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General and math vocabulary contributions to early numeracy skills in a large population-representative sample

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There are well-documented associations between numeracy development in preschool age children and general and math vocabulary, literacy, and executive functions. However, the studies have largely included small samples of children with parents with either predominantly low or higher socioeconomic background (SES); further, few studies have included measures of all these domains in the same analyses. In this current study, we examined how general vocabulary, math vocabulary, rhyme detection and three measures of executive functions are associated with numeracy development in a population representative sample of 2,931 Danish children 3–5 years (51% male, 89% white). Multi-level regressions controlling for child age and, sex, and in some analyses also parental education and income confirmed that general vocabulary ($\beta = 0.16$), math language ($\beta = 0.17$), rhyme detection ($\beta = 0.14$), attention shifting ($\beta = 0.09$), inhibitory control ($\beta = 0.05$), and working memory ($\beta = 0.09$) each are associated with numeracy after controlling for covariates when estimated in the same model. Analyses of extreme performance (low-high) suggests a much closer and symmetrical connection between math language and numeracy compared to that between general vocabulary and numeracy. Interestingly, family SES is weakly but significantly related to all measures, most strongly for the vocabulary measures, but does not influence the pattern of results from regression analyses. In conclusion, both general vocabulary and mathematics-specific vocabulary contribute substantially to early numeracy skills. Rhyme detection and executive functions are also associated with numeracy skills, but with a lower magnitude.

KEYWORDS

numeracy development, general and math language, literacy skills, executive functions, preschool aged children, associations

1 Introduction

Many of the skills, behaviors and dispositions that are important achievements in education are now understood to have critical roots in the preschool period, a conclusion that is supported by a large and diverse body of evidence from multiple fields of study including developmental science, neuroscience, molecular biology, and genomics (Institute of Medicine National Research Council, 2015). Among such skills are early math skills in the preschool years, which have been found to predict not only growth and achievement in math skills throughout elementary school, but also reading achievement (Duncan et al., 2007; Watts et al., 2014, 2018). Early numeracy skills have been found to be the domain that is most predictive of later math achievement (Nguyen et al., 2016). And as for many other developmental domains, children from homes with fewer socio-economic (SES) resources have been found to have weaker math skills before they enter school leading to poorer outcomes in elementary school (Duncan and Magnuson, 2011). Recent research also suggests that early numeracy skills are not acquired independently from other cognitive skills and are associated with general language and emergent literacy skills as well as executive functions (Purpura et al., 2011, 2019; Purpura and Ganley, 2014; Espinas and Fuchs, 2022). Additionally, research indicates that a subset of vocabulary, "math vocabulary", has special relevance for early numeracy. At the same time, current meta-analytic evidence estimates that early numeracy interventions targeting low-performing children aged 5–8 years have a moderate and significant positive effect on early numeracy achievement corresponding to approximately 0.6 of a standard deviation (SD) (Charitaki et al., 2021). However, large heterogeneity among student outcomes has been found. There is therefore a strong need to learn more about the development of early numeracy skills and their relation to other skills to be able to design developmentally appropriate interventions aligned well with other relevant dimensions.

In this study, we examine the relation between early numeracy and general vocabulary, math language, emergent literacy skills, and executive functions which have all been related to different aspects of mathematical development in an unselected and nationally representative large sample of 2931 3- to 5-year-old Danish children.

1.1 Development of early numeracy skills

The finding that early numeracy skills are not just associated with, but strongly predictive of later math achievement (Nguyen et al., 2016) has stimulated research on mapping the early development of numeracy skills. Expanding on earlier work such as Clements and Samara (2014) and Litkowski et al. (2020) identified fine-grained trajectories of preschool children's early numeracy development based on 801 3–5-year-old children from the United States (U.S.). Parents' reported level of education was similar to estimates of national distribution. Numeracy skills include abilities such as *numbering* (e.g., verbal counting, counting with one-to-one correspondence and cardinal number knowledge), *relations* (e.g., numerical identification, connecting numerical to quantity and numerical comparisons) and *arithmetic operations* (e.g., additional and subtraction). Litkowski et al. found that preschool aged children progress on all components of numeracy. In the domain of numbering, for example, at age 3 about half of the children can count to 10 and at 5 years, half of the children can count to 20. At age 4, more than 70% of children can complete one-to-one correspondences up to six items and by age 5 that number of correspondences increase to 11. In addition, children's performance on these numeracy skills were highly intercorrelated.

The substantial growth in early numeracy skills has led to examination of the possible role of other cognitive skills as associates and potential foundation for math development. Language and emergent literacy skills have been among the most intensively studied, given their own development in this period and the pervasive influence of language in many developmental domains. More general cognitive skills such as executive functions have also been associated with numeracy skills. Below we focus the relations between each of these skills and numeracy skills. LeFevre et al. (2010) integrated diverse evidence and evidence from their own longitudinal study into a model of mathematics development between 4 and 8 years of age. Their model posits three "pathways" of precursor developments: linguistic, quantitative, and spatial attention, which contribute independently to early numeracy, and somewhat differentially depending on the demands of specific mathematics tasks. Notably, the linguistic pathway, instantiated by receptive vocabulary and a phoneme deletion task, was the strongest and most stable of the three pathways. Later research has provided additional confirmation, including studies with younger children, which we will review below.

1.2 The relation between general and math-specific vocabulary and numeracy skills

In early childhood, vocabulary plays a central role in individual differences in development. It predicts not only other aspects of language, including grammar and pragmatics (Fenson et al., 2007), but development in other domains, including numeracy. For example, Purpura and Ganley (2014) found, based on a study of 199 preschool and kindergarten children from low- to middle-SES families, that, in contrast to working memory, general expressive vocabulary was strongly associated with a broad array of early numeracy skills such as verbal counting, one-to-one counting and cardinal knowledge. Schröder et al. (2022) found that a measure of general vocabulary at 18 months predicted natural number knowledge at 2.5 years in Swedish children ($N = 92$; no information about parental SES). Task-specific differences in the associations between vocabulary and math skills (e.g., Lin et al., 2021) and individual variation in mathematical and vocabulary knowledge (e.g., Ünal et al., 2021) suggests that vocabulary knowledge is important in the initial mathematical acquisition.

Beyond the association with general vocabulary, there is a subset of words with special relevance for mathematics (Purpura and Reid, 2016). The terms "math vocabulary" (Powell and Driver, 2015) and "math language" (Purpura and Reid, 2016; Turan and De Smedt, 2022) have been used to define mathematics-specific content terms and concepts. There is little consistency in the choice between these two category labels. Operationally, the presence of selected vocabulary defines the presence of this special language, though there are also syntactic forms that are often associated with them, e.g., *take away three from seven, what comes after eight, on top of.* We will use "math vocabulary" in most cases, regardless of the term used by study authors, in part because a key focus of this paper is the comparison of this special vocabulary category with the broader "general vocabulary" category.

As Turan and De Smedt (2022) note in their meta-analysis of math language/vocabulary and ability in the preschool period, several definitions of math vocabulary have been used. Math vocabulary in the *broad* sense includes the understanding and correct use of number words (e.g., one, two), quantitative words (e.g., *more, less*), comparative phrases (e.g., *smaller than, greater than*), spatial relations words (e.g., *above, beneath*), abbreviations (e.g., min for minutes) and symbols (e.g., + and =). The narrow sense includes quantitative and spatial terms that are typically more approximate in nature (e.g., *more, most, similar, few, before, near*). The distinction has both developmental and functional significance. A broad approach has been primarily developed and utilized in research on the elementary and middle school grades (Hassinger-Das et al., 2014; Powell and Driver, 2015; Ünal et al., 2021); whereas a more narrow framing of "math vocabulary" has been used primarily in the years prior to formal schooling (e.g., preschool; Purpura and Reid, 2016; Purpura et al., 2017b), though the broad approach has also sometimes been used (Turan and De Smedt, 2022). In the period prior to schooling, math vocabulary is foundational for later mathematics (Purpura et al., 2017b). However, the precise link between the learning of math vocabulary and the emergence of different types of mathematical skills is not clear.

The choice between the two definitions of math vocabulary will also depend in part on the purpose and design of the study. For example, the broader category is more relevant for studies of math-relevant parental and preschool-teacher input to young children, whereas the narrower category is particularly relevant for studies of the relation between math language and numeracy. Inclusion of words for number, which represent an overlapping category between vocabulary and numeracy, would artifactually inflate the correlation between the two.

The evidence on the relation of math vocabulary to early numeracy is largely consistent in supporting a distinct role beyond that of general vocabulary (Turan and De Smedt, 2022). Purpura and Reid (2016) used regression analysis to demonstrate additional variance accounted for by math vocabulary in preschool children ($N = 136$) in middle- to higher SES families. Toll and Van Luit (2014) utilized mediation analysis to confirm that math vocabulary substantially mediated the relation between children's broader language ability and early numeracy in Dutch preschool children based on a nationally representative population ($N =$ 1,030 children). Litkowski et al. (2020) utilized factor analysis to explore the relations among spatial ability, math language, and math ability skills, and found that the strongest relation was between math language and math skills. Hornburg et al. (2018) have mapped the relation between math vocabulary and more specific numeracy skills including verbal counting, one-to-one correspondence, numerical identification, cardinality, comparisons of sets and/or numerals, ordering numerals and storytelling. In a sample of 124 preschoolers with a range of different SES backgrounds, math vocabulary was found to be significantly related to all numeracy skills except subitizing and formal addition, which were also less related to general language ability (Hornburg et al., 2018). Turan and De Smedt (2022) conclude that "these studies provide converging evidence for a clear link between mathematical language and mathematical ability".

Although it is primarily focused on a somewhat older age range, a recent meta-analysis based on almost 8,000 children in the age range of 4–11 years of age ($M = 7.50$) is also illuminating. Lin et al. (2021) attempted to clarify the relation between math vocabulary and different types of mathematical tasks (foundational math tasks, e.g., number knowledge, number combinations, or operations vs. higher-order math tasks, e.g., solving word problems, fractions, and algebra) while controlling for both language comprehension and cognitive skills (non-verbal reasoning and working memory) as well as a range of other covariates (Lin et al., 2021). Similar moderate correlations between math vocabulary and both foundational math skills ($r = 0.48$) and higher-order math skills ($r = 0.50$) were found. However, when SES and other covariates were controlled, math vocabulary was more strongly correlated with higher-order mathematical tasks than foundational ones. Overall, Lin et al. (2021) suggest that a stronger math vocabulary enables the students to spend more cognitive resources on the processes of solving the tasks rather than simply understanding them. Mediation analyses indicated that language comprehension and cognitive skills accounts for 65% of the variance in the association between math vocabulary and math skills, but the associations between math vocabulary and mathematical skills were still significant when controlled for these two factors ($r = 0.17$), a finding which provides strong support for a unique contribution of math vocabulary.

Finally, a causal relation between math vocabulary and numeracy skills has been demonstrated utilizing intervention designs (Purpura et al., 2017a, 2021; Espinas and Fuchs, 2022). Intervention studies have also found that improving general language skills are not sufficient for increasing children's mathematical abilities but including a relatively broad range of mathematical vocabulary targets was more successful (Turan and De Smedt, 2022).

1.3 The relation between numeracy skills and emergent literacy skills

As reading and mathematical skills are related in elementary school (Duncan et al., 2007) it is not surprising that researchers focused on math have also investigated the role of skills which are foundational for reading development, particularly phonological awareness. Phonological awareness is manifested by a child's ability to detect and manipulate parts of the sound of words, for instance, matching, blending, and deleting parts of words (Wagner and Torgesen, 1987). Phonological awareness may facilitate numeracy development by enabling children to differentiate and manipulate individual words in the number sequence (Jordan et al., 2010). It is also possible that phonological awareness is a manifestation of (or proxy for) general metalinguistic and metacognitive ability which is especially useful in a domain such as mathematics which is not as fully embedded in the context as early language.

However, research results concerning the role of phonological awareness for numeracy development are mixed. For instance, in a study of 128 Filipino children from low-middle income families, phonological awareness (measured through a matching task and a syllable deletion task) explained variability in children's levels of early numeracy, which is consistent with the notion that phonological awareness is important for quantity–number competence in early numeracy development (Yang et al., 2021a). Similarly, a recent meta-analysis based on 94 studies documented overall significant relation between phonological processing and mathematical performance, which was moderated by phonological component, mathematical domain, task demand (accuracy, fluency), and participants' age (Yang et al., 2021b).

In contrast, a study of 69 1-to-5-year-old U.S. preschoolers from low-middle income families, which examined whether vocabulary, phonological awareness and print awareness predicted early numeracy development (as indexed by Preschool Early Numeracy Skills test), failed to show phonological awareness as a uniquely predictor of early numeracy development (Purpura et al., 2011). A study of 313 French preschoolers with diverse SES background ($M = 5.07$ years) evaluated the relation between early language and a broader range of literacy outcomes [letter knowledge, vocabulary, phonological awareness (syllable deletion and rhyme detection), and comprehension] and six domains of early numeracy (e.g., numeral identification, verbal counting and subitizing), and revealed that only letter knowledge–and not the other language and literacy domains–was a significant predictor of numeral identification and verbal counting (Thomas and Tazouti, 2021).

In summary, research linking early literacy skills with numeracy skills is at this point limited in quantity and mixed in its findings. Of particular interest would be studies that include a large and representative sample, as well as both language and emergent literacy measures as potential associates of numeracy.

1.4 The relation of executive functioning to vocabulary and early numeracy skills

Although the primary focus of the present study is the relationship of general and math vocabulary to early numeracy, executive functioning (EF) has considerable relevance, for several reasons. Executive function skills are a set of high-level cognitive processes, that enable individuals to regulate their thoughts and actions during goal-directed behavior (Friedman and Miyake, 2017). Like language, it is an early emerging ability that is foundational for later cognitive, social, and academic achievement (Bruce and Bell, 2022). Further and more specifically relevant, there is considerable evidence for links between EF and language, and between EF and mathematical abilities, suggesting that it may also play a role in the link between language and math. Following Miyake et al. (2000) and others, we consider EF to be a construct that—among other skills—includes three underlying cognitive processes: inhibitory control, (updating of) working memory, and cognitive flexibility. Below we consider in turn the links to vocabulary and to early math abilities.

An extensive literature (see Bruce and Bell, 2022, for a scoping review) has documented relations between executive functioning and vocabulary in early childhood. A variety of tasks have been used, though it has been argued that for children up to age 3, a one-factor model for diverse measures fits the EF data. Most studies have used a battery of EF tasks and composite scores.

It is well-established that EF and early math skills in preschool aged children are related (see Emslander and Scherer, 2022, for a systematic review and meta-analysis). Most of the studies and measures in studies of preschool aged children focus on early numeracy skills (Nelson and Mazzocco, 2020). Theoretically, EF skills are either hypothesized to underpin math development (Nguyen et al., 2019) or the relationship is hypothesized to be bidirectional (Fuhs et al., 2016). According to Bull and Lee (2014), specific components of EF influence math development in different ways; Attention shifting helps, for instance, children to swiftly shift between different math strategies, operations, and notations; inhibitory control helps children by suppressing irrelevant math strategies and retrieval of irrelevant numbers, and finally, working memory helps, for instance, children to keep quantities in mind, and manipulate them. Bidirectional relations between EF and math have been found in longitudinal studies (Fuhs et al., 2014; Schmitt et al., 2017; Chan and Scalise, 2022; Guedes and Cadima, 2022). For instance, in a longitudinal study where children were followed from preschool to kindergarten, Schmitt et al. (2017) found bidirectional relations between EF and math in preschool, whereas in kindergarten the relation became unidirectional with only EF predicting math. In a study of 125 3- to 5-year-old preschool children (Purpura et al., 2017b) found evidence that EF are differentially associated with numeracy skills. Response inhibition was related to most components of math and working memory to comparisons or combinations of numbers and quantities, whereas cognitive flexibility was related to more abstract math skills (Purpura et al., 2017b).

Focusing on numeracy skill, a recent study of 3- to 6-year-old children established concurrent relations between executive functions and four aspects of early numeracy skills, that is, set counting, numerical identification, number comparison and number line estimation (Chan and Scalise, 2022). There is also evidence of a causal relation between EF and numeracy skills which suggests that EF training could benefit math skills (Schmitt et al., 2015; McClelland et al., 2019), and that math intervention could improve working memory (Kroesbergen et al., 2014).

1.5 The relation of math vocabulary and early numeracy at low and high extremes

Extreme performance at both the high and the low end of the distribution of mathematical achievement has substantial consequences for individual children and for the larger society. Findings of significant and even substantial correlations do not guarantee a purely linear relation, and the relation of variables can vary greatly at the extremes, especially when outcomes rest on a multiplicity of foundational skills. For example, Bakker et al. (2023) found evidence for differences in the pattern of prediction to numeracy from other cognitive measures at the high end of the distribution with that in the low- and average-range. We are not aware of any research on either extreme of math vocabulary in the preschool period, although there is much evidence for very substantial variability in general vocabulary (Marchman and Dale, 2023). Research on math difficulties, which may include children with a diagnosed disability related to math or those struggling with instruction or both, begins in the kindergarten period at the earliest (Nelson and Power, 2018). Operational definitions of difficulty in research have been highly variable, e.g., ranging from the 10th to the 40th percentile, but the longitudinal studies reviewed by Nelson and Power confirm that children struggling with the earliest phases of math instruction in school are quite likely to have continuing difficulties in succeeding years. Whether even earlier measures have predictive validity is not yet known. However, the targeting, design, and interpretation of early intervention studies would benefit from greater understanding of the determinants, particularly general and math vocabulary, of extremely low and extremely high numeracy ability. Even less is known about extreme high performance in numeracy (see Bakker et al., 2023, for a review of what is known about variability across the full distribution of numerical ability).

1.6 The role of parental SES in early numeracy development

Individual differences in mathematical skills, like those in literacy and other domains, have important implications for children and for the society in which they are growing up, for instance, academic and vocational choice. Among the most consistent associates of individual differences in a wide range of cognitive and language skills is SES. With respect to the focus of the present paper, children from homes with less resources have been found to have less developed math skills before they enter school, leading to poorer outcomes in elementary school (Duncan and Magnuson, 2011; Purpura and Reid, 2016; Gjicali et al., 2019; Johnson et al., 2022). For instance, based on data from The Early Childhood Longitudinal Study, Kindergarten Class of 1998–99 (ECLS-K), Duncan and Magnuson estimated that when children at the bottom and top quintiles of SES were compared, the children from families with low-SES scored 1.3 standard deviations lower than their peers from families with high-SES. A recent systematic review suggested that, among other factors, parents' beliefs, practices, and language about math to some extent explain the development of gaps in math abilities during early childhood related to the SES-status of the parents (Elliott and Bachman, 2018). There is notably less SES variation between families in Denmark than in the U.S., not least because the welfare state subsidizes families with low-SES to a much higher extent than in the U.S. (Landersø and Heckman, 2017). According to 2015 data from the OECD Centre for Opportunity and Equality (http://www.oecd.org/social/inequality.htm), Denmark has the lowest Gini coefficient among OECD countries, and the lowest income difference ratio of 5.2 between the richest 10% and poorest 10% (cf. OECD mean = 9.6; United States = 18.8). There is very limited empirical information on Danish preschool children's numeracy skills (Sjoe et al., 2019) and no data on Danish children's math language development. Consequently, we explore the developmental trends of early math language and numeracy development and compare to similar U.S. studies.

1.7 The present study, and the Danish context

The current study assessed the extent to which preschooler's early numeracy skills are associated with general vocabulary, math language, phonological awareness, and several aspects of executive functioning. The study adds to the current knowledge basis in several respects. First, we investigate the relations among this particularly comprehensive set of measures in a large, population representative sample. As noted in the preceding review, the majority of published reports on early numeracy and its correlates have included a relatively small sample subject to the overrepresentation of high-SES families common in convenience samples, or conversely, a sample with a specific focus on children at risk for low performance. Given the likelihood of multiple, correlated influences on early numeracy, studies with small samples may be substantially underpowered. Second, and relatedly, in addition to evaluating the overall relationships among these measures, we specifically examined vocabulary correlates of low and high numeracy performance, as both extremes will be represented in the sample. Third, the children are developing in the context of a unique educational system where children have been enrolled in universal childcare since infant/toddlerhood. In 2017, ∼88% of 1-to-2-year-old children and 97% of 3-to-5-year-old children were in these nationally funded programs (European Commission/EACEA/Eurydice, 2019), thereby restricting the variation in children's numeracy (and other) experiences to be more uniform from an instructional perspective. Childcare in Denmark (the term "preschool" is seldom used) has traditionally, and uniformly, focused on supporting children's social skills rather than early academic skills and although some emergent literacy content has been introduced, there is less of a differential focus on literacy with respect to numeracy, a fact which may have implications for the relation of the two domains. Fourth, we address SES influences more explicitly, both in terms of its effects on individual variables and on the overall prediction of early numeracy.

The specific research questions which are addressed in the study are: (1) What are the developmental trends, including possible sex differences, of Danish children on numeracy, general vocabulary, math language, rhyme, and executive functions from age 3–5? (2) How well is numeracy statistically predicted by general vocabulary, math language, rhyme detection, and selected aspects of executive functioning, individually and collectively? (3) How well do general and math language, individually and together, statistically predict high and low numeracy performance? (4) How are measures of SES related to numeracy and the patterns of prediction addressed in question 2?

2 Material and methods

2.1 Participants and procedures

The participants were 2,931 (1,494 boys, 1,437 girls) 3- to 5-year-old children (M = 4.37 years, SD = 0.76), enrolled in 329 classrooms in 88 childcare centers in nine municipalities across Denmark. Approximately 11% of the children had immigrant status. Parent education and income were slightly below the national average: 13.1% of mothers and 14.4% of fathers had primary schools as highest education, 35.5% of mothers and 47.0% of fathers had high school as highest education, 38.9% of mothers and 27.7% of fathers had completed a graduate degree, whereas 12.5% of mothers and 10.7% of fathers had completed a postgraduate degree. Fathers' mean income were 307.000 DKK and mothers were 179.000 DKK which is comparable to the national average. Thus, overall, the sample was relatively population-representative of Danish children.

The study sample was drawn from a large-scale intervention study designed to improve school readiness skills in children 1 to 6 years from 2018 to 2019 (Bleses et al., 2020b). The sample was unusually large, as its size was based on the power needs of the intervention project. Intervention studies typically have much smaller expected effect sizes than assessment projects (Kraft, 2020) and hence require larger samples. In the present case, the large sample size increases confidence in the conclusions as well as enabling more complex analyses. The Danish Data Protection Agency approved the collection and treatment of data J.nr. 2015-57-0002, 741. The RCT study was not preregistered. The present study uses the pretests from the trial, and the sample used in this study was therefore not influenced by the RCT. The trial evaluated the effect of two professional development interventions to support the implementation of a school readiness intervention "We learn together" (Bleses et al., 2020a, 2021) compared to an active control group, that is, all three intervention groups implemented the intervention.

2.2 Instruments

A data coordinator made agreements with the childcare centers for testing all the children. Pretest data were collected by 57 trained student assistants in the children's childcare between August and September 2018. The student assistants, who mainly were university students in linguistics, psychology or cognitive science, were trained at a 4-h workshop before testing the children. The workshop was designed to provide the student with sufficient skills to administer the tests and to have appropriate behaviors with children. Every second week, there was a meeting where the students could ask questions.

2.2.1 Numeracy

Numeracy skills for each child were measured using the Danish adaption of Preschool Early Numeracy Skills Test—Brief Version (PENS-B, Purpura et al., 2015). PENS-B assesses the child's ability in set comparison, numeral comparison, one-to-one correspondence, number order, identifying numerals, ordinality, and number combinations. No modifications to the original U.S. version beyond translation were made. Numeracy was measured with 24 items [e.g., Count these dots (3)]. Internal consistency was high ($\alpha = 0.91$).

2.2.2 Math language/vocabulary

A Danish adaption of the instrument *The Preschool Assessment of Math Language* (PALM, Purpura and Reid, 2016) was created for this project. PALM assesses the child's understanding of key words used in early mathematics (e.g., *a lot, more, nearest, far away*). PALM assesses math vocabulary in the narrow sense, which is more appropriate than the broad sense for research on the relation of vocabulary to early numeracy, as described earlier. In adapting PALM to Danish, we first made raw translations of each item. Only one modification was then required by linguistic differences between Danish and U.S. English. In English *under* and *below* are two separate items but Danish has only one word, *under*. In this case *under* is used in both cases. The forms were then back translated to English to test the translation, but that process did not give rise to any changes. Scores represent the sum of correct answers. Math language was measured with 16 items (e.g., Point to the dot that is nearest to the boy). Internal consistency was good ($\alpha = 0.87$).

2.2.3 (General) vocabulary

For each child, we administered the expressive vocabulary scale from a published assessment instrument, Language Assessment of Children 3–6 years (LA 3–6, Bleses et al., 2010b; Haghish et al., 2021; Højen et al., 2022) to assess the child's general vocabulary skills. In order to keep the scale brief but developmentally appropriate across this age range, a master list of 51 words was selected, and four 25-word lists, partially overlapping, were chosen from that list, one for each age 3–6. The child was asked to name a picture [e.g., What is this? (a picture of a squirrel)]; in cases such as actions where it might not be obvious which of the multiple aspects of the picture is to be named, two contrasting pictures are given to help elicit the relevant word (e.g., The woman is eating. What is the man doing?) One point was given for each correct response. For children older than 3 a modified basal system was employed. For words at a given age level which were also on the list for the immediately younger level, a percentage correct was determined for that child, and that percentage was applied to all words at lower age-levels to determine the additional points credit. This system is more conservative than conventional basal systems, which give full credit for all easier words. Based on this system, the maximum score possible was 25, 35, 45, and 51 words for ages 3, 4, 5, and 6, respectively. Scores represent the sum of correct answers. Internal reliability was acceptable ($\alpha = 0.85$). Criterion validity of the general vocabulary subscale assessed, has been demonstrated through correlations with Danish versions of the Peabody Picture Vocabulary Test (Dunn and Dunn, 2007) and Expressive Vocabulary Test (Williams, 2007; Bleses et al., 2018).

2.2.4 Rhyme detection

For each child, we also administered the 15-item rhyme detection scale from the LA 3–6 assessment instrument (Bleses et al., 2010b; Haghish et al., 2021; Højen et al., 2022). Rhyme detection, an awareness of whether two syllables are identical after the initial consonant or consonant cluster, is a good proxy for phonological awareness overall because at the younger ages it is indistinguishable from other aspects of phonological awareness and in older ages is still highly correlated and largely overlapping with them (Anthony and Lonigan, 2004).

For rhyme detection, three pictures are shown, and the child is asked to point to the two pictures of objects whose names are words that rhyme. The administrator names the three pictures and asks the child to point to "the two things that sound alike: those that rhyme". Scores represent the sum of correct answers. Internal reliability was high ($\alpha = 0.94$).

In addition to these mathematical and language measures, the executive function measures from the *Early Years Toolbox were* used to assess children's attention shifting, inhibitory control and visual-spatial working memory (Howard and Melhuish, 2017). The Early Years Toolbox items were administrated to children aged 3–5 with three apps on a tablet.

2.2.5 Attention shifting ("the card sorting" task)

This task requires children to sort cards by a sorting dimension (color or shape) into one of two locations (blue rabbit or a red boat). The sorting rule changes through the test which consists of a demonstration trial, two practice trials and a post-switch phase, in which children are required to sort cards by the other sorting dimension. Scores represent the number of correct sorts after the pre-switch phase (see Howard and Melhuish, 2017 for a detailed description). Internal reliability was high ($\alpha = 0.95$).

2.2.6 Inhibitory control (the "go/no-go" task)

The task requires children to tap the screen on "go" trials ("catch the fish", 80% trials) and not tap the screen on "no-go" trials (avoiding sharks, 20% trials). After a practice trial, where auditory feedback is provided on all, the task consists of 75 stimuli divided evenly in three test blocks. Each stimulus (i.e., fish or shark) is presented for 1,500 ms, separated by a 1,000 ms interstimulus interval (see Howard and Melhuish, 2017 for a detailed description). Internal reliability was high ($\alpha = 0.92$).

2.2.7 Visual-spatial working memory ("Mr. Ant" task)

In the task children are asked to remember the spatial locations of "stickers" placed on a cartoon and identify the locations after a brief retention interval. There are three trials at each of the eight levels of increasing complexity (one to eight stickers). The task continues until completion of all 24 trials or failure on all three trials at the same level of difficulty (see Howard and Melhuish, 2017 for a detailed description). Internal reliability was high ($\alpha = 0.90$).

2.3 Covariates

Age and sex of child were obtained from Statistics Denmark and were used as covariates in most analyses. For examination of the role of demographic factors, a secondary set of analyses were conducted in which education and income of both parents were also treated as covariates. Background information with respect to mother's and father's education, family income, and parental immigrant status, and the child's immigrant status (i.e., whether the child had no immigrant background as opposed to the child or both parents being immigrants) was also obtained from Statistics Denmark, using the Danish Central Personal Number System. Based on categorization used by Statistics Denmark, mother's and father's education were divided into four categories: primary education (no education or elementary school as highest level), high school (high school as the highest level, upper secondary school, or vocational education as highest level), graduate degree (some post-secondary education or specialized training, such as teacher or nurse), and postgraduate (4-year post-secondary education or more as highest level).

2.4 Analytic strategy

We addressed *research question one* first by characterizing development across this age range with descriptive statistics of all measures by age, including *n*, means, standard deviations, and difference in scores across ages, expressed with Cohen's *d*, for each outcome. We also examined possible sex differences through correlations and regressions with child sex as a predictor. As a secondary analysis, we compared numeracy and math language performance of the Danish children with those reported for U.S. children.

Research question two examines how well general vocabulary, math language, rhyme detection and executive functions collectively statistically predict numeracy skills. As a preliminary analysis, we computed correlations, both zero-order and controlling for age and sex, for each of these with numeracy, as well correlations among the full set of measures. The main analyses were regressions. Children are nested within childcares; to account for the nested structure of the data we included childcares as a random effect in the regression models, following Raudenbush and Bryk (2002). Intraclass correlations (ICC) provided evidence for variance at each level to the extent that it warranted a multi-level model, though it was very modest for childcare (ICC = 0.02). Two-level standard multi-level mixed effects models were estimated using Stata 16, including child age and sex as control variables. A model was estimated for each predictor by itself, and then for the complete set of predictors. Conditional R^2 of the hierarchical regression models were computed to examine how much of the variation in early numeracy could be attributed to each outcome (Rights and Sterba, 2019). Note that it is conventional for regression results to be discussed in terms of "prediction" of an outcome variable by other measures. In the present study, all variables were assessed at approximately the same time, and therefore this is not a prediction study in the conceptual sense, although the analyses are the ones which would be used for true prediction. The terminology of prediction will be used only in the Section 3, for clarity.

To address *research question three,* which concerns the associates of high and low numeracy skills, we conducted multi-level mixed-effects logistic regressions with either general vocabulary or math language as the predictor, and either a highest-20% or lowest-20% criterion for the numeracy outcome. For ease of interpretation, we graphically represented the likelihood of being in the defined outcome category as a continuous function of each predictor. The distributions for these three variables are continuous, and, as noted earlier, there is no consensus criterion for low performance. As we acknowledge, with continuous variables and distributions, the choice of a cutoff is somewhat arbitrary. Histograms of the key variables show normality of general vocabulary, substantial negative skew with a moderate ceiling effect for math vocabulary, and substantial positive skew and slight floor effect for numeracy. These distributions make sense developmentally, in that these aspects of math vocabulary are learned relatively early, while numeracy is slower developing. (The general vocabulary measure, in contrast, has sufficient variance in difficulty to cover this age range more fully.) The floor and ceiling effects also provide some support for choosing a relatively generous criterion for extremes. We selected the cut-off for extreme performance at 20% based on prior research (Dale et al., 2023). The 20% cutoff was also chosen to balance the desire for sufficiently extreme performance to potentially affect future education, with the need for an adequate sample at the extremes for statistical analysis.

Finally, *research question four* addressed the role of SES in the language and numeracy measures individually, as well as its effect on overall regression-based associations with numeracy. For this purpose, we first examined the correlations of SES with individual behavioral measures. To visualize the associations between mother's education and numeracy and language/literacy measures, we graphically plotted the average score of all four outcomes by mother's education. To distinguish how SES is associated with numeracy based on a national representative sample, we compared the regressions in which only age and sex were controlled (in addressing research question two) with those (computed here) which also controlled for SES variables, to determine how much, if any difference, there was in the results.

Covariates had <6% missing data. For the analyses corresponding to research questions 2 and 3, multiple imputation was performed, using chained equation in Stata to fill in missing values. This method fills in missing values in multiple variables iteratively by using chained equations, a sequence of univariate imputation methods with fully conditional specification of prediction equations (Stata Technical Support, 2013). The imputation model was specified for each outcome measure since patterns of missing data varies by each dependent variable.

3 Results

3.1 Developmental trends

To examine our first research question—what are the developmental trends of Danish children in math language and numeracy from age 3 to 5—descriptive statistics (n, mean scores, standard deviations, and differences in score across ages, and between the sexes expressed with Cohens d) for each outcome are provided in Table 1. In these cross-sectional data, development between 4 and 5 as measured by d is generally lower than between ages 3 and 4.

To put the development of math language and numeracy in Danish children into comparative perspective, we compared the results to those in the original U.S. samples (Purpura et al., 2015; Purpura and Reid, 2016) in Figure 1. Danish children scored lower on the numeracy task at all three age levels ($d = 0.76$, 1.03, 1.44, respectively, based on pooled SD) compared to the U.S. children. At age 5 the mean numeracy score of U.S. children was almost twice as large ($M = 8.42$ vs. 15.36). In contrast, there were only small differences in math language between the two countries ($d = 0.38$, 0.27, and 0.33).

In *research question two* we explored how much of the variation in numeracy is associated individually and collectively with general vocabulary, math language, rhyme detection and executive functions, based on standard scores for all child outcomes. All six measures were available for 2,271 children. The first seven rows of Table 2 display the associations between the numeracy, language/literacy and executive functioning measures. Below the diagonal for those rows, the table shows the zero-order pairwise correlations among the child measures. Above the diagonal the partial associations between numeracy and the other measures (controlled for age and sex) are shown. The language measures are all weakly but significantly (given the large n) correlated with numeracy at approximately the same magnitude ($r = 0.13$–0.14). Numeracy is also weakly but significantly correlated with all three measures of executive functions ($r = 0.07$–0.10). General vocabulary and math language demonstrate the highest pairwise correlations among the measures. Notably, math language behaves more like numeracy than general vocabulary in relation to executive functions with significant partial correlations with all measures of executive functions. The ninth row of the table shows that child sex is not associated with numeracy, only with general vocabulary and all three measures of executive functions ($r = 0.06$–0.16).

Our major analyses for this research question are regressions. Table 3 presents seven models with child age and sex as covariates and adjusted for the nested structure of the data (in a later analysis parental SES variables are included to illuminate the specific role of SES). The first six models examine each of the predictors individually and the final model 7 includes all predictors. Each of the first six models predicted numeracy skills significantly, based on the individual measures. All language measures predicted numeracy in a similar rate; however, coefficients were highest for math language ($\beta = 0.41$). A one standard deviation increase in math language statistically predicted a 0.41 standard deviation increase in early numeracy ($p < 0.001$). Each of the executive functions also predicted numeracy when examined individually, but the magnitude was lower ($\beta = 0.17$–0.20). Model 7 included all six child measures in the standard multiple regression. Each of the child measures still made a unique, significant contribution to the prediction of numeracy. The contributions of language predictors were highest and of similar magnitude ($\beta = 0.15$–0.17). The contributions of the executive function predictors were also significant, but lower ($\beta = 0.05$–0.09). In total, the predictor variables in Model 7 explained 46% of the variation in early numeracy. Although sex was not a significant predictor in any of the individual predictor analyses, in the final model with all predictors, sex had a significant but small contribution.

TABLE 1 Descriptive statistics for child outcomes by age.

Age or sex group	Numeracy				Math language				Vocabulary				Rhyme			
	N	*Mean*	*SD*	*d*	*N*	*Mean*	*SD*	*d*	*N*	*Mean*	*SD*	*d*	*N*	*Mean*	*SD*	*d*
3-year-olds	1,001	2.25	2.37	Ref.	1,017	3.62	4.35	ref.	1,017	9.09	5.5	Ref.	1,002	7.68	3.76	Ref.
4-year-olds	1,145	5.39	3.88	0.96***	1,161	8.03	5.06	0.93***	1,161	15.27	7.56	0.92***	1,150	11.35	3.45	1.02***
5-year-olds	756	8.42	4.41	1.81***	759	11.17	3.72	1.84***	759	21.45	9.33	1.67***	756	13.4	2.49	1.75***
Male	1,496	5.07	4.43	Ref.	1,520	7.09	5.41	Ref.	1,420	14.29	8.79	Ref.	1,498	10.5	4.05	Ref.
Female	1,435	5.04	4.20	0.01	1,454	7.39	5.4	0.06	1,454	14.94	8.95	0.07*	1,439	10.62	4.11	0.03
	Attention shifting				Inhibitory control				Working memory							
	N	*Mean*	*SD*	*d*	*N*	*Mean*	*SD*	*d*	*N*	*Mean*	*SD*	*d*				
3-year-olds	838	3.00	3.70	Ref.	811	0.35	0.16	Ref.	846	1.92	1.94	Ref.				
4-year-olds	1,024	5.22	4.10	0.57***	1,010	0.53	0.20	0.91***	1,047	3.74	2.30	0.85***				
5-year-olds	669	6.91	3.75	1.05***	675	0.65	0.19	1.56***	687	5.28	2.00	1.70***				
Male	1,271	5.26	4.26	Ref.	1,231	0.54	0.22	Ref.	1,281	3.69	2.45	Ref.				
Female	1,294	4.51	4.03	0.17***	1,295	0.07	0.22	0.32***	1,332	3.37	2.51	0.13***				

***$p < 0.001$, *$p < 0.05$. Ref. denotes that the three-year-old children in all cases are the reference group for the comparisons. Group comparisons are estimated with *t*-tests. There is also a statistically significant, though smaller difference between 4- and 5-year-old children's skills for all measures (not shown in the table).

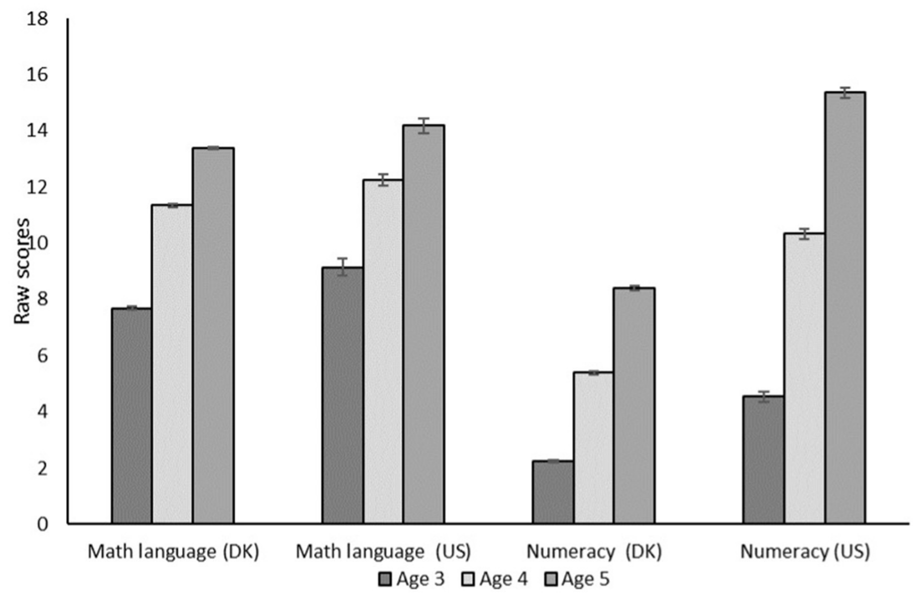

FIGURE 1
Comparisons of raw scores of math language and numeracy by age and language (US data is based on Purpura et al., 2015; Purpura and Reid, 2016).

3.2 Statistical prediction to high and low numeracy skills by general and math language

To explore research question three—Figure 2 illustrates the relations among general and math language and numeracy at the high and low extremes. The x-axis represents performance on the specific predictor (general or math language), while the y-axis represents the probability of being in the relevant numeracy outcome category at that level of the predictor. The steepness of the curves reflects the degree of relationship. Low vocabulary (gray diamond) is more strongly related to low numeracy than high vocabulary (black triangle) is to high numeracy. Low math language (black circle) is also very strongly related to low numeracy, but in this case high math language (gray square) is also strongly related to high numeracy. The curvilinear trends shown here suggest that the relations are even stronger at the extremes than the overall correlations in Table 2 would imply.

The difference in these two patterns for these two predictors led us to do an additional regression focused on the relation of general to math language. The results are illustrated in Figure 3. The prediction from general to math language is stronger at the low

TABLE 2 Zero-order and partial correlations (controlling for age and sex) between numeracy, math language, vocabulary and parental background characteristics ($N = 2{,}931$).

	Numeracy	Math language	Vocabulary	Rhyme	Attention shifting	Impulse control	Working memory
Numeracy	*1.00*	*0.13******	*0.14******	*0.13******	*0.07******	*0.10******	*0.10******
Math language	0.61**	1.00	*0.39******	*0.27******	*0.09******	*0.09******	*0.09******
Vocabulary	0.59***	0.73***	1.00	*0.24******	*0.10******	*−0.01*	*0.02*
Rhyme	0.67***	0.68***	0.57***	1.00	*−0.01*	*0.06******	*0.04*+
Attention shifting	0.38***	0.43***	0.43***	0.37***	*1.00*	*0.05**	*0.10******
Impulse control	0.44***	0.46***	0.42***	0.44***	0.32***	1.00	0.15***
Working memory	0.47***	0.51***	0.46***	0.46***	0.36***	0.45***	1.00
Age	0.59***	0.61***	0.58***	0.59***	0.40***	0.53***	0.54
Sex (ref. female)	0.00	−0.01	−0.04*	−0.03	–0.09***	−0.16***	−0.06*
Mother's education (ref. primary school)	0.13***	0.19***	0.19***	0.16***	0.10***	0.09***	0.09***
Father's education (ref. primary school)	0.11***	0.15***	0.17***	0.14***	010***	0.06***	0.08***
Mother's income	0.15***	0.25***	0.22***	0.19***	0.12***	0.12***	0.13***
Father's income	0.10***	0.18***	0.17***	0.11***	0.07***	0.05*	0.07***

$+p < 0.10$. $^{*}p < 0.05$. $^{**}p < 0.01$. $^{***}p < 0.001$. The correlations above the diagonal (in italics) are partial correlations controlled for age and sex.

TABLE 3 Mixed effects regression predicting early numeracy performance controlling for age and sex ($N = 2{,}271$).

	Model 1		Model 2		Model 3			
	β	SE	β	SE	β	SE		
Age	0.48***	(0.03)	0.44***	(0.03)	0.58***	(0.03)		
Sex	0.01	(0.03)	−0.01	(0.03)	−0.01	(0.03)		
Vocabulary	0.36***	(0.02)						
Math language			0.41***	(0.02)				
Rhyme					0.35***	(0.02)		
Attention shifting								
Inhibitory control								
Working memory								
Conditional R∧2	0.43		0.42		0.42			
	Model 4		**Model 5**		**Model 6**		**Model 7**	
	β	**SE**	**β**	**SE**	**β**	**SE**	**β**	**SE**
Age	0.68***	(0.02)	0.64***	(0.03)	0.63***	(0.03)	0.24***	(0.03)
Sex	−0.01	(0.03)	0.02	(0.03)	−0.01	(0.03)	0.06*	(0.03)
Vocabulary							0.16***	(0.02)
Math language							0.17***	(0.03)
Rhyme							0.15***	(0.02)
Attention shifting	0.17***	(0.02)					0.05***	(0.02)
Inhibitory control			0.18***	(0.01)			0.09***	(0.02)
Working memory					0.20***	(0.02)	0.08***	(0.02)
Conditional R∧2	0.37		0.37		0.37		0.46***	

Standard errors in parentheses. Child outcomes are computed as z-scores. $^{*}p < 0.05$, $^{***}p < 0.001$.

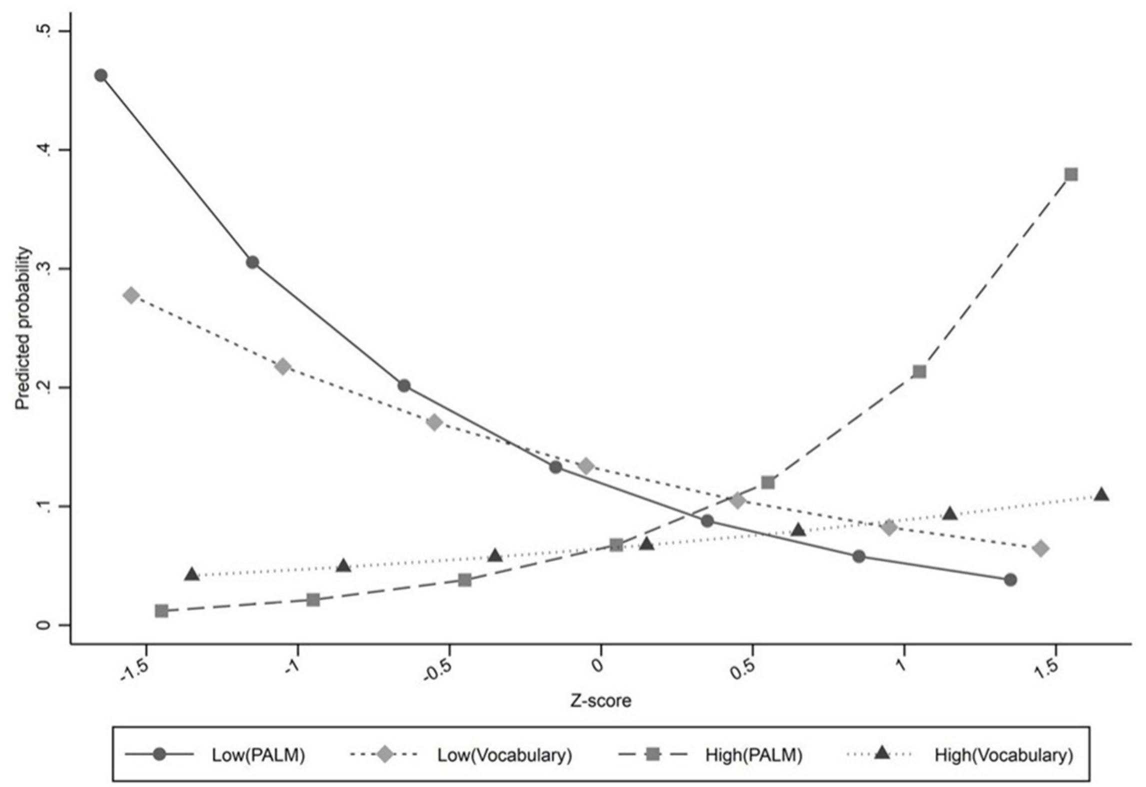

FIGURE 2
Prediction of low- and high-performing early numeracy from general vocabulary and PALM. Low/high numeracy is defined as the 20% lowest/highest percentile. The graph is based on a multilevel mixed-effects logistic regression model with multiple imputation. The y-axis displays the predicted probability of being among the lowest or highest performing children. Low(PALM), prediction from PALM to low numeracy (PENS); Low(Vocabulary), prediction from Vocabulary to low numeracy (PENS); High(PALM), prediction from PALM to high numeracy (PENS); High(Vocabulary), prediction from Vocabulary to high numeracy (PENS).

extreme than at the high extreme, a pattern which is similar to that shown in Figure 2 for the prediction from low general vocabulary to low numeracy.

3.3 The role of parental SES

Finally, in research question four, we address the role of parental SES (education and income) in prediction of numeracy skills based on this representative sample. As a first step, we computed correlations with the parental SES measures. These are shown rows 10–13 of Table 2. All correlations are significant, though weak ($r = 0.10$–0.15). The correlations are consistently slightly highest for mother's income. A visualization of the relation of skills to background characteristics is presented in Figure 4, which is based on standardized scores ($M = 0$, $SD = 1$) for numeracy, math vocabulary, general vocabulary, and rhyme detection. Across all four child measures children's scores increased with mother's educational attainment ($p < 0.05$). The difference in math vocabulary between children with mothers with only a primary education and those whose mothers have a postgraduate degree is $d = 0.68$, which is substantial. The difference in numeracy is $d = 0.45$, which is lower, but still considerable.

The apparent discrepancy between the large SES differences based on mean scores for each educational group (Figure 4) and the low correlations for SES (Table 2) is due to the fact that the most substantial SES effect is at the low extreme of parental education (see Figure 4), which was much less common (13% of mothers and 14% of fathers) than the two middle groups. The low frequency leads to less influence on the correlation. Thus, these two analyses suggest that overall SES effects are modest, but most substantial at the low extreme of parental education.

To investigate the question further, we ran seven additional regression models, adding the SES variables as co-variates (see Table 4). A comparison between the models (Tables 3, 4) shows no differences in the total amount of variance explained between the models with or without parent education and income. Only trivial changes are present for individual predictors in models one to six. Both age and sex continue to significantly predict numeracy skills in the full model.

Thus, mothers' education is statistically associated with numeracy outcomes in all analyses except the final model where all predictors are included; in this model mothers' education is only marginally associated with numeracy. Fathers' education and the income variables are not related to numeracy. Taken together, these results suggest a very limited role for SES variables in this large, Danish-representative sample. For comparability with other

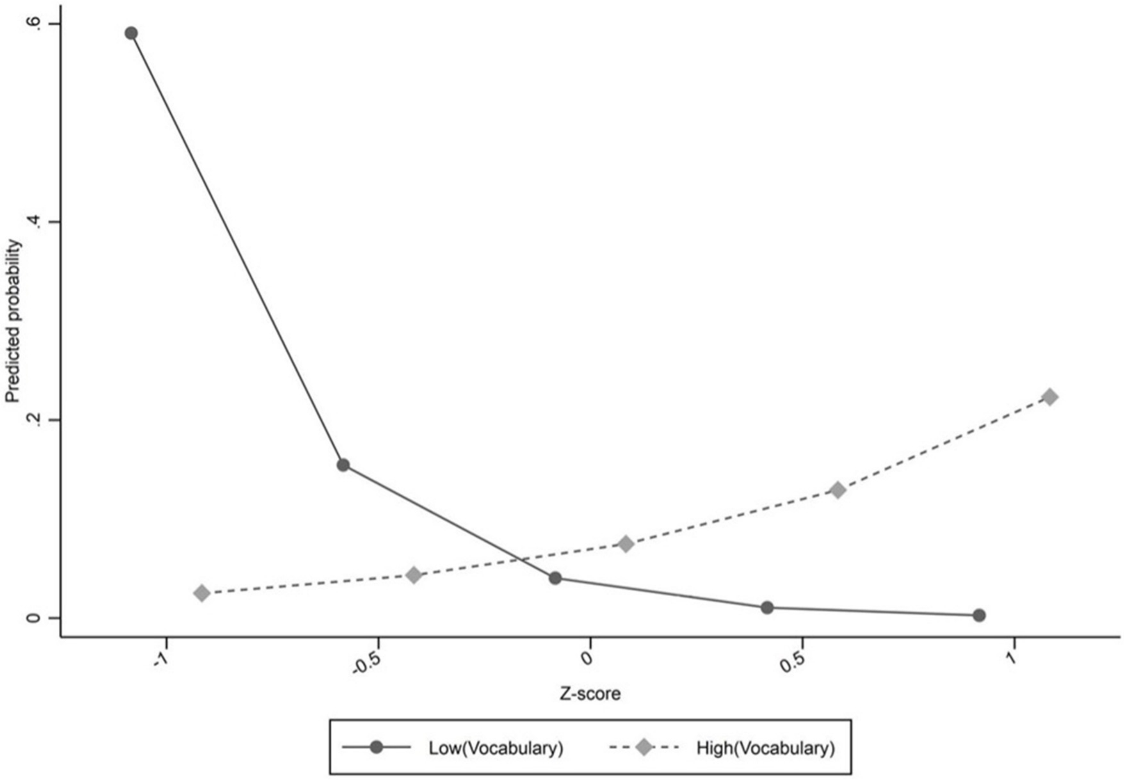

FIGURE 3
Prediction of low- and high-performing math language. Low/high math language is defined as the 20% lowest/highest percentile. The graph is based on a multilevel mixed-effects logistic regression model with multiple imputation. The y-axis displayed the predicted probability of being among the lowest or highest performing children on math language. Low(Vocab), prediction from general vocabulary to low math language; High(Vocab), prediction from general vocabulary to high math language.

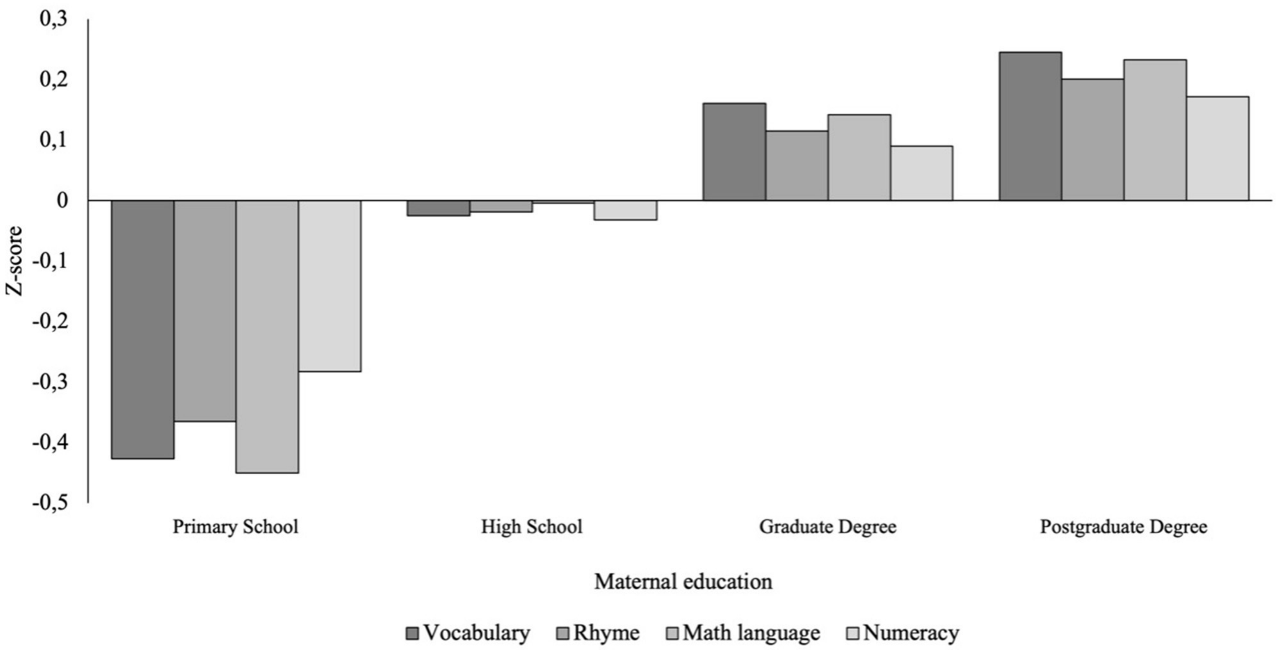

FIGURE 4
Associations of general vocabulary, rhyme, math language, and numeracy with maternal education.

TABLE 4 Mixed effects regression predicting early numeracy performance controlling for child age, sex and parental SES ($N = 2,271$).

	Model 1		Model 2		Model 3			
	β	SE	β	SE	β	SE		
Age	0.50***	(0.02)	0.46***	(0.02)	0.51***	(0.03)		
Sex	0.01	(0.03)	−0.01	(0.03)	−0.01	(0.03)		
MOT education	0.06***	(0.02)	0.06***	(0.02)	0.07***	(0.02)		
FAT education	0.01	(0.02)	0.03	(0.02)	0.01	(0.02)		
MOT income	0.01	(0.02)	0.01	(0.01)	0.01	(0.01)		
FAT income	0.01	(0.01)	0.01	(0.01)	0.01	(0.01)		
Vocabulary	0.34***	(0.02)						
Math language			0.39***	(0.02)				
Rhyme					0.34***	(0.02)		
Attention shifting								
Inhibitory control								
Working memory								
Conditional R∧2	0.43		0.43		0.42			
	Model 4		Model 5		Model 6		Model 7	
	β	SE	β	SE	β	SE	β	SE
Age	0.68***	(0.03)	0.64***	(0.03)	0.64***	(0.03)	0.25***	(0.03)
Sex	−0.01	(0.03)	0.02	(0.03)	−0.01	(0.03)	0.06*	(0.03)
MOT education	0.09***	(0.02)	0.09***	(0.02)	0.09***	(0.02)	0.04+	(0.02)
FAT education	0.03	(0.02)	0.05+	(0.01)	0.04+	(0.02)	0.01	(0.02)
MOT income	0.02	(0.01)	0.01	(0.01)	0.01	(0.01)	−0.01	(0.01)
FAT income	0.01	(0.01)	0.01	(0.01)	0.01	(0.01)	−0.01	(0.01)
Vocabulary							0.16***	(0.02)
Math Language							0.17***	(0.03)
Rhyme							0.14***	(0.03)
Attention shifting	0.15***	(0.02)					0.05***	(002)
Inhibitory control			0.11***	(0.01)			0.09***	(0.02)
Working memory					0.18***	(0.02)	0.08***	(0.02)
Conditional R∧2	0.38		0.38		0.38		0.461	

Standard errors in parentheses. Child outcomes are computed as *z*-scores. $^{+}p < 0.10$, $^{*}p < 0.05$, $^{***}p < 0.001$.

research studies, we ran a model with mothers' education only, and the results are essentially identical.

4 Discussion

The purpose of the present study was to determine how strongly early numeracy skills were associated with general vocabulary, math language, and phonological awareness (as measured by rhyme detection) as well as three measures of executive functions, individually and collectively, in Danish preschool aged children. Although these relations have been examined in other studies, this current study is based on a large, heterogeneous sample and in a cultural setting with universal childcare and less SES variation than in U.S. studies. Although early math skills are not a focus of early childcare programs in either country, there is much more of an emphasis on early language and emergent literacy skills (e.g., rhyming, letter naming) in the US.

4.1 Developmental trends on math language, numeracy, and other skills

The first finding was that a clear developmental trend occurs for math language and numeracy in Danish children from ages 3 to 5. For both domains, mean scores increased with age and so did the variation among children. These findings were based on Danish adaptations of the PALM and PENS-B measures. Although there is no "gold standard" with which to assess concurrent validity of these measures, the relation of both numeracy and math language scales to development, and to SES measures suggests that these instruments are valid measures of math language and numeracy in Danish children. No differences in math language or numeracy were associated with sex in this study, in contrast to the finding of significantly, though only trivially higher numeracy scores ($d = 0.11$) for girls based on the TEAM measure (Sjoe et al., 2019). This difference might reflect differences in the measure, as TEAM also includes computational items of adding and subtracting. Sample size is also highly relevant; the sample in Sjoe et al. was twice the size of the present study. Purpura et al. (2015) also found no significant sex differences, based on a sample size of 124. The three studies together suggest that if a sex difference exists, it is of trivial magnitude.

Second, the DK-US comparison revealed that there were differences in numeracy acquisition (Danish children scored lower) but only very small differences in math language. One explanation for the country difference in numeracy may be SES differences in the samples; the more representative Danish sample has lower average SES than the US sample where more than 40% of the families have at least one parents with a postgraduate degree. In contrast, this is only true for 12% of the families in the Danish sample. However, it is notable that the same pattern is not seen for math language. An alternative explanation for the comparatively scores on lower numeracy is that Danish children may experience less supportive math environment in their families and childcare settings to U.S. children. In the Danish context, most childcares have a child oriented educational strategy and emphasize social development with less attention to academic skills which may reduce math related stimulation (Jensen, 2017). It can only be speculated that this difference is less substantial for math language than for numeracy itself. In any case, however, to our knowledge Danish children's math opportunities in childcares and homes have not been systematically studied yet, so further research is needed to examine this potential mechanism for the divergence in numeracy skills.

4.2 The relation of numeracy with general and math-specific vocabulary, emergent literacy and executive functions

We also utilized regression analyses to investigate the correlation of numeracy with general vocabulary, math language, rhyme detection and three measures of executive functions. The main result was that when adding all outcome measures in the same model (controlling for age and sex), general and math language, literacy and executive function each measures contributed uniquely to the total correlation with of numeracy skills. General and math language had the highest, and the executive functions that lowest contribution to numeracy skills; altogether, the six measures accounted for 46% of the variation in early numeracy. These findings are largely consistent with international studies of the connections among early numeracy, math language and general vocabulary (Purpura and Reid, 2016). There was no difference in statistical prediction between the two measures of vocabulary, which contrasts with Litkowski et al. (2020) who found that the stronger relation was between math language and numeracy skills.

In the current study, phonological awareness (rhyme detection) was also correlated with numeracy, though not with the same magnitude as the correlations with general vocabulary and math language. Nevertheless, the finding was consistent with the conclusions of a meta-analysis by Yang et al. (2021b). However, this result contrasts with those of Purpura et al. (2011), that failed to show phonological awareness as a unique predictor of early numeracy development. It is unclear to what extent the difference in results may be due to the more representative Danish sample (perhaps higher SES children all have phonological awareness at a level useful for numeracy), differences in the phonological awareness measure (Purpura et al. did not include a rhyme detection task), or the distinctive features of Danish phonology which make the segmentation task especially challenging and therefore more revealing of relevant individual differences (Bleses et al., 2010a). Further research is needed to clarify these findings.

Consistent with the substantial literature that has linked EF with numeracy skills (see e.g., Fuhs et al., 2014; Purpura et al., 2017b; Schmitt et al., 2017), we also identified all three measures of EF as correlates of numeracy skills, though the magnitude of the unique contributions were lower than for the language measures, in particular for working memory.

Taken together, the results show that when tested in the same model in a large and representative sample of preschooler, language, literacy, and executive functions are all significant correlates of numeracy skills (Bruce and Bell, 2022). In general, findings from previous research are replicated here, providing substantial evidence for the validity of previous conclusions. In particularly, our findings are consistent with models of numeracy development that propose three pathways of precursor developments: linguistic, quantitative, and spatial attention (LeFevre et al., 2010). The use of a large and nationally representative sample allows for broader generalizations, as the effects are less likely to be sample specific.

4.3 The relation of general and math language and early numeracy as shown at the extremes

The results illustrated in Figure 3 show that low vocabulary is very seldom accompanied by high numeracy (gray diamonds), whereas high vocabulary may or may not be accompanied by high numeracy. Although the present data are only cross-sectional, this pattern is consistent with a relationship in which substantial general vocabulary is developmentally prior to and perhaps even a necessary condition for substantial numeracy development. However, it is not a sufficient condition, in that high general vocabulary is not always accompanied by high numeracy. In contrast, the strong relation between math language to both extremes of numeracy (gray squares)—low to low and high to high—suggests a much closer and symmetrical connection, suggesting possible bidirectional effects. The results illustrated in Figure 4 is similar to that observed for the relation of general vocabulary and numeracy, i.e., as general vocabulary appears to be a necessary but not sufficient condition for math language. Thus, the relations among the skills differ developmentally. General vocabulary—or the skills that underly general vocabulary—provide a necessary basis for learning math-specific vocabulary, but they do not guarantee it. It is likely that environmental factors, such as the presence of math language in parent-child discourse, play an important role here. The relation between math language and numeracy is qualitatively different, with a connection that appears strong across the entire distribution. In this case, there may be bidirectional effects, with language and conceptual development mutually affecting each other. These interpretations, however, are only speculative hypotheses and need to be investigated in longitudinal and/or intervention studies.

4.4 The role of parents' SES for numeracy development

Even though there were considerable SES-related differences in numeracy scores when the associations were examined in isolation through comparison of SES categories, the zero-order correlations only showed weak correlations of education and income measures with numeracy development. The relation was even weaker in the regressions reported in Table 4, which included child age and sex as covariates. Only maternal education was correlated in those analyses. Thus, a central result of the study is that parental education level and income are not substantially correlated with numeracy in the present sample. This finding contrasts with that of some previous studies (e.g., Duncan and Magnuson, 2011; Gjicali et al., 2019). It is possible that that the role of SES for numeracy skills in preschool aged children depends, at least in part, on the cultural, economic and educational context of the sampling. In this view, the lower degree of variation in SES in Denmark compared to the U.S. may have contributed to the finding (Landersø and Heckman, 2017). However, the consistently (and significantly) higher correlations of both general and math vocabulary with maternal education and income suggests that this is not the entire story, and that there are numeracy-specific aspects of the environment not as closely linked to SES that are important. SES differences have been associated with both the quantity and quality of parents' math support and activities. A recent meta-analysis of parents' math practices in the home (such as counting activities and playing board games) has shown relations with math achievement in children ($r = 0.13$); however, the magnitude of the relation is relatively weak (Daucourt et al., 2021). Elliott and Bachman (2018) argue that parents' cognitions, practices, and language use may mediate associations between SES and children's math skills. Besides a lower degree of variation in SES in Denmark, there are also some cultural differences between parents in Denmark and in the US. A comparative study of home learning in the US and Denmark indicated that parents in both countries held similar ability and effort mindsets, but differed significantly in home learning activities, with Danish parents providing significantly fewer family learning activities, learning extensions, and parental time investment than US parents (Justice et al., 2020). In addition, higher levels of effort mindset were not associated with higher levels of parental time investment in Danish parents. Moreover, the cornerstone of educational practice in Danish ECE settings is less focused on learning. Rather, there is a child-centered approach emphasizing child-initiated play, the right to a childhood as a period of life in itself, and children's rights and influence on their own daily everyday life (Kragh-Müller, 2017). These cultural differences between the US and Denmark suggests that although Denmark has a lower degree of variation in SES, the lack of focus on learning has the consequence that the developmental trajectories of children are not very different from those of US children.

4.5 Limitations and future directions

The most important limitation of this study is the use of cross-sectional information to infer developmental changes. This precludes study of individual differences in the trajectory of development, and even more importantly, of using statistical models which are more appropriate for evaluating causal models of the relations among general and math vocabulary and numeracy, such as cross-lagged panel designs. Longitudinal designs would substantially increase the conceptual power of conclusions about language and numeracy. Another limitation is the use of numeracy alone as the math measure, even though the math language measure includes both numeracy and geometry concepts. This difference in scope between the measures may obscure stronger, domain-specific connections.

The present study is based to an important extent on the distinction between general and math vocabulary. The same distinction may also be relevant for studies of the home and childcare environments. In characterizing the environment, rather than assessing specific abilities in the child, it would be appropriate to also include elements of the broader sense of math language, such as syntactic forms.

4.6 Implications for research and practice

The current study shows that both general and math specific vocabulary, emergent literacy and executive functions are correlated with numeracy development in preschool aged children. The link between general and math language at the one hand and numeracy skills on the other hand proved to be strong across the whole distribution and suggests the presence of bidirectional effects. This leads us to the speculation that a broad focus of skill development in preschool is important; beyond general vocabulary and preliteracy, which are already in focus in preschools, adding math vocabulary facilitation activities may be uniquely beneficial across the developmental spectrum at the preschool level. The associations between executive functions and numeracy development similarly suggests that supporting EF in preschools might also benefit children's numeracy development, though this is even more speculative. Such a proposal is further supported by evidence from a new randomized trial that demonstrated that training EF facilitated both EF and math development in preschoolers (Prager et al., 2023).

However, relations between language and early numeracy skills are presumed to begin before the preschool age. A recent study with toddlers indicated moderate to strong concurrent correlations and stability in growth over time of general vocabulary, math language and numeracy skills (Slot et al., 2020). Similarly, Seitz and Weinert (2022) found small but significant positive predictions from 17-month numeracy skills to age 4 mathematical competence. Findings like these suggest that numeracy skill should begin *before* preschool and that a broader approach targeting general vocabulary, math language, phonological awareness and numeracy skills would be optimal. Recent universal school readiness intervention studies targeting both language and numeracy skills in infant/toddler classrooms has demonstrated that general vocabulary, math language as well as early numeracy skills can be improved even before preschool (Bleses et al., 2020a, 2021) but long-term effects on numeracy skills in preschools and later educational achievement have yet to be established.

Data availability statement

Data cannot be shared publicly because part of the data used in this study is from administrative registers provided by Statistics Denmark. Because the data include confidential information on individual citizens, they are not publicly available but placed on a secured server hosted by Statistics Denmark. Data security policy means that independent researchers can apply to Statistics Denmark for access and are required to complete a test on data security policy and rules. TrygFonden's Centre for Child Research will provide assistance for researchers who are interested in getting access to the data. Please contact the centers secretary, Mette Vad Andersen, for more information (mvandersen@econ.au.dk).

Author contributions

DB: Conceptualization, Formal analysis, Funding acquisition, Investigation, Methodology, Project administration, Writing—original draft. MM: Conceptualization, Data curation, Formal analysis, Methodology, Writing—original draft. DP: Conceptualization, Methodology, Writing—review & editing. PD: Conceptualization, Methodology, Writing—original draft.

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Conflict of interest

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

Publisher's note

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