital sector.

The third step: Calculate the Shapley values of risk sharing parties.

In order to quantify the cost of risk disposal in consideration of the differences in risk-management capabilities between government departments and social capital partners, the cost of individual risk disposal and the cost of collaborative risk disposal are evaluated. The Shapley values calculation formula, as shown in Eq. 30, is used to calculate the Shapley values for both government departments and social capital partners.

$$\varphi_i(c) = \frac{(n - |S|)! (|S| - 1)!}{n!} (c(S) - c(S \setminus i)) \quad (30)$$

Where: i represents the subject sharing the risk and can take values from 1 to n . n is the total number of risk-sharing subjects, with $n = 2$. $c(S)$ denotes the cost saved when both parties collaborate in risk management. $c(S \setminus i)$ represents the cost saved by the subject sharing the risk. $\varphi_i(c)$ indicates the Shapley values of the risk-sharing subject.

The last step: By modifying the sharing ratio calculated in the Utility Theory with the Shapley values, the desired sharing ratio and sharing amounts of risks for both parties are obtained.

Based on the calculation results of Steps (2) and (3), the revised risk-sharing amounts and sharing ratios are obtained with Eqs 31, 32:

$$\varphi_i(c) = \varphi_i(c) + (K_i - 1/n) \times c(S \setminus i) \quad (31)$$

$$K_i = \varphi_i(c) / \sum_{i=1}^n \varphi_i(c) \quad (32)$$

In the above equations, i represents the risk-sharing subject and can take values of $i = 1, 2, \dots, n$; n represents the number of risk-sharing subjects; K_i is the risk-sharing ratio of the risk-sharing party i calculated by the Utility Theory; K_i is the risk-sharing ratio of the risk-sharing party i modified with the Shapley Value; $\varphi_i(c)$ is the Shapley value of the risk-sharing party i ; $\varphi_i(c)$ is the revised risk-sharing amount of the risk-sharing party i ; and $c(S \setminus i)$ is the contribution of risk sharing in the alliance S after excluding i .

5 Example analysis

5.1 Project funding

The Chongqing Fengsheng Reservoir project construction-and-operation company has a registered capital of 10 million yuan. Social investors who have won the bidding hold a 75% stake, while the PPP project implementation agencies hold the rest 25% stake. The construction of the reservoir constitutes a significant portion of the overall investment, with a preliminary estimated budget of 574.74 million yuan. The project construction will be carried out based on the total amount of investment determined through the bidding process.

To support the project, the successful social investors will contribute 114.94 million yuan, which accounts for 20% of the total project investment. This amount will be injected as a one-time cash investment into the project construction-and-operation company, specifically for expenses related to land acquisition, resettlement, and construction costs. Additionally, the central and municipal governments will provide investment subsidies amounting to 459.8 million yuan.

5.2 Risk-sharing ratio calculation

The Chongqing Fengsheng Reservoir project involves the participation of two entities: the government departments and the social capital party. A preliminary risk-sharing scheme for the project is outlined in Table 1. Among the various risks associated with the project, those that can be controlled by both the public and private parties will be managed through individual risk-control methods, with each party independently bearing the losses incurred by such risks.

For risks that require shared responsibility, it is essential to establish a scientifically and reasonably determined risk-sharing ratio. For the present analysis, a typical example of a risk shared by both parties within the sharing scheme is examined and the risk-sharing ratio between the two entities is analyzed.

A scenario is now assumed where a risk occurs, resulting in a loss cost of 12 million yuan for the government department and 20 million yuan for the social capital party, with the total loss cost of the project amounting to 32 million yuan. If the government department handles the risk independently, due to its special position and power in risk management, it would need to spend 20 million yuan while saving 12 million yuan.

On the other hand, if the social capital party handles the risk independently, leveraging its extensive experience and high management efficiency, it would need to spend 12 million yuan while saving 20 million yuan.

Alternatively, if both parties collaborate and jointly address the risk, they would need to spend 10 million yuan while saving 22 million yuan. These figures illustrate the potential costs and savings respectively associated with each party's independent risk-management efforts as well as their collaborative approach.

(1) Calculating the sharing ratio obtained from the Utility Theory

During the risk-control negotiation process, due to the differences in contribution between the two parties, the weighting coefficients for the government department, representing the advantaged party, and the social capital side, representing the disadvantaged party, are 0.75 and 0.25 respectively. Assuming utility function of the risk for the government department is:

$$U_1 = -\frac{1}{580}C_1^2 + V_1 + 1 \quad (33)$$

And assuming utility function of the risk for the social capital side is:

$$U_2 = -\frac{1}{400}C_2^2 + V_2 + 1 \quad (34)$$

Here, U_1 , V_1 , and C_1 represent the utility function, benefits, and risk-bearing cost of the government department, respectively. U_2 , V_2 , and C_2 represent the utility function, benefits, and risk-bearing cost of the social capital party. The values of a are assumed to be 580 and 400 in U_1 and U_2 , respectively.

The actual cost of this risk is C million yuan ($C \geq 0$), and the initial utility values for the government department and social capital side, excluding risk impact, are $U_1^0 = 1$ and $U_2^0 = 1$ respectively.

Based on Eq. 22 in the risk-sharing model, the average expected cost for both parties is 16 million RMB. Assuming the risk-sharing ratio for the government department is denoted as K , by substituting $a_1 = 0.75$ and $a_2 = 0.25$ into Eq. 23 and performing rearrangements. We obtain the following expression:

$$\max_f (U_1, U_2) = \max \{0.75[U_1(K \cdot (1200 - 1600), K \cdot 1600) - 1] + 0.25[U_2((1 - K) \cdot (2000 - 1600), (1 - K) \cdot 1600) - 1]\} \tag{35}$$

By substituting the risk utility function and by setting $\frac{\partial F}{\partial K} = 0$, we can solve for K as follows:

$$\frac{\partial F}{\partial K} = \frac{186200}{29} - \frac{284800K}{29} = 0 \tag{36}$$

Solving this equation yields $K = 0.3462$. Consequently, the risk-sharing ratio for the social capital side is given by $1 - K = 1 - 0.3462 = 0.6538$. Hence, the project's risk will be shared with the government department assuming 34.62% of the risk, while the social capital side will bear 65.38% of the risk.

(2) Calculating the Shapley values

In the calculation of risk sharing in PPP projects, controlling risks in a reasonable manner can lead to a reduction in project losses, which turns into the increase of project returns. The initial Shapley values calculation formula for the Fengsheng Reservoir PPP project in Chongqing is as follows:

$$\varphi_i(c) = \sum_{s \subseteq N} \frac{(n - |S|)! (|S| - 1)!}{n!} \tag{37}$$

Where: $i = 1$ represents the government side; $i = 2$ represents the social capital side; $n = 2$; $c(S) = 22$ million RMB; $c(S \setminus 1) = 12$ million RMB; $v(S \setminus 2) = 20$ million RMB. we can solve for Shapley value as follows:

$$\varphi_1 = \frac{(2 - 2)! (1 - 1)!}{2!} (2000 - 0) + \frac{(2 - 2)! (2 - 1)!}{2!} (2200 - 1000) = 16 \text{ million} \tag{38}$$

$$\varphi_2 = \frac{(2 - 1)! (1 - 1)!}{2!} (1200 - 0) + \frac{(2 - 2)! (2 - 1)!}{2!} (2200 - 2000) = 7 \text{ million} \tag{39}$$

Therefore, the initial Shapley value for the government side of the Fengsheng Reservoir PPP project in Chongqing is 16 million RMB, and the initial Shapley value for the social capital side is 7 million RMB.

(3) Modify the Utility-Theory-based sharing ratio with the Shapley Value

Integration of the Shapley Value and the Utility Theory can provide a more comprehensive approach to determining the risk-sharing ratio. While the Utility Theory considers factors like proportion of capital contribution and bargaining positions, it may not account for the varying abilities of the parties to handle

risks. On the other hand, the Shapley values calculation precisely considers the costs and abilities of each party in managing specific risks, thereby reflecting their differential risk-management capabilities.

By combining the two theories, the risk-sharing ratio can be adjusted. The Shapley Value can help modify the Utility-Theory-based ratio by considering the parties' abilities to handle risks. This approach provides a more nuanced perspective that incorporates both the overall contribution and bargaining positions of the involved parties, as well as their risk-management capabilities.

According to the above Utility-Theory formula (23), it is calculated that the government department of the Chongqing Fengsheng Reservoir PPP project bears the proportion $K_1 = 0.3462$ and the social capital party bears the proportion $K_2 = 1 - K_1 = 1 - 0.3462 = 0.6538$.

From the Shapley value Eq. 30, we calculate that Initial Shapley value $\varphi_1 = 16$ million for the government side and $\varphi_2 = 7$ million for the social capital side.

Calculations are performed according to Eqs 31, 32 to derive the final risk-sharing amount and sharing ratio of each party K' .

$$\varphi_1(c) = 1600 + (0.3462 - 0.5) \times 2200 = 12.6164 \text{ million} \tag{40}$$

$$\varphi_2(c) = 700 + (0.6538 - 0.5) \times 2200 = 9.3836 \text{ million} \tag{41}$$

$$K_1' = \varphi_1(c)' / (\varphi_1(c)' + \varphi_2(c)') = 0.5735 \tag{42}$$

$$K_2' = \varphi_2(c)' / (\varphi_1(c)' + \varphi_2(c)') = 0.4265 \tag{43}$$

The calculation results are shown in Table 3, the government department bears the amount $\varphi_1(c) = 12.6164$ million RMB, the social capital party bears the amount $\varphi_2(c) = 9.3836$ million RMB, the government department bears the risk ratio of 0.5735, and the social capital party bears the ratio of 0.4265.

5.3 Analysis of risk-sharing results

As the expected loss of the government sector to the risk is smaller than the social capital party, the government's contribution ratio in the project is larger than that of the social capital party, the government sector as a management organization often occupies an advantageous position in the negotiation on risk sharing, while the utility theory mainly takes into account the two sides of the capital contribution ratio, negotiation position and other factors, so when a single use of the utility theory of the Chongqing Fengsheng Reservoir PPP project, both sides of the risk-sharing ratio When calculating, the results are often favorable to the government sector. As can be seen in Table 4.3, the proportion of the risk loss borne by the government sector calculated using the utility theory is 34.62%, and the proportion borne by the social capital party is 65.38%, and the proportion of the risk borne by the government sector is much smaller than that borne by the social capital party. Consistent with the previously expected results, this result is unfair to the social capital side, and the use of a single utility theory to calculate the risk-sharing ratio will hinder the enthusiasm of social capital to participate in the quasi-public welfare water conservancy PPP project.

On the contrary, when government departments deal with risks, their processing capacity is often weaker than that of social capital,

and for various reasons, their cost is larger than that of social capital, while Shapley value theory considers more factors such as the cost and ability of both parties to deal with a certain risk. Therefore, when Shapley value theory is used alone to calculate the risk-sharing ratio of both parties in Chongqing Fengsheng Reservoir PPP project, the results are often beneficial to social capital. It can be seen from [Figure 4](#) that the proportion of government departments to bear the risk loss calculated by Shapley value theory is 69.57%, and the proportion of social capital is 30.43%. The proportion of government departments to bear the risk is much higher than that of social capital. Consistent with the previous expected results, this result is unfair to government departments. Using a single Shapley value theory to calculate the risk-sharing ratio will lead to excessive burden on government departments and cannot give full play to the advantages of the PPP model.

However, the introduction of the PPP model for quasi-public welfare water projects can bring the advantages of financial support, professional management, risk sharing, technological innovation and other aspects, which can help promote the smooth implementation and long-term development of the project. Facilitating quasi-public welfare water PPP projects requires full consideration of the responsibilities and interests of the partners to ensure that the cooperation is fair and equitable in order to achieve a win-win situation. A single risk-sharing calculation method using utility theory or Shapley value theory cannot ensure fairness for all parties.

Therefore, in order to improve the enthusiasm of the partners of Chongqing Fengsheng Reservoir PPP project and reduce the overall risk of the project, this study is based on a multi-party perspective, coupling the Shapley value with the utility theory, amending the risk sharing ratio of the utility theory with the Shapley value, and taking into account the differences in the proportion of the parties' contributions, negotiation status, risk disposal ability, etc. The Shapley value is used to modify the risk sharing ratio of utility theory, and the risk sharing calculation is carried out by taking into account the contribution ratio, negotiation status, risk disposal ability difference and other factors of each party. The results of the study show that: when the risk sharing model based on Shapley value and utility theory is used, the proportion of risk borne by the government is 57.35%, which is slightly higher than that of the social capital party (42.65%), and the calculation result is in between the results of the single utility theory or the Shapley value theory. The risk sharing model demonstrates significant advantages over the single method in considering various factors among all parties, which supports the previously proposed hypothesis. The derived risk sharing ratio is shown to be fairer and more reasonable, effectively mitigating the issue of unfairness arising from the use of a single method to calculate results. The model thoroughly accounts for the distinct characteristics and strengths of government departments and social capital parties, leading to a more rational and widely accepted risk-sharing approach. Consequently, it successfully mobilizes the enthusiasm and benefits of all involved parties, thereby reducing overall project risk.

The risk sharing model brings various benefits to quasi-public water resources PPP projects. Firstly, a fair and reasonable risk sharing ratio facilitates the formation of cooperation consensus among PPP project stakeholders. During the initial stages of project collaboration, mutual recognition of the fairness in risk sharing serves as the foundation for establishing cooperation

consensus. Consequently, acceptance of the equitable risk allocation reduces cooperation concerns and promotes successful project implementation.

Secondly, it encourages active participation of both social capital parties and government departments in quasi-public water resources PPP projects. The risk sharing model takes into account the characteristics and advantages of both parties, balancing their interests to derive a more equitable and acceptable risk sharing proportion.

Lastly, the model enhances the success rate of quasi-public water resources PPP projects. The fair and reasonable risk sharing mechanism reduces disputes and dissatisfaction over risk allocation, consequently improving the project's success rate. When all parties are satisfied with and perceive the risk sharing ratio as reasonable, they are more likely to fully commit to project implementation, thus reducing potential risks and uncertainties and increasing the likelihood of project success.

The successful application of this model provides robust support for the sustainable development of quasi-public water resources PPP projects and offers new research ideas and approaches for risk sharing in similar collaborative projects. By promoting active participation of all stakeholders through equitable risk sharing, the model achieves the dual objectives of risk reduction and project success. Hence, this model holds significant applicability value for future projects with similar characteristics.

6 Conclusion

In the context of quasi-public-welfare water conservancy PPP projects, both government departments and social capital parties encounter significant risks during the project implementation phase. This chapter aims to analyze the fundamental risk-sharing principles and evaluate the advantages and disadvantages of various risk-sharing methods. Furthermore, it establishes a risk-sharing model based on the Shapley Value and the Utility Theory, considering factors such as the involved parties' proportion of capital contributions, bargaining positions, and disparities in their risk management capacities. A risk study was conducted using the Chongqing Fengsheng Reservoir PPP project as a case study to analyze the risk sharing scheme. In light of a particular shared risk existed in the project, utility theory was employed to calculate the risk sharing ratio and Shapley values for different stakeholders. The Shapley values were then used to modify the utility theory-based risk sharing ratio. Based on these analyses, the following conclusions were drawn:

First, compared with the traditional cooperation model and risk sharing mechanism dominated by one party, this study proposes a fairer risk sharing mechanism in line with the characteristics of PPP projects for quasi-public welfare water conservancy PPP projects, in which all participants are on equal footing and need to take into account the interests of all parties and risk preferences at the same time.

Secondly, a quasi-public welfare water conservancy PPP project risk sharing model based on Shapley value theory and utility theory is constructed. Compared with the single use of shapley value theory, the government department bears 69.57% of the risk loss, and the single use of utility theory, the government department bears 34.62% of the risk loss. The calculation result of the coupling of Shapley value theory and utility theory is 57.35% of the risk loss borne by the

government department, which is between the results of the single method, and the result is more fair and reasonable. It enriches the risk management methods and theories of quasi-public welfare water conservancy PPP projects.

The research in this paper also has limitations and shortcomings. First of all, the parameter a in the risk utility function is only assumed to be brought into the calculation. In practical engineering applications, it is necessary to fit the actual observed utility data into the risk utility function by constructing mathematical models and optimization algorithms based on empirical research and specific problems. Otherwise, Risk sharing is based on risk identification, due to the lack of experience in quasi-public welfare water conservancy PPP projects, the construction of the risk evaluation index system has not yet been perfected, and the field should be further supplemented and improved in the next research. In addition, the research in this paper is a further optimization of the quantitative methods of risk sharing based on Shapley value theory and utility theory proposed by previous scholars, and the conclusions obtained are only superior to the single use of a particular method. If other new risk sharing methods emerge, it is necessary to conduct a comparative study of risk sharing methods.

Data availability statement

The original contributions presented in the study are included in the article/supplementary material, further inquiries can be directed to the corresponding author.

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Author contributions

DL: methodology, writing original, and resources. LL: review and editing, investigation, and funding acquisition. XT: review and provided advice for analyzing the data and figures. All authors contributed to the article and approved the submitted version.

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Conflict of interest

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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