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# Upper bound analysis of the anti-seismic stability of slopes considering the effect of the intermediate principal stress

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Considering the intermediate principal stress effect of soils, the unified strength theory was applied to correct the shear strength index of soils. Based on a mechanical model of the anti-seismic stability of slopes, the analytical formula of the stability coefficient was deduced by the upper bound method and the quasi-static method. Then the optimal solution of the stability coefficient was figured out via the Matlab software. The result shows that, if the intermediate principal stress is ignored, the shear strength of soils would obviously be underestimated; this is also the case with the slope stability coefficient, the relative error of which could reach 39.87%. The horizontal and vertical seismic forces significantly affect slope stability. When the horizontal seismic force is considered, the slope stability coefficient  $N_s$  is reduced by 79.82%. Similarly, if the seismic effect is not neglected, the stability of the slope would be seriously overestimated. Slope cutting can significantly improve slope stability. When the slope angle is reduced from 90° to 50°, the stability factor increases by 279.82%. The suitable design angle of the slope is between 50° and 60° without taking into account additional elements like groundwater level and stratum structure.

#### KEYWORDS

unified strength theory, intermediate principal stress, upper bound analysis, seismic force, stability coefficient

# **1** Introduction

Earthquakes not only demolish surface structures, but also cause serious disasters like landslides which threaten human existence and the safety of property. For instance, the 5.12 Wenchuan earthquake in 2008 caused a large-scale landslide that left 20,000 people who died or disappeared. In the "4-14 Yushu earthquake in 2010," landslides and debris flows directly hit residential areas and killed more than 3,000 people. In 2015, a 7.8 magnitude earthquake hit Afghanistan, which caused a landslide that killed about 180 people. A 7 magnitude earthquake occurred in Luzon Island, Philippines in 2022 and triggered landslides that killed four people and wounded 60. As can be seen, the influence of earthquakes on slope stability cannot be overlooked in earthquake-prone locations. Therefore, the study of slope seismic stability has significant engineering value and research significance.

Nowadays there are numerous ways to study the seismic stability of slopes, including theoretical analysis, simulation test, and numerical analysis. Qin and Chian (2020) assessed the seismic stability of a reinforced slope using the quasi-dynamic approach. The seismic stability of an underwater slope was assessed by Yang et al. (2021) using a modified version of the Newmark technique. Srilatha et al. (2013) conducted a comparative study of unreinforced and reinforced soil slopes by a shaking table test. With the development of computer technology, numerical analysis has become increasingly significant in the seismic stability of slopes. Zhang Z. L. et al. (2017) tested the dynamic behavior and failure mechanism of a slope during an earthquake using the FLAC 2D numerical simulation program. However, the above method requires a significant amount of labor and a lengthy time period, which is not conducive to its application in engineering. Since Terzaghi first applied the quasi-static method to the geotechnical engineering field, the method has been widely used in engineering and has been incorporated into corresponding codes and guidance manuals. Therefore, using the quasi-static method is reliable for analysis and calculation. Michalowski and Martel (2011) introduced the seismic force simplified by the quasi-static method into the calculation of the slope, and obtained the upper limit solution of the stability factor. Nian et al. (2016) analyzed the seismic stability of a slope reinforced with row piles via the quasi-static method. The analytical expression of the stability factor was derived based on the combination of limit analysis and strength reduction. Moreover, the influence of the horizontal seismic coefficient on the horizontal stability force and optimal pile position in a seismic area was studied. Su et al. (2018) utilized the upper bound method of limit analysis and analyzed the seismic stability of a slope considering the influence of bolts. Wang et al. (2019) analyzed the seismic stability of unsaturated soil slopes using a semi-analytical method. Zhang et al. (2021a) introduced the quasi-static method into the calculation of slope stability, calculated the optimal solution of the stability factor of a saturated soft soil slope based on the upper limit method of limit analysis, and analyzed the stability factor of the saturated soft clay slope affected by the horizontal and vertical seismic forces. Xu and Yang (2018) analyzed the seismic stability of heterogeneous slopes using the quasi-static method, and studied the effects of reinforcement, slope angle, seismic force, and soil properties on slope stability.

The aforementioned scholars studied slope stability under seismic activity in great detail, but the majority of the strength criteria they employed were based on the M-C criterion, which is only considered for major stress and minor principal stress, and ignores the impact of intermediate principal stress on the soil. Yu et al. (1983) proposed the theory of twin shear stress strength based on the twin shear stress yield criterion. Although the theory considered the influence of intermediate principal stress, it is applicable to a single material. Based on this, Yu (2002) proposed the unified strength theory based on the unified yield criterion, which not only takes the influence of large and small principal stresses into account, but also considers the influence of intermediate principal stresses on the soil. It includes the M-C strength theory, weighted twin shear strength theory, maximum tensile strain strength theory, and other strength theories, which can be basically applied to all kinds of geotechnical materials. Thus, it is recognized by scholars in the geotechnical engineering field. Deng et al. (2019) studied the influence of unified strength theory on the failure characteristics and bearing capacity of strip foundations, and determined the appropriate intermediate principal stress influence parameters via model tests and numerical simulations. Zhao et al. (2016) obtained the unified solution of Rankine Earth pressure based on the twin shear unified strength theory by considering the intermediate principal stress, which proved that the potential strength of soil could be better used with consideration of the intermediate principal stress effect.

In fact, the shear strength of soil will be underestimated without considering the Mohr-Coulomb strength of the intermediate principal stress effect, and the stability of a slope cannot be correctly evaluated. Based on the former study, this work takes the intermediate principal stress effect on slope stability under seismic forces into account, and the unified strength theory is introduced into the slope seismic stability analysis. The seismic stability coefficient is obtained using the upper limit method of plastic theoretical limit analysis and the quasi-static method, and the intermediate principal stress influences on parameter b, slope angle, and seismic stability coefficient are sufficiently studied. Therefore, using unified strength theory in slope stability analysis may fully use the potential of soil shear strength, properly evaluate the stability of rock and soil slopes, and provide a reference for engineering practice.

# 2 Theory and method

### 2.1 Unified strength theory

The unified strength theory is based on the twin shear theory which includes many strength theories. The unified strength theory parameter b is between 0 and 1, which is the convex theory. Parameter b also reflects the degree of influence of intermediate principal stress on material failure, so it is also known as the influence parameter of intermediate principal stress. The unified strength theory expressed by the initial cohesion of soil and the initial angle of internal friction is as follows (Yu, 2011):

When 
$$\sigma_2 \leq \frac{\sigma_1 + \sigma_3}{2} - \frac{\sigma_1 - \sigma_3}{2} \sin \varphi_0$$
,  
 $F_1 = \sigma_1 (1 - \sin \varphi_0) - \frac{1}{1 + b} (b\sigma_2 + \sigma_3) (1 + \sin \varphi_0) = 2c_0 \cos \varphi_0$ 
(1)

When  $\sigma_2 \ge \frac{\sigma_1 + \sigma_3}{2} - \frac{\sigma_1 - \sigma_3}{2} \sin \varphi_0$ ,  $F_2 = \frac{1}{1+b} \left(\sigma_1 + b\sigma_2\right) \left(1 - \sin \varphi_0\right) - \sigma_3 \left(1 + \sin \varphi_0\right) = 2c_0 \cos \varphi_0$ (2)

The intermediate principal stress for soil entering the plastic is stated as

$$\sigma_2 = \frac{\sigma_1 + \sigma_3}{2} \tag{3}$$

where  $c_0$ ,  $\varphi_0$  is the initial cohesion and internal friction angle of the soil; *b* is the value range: (-1, 1); and  $\sigma_1$ ,  $\sigma_2$ ,  $\sigma_3$  are the major, intermediate, and minor principal stress, respectively.

The limit equilibrium condition in the soil may be calculated *via* the ultimate Mohr circle and shear strength envelope as follows:

$$\sigma_1 = \sigma_3 \frac{1 + \sin \varphi}{1 - \sin \varphi} + 2c \frac{\cos \varphi}{1 - \sin \varphi} \tag{4}$$

Under the intermediate principal stress effect, the unified cohesion *c* and the internal friction angle  $\varphi$  of the soil can be described as (Tang et al., 2022)

$$\varphi = \arcsin\left[\frac{2\left(1+b\right)\sin\varphi_{0}}{2+b\left(1+\sin\varphi_{0}\right)}\right]$$
(5)

$$c = c_0 \frac{2(1+b)\sqrt{\sin\varphi_0 + 1}}{\sqrt{(2+b)[2+b+(2+3b)\sin\varphi_0]}}$$
(6)

### 2.2 Upper bound limit analysis method

The upper bound limit analysis method is an effective method to study geotechnical structure stability (Zhang B. et al., 2020; Zhang J. H. et al., 2020). It points out that the external power equals the internal loss power, and the obtained load is not less than the real failure load, as seen below:

$$\int_{S} T_{i} v_{i} dS + \int_{V} F_{i} v_{i} dV = \int_{V} \sigma_{ij}^{*} \varepsilon_{ij}^{*} dV$$
(7)

where  $T_i$  is the surface force acting on the boundary and surface of the object;  $F_i$  is the volume force acting on the object, respectively;  $\sigma_{ij}^*$  and  $\varepsilon_{ij}^*$  are the actual stress and strain rate in the motion allowable velocity field, separately; and *S* and *V* are the surface area and volume.

# 2.3 Quasi-static method

The current common method for quantifying seismic effects is the quasi-static method (Zhang D. B. et al., 2021), which is adopted by engineers due to its clear mechanics concept and easy calculation.



The quasi-static method regards instantaneous seismic action as static action on the center of gravity of the structure. The expressions for the vertical static force and the horizontal static force are as follows:

$$\begin{cases} F_{\rm h} = k_{\rm h}G \\ F_{\rm v} = k_{\rm v}G \end{cases}$$
(8)

where the horizontal acceleration coefficients  $k_h$  under earthquakes are generally 0, 0.1, 0.2, 0.3;  $k_v$  is equal to  $\zeta k_h$ ; the value of the proportion coefficients of vertical seismic acceleration  $\zeta$  are -1, -0.5, 0, 0.5, 1 (Zhang B. et al., 2021b; Zhang J. H. et al., 2022); and *G* is the gravity value of the structure.

## 3 Mechanical model

Different from the failure characteristics of rock mass, the failure of soils generally has a curved shape (Wu et al., 2021; Wang et al., 2022). The research shows that when a soil slope is damaged, the actual failure surface approximates the logarithmic spiral curve (Michalowski and Drescher 2009; Utili 2013). According to the results of academic research, a slope calculation model is constructed. In Figure 1, the sliding mechanism rotates clockwise around the point *O* with an angular velocity  $\omega$ , and the angle between the velocity and the discontinuities AC is  $\varphi$ , where  $\varphi$  is the friction angle of the soil, and the failure surface AC is the logarithmic spiral curve. The slope top is level, the slope height is *H*, the slope angle is  $\beta$ ,  $OA=r_0$ , AB=L,  $\theta_0$ ,  $\theta_h$  are the angle between any diameter of *O* origin and the horizontal plane, and  $\theta$  is the angle between any diameter of *O* origin and the horizontal datum line.



The expression for *AC* is

$$r = r_0 e^{(\theta - \theta_0) \tan \varphi} \tag{9}$$

Therefore, the length of OC is

$$r_{\rm h} = r_0 e^{(\theta_{\rm h} - \theta_0) \tan \varphi} \tag{10}$$

It is derivable from the geometric relationship:

$$\frac{H}{r_0} = e^{(\theta_h - \theta_0) \tan \varphi} \sin \theta_h - \sin \theta_0 \tag{11}$$

$$\frac{L}{r_0} = \cos\theta_0 - \left[\frac{e^{(\theta_h - \theta_0)\tan\varphi}\sin\theta_h - \sin\theta_0}{\tan\beta} - e^{(\theta_h - \theta_0)\tan\varphi}\cos(\pi - \theta_h)\right]$$
(12)

# 4 Stability calculation

### 4.1 Fundamental assumptions

When the anti-seismic stability of slopes is assessed with the upper bound method, the following presumptions must be satisfied (Wu et al., 2019; Zhang D. B. et al., 2019): 1) the soil is an ideal plastic material, and obeys the associated flow law; 2) seismic action does not cause the effect of soil liquefaction; 3) the soil yield surface is convex in the stress space.

### 4.2 External power

The external power of a slope under seismic activity is divided into two sections. One is the power exerted by the weight force of the sliding body, the other is the power exerted by the seismic force. For the convenience of calculation, the external power produced by the *ABC* sliding body is regarded as the power produced by the *ACO* block minus the power produced by the *ABO* and *BCO* blocks.

TABLE 1 c and $\varphi$	under the	unified	strength	theory.
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No.	b	c₀/kPa	$\varphi_0/^\circ$	c/kPa	$\Delta_1$	$arphi/^{\circ}$	$\Delta_2$
1	0	20	20	20	0	20	0
2	0.25	20	10	21.87	8.53%	10.91	8.35%
3	0.25	20	15	21.73	7.97%	16.23	7.59%
4	0.25	20	20	21.62	7.49%	21.48	6.87%
5	0.25	15	25	16.14	7.08%	26.65	6.19%
6	0.25	20	25	21.52	7.08%	26.65	6.19%
7	0.25	25	25	26.90	7.08%	26.65	6.19%
8	0.5	20	10	23.32	14.24%	11.62	13.93%
9	0.5	20	15	23.07	13.31%	17.18	12.67%
10	0.5	20	20	22.86	12.52%	22.59	11.47%
11	0.5	15	25	17.02	11.85%	27.88	10.33%
12	0.5	20	25	22.69	11.85%	27.88	10.33%
13	0.5	25	25	28.36	11.85%	27.88	10.33%
14	0.75	20	10	24.49	18.32%	12.18	17.91%
15	0.75	20	15	24.14	17.14%	17.92	16.29%
16	0.75	20	20	23.85	16.15%	23.46	14.76%
17	0.75	15	25	17.71	15.30%	28.84	13.30%
18	0.75	20	25	23.61	15.30%	28.84	13.30%
19	0.75	25	25	29.52	15.30%	28.84	13.30%
20	1	20	10	25.44	21.39%	12.64	20.90%
21	1	20	15	25.01	20.02%	18.52	19.02%
22	1	20	20	24.65	18.88%	24.16	17.23%
23	1	15	25	18.27	17.91%	29.60	15.53%
24	1	20	25	24.36	17.91%	29.60	15.53%
25	1	25	25	30.45	17.91%	29.60	15.53%

Note:  $\Delta_1 = \left|\frac{c-c_0}{c}\right| \times 100\% \ \Delta_2 = \left|\frac{\varphi-\varphi_0}{\varphi}\right| \times 100\%$ 

An area element in the logarithmic spiral ACO region was taken, and the external power generated in this region was obtained as





$$W_{ACO} = \int_{\theta_0}^{\theta_h} \frac{1}{3} \gamma \omega r^3 \cos \theta d\theta$$
  
=  $\gamma \omega r_0^3 \int_{\theta_0}^{\theta_h} \frac{1}{3} e^{[3(\theta - \theta_0) \tan \varphi]} \cos \theta d\theta$  (13)  
=  $\gamma \omega r_0^3 f_1$ 

The external power generated by the block ABO is

$$W_{ABO} = \frac{1}{6} \gamma \omega r_0^3 L \left( 2 \cos \theta_0 - \frac{L}{r_0} \right) \sin \theta_0$$

$$= \gamma \omega r_0^3 f_2$$
(14)

The external power generated by the block BCO is

$$W_{BCO} = \frac{1}{6} \gamma \omega r_0^3 \frac{H}{r_0 \sin \beta} e^{(\theta_h - \theta_0) \tan \varphi} \sin (\theta_h + \beta)$$
$$\times \left[ \cos \theta_0 - \frac{L}{r_0} - e^{(\theta_h - \theta_0) \tan \varphi} \cos (\pi - \theta_h) \right] \qquad (15)$$
$$= \gamma \omega r_0^3 f_3$$

Therefore, the external power made by the sliding body ACB is

$$W_{\rm g} = \gamma \omega r_0^3 (f_1 - f_2 - f_3) \tag{16}$$

where



$$f_1 = \frac{(3\tan\varphi\cos\theta_{\rm h} + \sin\theta_{\rm h})e^{3(\theta_{\rm h} - \theta_0)\tan\varphi} - 3\tan\varphi\cos\theta_0 - \sin\theta_0}{3(1 + 9\tan^2\varphi)}$$

(17)

$$f_{2} = \frac{1}{6} \frac{L}{r_{0}} \left( 2\cos\theta_{0} - \frac{L}{r_{0}} \right) \sin\theta_{0}$$
 (18)

$$f_{2} = \frac{1}{6} \frac{H}{r_{0} \sin \beta} e^{(\theta_{h} - \theta_{0}) \tan \varphi} \sin (\theta_{h} + \beta) \bigg[ \cos \theta_{0} - \frac{L}{r_{0}} - e^{(\theta_{h} - \theta_{0}) \tan \varphi} \cos (\pi - \theta_{h}) \bigg]$$
(19)

The total power generated by the horizontal and vertical seismic forces is

$$W_{\rm e} = \gamma \omega r_0^3 [k_{\rm v} (f_1 - f_2 - f_3) + k_{\rm h} (g_1 - g_2 - g_3)]$$
(20)

in which  $k_{\rm h}$  is the horizontal acceleration coefficient under the earthquake, and  $k_{\rm v}$  is the vertical seismic acceleration coefficient under the earthquake.

The total external power is shown as follows:

$$W = W_{\rm g} + W_{\rm e}$$
  
=  $\gamma \omega r_0^3 [(1 + k_{\rm v})(f_1 - f_2 - f_3) + k_{\rm h}(g_1 - g_2 - g_3)]$  (21)

where

$$g_{1} = \frac{\left(3 \tan \varphi \cos \theta_{\rm h} - \cos \theta_{\rm h}\right) e^{3 (\theta_{\rm h} - \theta_{\rm 0}) \tan \varphi} - 3 \tan \varphi \sin \theta_{\rm 0} + \cos \theta_{\rm 0}}{3 \left(1 + 9 \tan^{2} \varphi\right)}$$

$$g_2 = \frac{1}{3} \frac{L}{r_0} \sin^2 \theta_0$$
 (23)

$$g_{3} = \frac{e^{(\theta_{h} - \theta_{0})\tan\varphi} \cdot \sin(\theta_{h} + \beta)H}{6r_{0}\sin\beta} \left[2\sin\theta_{h}e^{(\theta_{h} - \theta_{0})\tan\varphi} - \frac{H}{r_{0}}\right]$$
(24)

# 4.3 Internal energy dissipation rate

The internal energy loss rate  $P_s$  along the sliding plane is

TABLE 2 Stability	factor	under	the	influence	of	parameter	b	(ζ=0.5,
k <sub>h</sub> =0.2).								

$$b \beta/^{\circ} N_{\rm s}$$

		$\varphi_0=10^\circ$	$\varphi_0=15^{\circ}$	$\varphi_0=20^\circ$	$\varphi_0=25^{\circ}$	φ <sub>0</sub> =30°
0	40	6.11	7.70	9.95	13.39	19.46
	50	5.48	6.56	7.94	9.78	12.33
	60	4.93	5.70	6.62	7.76	9.21
	70	4.41	4.96	5.59	6.33	7.21
	80	3.91	4.30	4.72	5.20	5.75
	90	3.42	3.68	3.97	4.28	4.61
0.25	40	6.36	8.18	10.82	15.00	22.94
	50	5.66	6.87	8.43	10.52	13.49
	60	5.06	5.91	6.93	8.20	9.82
	70	4.51	5.11	5.79	6.60	7.57
	80	3.98	4.40	4.86	5.38	5.97
	90	3.47	3.75	4.06	4.38	4.74
0.5	40	6.57	8.57	11.55	16.42	26.08
	50	5.80	7.12	8.82	11.13	14.49
	60	5.17	6.08	7.18	8.55	10.31
	70	4.58	5.22	5.95	6.81	7.84
	80	4.03	4.48	4.97	5.51	6.13
	90	3.50	3.80	4.12	4.46	4.83
0.75	40	6.74	8.91	12.16	17.66	29.49
	50	5.92	7.33	9.15	11.65	15.33
	60	5.25	6.21	7.38	8.84	10.72
	70	4.64	5.31	6.08	6.99	8.07
	80	4.07	4.54	5.05	5.62	6.26
	90	3.53	3.85	4.18	4.53	4.91
1	40	6.89	9.19	12.70	18.85	32.37
	50	6.02	7.50	9.43	12.09	16.10
	60	5.32	6.33	7.55	9.08	11.06
	70	4.69	5.39	6.19	7.13	8.26
	80	4.11	4.59	5.12	5.71	6.37
	90	3.56	3.88	4.22	4.58	4.97

$$P_{s} = \int_{\theta_{0}}^{\theta_{h}} cvrd\theta$$

$$= \frac{cr_{0}^{2}\omega}{2\tan\varphi} e^{\left[2(\theta_{h}-\theta_{0})\tan\varphi-1\right]}$$
(25)

### 4.4 Analytical solution

According to the energy conservation theorem,

$$W = P_s \tag{26}$$

The critical height of slopes can be expressed as

$$H_{\rm c} = \frac{c \left[ e^{2(\theta_{\rm h} - \theta_0) \tan \varphi - 1} \right] \left[ \sin \theta_{\rm h} e^{(\theta_{\rm h} - \theta_0) \tan \varphi} - \sin \theta_0 \right]}{2\gamma \tan \varphi \left[ (1 + k_{\rm v}) \left( f_1 - f_2 - f_3 \right) + k_{\rm h} \left( g_1 - g_2 - g_3 \right) \right]}$$
(27)

Then the stability factor can be written as

$$N_{\rm s} = \frac{\gamma H_{\rm c}}{c} \tag{28}$$

# 5 Results analysis

### 5.1 Shear strength index

From Figures 2A,B and Table 1, as the influence parameter *b* of the intermediate principal stress increases, the internal friction angle  $\varphi$  and the cohesion *c* show an increasing trend. This is when the intermediate principal stress (b=0) is ignored, that is, the soil obeys the M-C strength criterion. At this point, the soil cohesion *c* and the internal friction angle  $\varphi$  are the initial cohesion  $c_0$  and the initial internal friction angle  $\varphi_0$ . When the intermediate principal stress (b=0.25, 0.5, 0.75, 1) was considered, a set of initial soil shear strength indicators ( $c_0=20$  kPa,  $\varphi_0=20^\circ$ ) were taken. When b=0.25, the cohesion force c=21.62 kPa, and the internal friction angle  $\varphi$ =21.48°, the relative error reached 7.49% and 6.87%, respectively. When b=0.5, the cohesion force c=22.86 kPa, and the internal friction angle  $\varphi=22.59^{\circ}$ , the relative error reached 12.52% and 11.47%, respectively. When b=0.75, the cohesive force c=23.85 kPa, and the internal friction angle  $\varphi$ =23.46°, the relative error reached 16.15% and 14.76%, respectively. When b=1, the cohesive force c=24.65 kPa, and the internal friction angle  $\varphi$ =24.16°, the relative error reached 18.88% and 17.23%, respectively. Therefore, the shear strength of soil will obviously be underestimated and underutilized if the intermediate principal stress is ignored.

As the initial internal friction angle  $\varphi_0$  of the soil increases from 10° to 20°, the internal friction angle  $\varphi$  increases from 12.64° to 24.16°; when the intermediate principal stress is considered (*b*=1), the relative error decreases from 20.90% to 17.23%. As the initial cohesion  $c_0$  of the soil increases from 15 kPa to 25 kPa ( $\varphi_0$ =25°), the cohesion *c* increases from 18.27 kPa to 30.45 kPa; when the principal stress is considered (*b*=1), the relative error remains unchanged at 15.53%. When the influence of intermediate principle stress is considered, with the increase of initial internal friction angle  $\varphi_0$ , the relative error of internal friction angle  $\varphi$  decreases gradually. However, when the initial internal friction angle is constant, the relative error of cohesion *c* remains unchanged with the increase of initial cohesion *c*<sub>0</sub>.

In order to more intuitively reflect the influence of the change of the intermediate principal stress influence parameter b on the soil shear strength index, three different soils were selected for comparative analysis (Jiang et al., 2022). In Figures 3A,B, the



change curve of the friction angle  $\varphi$  in the three soils is relatively steep, while the change curve of the soil cohesion *c* is relatively gentle. Therefore, the intermediate principal stress has a greater notable effect on the internal friction angle of the soil. Likewise, among the three types of soil, the intermediate principal stress influence on parameter *b* has the most obvious influence on the shear strength index of strongly weathered mudstone, followed by silty clay, and the effect on the mixed fill is slight.

### 5.2 Stability factor

# 5.2.1 Influence of principal stress on stability factor

As shown in Figures 4A,B; Table 2, with the increase of the influence parameter *b* of intermediate principal stress, the stability factor  $N_{\rm s}$  increases linearly. When the initial internal friction angle  $\varphi_0$  increases, the variation curve slope of the stability factor  $N_{\rm s}$  increases, while the slope angle  $\beta$  increases

and the variation curve slope of the stability factor  $N_s$  decreases. When the intermediate principal stress is ignored (b=0,  $\zeta=0.5$ ,  $k_h=0.2$ ,  $\varphi_0=30^\circ$ ,  $\beta=50^\circ$ ), the stability factor  $N_s$  of the slope is 12.33, and while other conditions remain unchanged, the intermediate principal stress is considered. When b=0.25 and the stability factor  $N_s$  is 13.49, the relative error reached 8.06%, while when b=0.5 and the stability factor  $N_s$  is 14.49, the relative error reaches 14.86%. When b=0.75 and the stability factor  $N_s$  is 15.33, the relative error reaches 19.56%, while b=1 and the stability factor  $N_s$  is 16.10, the relative error reaches 23.41%. Consequently, the slope stability will obviously be underestimated if the intermediate principal stress is ignored.

The initial internal friction angle  $\varphi_0$  is 15°, 20°, 25°, and 30° when the influence of the intermediate principal stress (*b*=0.5,  $\beta$ =40°) is considered. The stability factor  $N_s$  is 8.57, 11.55, 16.42, and 26.08, which increases by 30.50%, 58.08%, 149.92%, and 296.94%, respectively, compared with  $\varphi_0$ =10°. According to the aforementioned analysis results, slope stability is greatly sensitive to changes in the initial internal friction angle  $\varphi_0$ . Thus,

μ,	Ψ0/							
		$k_{\rm h}$ =0	<i>k</i> <sub>h</sub> =0.1	<i>k</i> <sub>h</sub> =0.2	<i>k</i> <sub>h</sub> =0.3			
40	10	10.30	7.75	6.11	5.01			
	20	20.00	13.47	9.95	7.86			
	30	58.27	29.47	19.46	14.53			
50	10	8.52	6.75	5.48	4.54			
	20	13.63	10.18	7.94	6.41			
	30	25.41	16.94	12.33	9.58			
60	10	7.26	5.93	4.93	4.16			
	20	10.39	8.18	6.62	5.48			
	30	16.04	11.87	9.21	7.39			
70	10	6.25	5.22	4.41	3.78			
	20	8.30	6.74	5.59	4.71			
	30	11.49	8.96	7.21	5.94			
80	10	5.38	4.56	3.91	3.39			
	20	6.75	5.60	4.72	4.04			
	30	8.68	6.99	5.75	4.83			
90	10	4.58	3.94	3.42	3.00			
	20	5.51	4.64	3.97	3.44			
	30	6.69	5.50	4.61	3.93			

TABLE	3 Stability	factor	with	different	slone	angles	(b=0.5)	(=0 5)
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TABLE 4 Stability factor with different  $k_{\rm h}$  ( $\zeta$ =0.5,  $\beta$ =70°).

b  $k_{\rm h}$  $N_{\rm s}$ 

		$\varphi_0=10^\circ$	$\varphi_0=15^{\circ}$	$\varphi_0=20^\circ$	$\varphi_0=25^{\circ}$	<i>φ</i> <sub>0</sub> =30°
0	0	6.25	7.18	8.30	9.70	11.49
	0.1	5.22	5.92	6.74	7.74	8.96
	0.2	4.41	4.96	5.59	6.33	7.21
	0.3	3.78	4.22	4.71	5.28	5.94
0.25	0	6.41	7.43	8.68	10.24	12.24
	0.1	5.34	6.11	7.02	8.11	9.47
	0.2	4.51	5.11	5.79	6.60	7.57
	0.3	3.86	4.34	4.87	5.49	6.21
0.5	0	6.53	7.64	8.98	10.67	12.85
	0.1	5.43	6.26	7.23	8.41	9.87
	0.2	4.58	5.22	5.95	6.81	7.84
	0.3	3.92	4.43	5.00	5.65	6.41
0.75	0	6.63	7.80	9.23	11.03	13.35
	0.1	5.51	6.38	7.41	8.65	10.19
	0.2	4.64	5.31	6.08	6.99	8.07
	0.3	3.97	4.50	5.10	5.78	6.57
1	0	6.72	7.94	9.44	11.32	13.77
	0.1	5.57	6.48	7.56	8.85	10.47
	0.2	4.69	5.39	6.19	7.13	8.26
	0.3	4.01	4.56	5.18	5.89	6.71

increasing the initial internal friction angle  $\varphi_0$  of the soil can significantly improve the slope stability.

#### 5.2.2 Influence of slope angle on stability factor

As shown in Figures 5A,C, as the slope angle decreases, the slope stability factor  $N_s$  increases nonlinearly, and the curve of stability coefficient  $N_s$  has an obvious inflection point at the slope angle  $\beta$ =50°. Combining with Table 3, the stability factor  $N_s$  is 6.69 when  $\beta = 90^{\circ}$  ( $k_h = 0, \zeta = 0, \varphi_0 = 30^{\circ}$ ). By cutting the slope, when the slope angle  $\beta$  decreases from 90° to 40°, the stability factor  $N_s$  is 58.27, which increases by 771%. When  $\beta$  decreases from 90° to 50°, the stability factor  $N_{\rm s}$ is 15.41, which increases 279.92%. As  $\beta$  decreases from 90° to  $60^{\circ}$ , the stability factor  $N_{\rm s}$  is 16.04, an increase of 139.76%. As  $\beta$  decreases from 90° to 70°, the stability factor  $N_{\rm s}$  is 11.49, an increase of 71.74%. As  $\beta$  decreases from 90° to 80°, the stability factor  $N_{\rm s}$  is 8.68, with an increase of 29.74%. Consequently, slope cutting is beneficial to slope stability, and the slope angle reduced to 40°-60° can significantly improve slope stability. The computation outcomes are substantially influenced by the change in slope angle, which is very sensitive to the slope sliding stability.

From Figure 5B, when  $k_{\rm h}$ =0.2,  $\varphi_0$ =20° and  $\zeta$ =0.5 and the slope angle  $\beta$ =90°, the stability factor  $N_s$  is about 4.2, which is slightly affected by the intermediate principal stress parameter b. In

Figure 5D, under the conditions of  $\varphi_0=20^\circ$ , b=0.5, and  $k_{\rm b}=0.2$ , with the slope angle  $\beta$  decreasing, the variation law of the stability factor  $N_{\rm s}$  curve under different vertical seismic force acceleration proportional coefficient  $\zeta$  is almost the same.

### 5.2.3 Influence of earthquake force on stability factor

From Figures 6A,C, the slope stability factor  $N_s$  shows a decreasing trend as the horizontal seismic force acceleration coefficient  $k_{\rm h}$  increases. In Tables 4, 5, under the conditions of  $\varphi_0=20^\circ$ ,  $\beta=70^\circ$ , b=0.5,  $\zeta=0.2$ , when the horizontal seismic force  $(k_{\rm h}=0)$  is overlooked, the stability factor  $N_{\rm s}$  is 8.98. While the horizontal seismic force ( $k_h$ =0.1, 0.2, 0.3) is considered, the stability factor N<sub>s</sub> is 7.23, 5.95, and 5.00, respectively. Compared with overlooking the horizontal seismic force ( $k_{\rm h}$ =0), the stability factor  $N_{\rm s}$  is reduced by 24.22%, 50.90%, and 79.82%, respectively. Therefore, the slope stability is significantly impacted by the horizontal seismic force. If the horizontal seismic force is disregarded, the slope stability will be seriously overestimated, resulting in a potential landslide risk.

From Figures 6B,D, with the vertical seismic force acceleration proportional coefficient  $\zeta$  ( $k_{\rm h} \neq 0$ ) increasing, the TABLE 5 Stability factor with different  $\zeta$  ( $k_{\rm h}$ =0.2,  $\beta$ =70°).

 $b \zeta N_s$ 

		$\varphi_0=10^\circ$	$\varphi_0=15^{\circ}$	$\varphi_0=20^\circ$	$\varphi_0=25^{\circ}$	φ <sub>0</sub> =30°
0	-1	5.52	6.16	6.89	7.73	8.72
	-0.5	5.10	5.71	6.40	7.21	8.16
	0	4.73	5.31	5.97	6.74	7.66
	0.5	4.41	4.96	5.59	6.33	7.21
	1	4.13	4.65	5.25	5.95	6.81
0.25	-1	5.63	6.33	7.13	8.04	9.11
	-0.5	5.21	5.87	6.63	7.50	8.54
	0	4.83	5.46	6.18	7.02	8.03
	0.5	4.51	5.11	5.79	6.60	7.57
	1	4.22	4.79	5.44	6.22	7.15
0.5	-1	5.72	6.47	7.31	8.28	9.41
	-0.5	5.29	6.00	6.80	7.73	8.84
	0	4.91	5.58	6.35	7.25	8.32
	0.5	4.58	5.22	5.95	6.81	7.84
	1	4.29	4.90	5.60	6.42	7.42
0.75	-1	5.79	6.58	7.46	8.47	9.66
	-0.5	5.36	6.10	6.94	7.92	9.08
	0	4.98	5.68	6.49	7.43	8.55
	0.5	4.64	5.31	6.08	6.99	8.07
	1	4.35	4.99	5.72	6.59	7.64
1	-1	5.85	6.67	7.58	8.63	9.86
	-0.5	5.41	6.19	7.06	8.08	9.28
	0	5.03	5.76	6.60	7.58	8.74
	0.5	4.69	5.39	6.19	7.13	8.26
	1	4.40	5.06	5.83	6.73	7.82

slope stability factor  $N_s$  decreases linearly. In Tables 4, 5, under the conditions of  $\varphi_0=20^\circ$ ,  $\beta=70^\circ$ , and b=0.5, when just the horizontal seismic force ( $k_h=0.2$ ,  $\zeta=0$ ) is taken into account, the stability factor  $N_s$  is 6.35. While both horizontal and vertical seismic forces ( $k_h=0.2$ ,  $\zeta=1$ ) are considered, the stability factor  $N_s$  is 5.60, which is reduced by 13.39%, compared to just considering the horizontal seismic force. Thus, the influence of the vertical seismic force on slope stability cannot be ignored. If the vertical seismic force is disregarded, the slope stability will be overestimated to some extent.

# 6 Conclusion

1) If the intermediate principal stress of soils is ignored, the shear strength of soils would obviously be reduced, which resulted in a serious underestimation of slope stability. Compared with considering the effect of intermediate principal stress (b = 1), when the effect of intermediate principal stress is not considered (b = 0). relative difference of the cohesion c and the internal friction angle  $\varphi$  reaches 21.39% and 20.90% respectively ( $\varphi_0 = 10^\circ$ ,  $c_0 = 20$  kPa). Similarly, the relative difference of stability factor *Ns* of slopes reaches 23.41% with the conditions of  $\varphi_0 = 30^\circ$ ,  $\beta = 50^\circ$ ,  $k_h = 0.2$  and  $\zeta = 0.5$ . Accordingly, a suitable strength theory should be used to properly take into account the impact of the intermediate principal stress for the stability analyses of different soil slopes.

- 2) Slope cutting is an effective way to strengthen slope stability. When the slope angle decreases from 90° to 60°, 50°, and 40°, the stability factor  $N_{\rm s}$  increases by 139.76%, 279.92%, and 771%, respectively. For slope stability, the sensitivity of physical parameters from large to small is as follows: slope angle  $\beta$ , initial internal friction angle  $\varphi_0$ , horizontal seismic force parameter  $k_{\rm h}$ , medium principal stress influence parameter b, and vertical seismic force parameter  $\zeta$ . The value of slope angle beta has the greatest influence on the calculation results. Without taking into account additional elements like groundwater level and stratum structure, the design slope angle is between 50° and 60°, which balances economy and safety.
- 3) The slope stability would be seriously overstated if the seismic force was disregarded. When the horizontal seismic force acceleration coefficient  $k_{\rm h}$  increases from 0 to 0.3 ( $\varphi_0=20^\circ$ ,  $\beta=70^\circ$ , b=0.5, and  $\zeta=0.5$ ), the stability factor  $N_{\rm s}$  reduces by 79.82%. While the vertical seismic force acceleration proportional coefficient  $\zeta$  increases from 0 to 1 ( $\varphi_0=20^\circ$ ,  $\beta=70^\circ$ , b=05, and  $k_{\rm h}=0.2$ ), the stability factor  $N_{\rm s}$  decreases by 13.39%. It is clear that the slope stability would be obviously exaggerated if the horizontal seismic force and the vertical seismic force on the stability of slopes is not as great as the effect of horizontal seismic force. However, neglecting these two effects will greatly increase the risk of slope landslide.
- 4) There is a certain inaccuracy with the actual scenario since the slope is treated as a two-dimensional plane for analysis in this work, which ignores the influence of slope width and boundary conditions. As a result, future research work could focus on the threedimensional influence and boundary conditions of a slope.

# Data availability statement

The original contributions presented in the study are included in the article/Supplementary Material; further inquiries can be directed to the corresponding author.

# Author contributions

This study's goals and ideas are offered by BZ. BZ derived the formula, analyzed the original data, and wrote the first manuscript. QS revised and polished the manuscript. All of the listed authors contributed substantially to the research and gave approval for the paper's publication.

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# **Conflict of interest**

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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