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Study on coalbed methane flow characteristics based on fractal bifurcation fracture network model

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The geometric structure and distribution of the fracture network significantly impact the coalbed methane flow characteristics. The indoor optical microscope test is utilized to analyze the distribution and structural characteristics of natural fractures in coal. The results indicate that the fracture network in coal consists primarily of irregular bifurcated fractures, but the influence of the bifurcation fracture network's structural characteristics on permeability remains unclear. Therefore, the fracture network geometric structure characteristic parameters are considered in accordance with the fractal theory, and the analytical formula of the bifurcation fracture network permeability is established. Meanwhile, the bifurcation fracture network geometric model with varied structural parameters is reconstructed using the pixel probability decomposition algorithm. Finally, the influence of the key parameters of the reconstructed bifurcation fracture network on the coal seam permeability is analyzed through numerical simulation. The results indicate that the permeability of the bifurcated fracture network increases with the increase of fracture porosity ϕ_f , aperture ratio χ , and proportionality coefficient η , and decreases with the increase of tortuosity fractal dimension DT , bifurcation angle θ , fractal dimension D_f , and bifurcation level n . Among them, fracture porosity and proportionality coefficient have the greatest influence on permeability, followed by tortuosity fractal dimension, aperture ratio.

KEYWORDS

fractal geometry, permeability model, bifurcation fracture, structural characteristic, coalbed methane

1 Introduction

Coal bed methane (CBM) is a clean, environmentally friendly high-quality energy source. Efficient exploitation of CBM profoundly impacts the healthy and sustainable development of the world's resources (Li and Jin 2019; Li et al., 2022). The productivity of underground oil and gas reservoirs is mainly determined by a large number of their randomly distributed fractures, and their microstructure and distribution characteristics significantly impact the fluid flow characteristics (Xu et al., 2006; Liu et al., 2015; Wang et al., 2018). At present, researchers still have insufficient understanding, characterization, and model establishment of the structural characteristics of the bifurcation fracture network. It is crucial to study the structural characteristics of the bifurcation fracture networks to accurately construct the permeability model of coal reservoirs and improve the low permeability of coalbed methane.

Studies have shown that bifurcation is the key characteristic of fractures in porous media. Naturally, fracture networks are often interconnected to form multistage bifurcation fracture

networks (Xu et al., 2016). This paper believes that coal, as a typical fractured porous medium, has similar topological structures in its fracture network. Bobaru and Zhang (2015) analyzed the causes of fracture bifurcation and its problem in homogeneous, isotropic materials using the near-dynamic brittle fracture model, such as fracture morphology, bifurcation angle, and branch expansion velocity. The authors pointed out that bifurcation is the normal state of fracture and analyzed the structural characteristics of the bifurcation fracture but did not study the flow characteristics of the fluid in fracture. Through the CT scanning test, Fu et al. (2020) observed that coal samples contained many fractures of different scales, and their structure and distribution characteristics were difficult to describe. Therefore, it was necessary to simplify the structure characteristics reasonably. For example, Jin et al. (2020) and Zhao et al. (2020) used fractal theory to describe the complex microstructure of porous media and eases the modeling of multi-scale microstructures significantly. In addition, some scholars have used a tree-like bifurcation fracture network to characterize the complex fracture network in underground reservoirs. Zhang et al. (2021) compared the fracture network in porous media to a tree-like bifurcation fracture, established the expression of effective thermal conductivity according to the fractal characteristics of pores' diameter and fracture size, and analyzed the influence of microstructure parameters of porous matrix and bifurcation fracture networks on thermal conductivity. Zhu et al. (2021) used tree-like bifurcation fracture networks to characterize fracturing fractures, established a simulation model of tree-like bifurcation fracture networks, and discussed the influence of fracture opening, bifurcation angle, and tortuosity degree on the imbibition of the bifurcation fracture network.

Much work on the structural characteristics of bifurcated fracture networks has been done through theoretical analysis and numerical simulation. Establishing an effective fracture network permeability model is critical to revealing the fluid flow characteristics (Jafari and Babadagli 2016). Miao et al. (2015a), Miao et al. (2015b) takes into account the microstructure parameters of fractures, such as opening, length, dip angle, density, etc., and establishes the fractal permeability model of dual-porosity media. Jin et al. (2017) believe that the fractal behavior of pore networks in natural tight reservoirs has a significant influence on the transport property of the reservoir. They study the relationship between the four fractal dimensions and establish the mathematics to describe the flow of fluid in fractal tight porous media. Li et al. (2021) took into account the influence mechanism of the distribution characteristics of coal surface fracture structure on coal permeability and developed a fractal permeability model for coal and rock. Shi et al. (2022) proposed a fractal permeability model for fractured rocks that comprehensively considered the geometric fracture characteristics and the fluid transport mechanism. The permeability of fractured rock with different geometric parameters and roughness is analyzed. However, their study neglected the connections between fractures and did not consider the influence of the bifurcation structures of the fracture network on fluid seepage. Xu et al. (2008) utilized the dual-domain model to simulate bifurcated fractured porous media, established the fractal analytical expression of permeability, and demonstrated that the fractal dimension of pore distribution has a significant impact on seepage characteristics. Miao et al. (2018) takes into account the aperture ratio, length ratio, branch number, and bifurcation level of fractures and proposes an analytical expression for dimensionless permeability of damaged tree bifurcation

networks. Wang and Cheng (2020) established a fractal permeability model of a two-dimensional curved tree fracture network. However, the relationship between the structural parameters of the bifurcation fracture network and permeability is unclear in their study.

In conclusion, the structural parameters of bifurcation fracture networks, the establishment of permeability models, and the quantitative relationship between the structural parameters and permeability need to be studied. Therefore, based on the fractal geometry theory, the geometric structure characteristics of bifurcation fracture networks are considered, and the analytical expression of permeability of bifurcation fracture networks is derived. Afterward, the geometric model of bifurcation fracture networks with different structural characteristics was reconstructed by the pixel probability decomposition algorithm, and the model was imported into COMSOL Multiphysics numerical simulation software. Lastly, the influence of vital geometric parameters on the bifurcation fracture network's permeability is analyzed.

2 Structural characteristics of bifurcation fracture networks

2.1 Sample preparation and experiment introduction

In order to observe and analyze the geometric structure and distribution of the primary fracture networks in coal rocks, several samples were collected from Xichenzhuang Coal Mine, Jincheng city, Shanxi Province, China (Supplementary Figure S1A). Some were processed into 50 mm × 50 mm specimens, and their macroscopic natural fracture networks were observed by stereomicroscope (Supplementary Figure S1B). Due to the wide observation range and large acceptable size of the specimens, many fracture networks can be observed by stereomicroscopes. However, the magnification is limited ($\leq 50\times$), and most microscopic fracture structures cannot be observed.

In order to more accurately observe the fracture networks' morphology, the rest were processed into irregular samples of about 8 mm for scanning electron microscopy (SEM) observation (Supplementary Figure S1C) to obtain the microstructure of the fracture networks. The scanning electron microscope's accuracy ($\leq 25,000,00\times$) is much greater than that of the stereographic one, but the observation range is small, and the fracture's structure is limited. Therefore, combining the two can reflect the structural characteristics of coal and rock fracture networks more intuitively and clearly.

2.2 Structural characteristics of the fracture networks

Combining microscope (Supplementary Figure S2) and SEM (Supplementary Figure S2) results, it is observed that the natural fracture network in coal consists primarily of a large number of irregularly curved bifurcate fractures, followed by a limited number of single fractures with poor connectivity. Compared to bifurcate fractures, single ones' contribution to the seepage is minimal; therefore, this paper mainly focuses on the seepage characteristics in the coal-rock bifurcation fracture networks.

2.2.1 Bifurcation and curved characteristics of fracture networks

Studies have shown that fracture networks' tortuosity and bifurcation properties significantly affect the reservoirs' permeability (Yu and Cheng 2002; Xu et al., 2016). The fracture networks' structural features and bifurcation are depicted in Supplementary Figure S2C and Supplementary Figure S3B. In the fracture network, the main fracture usually extends into two branches, and the branches will continue to bifurcate to form lower-level bifurcation fractures. The level of the branch is denoted by k , and the maximum order of the branch is denoted by n . In this study, the angle between two branches of the same level is defined as the bifurcation angle, denoted by θ .

In addition, the fractures' curved nature is usually characterized by tortuosity τ . As depicted in Supplementary Figure S4A, L_t is the bending length of the fracture, L_0 is the linear length of the fracture, and tortuosity is equal to the ratio of the two.

In the figure, a is the fracture opening, h is the fracture depth, and the relationship between L_t and L_0 can be expressed as follows (Yu and Li 2004):

$$L_t(\alpha) = a^{1-D_T} L_0^{D_T} \tag{1}$$

Where D_T is the fractal dimension characterizing the fractures' curved nature, the curvier the fracture, the greater is τ ; that is, the greater the value of D_T . When $D_T=1$, the fracture is linear, with $1 < D_T < 2$ in two-dimensional space and $1 < D_T < 3$ in three-dimensional space.

From equation (1), D_T can be calculated by the following equation (Yu and Li, 2004; Yu 2008):

$$D_T = 1 + \frac{\ln \tau_{av}}{\ln(L_0/a_{av})} \tag{2}$$

Where τ_{av} is the average tortuosity of fractures; a_{av} is the average opening of the fracture.

2.2.2 Characteristic relationship between the opening and the length of bifurcation fractures

According to Murray's law inference, in fracture networks with perfect fluid flow, all branched fractures' cross-sectional sum should be greater than the primary fracture, and the main fracture radius of the cubic must approximately be equal to the sum of the cubic bifurcation fractures' radius. Therefore, fractures' aperture will decrease during the bifurcation process illustrated in Supplementary Figure S2, Supplementary Figure S3, and observations. At the same time, through plenty of observations and analysis, it is found that the length of all the fractures in the bifurcation fracture network is always random, and the length of the bifurcation fractures is considered to meet the fractal distribution. In order to deeply analyze the structural characteristics of the bifurcated fracture networks, the length ratio β , the opening ratio χ , and the proportion coefficient η are introduced to represent the relationship between the opening and the length of the bifurcated fracture network:

$$\beta = \frac{l_{k+1}}{l_k} \tag{3}$$

$$\chi = \frac{a_{k+1}}{a_k} \tag{4}$$

$$\eta = \frac{a_k}{l_k} \tag{5}$$

Where β is the length ratio, expressed as the ratio between the length of the secondary fracture and the upper one; χ is the opening ratio, expressed as the ratio of the opening degree of the secondary fracture to the upper one's, and $0 < \chi < 1$; l_0 and a_0 are the length and the opening of the main fractures, respectively. L_k and a_k ($k=0,1,..,n$) are the length and the opening of the k -level bifurcation fracture, respectively, and the length and opening ratio in the bifurcation fracture networks are considered unchanged (Xu and Yu 2001). η is the proportionality coefficient. For the fracture network satisfying the self-similar distribution, the ratio between the opening and the length of a single fracture is usually between 0.1 and 0.001 (Klimczak et al., 2010; Luo et al., 2021).

3 Fractal permeability model of bifurcation fracture networks

3.1 Fractal characteristics of bifurcation fracture networks

A large number of random fracture networks are often analyzed by fractal geometry, and fractal dimension is often used as a quantitative parameter to characterize the distribution complexity of fracture networks (Miao et al., 2015a). In this study, both the length and opening of the bifurcation fracture network meet the fractal scaling law (Yu and Li 2001; Wu et al., 2021). The total number of bifurcation fracture networks whose main fracture length is greater than or equal to l_0 satisfies the following fractal scaling relation (Liu et al., 2016; Xu et al., 2016):

$$N(L \geq l_0) = (l_{0max}/l_0)^{D_f} \tag{6}$$

Where N is the number of bifurcation fracture networks; l_0 is the length of the main fracture; l_{0max} is the maximum length of the main fracture; D_f is the fractal dimension representing the distribution of fractures, which is usually obtained by the box-counting method, $0 < D_f < 2$ in two-dimensional space and $0 < D_f < 3$ in three-dimensional space. In addition, Xu et al. (2016) proved that the fractal dimension of fracture length is equal to the box dimension D_f .

In general, Equation (6) is continuously differentiable. By differentiating l_0 , we can obtain:

$$-dN(l_0) = D_f l_{0max}^{D_f} l_0^{-(D_f+1)} dl_0 \tag{7}$$

3.2 The establishment of the fractal permeability model

Based on the fractal geometry theory and considering the structural characteristics of the fracture network, this paper establishes the permeability model of bifurcation fracture networks. During this process, the adjacent bifurcation fractures' impact on one another is ignored, and the energy loss at the bifurcation is not considered.

The fluid flow in a single fracture is usually described by the cubic law (Miao et al., 2015a; Miao et al., 2015b):

$$q = \frac{a^3 h \Delta P}{12 \mu L_0} \tag{8}$$

where a is the fracture's opening; h is its depth; μ is the hydrodynamic viscosity coefficient; and ΔP is the pressure difference between the two ends of the fracture.

Since fractures are curved in natural conditions, Wang and Cheng (2020) obtained the flow rate in a single curved fracture considering the curved characteristics and combined it with Newton's law of viscosity, satisfying the fractal distribution as follows:

$$q = \int_{(a/2)}^{(a/2)} v dA = 2 \int_0^{(a/2)} v h d\alpha = \frac{\Delta P a^{2+D_T} h}{2^{1+D_T} \mu L_0^{D_T} (2 + D_T)} \quad (9)$$

According to Equation (9), the flow rate of a single k-level fracture is:

$$q_k = \left(\frac{1}{2}\right)^k q = \frac{\Delta P_k a_k^{2+D_T} h}{2^{1+D_T} \mu_k^{D_T} (2 + D_T)} \quad (10)$$

Therefore, the pressure difference between the two ends of the k-level fracture is:

$$\Delta P_k = \frac{q_k 2^{1+D_T} \mu_k^{D_T} (2 + D_T)}{a_k^{2+D_T} h} \quad (11)$$

Ignoring the pressure loss at the bifurcation, the total pressure difference of the whole bifurcation fracture is:

$$\Delta P = \sum_{k=0}^n \Delta P_k \quad (12)$$

By substituting Equations 3 and 4 and Equation 5 into Equations 11 and 12, we can obtain:

$$\Delta P = \frac{2^{1+D_T} \mu (2 + D_T) q}{h a_0^2 \eta^{D_T}} \frac{1 - \left(\frac{\beta^{D_T}}{2\chi^{2+D_T}}\right)^{n+1}}{1 - \frac{\beta^{D_T}}{2\chi^{2+D_T}}} \quad (13)$$

Therefore, the total flow rate of a bifurcation fracture is):

$$q_Y = \frac{\Delta P h a_0^2 \eta^{D_T}}{2^{1+D_T} \mu (2 + D_T)} \frac{1 - \frac{\beta^{D_T}}{2\chi^{2+D_T}}}{1 - \left(\frac{\beta^{D_T}}{2\chi^{2+D_T}}\right)^{n+1}} \quad (14)$$

Assuming that the fracture area in each section of the study object is equal, then:

$$\begin{aligned} A_f &= \int_{l_0 \min}^{l_0 \max} l_0 a_0 dN(l_0) \\ &= - \int_{l_0 \max}^{l_0 \min} h a_0 dN(l_0) \\ &= \frac{\eta D_f}{2 - D_f} l_0 \max^{2D_f - 2} \left[1 - \left(\frac{l_0 \min}{l_0 \max}\right)^{2 - D_f} \right] \end{aligned} \quad (15)$$

Therefore,

$$A = A_f / \varphi_f = \frac{\eta D_f}{\varphi_f (2 - D_f)} l_0 \max^{2D_f - 2} \left[1 - \left(\frac{l_0 \min}{l_0 \max}\right)^{2 - D_f} \right] \quad (16)$$

where A_f is the fracture area; A is the total area of the study section; and φ_f is the fracture porosity, which is numerically equal to the ratio of A_f to A .

After integrating equation (7) and substituting equation (15), the total flow of the bifurcation fracture network is obtained as follows:

$$\begin{aligned} Q &= \int_{l_0 \min}^{l_0 \max} q_Y dN(l_0) \\ &= \frac{\eta^{2-D_f} \Delta P D_f l_0 \max^{2D_f - 3}}{2^{1+D_T} \mu (2 + D_T) (3 - D_f)} \frac{1 - \frac{\beta^{D_T}}{2\chi^{2+D_T}}}{1 - \left(\frac{\beta^{D_T}}{2\chi^{2+D_T}}\right)^{n+1}} \left[1 - \left(\frac{l_0 \min}{l_0 \max}\right)^{3 - D_f} \right] \end{aligned} \quad (17)$$

Since $l_0 \min / l_0 \max < 10^{-2}$ and $0 < D_f < 2$ stand in two-dimensional space $(l_0 \min / l_0 \max)^{3 - D_f} \approx 0$, Eq. 17 can be simplified as follows.

$$Q = \frac{\eta^{2-D_f} \Delta P D_f l_0 \max^{2D_f - 3}}{2^{1+D_T} \mu (2 + D_T) (3 - D_f)} \frac{1 - \frac{\beta^{D_T}}{2\chi^{2+D_T}}}{1 - \left(\frac{\beta^{D_T}}{2\chi^{2+D_T}}\right)^{n+1}} \quad (18)$$

According to the geometric relationship of bifurcation fractures, the equivalent linear length L of the curved bifurcation fracture networks along the flow direction (horizontal) is:

$$L = l_0 \left[1 + \frac{\sin \theta}{\sqrt{2(1 - \cos \theta)}} \frac{\beta(1 - \beta^n)}{1 - \beta} \right] \quad (19)$$

Therefore, by substituting equations (16) and (18) and (19) into Darcy's law ($k = (Q\mu L) / (A\Delta P)$) for calculation, the analytical expression of bifurcation fracture networks' permeability satisfying the fractal distribution can be obtained as follows:

$$K_f = \frac{\eta^{1+D_T} \varphi_f (2 - D_f)}{2^{1+D_T} (2 + D_T) (3 - D_f)} \frac{1 - \frac{\beta^{D_T}}{2\chi^{2+D_T}}}{1 - \left(\frac{\beta^{D_T}}{2\chi^{2+D_T}}\right)^{n+1}} \left[1 + \frac{\sin \theta}{\sqrt{2[1 - \cos \theta]}} \frac{\beta(1 - \beta^n)}{1 - \beta} \right] \quad (20)$$

3.3 Validity verification of the model

In order to verify the reliability of the fractal permeability model of bifurcation fracture networks in this study, the data in Supplementary Table S1 are put into Expression (20) and compared with the fractal permeability model of fractured porous media proposed by Miao and Yang (2015a). Supplementary Figure S5 shows the comparison results.

In this research, bifurcations and flexural characteristics are accounted for in Equation (20), and the results indicate that Miao et al. overestimated permeability. The reason is that the flexural and bifurcation properties of fractures hinder the seepage of coal seams, which is consistent with the actual situation. The fractal permeability model presented in this paper is therefore more effective.

3.4 Reconstruction of the geometric model of bifurcation fracture network

According to Equation (20), the permeability of the bifurcation fracture network is affected by the geometrical structural characteristics of the fracture, such as the fractal dimension of tortuosity, length and opening ratio, proportionality coefficient, fracture porosity, fractal dimension, bifurcation angle, and bifurcation level. In order to quantitatively describe their relationship with CBM flow characteristics, the complex fracture networks in natural coal seams were simplified for subsequent

modeling analysis. In the past, researchers often used CT imaging to reconstruct the network model (Song et al., 2020; Song et al., 2021; Song et al., 2022) present a novel pore scale modeling on dissociation and transportation mechanism of MH in porous sediments. But the reconstructed model was too complex and not targeted. Therefore, based on the basic theory of the two-dimensional pixel spatial probability decomposition method, this paper used a two-dimensional random fracture network generation program to generate multiple fracture networks with different tortuosity (Xia et al., 2021a; Wu et al., 2021) and rebuilt multiple bifurcation fracture network models with different characteristic parameters.

Multiple fractures with specific statistical characteristics are generated in the two-dimensional pixel space. The growth probability values of the eight growth directions of the fractures to be determined and the tortuosity of the generated random fracture network are shown in Supplementary Table S2. The relevant geometric parameters of the reconstructed bifurcation fracture network model are shown in Supplementary Table S3.

4 Numerical modeling

4.1 Basic assumptions

The flow situation of underground coalbed methane is very complex. In order to make the model converge better, the following assumptions should be made first (Wang et al., 2016; Xia et al., 2021a):

The coal reservoir is simplified as a dual pore medium consisting of matrix and fracture network systems. The transport of coalbed methane mainly occurs in bifurcation fracture networks.

- 1) The flow of coalbed methane in coal seams meets Darcy's law (single-phase flow);
- 2) The fluid seepage direction is parallel to the main fracture of the bifurcation fracture;
- 3) The interaction between fluid flow in fractures and fluid flow in matrix is not taken into account;
- 4) The seepage field is simulated regardless of the deformation of the coal bed and the surrounding mechanical field.

4.2 Fluid governing equations and boundary conditions

The continuity equation of CBM flow in coal seams can be expressed by Darcy's law combined with mass conservation (Xia et al., 2021b; Ren et al., 2021):

$$\nabla \cdot (\rho_g \mathbf{u}) = 0, \rho_g = \frac{M_g}{RT} \quad (21)$$

$$\mathbf{u} = -\frac{K}{\mu} \nabla P, \nabla P = \frac{\Delta P}{L} \quad (22)$$

Where ρ_g is the gas density, Kg/m^3 ; u is the flow rate, m/s ; T is the coal reservoir's temperature, $^\circ\text{C}$; M_g is the gas's molar mass, g/mol ; R is the gas constant.

Related parameters of fluid and matrix in the simulation are shown in Supplementary Table S4. The model's boundary conditions are shown in Supplementary Figure S6.

4.3 Parameter analysis and discussion

The reconstructed bifurcation fracture network model was imported into Comsol Multiphysics numerical simulation software, and Equation (20) was solved according to the boundary conditions in Supplementary Figure S6 and relevant parameters in Supplementary Table S4. Afterward, the velocity distribution cloud diagram of coalbed methane in the bifurcation fracture network with different structural characteristic parameters was obtained (Supplementary Figure S7). Then, the flow velocity at the outlet was counted. Subsequently, the permeability value was calculated using Darcy's law.

The simulation results show that the gas velocity in the bifurcation fracture is significantly higher than that of in the matrix, the coalbed methane migration mainly occurs in the bifurcation fracture networks, and the matrix contributes little to the permeability of the coal reservoirs, which is consistent with the previous findings of (Yu and Cheng, 2002; Yu, 2008).

In addition, due to the presence of bifurcation structures in the fracture the velocity of the fluid in the fracture decreases gradually with the occurrence of bifurcation. The gas velocity in the main fracture is significantly higher than that of the secondary fracture, and the higher the bifurcation level is, the smaller the fluid velocity. Therefore, the bifurcation of the fracture has an inhibitory effect on the gas seepage.

4.3.1 Relationship between tortuosity fractal dimension D_T and bifurcation fracture network's permeability K_f

In order to obtain the relationship between the fracture's tortuosity and permeability, other geometric structural parameters were fixed, and the fractal dimension's tortuosity's value was increased from 1.0012 to 1.0548. The structural parameters of 12 bifurcated fracture network models with different tortuosity established in the study are shown in Supplementary Table S3 (Case. I). Supplementary Figure S7A shows the velocity distribution cloud of the bifurcation fracture network model with different tortuosity.

Supplementary Figure S8 compares and analyzes the numerical simulation and the theoretical prediction results of the bifurcated fracture network's permeability as the fractal dimension of tortuosity increases. The results show that the larger the fractal dimension of tortuosity, the smaller the permeability is. The reason is that when the fluid flows in the fracture, the curvier (rough) the fracture wall, the greater the flow resistance, and the longer the fluid flow path is. In addition, the collision between the fluid particles and the convex part of the fracture wall increases the fluid's energy loss, and the seepage velocity of the fluid decreases. This conclusion is consistent with previous research results (e.g., Yu and Cheng 2002; Liu et al., 2015).

4.3.2 Relationship between fracture porosity ϕ_f , bifurcation level n , and the bifurcation fracture network's permeability K_f

As observed in Supplementary Figure S9, when the bifurcation level $n = 1, 3, \text{ and } 5$, the permeability and fracture porosity both exhibit a stable linear growth relationship, and the difference between the permeability values at the same fracture porosity is very small. Therefore, the permeability of the bifurcation fracture network is almost not affected by the bifurcation level.

In order to determine the quantitative relationship between the fracture porosity and its network permeability, it was increased from

0.0078 to 0.0447, the bifurcation level was increased from 1 to 5, and other structural parameters were not altered. The geometric structural parameters of 10 groups of bifurcation fracture network models with different fracture porosities are shown in Table 3 (Case II). The simulation results are shown in [Supplementary Figure S7B](#).

[Supplementary Figure S7B](#) shows that fracture porosity positively correlates with permeability. For example, although the maximum fracture porosity is only increased by less than 0.04, the maximum seepage velocity of the CBM in the bifurcated fracture network is increased from 7.14×10^{-4} (m/s) to 1.15×10^{-2} (m/s) indicating that the fracture porosity plays a vital role in the fracture network's permeability. The study area's fracture porosity, defined as the surface area of the fracture network as a percentage of its total surface area, indicates that the permeability of the fracture network increases with the fracture porosity, i.e., the number of fractures that serve as the main channel for fluid flow. Our results and ([Miao et al., 2015a](#); [Miao et al., 2015b](#)) were consistent with ([Xu et al., 2016](#)) et al.'s findings.

[Supplementary Figure S10](#) shows the relationship between the fracture porosity φ_f and the fractal dimension D_f of the reconstructed bifurcation fracture network model. The fractal dimension increases exponentially with increasing fracture porosity. The analysis above shows that the fracture porosity of the fracture network has a significant positive impact on the permeability. However, according to Equation (20), the bifurcation fracture network's permeability negatively correlates with the fractal dimension. Therefore, it can be inferred that although the bifurcation fracture network's fractal dimension increases indirectly with increasing the fracture porosity, the impact of the fracture porosity on permeability is much greater than that of the fractal dimension on permeability.

4.1.3 Relationship between bifurcation angle, θ , and bifurcation fracture network's permeability, Kf

[Supplementary Figure S7C](#) demonstrates that the seepage velocity within the main fracture appears to be stable. With the occurrence of bifurcation, however, the seepage velocity in the low-level fracture gradually decreases, and the greater the bifurcation angle, the slower the seepage velocity in the secondary fracture at the same level. Additionally, it has been demonstrated that the larger the bifurcation level, the more apparent the change. This is because the angle between the low-level fracture and the main fracture (the direction of fluid flow) becomes closer to vertical as the bifurcation angle increases. Consequently, the fluid flowing through the fracture experiences greater resistance, the local losses at the bifurcation increase, and the fluid seepage velocity decreases. [Supplementary Figure S11](#) compares the simulation results with the theoretically predicted ones of permeability at different bifurcation angles. It can be seen from the figure that the fitting curve of the simulation results is consistent with the theoretically predicted one.

4.1.4 Relationship between the opening ratio χ , the proportion coefficient η . And the permeability Kf of the bifurcation fracture network

In order to determine the influence of the opening ratio and the bifurcation fracture network's proportionality coefficient on permeability, the permeability values with the opening ratio increased from 0.55 to 1.00, and other parameters fixed are obtained when the proportionality coefficient is 0.4, 0.5 and 0.0625,

respectively. The geometric structural parameters of the bifurcation fracture network model in the simulation are shown in [Supplementary Table S3](#) (Case. VI), and the simulation results are shown in [Supplementary Figure S7D](#). The result show that as the opening ratio increases, bifurcations in the fracture network fluid flow rate increase. The reason is that the opening ratio indirectly increases the fluid seepage channel.

[Supplementary Figure S12](#) provides a quantitative analysis of the relationship among the three parameters. Under different proportional coefficients, the permeability and the bifurcation fracture network's opening ratio have a non-linear growth relationship with the same growth trend. In addition, the longitudinal comparison reveals that, under similar opening ratio conditions, the greater the proportional coefficients, the greater the permeability of the bifurcation fracture network.

But the influence of the proportionality coefficient is greater than that of the openness ratio. For example, when the scaling coefficient η increases from 0.04 to 0.05, the fracture network's permeability increases from $2.886 \times 10^{-13} \text{ m}^2$ to $1.0274 \times 10^{-12} \text{ m}^2$ when the opening ratio is at $\chi=0.8$. However, when the opening ratio χ is increased by 0.1 with the scaling coefficient η remaining unchanged, the permeability only increases by about $3.5 \times 10^{-13} \text{ m}^2$.

5 Conclusion

- 1) Based on the fractal geometry theory combined with the bifurcation fracture network's structural parameters, an analytical expression of the bifurcation fracture network's permeability was established. The pixel probability algorithm and mathematical software were then used to reconstruct multiple sets of bifurcation fracture geometric models with distinct structural parameters.
- 2) The permeability analytical formula (20) was solved using COMSOL Multiphysics numerical simulation software. The simulated result was nearly consistent with the theoretically predicted trend.
- 3) The fracture flexural and bifurcation properties inhibit the flow of the coalbed methane in the bifurcation fracture network to different degrees. That is, the permeability of the fracture network decreases with the increase of the tortuosity fractal dimension D_T , bifurcation angle θ , and bifurcation level n . The fracture degrees and openings of bifurcation fractures play a prominent role in promoting the coalbed methane's transport. That is, the permeability of bifurcation fracture networks increases with the increase of the fracture porosity φ_f , pore size ratio χ , and proportion coefficient η .
- 4) The intricate geometric structures of fracture networks significantly influence coalbed methane migrations. According to sensitivity analysis, the degree to which structural parameters of a bifurcation fracture network affect permeability is as follows: $\varphi_f > \eta > D_T > \chi > D_f > \theta > n$.

Data availability statement

The original contributions presented in the study are included in the article/[Supplementary Material](#), further inquiries can be directed to the corresponding author.

Author contributions

SJ carried out the experimental and manuscript writing work. BX, JP, TZ offered theoretical guidance and carried out the modification of the manuscript.

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Conflict of interest

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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Supplementary material

The Supplementary Material for this article can be found online at: <https://www.frontiersin.org/articles/10.3389/feart.2022.1108786/full#supplementary-material>

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