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Event-Based Resilient Control of Multi-Agent Systems in Non-Ideal Communication Networks

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This article focuses on the consensus problem of linear multi-agent systems under denial-of-service attacks and directed switching topologies. With only intermittent communication, the leader-following consensus can be preserved by fully distributed event-triggered strategies. Theoretical analysis shows that the proposed event-triggered resilient controller guarantees the exponential convergence in the presence of denial-of-service attacks and the exclusion of Zeno behavior. Compared to the existing studies where continuous communication between neighboring agents is required, the event-triggered data reduction scheme is provided to tackle the effects of denial-of-service attacks on directed switching topology as well as to avoid continuous communication and reduce energy consumption. The obtained results can be extended to the scenario without a leader. Numerical simulations are finally given to illustrate the effectiveness of the proposed method.

Keywords: event-triggered control, multi-agent systems (MASs), denial-of-service (DoS) attacks, switching topologies, zeno behavior

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1 INTRODUCTION

In past decades, cooperative control of multi-agent systems (MASs) has become one of the significant research interests in many areas, such as artificial intelligent, mathematics, biology, and control engineering (Olfati-Saber and Murray, 2004; Ren et al., 2007; Mei et al., 2016). Consensus control, one of the most important cooperative controls, is to guarantee the agents to achieve an agreement ultimately *via* neighborhood communication. Many related works on leaderless consensus and leader-following consensus (also known as consensus tracking) have been reported by Li et al., (2016),Qin et al., (2016), Li et al., (2018), and Zhu et al., (2021).

In Ma and Yang, (2016), Wang and Yang, (2016), Wang et al., (2020), and Zhang et al., (2021), continuous communication of neighbor agents and control is required to achieve the consensus for MASs. Recently, many efforts were devoted to the consensus problem for MASs with the event-triggered strategies. The event-triggered control method has obvious advantages which can effectively avoid continuous communication and reduce energy consumption and is widely applied in dealing with the scenario in which the bandwidth of the communication channel in a multi-agent system is limited. To design appropriate event-based control laws and the triggering functions are the main works in the event-triggered consensus problem. Event-triggered and self-triggered control schemes for single- or double-integrator agents have been developed by Dimarogonas et al., (2012), Meng and Chen, (2013), Seyboth et al., (2013), and Fan et al., (2015). For MASs with general linear dynamics, Yang et al., (2016), Cheng et al., (2017), Cheng and Li, (2019), Hu et al., (2020), and Xu and Wu, (2021) considered the consensus problem by designing event-triggered communication and control schemes. Specifically, Cheng et al., (2017) studied the leader-following consensus problem and eliminated the need of continuous communication between neighboring agents, where fixed and switching topologies are considered.

Among the studies on consensus of MASs, the security problem has attracted significant attention, as surveyed by Wang and Yang, (2018) and Wang et al., (2021), due to MASs being vulnerable to malicious attacks. The denial-of-service (DoS) attack is one of the common attacks on communication networks (Qin et al., 2018; Pelechrinis et al., 20212010), in which the legitimate members of MASs are unable to access information resources due to the malicious cyber threat actors. Some interesting results for MASs under DoS attacks have been obtained by Wang and Yang, (2018), Wang et al., (2021), Lu and Yang, (2018), and Zhang et al., (2019). To effectively alleviate needless waste of network resources, the event-triggering strategies for MASs under DoS attacks are proposed by Xu et al., (2019), Feng and Hu, (2020), Sun and Yang, (2020), and Tand et al., (2021). Note that the aforementioned results studied in the fixed topologies, though, for the unreliable communication environment under DoS attacks, the research results of using event-triggered methods to the systems under DoS attacks and directed switching topologies are relatively few.

Motivated by the aforementioned discussion, this article addresses the distributed event-triggered communication and control of linear MASs under DoS attacks, where directed switching topologies are considered. Although the control strategy under DoS attack is also studied by Wang et al., (2021), the continuous controller update is required. Moreover, unlike the result in Feng and Hu, (2020), assuming that the connection network is a fixed topology, a switching topology network including intermittent communication is considered, which is more reasonable in practical application. The contribution in this article is summarized as follows: First, an event-triggered secure approach, for solving the leader-following consensus problem of tracking a time-varying state of the leader agent in the presence of DoS attacks, has been developed. Moreover, while event-triggered control problem under DoS attack is considered in this article, the analysis is focusing on the switching topologies case. Furthermore, it is shown that the event-based control strategy is capable of reducing the frequency of communication and controller updates as well as excluding the Zeno behavior and event-triggered controller for leaderless multi-agent systems is further introduced.

The rest of the article is organized as follows: In **Section 2**, the necessary preliminaries are introduced. The achievement of the consensus and the exclusion of the Zeno behavior are presented in **Section 3**. **Section 4** gives the simulation results, and finally the conclusions are drawn in **Section 5**.

2 PRELIMINARIES AND PROBLEM STATEMENT

2.1 Switching Communication Topologies

Among the $N+1$ agents, the communication topology is modeled by a directed switching graph $\mathcal{G}^{\sigma(t)}$, of which one member is the leader (labeled by 0) and the rest N agents are the followers. $\mathcal{G}^{\sigma(t)} = \{\mathcal{G}^s : s \in \mathcal{S}\}$ is a set containing all possible directed connected graphs, where $\sigma(t)$: $[0,+\infty) \to \mathcal{S}$ is defined as the switching signal with an index set $\mathcal{S}$. $\mathcal{A}^{\sigma(t)} = [a_{ij}^{\sigma(t)}] \in \mathbb{R}^{(N+1)\times(N+1)}$ is an adjacency matrix associated with nonnegative adjacency element a_{ij} and zero diagonal elements. Directed edge $(j,i) \in \mathcal{E}^{\sigma(t)}$ denotes that the agent i can obtain information from the agent j but not vice versa. $\mathcal{L}^{\sigma(t)} = [l_{ij}^{\sigma(t)}] \in \mathbb{R}^{(N+1)\times(N+1)}$ is a Laplacian matrix of $\mathcal{G}^{\sigma(t)}$, with $l_{ii}^{\sigma(t)} = \sum_{j\neq i} a_{ij}^{\sigma(t)}$ and $l_{ij}^{\sigma(t)} = -a_{ij}^{\sigma(t)}, i \neq j$. We partition the Laplacian matrix $\mathcal{L}^{\sigma(t)} \in \mathbb{R}^{(N+1)\times(N+1)}$ as $\mathcal{L}^{\sigma(t)} = [0, \mathbf{0}; \mathcal{L}_1^{\sigma(t)}, \mathcal{L}_2^{\sigma(t)}]$, where $\mathcal{L}_1^{\sigma(t)} = [\mathcal{L}_{1ij}^{\sigma(t)}] \in \mathbb{R}^{N\times 1}$ and $\mathcal{L}_2^{\sigma(t)} = [\mathcal{L}_{2ij}^{\sigma(t)}] \in \mathbb{R}^{N\times N}$. Note that $\mathcal{L}_2^s$ $(s \in \mathcal{S})$ is a nonsingular M-matrix that satisfies $\Sigma^s \mathcal{L}_2^s + \mathcal{L}_2^{sT} \Sigma^s > 0$, $\forall s \in \mathcal{S}$, where $\Sigma^s = \mathrm{diag}\{\xi_1^s, \xi_2^s, \ldots, \xi_N^s\}$, $\xi^s = [\xi_1^s, \xi_2^s, \ldots, \xi_N^s]^T = (\mathcal{L}_2^{sT})^{-1}\mathbf{1}_N$.

Assumption 1: *The connection digraph $\mathcal{G}^{\sigma(t)}$ is fixed on every interval $[t_m, t_{m+1})$, $m = 0, 1, 2, \cdots$. The leader node exists as a directed path to at least one follower.*

Assumption 2: Topology-dependent average dwell time τ_{as} satisfies

$$N_\sigma^s(t,T) \leq N_0^s + \frac{T^s(t,T)}{\tau_{as}}, \tag{1}$$

where $N_\sigma^s(t,T)$ represents the number of switching of the sth topology over (t, T), $s \in \mathcal{S}$, $N_0^s > 0$ denotes the mode-dependent chattering bounds, and $T^s(t, T)$ is the total running time of $\mathcal{G}^s$.

2.2 Intermittent DoS Attack Constraints

DoS attack refers to a class of attacks where an adversary renders certain or all components of an inaccessible control system. The DoS attacks can simultaneously affect both the measurement and control channels, which leads to the loss of data availability. One basic assumption of this article is that during the time DoS is active, only the exchange information among the group agents is interrupted, and thus the interaction information is inaccessible. The time series is shown in **Figure 1**.

There are two states in a complete time sequence. One is the normal communication period without any malicious actions, and the other is the period interfered by DoS attacks. Here, the two states are defined as follows: when $t \in \Omega_I$, intermittent communication failure occurs in the switching topologies, and thus the agent cannot obtain neighbor information; and when $t \in \Omega_C$, all agents can receive neighbor information, in which $\Omega_I = \cup_{h\in\mathbb{N}_+}[t_h, t_h^1)$, $\Omega_C = \cup\, [t_0, t_1) \cup_{h\in\mathbb{N}_+} [t_h^1, t_{h+1})$, and there exist the non-overlapping subintervals $[t_h^0, t_h^1), \ldots, [t_h^\ell, t_h^{\ell+1}), \ldots, [t_h^{l_h-1}, t_h^{l_h})$ in each interval $[t_h, t_{h+1})$, with $t_h^0 = t_h, t_h^{l_h} = t_{h+1} = t_{h+1}^0$, and $t_h^\ell\,(\ell = 0, 1, \ldots, l_h)$ are the topology switching time instants.

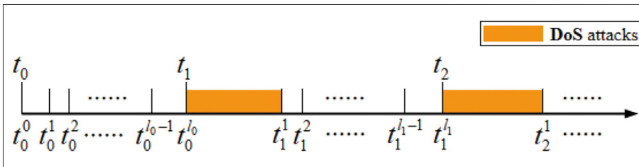

FIGURE 1 | Time series.

2.3 Problem Statement

Consider a group of $N + 1$ agents consisting of N followers and one leader. The dynamics of the ith follower agents are described by

$$\dot{x}_i(t) = Ax_i(t) + Bu_i(t), i = 1, \ldots, N, \tag{2}$$

where $x_i(t) \in \mathbb{R}^n$ and $u_i(t) \in \mathbb{R}^m$ are the state and the control input of agent i, respectively. Matrices A and B are constant matrices with compatible dimensions. It is assumed that the matrix pair (A, B) is stabilizabie.

The dynamics of the leader agent are expressed as

$$\dot{x}_0(t) = Ax_0(t), \tag{3}$$

where $x_0(t) \in \mathbb{R}^n$ is the state of the leader.

In this article, for the directed switching topological structure, our objective is to cope with the leader-following consensus problem of multi-agent systems under DoS attacks by designing event-triggered information transmission and control scheme, which leads $x_i(t) - x_0(t)$ exponentially to converge to 0, and exclude the Zeno behavior.

3 MAIN RESULTS

In this section, we study the consensus problem of multi-agent systems with directed switching topologies in the presence of DoS attacks and present the design of an event-triggered control scheme.

For each follower i, the state estimate is defined as $\hat{x}_i(t) = e^{A(t-t_k^i)}x_i(t_k^i)$, $\forall t \in [t_k^i, t_{k+1}^i)$. The event-triggered controller for each follower agent i is proposed as

$$u_i(t) = \varrho K\left[\sum_{j=1}^{N} a_{ij}^{\sigma(t)}(\hat{x}_i(t) - \hat{x}_j(t)) + a_{i0}^{\sigma(t)}(\hat{x}_i(t) - x_0(t))\right] \tag{4}$$

with

$$\varrho = \begin{cases} 1, & if\ t \in \Omega_C, \\ 0, & if\ t \in \Omega_I, \end{cases} \tag{5}$$

where $K \in \mathbb{R}^{m\times n}$ is the feedback gain matrix to be determined and $\hat{x}_0(t) = x_0(t)$.

The measurement error $e_i(t)$ is defined as

$$e_i(t) = \hat{x}_i(t) - x_i(t), \quad i = 1, \ldots, N. \tag{6}$$

The triggering time instant t_{k+1}^i is determined by the following triggering mechanism:

$$t_{k+1}^i = \begin{cases} t_h^{\ell}, & if\ \mathcal{G}^{\sigma(t)} switches, \\ \inf\{t > t_k^i:\ f_i^{\sigma(t)}(t) > 0\}, & otherwise, \end{cases} \tag{7}$$

where the triggering function is given by

$$f_i^{\sigma(t)}(t) = \|e_i(t)\| - c\|\sum_{j=1}^{N} a_{ij}^{\sigma(t)}(\hat{x}_i(t) - \hat{x}_j(t)) + a_{i0}^{\sigma(t)}(\hat{x}_i(t) - x_0(t))\|, \tag{8}$$

with c is a positive constant to be determined.

Remark 1. The expression (**Eq. 7**) shows that agents update their communication when topology switches or event-triggering condition is satisfied.

Theorem 1. Consider the leader-following multi-agent systems (2) and (3). Suppose that controller (4) and event-triggered function (8) are applied with $K = WP^{-1}$ and $0 < c < \sqrt{\frac{1}{\lambda_{max}(P^{-1})}}$. Then, the leader-following consensus problem can be solved if the following condition holds:

1) There exist matrices $P > 0$ and W such that

$$\begin{bmatrix} \Xi^s & \Sigma^s\mathcal{L}_2^s \otimes BW & 0 & \mathcal{L}_2^{sT} \otimes P \\ * & -I_N \otimes P & \mathcal{L}_2^{sT} \otimes P & 0 \\ * & * & -\frac{1}{2N}I_{N\times n} & 0 \\ * & * & * & -\frac{1}{2N}I_{N\times n} \end{bmatrix} < 0 \tag{9}$$

and

$$AP + PA^T - \alpha P < 0, \tag{10}$$

with $\Xi^s = \Sigma^s \otimes AP + \Sigma^s \otimes PA^T + \Sigma^s\mathcal{L}_2^s \otimes BW + \mathcal{L}_2^{sT}\Sigma^s \otimes W^TB^T + \lambda_s(\Sigma^s \otimes P)$, $\lambda_s > 0$ *and* $\alpha > 0$.

2) *Choose* $\mu_s > b_0/a_0$ *such that* $\sum_{s\in S}(\lambda_s - \frac{\ln\mu_s}{\tau_{as}})T^s(t_0, t_1) - \ln\frac{b_0}{a_0} > 0$ *and* $\sum_{s\in S}(\lambda_s - \frac{\ln\mu_s}{\tau_{as}})T^s(t_h^1, t_{h+1}) - \ln\frac{b_0}{a_0} > \alpha(t_h^1 - t_h)$, *where* a_0 *and* b_0 *will be given later.*

Proof. Define the tracking error of each follower i as $\delta_i(t) = x_i(t) - x_0(t)$. Let $\delta(t)$ and $e(t)$ be the column stack vectors of $\delta_i(t)$ and $e_i(t)$, respectively. By combining **Eq. 2**, **Eq. 3**, and **Eq. 4**, the dynamics of the error closed-loop system can be expressed as

$$\dot{\delta}(t) = \varrho((I_N \otimes A + \mathcal{L}_2^{\sigma(t)} \otimes BK)\delta(t) + (\mathcal{L}_2^{\sigma(t)} \otimes BK)e(t)) \\ (1-\varrho)(I_N \otimes A)\delta(t). \tag{11}$$

Consider the multiple topology-dependent Lyapunov function candidate

$$V_{\sigma(t)}(t) = \varrho\delta^T(t)(\Sigma_{\sigma(t)} \otimes \bar{P})\delta(t) + (1-\varrho)\delta^T(t)(I_N \otimes \bar{P})\delta(t), \tag{12}$$

with $\bar{P} = P^{-1}$.

Denote $a_s = \lambda_{min}(\Sigma^s)$, $a_0 = \min_{s\in S}\{a_s\}$, $\bar{a} = a_0\lambda_{\min}(\bar{P})$, $b_s = \lambda_{max}(\Sigma^s)$, $b_0 = \max_{s\in S}\{b_s\}$, and $\bar{b} = b_0\lambda_{\max}(\bar{P})$. It follows from **Eq. 12** that $\bar{a}\|\delta(t)\|^2 \le a_0\delta^T(t)(I_N \otimes \bar{P})\delta(t) \le a_s\delta^T(t)(I_N \otimes \bar{P})\delta(t) \le V_s(t) \le b_s\delta^T(t)(I_N \otimes \bar{P})\delta(t) \le b_0\delta^T(t)(I_N \otimes \bar{P})\delta(t) \le \bar{b}\|\delta(t)\|^2$.

For $t \in \Omega_C$, we assume $\sigma(t) = s$; at this time, the network communication is not subjected to attacks. Note that $V_{s1}(t) \le b_{s1}\delta^T(t)(I_N \otimes \bar{P})\delta(t) \le \mu_{s1}V_{s2}(t)$, where $s1, s2 \in \mathcal{S}$.

Let $\zeta(t) = [\delta^T(t), e^T(t)]^T$. The derivative of $V_s(t)$ along the trajectories of **Eq. 11** can be expressed as

$$\begin{aligned}\dot{V}_s(t) &= 2\delta^T(t)(\Sigma^s \otimes \bar{P})\left[(I_N \otimes A)\delta(t) + (\mathcal{L}_2^s \otimes BK)\delta(t) + (\mathcal{L}_2^s \otimes BK)e(t)\right] \\ &= 2\delta^T(t)(\Sigma^s \otimes \bar{P}A)\delta(t) + 2\delta^T(t)(\Sigma^s\mathcal{L}_2^s \otimes \bar{P}BK)\delta(t) + 2\delta^T(t)(\Sigma^s\mathcal{L}_2^s \otimes \bar{P}BK)e(t) \\ &\leq \zeta^T(t)M_1^s\zeta(t) - \lambda_s\delta^T(t)(\Sigma^s \otimes \bar{P})\delta(t) + \lambda_{max}(\bar{P})e^T(t)e(t),\end{aligned} \tag{13}$$

where

$$M_1^s = \begin{bmatrix} \Xi_1^s & \Sigma^s\mathcal{L}_2^s \otimes \bar{P}BK \\ * & -I_N \otimes \bar{P}\end{bmatrix},$$
$$\Xi_1^s = \Sigma^s \otimes \bar{P}A + \Sigma^s \otimes A^T\bar{P} + \Sigma^s\mathcal{L}_2^s \otimes \bar{P}BK + \mathcal{L}_2^{sT}\Sigma^s \otimes K^TB^T\bar{P} + \lambda_s(\Sigma^s \otimes \bar{P}).$$

By **Eq. 7**, one has

$$\|e_i(t)\| \leq c\|\sum_{j=1}^N a_{ij}^s(e_i(t) - e_j(t)) + a_{i0}^s e_i(t) + \sum_{j=1}^N a_{ij}^s(\delta_i(t) - \delta_j(t)) + a_{i0}^s\delta_i(t)\|, \tag{14}$$

which implies $\sum_{i=1}^N \|e_i(t)\|^2 \leq 2Nc^2(\|(\mathcal{L}_2^s \otimes I_n)e(t)\|^2 + \|(\mathcal{L}_2^s \otimes I_n)\delta(t)\|^2)$. Furthermore, it follows that

$$\|e(t)\|^2 \leq \zeta^T(t)\begin{bmatrix} 2Nc^2(\mathcal{L}_2^{sT}\mathcal{L}_2^s \otimes I_n) & 0 \\ 0 & 2Nc^2(\mathcal{L}_2^{sT}\mathcal{L}_2^s \otimes I_n)\end{bmatrix}\zeta(t). \tag{15}$$

According to the fact $0 < c^2 < \frac{1}{\lambda_{max}(P)}$, it can be derived from **Eq. 13** and (**Eq. 15**) that

$$\dot{V}_s(t) \leq \zeta^T(t)M_2^s\zeta(t) - \lambda_s\delta^T(t)(\Sigma^s \otimes \bar{P})\delta(t), \tag{16}$$

where

$$M_2^s = \begin{bmatrix} \Xi_2^s & \Sigma^s\mathcal{L}_2^s \otimes \bar{P}BK \\ * & -I_N \otimes P + 2N(\mathcal{L}_2^{sT}\mathcal{L}_2^s \otimes I_n)\end{bmatrix},$$
$$\Xi_2^s = \Sigma^s \otimes \bar{P}A + \Sigma^s \otimes A^T\bar{P} + \Sigma^s\mathcal{L}_2^s \otimes \bar{P}BK + \mathcal{L}_2^{sT}\Sigma^s \otimes K^TB^T\bar{P} + \lambda_s(\Sigma^s \otimes \bar{P}) + 2N(\mathcal{L}_2^{sT}\mathcal{L}_2^s \otimes I_n).$$

Multiplying both sides of M_2^s by $diag\{I_N \otimes P, I_N \otimes P\}$, one gets that $M_2^s < 0$ is equivalent to $M_3^s < 0$ with $M_3^s = [\Xi_3^s, \Sigma^s\mathcal{L}_2^s \otimes BW; *, -I_N \otimes P + 2N(\mathcal{L}_2^{sT}\mathcal{L}_2^s \otimes PP)]$ and $\Xi_3^s = \Sigma^s \otimes AP + \Sigma^s \otimes PA^T + \Sigma^s\mathcal{L}_2^s \otimes BW + \mathcal{L}_2^{sT}\Sigma^s \otimes W^TB^T + \lambda_s(\Sigma^s \otimes P) + 2N(\mathcal{L}_2^{sT}\mathcal{L}_2^s \otimes PP)$. According to Schur complement lemma, from **Eq. 9** one has $M_3^s < 0$. Thus,

$$\dot{V}_s(t) \leq -\lambda_s V_s(t). \tag{17}$$

For $t \in [t_0, t_1)$, when the communication network switches among finite digraphs, it follows from **Eq. 17** that

$$V_{\sigma(t_0^{l_0-1})}(t_1^-) \leq \prod_{\ell=0}^{l_0-1}\mu_{\sigma(t_0^\ell)}\exp\left\{-\sum_{\ell=0}^{l_0-1}\lambda_{\sigma(t_0^\ell)}(t_0^{\ell+1} - t_0^\ell)\right\} \times V_{\sigma(t_0)}(t_0). \tag{18}$$

For $t \in [t_h^1, t_h^{l_h})$, it follows from **Eq. 17** that

$$V_{\sigma(t_h^{l_h-1})}(t_h^{l_h-}) \leq \prod_{\ell=1}^{l_h-1}\mu_{\sigma(t_h^\ell)}\exp\left\{-\sum_{\ell=1}^{l_h-1}\lambda_{\sigma(t_h^\ell)}(t_h^{\ell+1} - t_h^\ell)\right\} \times V_{\sigma(t_h^1)}(t_h^1). \tag{19}$$

When $t \in \Omega_I$, the communication topology of MASs will be affected by DoS attacks, and thus the corresponding communication topology of the MASs will become paralyzed. Then, the time derivative of **Eq. 12** along **Eq. 11** becomes

$$\dot{V}_{\sigma(t)}(t) = \delta^T(t)[I_N \otimes (A^T\bar{P} + \bar{P}A)]\delta(t) \leq \alpha V_{\sigma(t)}(t). \tag{20}$$

It follows from **Eq. 20** that

$$V_{\sigma(t_h)}(t_h^{1-}) \leq \exp\{\alpha(t_h^{1-} - t_h)\}V_{\sigma(t_h)}(t_h). \tag{21}$$

Next, analysis is given for $t \in [t_0, t_{h+1}]$. Note that $V_{\sigma(t_{h+1})}(t_{h+1}) \leq \frac{1}{a_0}V_{\sigma(t_h^{l_h-1})}(t_h^{l_h-})$ and $V_{\sigma(t_h^1)}(t_h^1)) \leq b_0 V_{\sigma(t_h)}(t_h^{1-})$. Then, one gets

$$V_{\sigma(t_{h+1})}(t_{h+1}) \leq \exp\left\{\ln\frac{b_0}{a_0} + \alpha(t_h^1 - t_h) + \sum_{\ell=1}^{l_h-1}\ln\mu_{\sigma(t_h^\ell)} - \sum_{\ell=1}^{l_h-1}\lambda_{\sigma(t_h^\ell)}(t_h^{\ell+1} - t_h^\ell)\right\}V_{\sigma(t_h)}(t_h). \tag{22}$$

Let $\nu_0 = \exp\{\sum_{s\in S}N_0^s\ln\mu_s\}$, $\phi_h = \sum_{s\in S}(\lambda_s - \frac{\ln\mu_s}{\tau_{as}})T^s(t_h^1, t_{h+1}) - \ln\frac{b_0}{a_0} - \alpha(t_h^1 - t_h)$, and $\phi_0 = \sum_{s\in S}(\lambda_s - \frac{\ln\mu_s}{\tau_{as}})T^s(t_0, t_1) - \ln\frac{b_0}{a_0}$. For $t \in [t_0, t_{h+1}]$, based on the aforementioned analysis, one gets

$$V_{\sigma(t_{h+1})}(t_{h+1}) \leq \nu_0\exp\left\{-\sum_{h=1}^{h}\phi_h - \phi_0\right\}V_{\sigma(t_0)}(t_0). \tag{23}$$

Finally, for any $t \geq 0$, there exists a constant $\hbar \geq 0$ such that $t_\hbar \leq t \leq t_{\hbar+1}$. Let $\varrho_0 = \max_{\hbar\in\mathbb{N}}\{t_{\hbar+1} - t_\hbar\}$ and $\underline{\phi} = \min_{\hbar\in\mathbb{N}}\{\phi_\hbar\}$, and then one has

$$\begin{aligned}V_{\sigma(t_\hbar)}(t) &\leq \nu_0\exp\{\alpha(t - t_\hbar)\}V_{\sigma(t_\hbar)}(t_\hbar) \\ &\leq \nu_0\exp\{\alpha\varrho_0\}\exp\{-\hbar\underline{\phi}\}V_{\sigma(t_0)}(t_0), \\ &\leq \nu\exp\{-\chi(t - t_0)\}V_{\sigma(t_0)}(t_0),\end{aligned} \tag{24}$$

where $\nu = \nu_0\exp\{\alpha\varrho_0 + \underline{\phi}\}$ and $\chi = \underline{\phi}/\varrho_0$. Moreover, one obtains $\|\delta(t)\|^2 \leq \frac{\nu\bar{b}}{\bar{a}}e^{-\chi(t-t_0)}\|\delta(t_0)\|^2$, which implies each follower agent follows the leader agent exponentially.

Remark 2. Note that under directed switching topologies and DoS attacks, the triggering mechanism

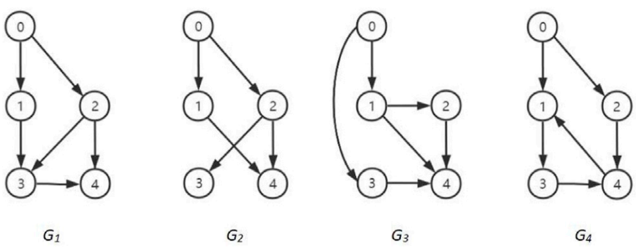

FIGURE 2 | Switching direct graphs among one leader and four follower agents.

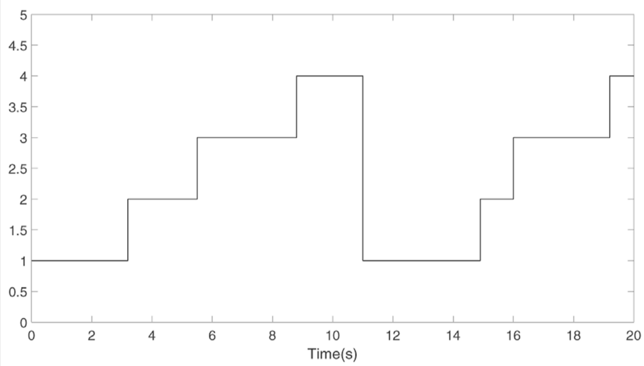

FIGURE 3 | Topology switching signal.

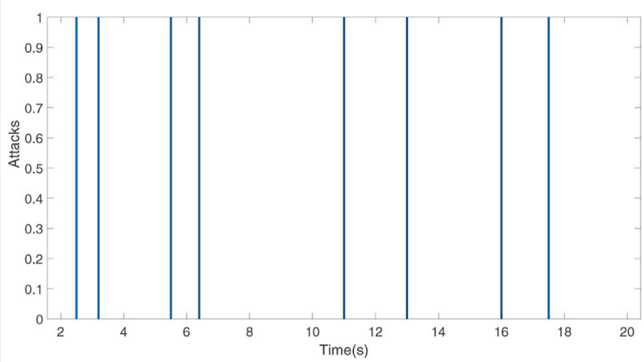

FIGURE 4 | DoS attack signal.

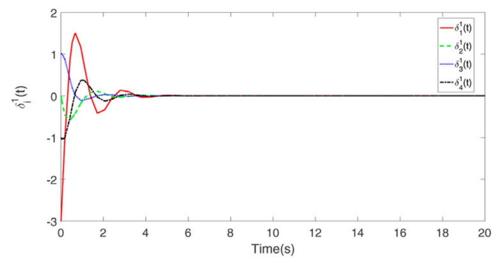

FIGURE 5 | Trajectories of tracking errors $\delta_i^1, i = 1, \ldots, 4$.

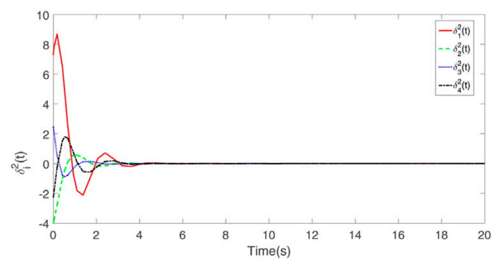

FIGURE 6 | Trajectories of tracking errors $\delta_i^2, i = 1, \ldots, 4$.

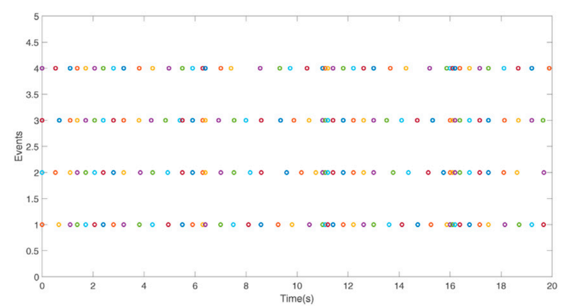

FIGURE 7 | Triggering instants time for the agent i, i = 1, 2, 3, 4.

design and the stability analysis of close-loop systems become even more difficult. Theorem 1 shows that the leader-following consensus problem is solved, for the situation with more practical significance; network intermittent DoS attacks are addressed in the directed switching topologies.

In the following, we show the feasibility of the proposed event-triggered protocol.

Theorem 2 There exists no Zeno behavior in the multi-agent systems (2) and (3).

PROOF. Due to the average dwell-time property, they do not have Zeno behavior when the communication topology switches.

Then, considering two consecutive triggers which are induced by the triggering condition, let $\zeta(t) = \frac{\|e(t)\|}{\|q^{\sigma(t)}(t)\|}$ and the time derivative of $\frac{\|e(t)\|}{\|q^{\sigma(t)}(t)\|}$ for $t \in [t_k^i, t_{k+1}^i)$ is estimated to obtain $\dot{\zeta}(t) \le \frac{\|\dot{e}(t)\|}{\|q^{\sigma(t)}(t)\|} + \frac{\|e(t)\|\|\dot{q}^{\sigma(t)}(t)\|}{\|q^{\sigma(t)}(t)\|^2}$ with the fact that $\|\dot{e}(t)\| \le \|I_N \otimes A\|\|e(t)\| + \|I_N \otimes BK\|\|q^{\sigma(t)}(t)\|$ and $\|\dot{q}^{\sigma(t)}(t)\| \le \|I_N \otimes A\|\|q^{\sigma(t)}(t)\|$. It follows that $\dot{\zeta}(t) \le c_1\zeta(t) + c_2$, where $c_1 = 2\|I_N \otimes A\|$, $c_2 = \|I_N \otimes BK\|$. Then, $\zeta(t)$ satisfies the bound $\zeta(t) \le \varphi(t, \varphi_0)$, in which $\varphi(t, \varphi_0)$ is the solution of $\dot{\varphi} = c_1\varphi + c_2, \varphi(0, \varphi_0) = \varphi_0$, given by $\varphi(t, \varphi_0) = -\frac{c_2}{c_1} + (\frac{c_2}{c_1} + \varphi_0)e^{c_1 t}$. Then, the inter-execution times are bounded by the solution $\iota \in R^+$ of $\varphi(\iota, 0) = c\sqrt{N}$. Since $\varphi(\iota, 0) = \frac{c_2}{c_1}(e^{c_1\iota} - 1)$, we obtain $\iota = \frac{\ln((c_1/c_2)c\sqrt{N}+1)}{c_1}$, which is strictly positive. So, Zeno behavior is excluded for the follower agent i.

Finally, the case that two consecutive triggers are brought by both switching topology and the condition of the triggers is studied. Based on the assumption that the triggering occurs when the topology switches and $e_i(t_h^\ell)$ is reset to zero, so t_h^ℓ can be considered as the triggering time t_k^i. Thus, this interval $[t_h^\ell, t_{k+1}^i)$ can be proven to be lower bounded by a positive constant. The inter-event interval $[t_k^i, t_h^\ell)$ cannot be proven to be lower bounded since the switching may happen immediately after triggers induced by the condition of the triggers.. Assume that the next triggering times be $t_h^\ell, \ldots, t_h^s, t_{k+1}^i$ with $\ell \le s < l_h - 1$. We claim that no Zeno behavior occurs since the interval $[t_h^s, t_{k+1}^i)$ has a positive lower bound as discussed previously.

Remark 3. Note that the control strategies under DoS attack are also studied in many references, and the continuous controller update is required. In this article, an event-triggered control scheme is provided for MASs in the presence of denial-of-service attacks on directed switching topology.

With the aforementioned notations, this subsection is concluded by summarizing the algorithm for generating the $u_i(t)$.

Algorithm 1. Initialization

(1) Set $t_k^i = 0$ and $k = 0$;
(2) send $x_i(t_k^i)$ to its neighbors;
(3) compute $u_i(t) = Kq_i^{\sigma(t)}(t_0)$;
while $t < T$, T is the desired lifespan of the system.
If $t \in \Omega_C$,
send $x_i(t)$ to its neighbors;
if $\|e_i(t_k^i)\| > c\|q_i^{\sigma(t)}\|$ or $\mathcal{G}^{\sigma(t)}$ switches,
(1) update $k = k + 1$ and $t_k^i = t$; and
(2) update $u_i(t) = Kq_i^{\sigma(t)}(t_k^i)$;
end if
else ($t \in \Omega_I$)
$u_i(t) = 0$;
end if
end while

Remark 4. The obtained results are then extended to achieve consensus for multi-agent systems without a leader. For a leaderless consensus problem, we assume that the undirected switching graph $\mathcal{G}^{\sigma(t)}$ is fixed and connected to each time interval $[t_m, t_{m+1})$, $t_0 = 0$, $m \in N$. The adjacency matrix and Laplacian matrix are denoted by $\mathcal{A}^{\sigma(t)} = [a_{ij}^{\sigma(t)}]_{N\times N}$ and $\mathcal{L}^{\sigma(t)} = [l_{ij}^{\sigma(t)}]_{N\times N}$, respectively.

The event-triggered controller for each follower agent i is proposed as

$$u_i(t) = \varrho K \sum_{j=1}^{N} a_{ij}^{\sigma(t)}(\hat{x}_i(t) - \hat{x}_j(t)), \tag{25}$$

where $\hat{x}_i(t) = e^{A(t-t_k^i)}x_i(t_k^i)$, $\forall t \in [t_k^i, t_{k+1}^i)$, ϱ is defined in **Eq. 5**, and $K \in \mathbb{R}^{m\times n}$ is the feedback gain matrix to be determined.

The triggering time instant t_{k+1}^i is determined by the following triggering mechanism:

$$t_{k+1}^i = \begin{cases} t_h^\ell, & if\ \mathcal{G}^{\sigma(t)} switches \\ \inf\{t > t_k^i:\ f_i^{\sigma(t)}(t) > 0\}, & otherwise. \end{cases}, \tag{26}$$

where the triggering function is given by

$$f_i^{\sigma(t)}(t) = \|e_i(t)\| - c\|\sum_{j=1}^{N} a_{ij}^{\sigma(t)}(\hat{x}_i(t) - \hat{x}_j(t))\|, \tag{27}$$

with $e_i(t) = \hat{x}_i(t) - x_i(t)$ and $0 < c < \sqrt{\frac{1}{\lambda_{max}(P^{-1})}}$.

The feedback gain matrix is defined as $K = WP^{-1}$. If there exist matrices $p > 0$ and W such that the following inequalities hold:

$$\begin{bmatrix} \Xi^s & (\mathcal{L}^s)^2 \otimes BW & 0 & \mathcal{L}^s \otimes P \\ * & -I_N \otimes P & \mathcal{L}^s \otimes P & 0 \\ * & * & -\frac{1}{2N}I_{N\times n} & 0 \\ * & * & * & -\frac{1}{2N}I_{N\times n} \end{bmatrix} < 0 \tag{28}$$

and

$$AP + PA^T - \alpha P < 0, \tag{29}$$

where $\Xi^s = \mathcal{L}^s \otimes AP + \mathcal{L}^s \otimes PA^T + (\mathcal{L}^s)^2 \otimes BW + (\mathcal{L}^s)^2 \otimes W^T B^T + \lambda_s(\mathcal{L}^s \otimes P)$.

As well as, we choose $\mu_s > b_0/a_0$ such that $\sum_{s\in S}(\lambda_s - \frac{\ln\mu_s}{\tau_{as}})T^s(t_0, t_1) - \ln\frac{b_0}{a_0} > 0$ and $\sum_{s\in S}(\lambda_s - \frac{\ln\mu_s}{\tau_{as}})T^s(t_h^1, t_{h+1}) - \ln\frac{b_0}{a_0} > \alpha\ (t_h^1 - t_h)$, where $a_0 = \min_{s\in S}(\lambda_{\min}(\mathcal{L}_s))$ and $b_0 = \max_{s\in S}(\lambda_{\max}(\mathcal{L}_s))$. Then, there exists the event-triggered controller (25) such that leaderless consensus objective is ensured.

Denote $\xi(t) = (\xi_1^T(t), \ldots, \xi_N^T(t))^T$, where $\xi_i(t) = x_i(t) - \frac{1}{N}\sum_{j=1}^N x_j(t)$, then one has $\xi(t) = (M \otimes I_n)x(t)$, where $x = [x_1^T, \ldots, x_N^T]^T$ and $M = I_N - \frac{1}{N}11^T$. Denote $e(t) = (e_1^T(t), \ldots, e_N^T(t))^T$. The dynamics of the error closed-loop system can be further expressed as:

$$\begin{aligned}\dot{\xi}(t) = &\varrho((I_N \otimes A + \mathcal{L}^{\sigma(t)} \otimes BK)\xi(t) + (\mathcal{L}^{\sigma(t)} \otimes BK)e(t)) \\ &(1-\varrho)(I_N \otimes A)\xi(t),\end{aligned}. \tag{30}$$

Let $P = \bar{P}^{-1}$. Using the following Lyapunov function candidate,

$$V_{\sigma(t)}(t) = \varrho\xi^T(t)(\mathcal{L}^{\sigma(t)} \otimes \bar{P})\xi(t) + (1-\varrho)\xi^T(t)(I_N \otimes \bar{P})\xi(t) \tag{31}$$

and following the similar step in the proof-of-leader-following consensus problem, the leaderless consensus of the multi-agent system can be obtained, and the Zeno behavior can be excluded.

4 SIMULATION EXAMPLE

In this section, we present an example to verify the effectiveness of the theoretical results. A network of five agents with a communication graph $\mathcal{G}_{\sigma(t)}$ is shown in **Figure 2**.

The topology switching signal and the DoS attack signal are determined by **Figures 3**, **4**, respectively.

The parameters are selected as $A = [-1.175, 0.987\ 1; -8.458, -0.877\ 6]$ and $B = [-0.194, -0.035\ 93; -19.29, -3.803\ 6]$. The initial conditions are given as $x_0(0) = [0,-0.5]^T$, $x_1(0) = [-3,-6.8]^T$, $x_2(0) = [0,-4.5]^T$, $x_3(0) = [1, 2]^T$, and $x_4(0) = [-1,-2.8]^T$. The matrix K by solving **Eq. 9** and **Eq. 10** in Theorem 1 is derived as $K = [23.792\ 3, -0.189\ 8; -120.689\ 0, 1.020\ 4]$, and the related parameters with controller design are selected as $\lambda_1 = 1.45$, $\lambda_2 = 1.53$, $\lambda_3 = 1.62$, $\lambda_4 = 1.58$, $\alpha = 2.5$, and $c = 0.15$.

The consensus tracking errors are exhibited in **Figures 5**, **6**, in which the multi-agent systems (2) and (3) are subjected to DoS attacks. The triggering instants of each agent are shown in **Figure 7**. Based on the aforementioned analysis, the event-triggered controller adopted in this article can effectively reduce the number of communications, and the leader-follower consensus can be achieved under DoS attack.

5 CONCLUSION

One kind of the typical consensus problem of the linear multi-agent system under DoS attacks has been developed in this article. With the directed switching communication topologies, fully distributed event-triggered strategies are proposed. It is shown that secure leader-following consensus can be achieved, and the Zeno behavior is ruled out in the presence of DoS attacks. The obtained results are then extended to achieve consensus for multi-agent systems without a leader. Future work will try to extend the event-based consensus control results to the nonlinear multi-agent system with a leader of nonzero inputs under DoS attack.

DATA AVAILABILITY STATEMENT

The original contributions presented in the study are included in the article/Supplementary Material; further inquiries can be directed to the corresponding author.

AUTHOR CONTRIBUTIONS

JL contributed to methodology and writing—original draft preparation; LS contributed to conceptualization, methodology, and writing—original draft preparation; XW contributed to supervision and writing—review and editing; and DG contributed to software and validation.

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Conflict of Interest: The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.