



Epistemic Perspectives and Communicative Acts

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Searle (Speech Acts, 1969) introduced his famous distinction between constitutive and regulative rules that together define felicity conditions of speech acts. Regulative rules are normative rules, whereas constitutive rules define what counts as a performance of a speech act. In this paper we demonstrate with the example of assertions and referential uses of definite description that simple regulative rules can be given to speech acts that hold only on a core of well-behaved utterance situations. From this core, extended uses can be derived based on epistemic paths that are defined by the epistemic perspectives of speaker and hearer. As the use of speech acts get extended to a wider class of utterance situations, conflicts with the constitutive rules can emerge. We show that the extended uses are nevertheless felicitous. We represent epistemic relations in a possible worlds framework, and take an interactional approach that considers speech acts as part of joint communicative acts.

Keywords: speech acts, common knowledge, epistemic perspective, referential acts, assertions

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1 INTRODUCTION

Pragmatics is often defined as the study of language in context (see Kortt and Perry, 2020, Sec. 4), and, in particular, the study of the *relation of signs to interpreters* (Morris, 1938, p. 6). *Context* is a multifaceted concept that includes, among other things, the physical environment, social relations, the dialogue history, and epistemic states of the interlocutors. In this article, we address the dependencies between felicity conditions of communicative acts and the epistemic relations between interlocutors, i.e., their knowledge about each other and the facts of the world. Central to our approach will be the assumption that communicative acts are organized as action–response pairs (*joint projects*, Clark, 1996) that need to be coordinated between speaker and hearer.

Suppose one undertakes it to define felicity conditions of, for example, the speech act of *asserting*, then the question arises whether the requirement that speakers know that *p*, if they assert *p*, is part of the definition of the speech act, or merely a normative rule imposed by general requirements about cooperative communication (Grice, 1975). Searle (1969) introduced the famous distinction between *regulative* and *constitutive* rules that govern the use of speech acts. Constitutive rules are defining rules that say which linguistic utterances count as performances of a certain speech act type. Regulative rules are normative rules that say how a speech act *should* be performed. In Searle's classification, regulative rules include constraints on the speaker's or hearer's information state. For example, the act of asserting is subject to the regulative rule that speakers must believe what they assert to be true (Searle, 1969, p. 66). As constitutive rules state requirements particular of certain speech acts, and regulative rules general requirements of rational behavior, one would like constitutive requirements to be weak, and regulative rules to be powerful and applicable to as wide a range of speech acts as possible. In this article, we concentrate on two communicative acts that seem, at first, little related to each other: the illocutionary act of *asserting* and the locutionary act of

referring to a specific object with a definite description. We assume that they are constituted by the following minimal rules:

- 1) **Assertion.** The utterance of a sentence expressing proposition φ is a legitimate communicative act given the state of affairs represented by a model m if, and only if φ is true in m .
- 2) **Referential definite description.** The utterance of a definite description ‘*the φ* ’ with the aim of referring to a referent r is a legitimate communicative act given the state of affairs represented by a model m if, and only if $\varphi(r)$ is true in m .

Clearly, these rules on their own cannot guarantee the felicity of their communicative acts. Clark and Marshall (1981) argued that successful referential uses of definite descriptions require that $\varphi(r)$ is common knowledge between speaker and hearer. Furthermore, there should only be one object for which $\varphi(r)$ holds. For assertions the speaker should know that φ holds, and the hearer should believe it at least possible. Otherwise, asserting φ may lead to false beliefs, or fail to convince the hearer of the truth of φ . However, the speaker may assert φ exactly for the purpose of creating such a false belief, and the hearer may see through it and keep quiet. Assuming that ‘*tell*’ refers to an act of assertion, the following examples seem to be in conflict with the constitutive rule (1).

- 3) Leo told me that it is snowing in the Alps, but I knew that the snowing had stopped.
- 4) Leo told me that it is snowing in the Alps, but I knew that she is lying.

If (1) is correct, then no utterance of a sentence with propositional content φ should count as an assertion if φ is false. However, in (3) it seems fine to report that someone (Leo) asserted a proposition φ (snowing in the Alps) although the person reporting this act knows that φ is false. Example (4) shows that an utterance with meaning φ can be reported as an assertion even if the person uttering it is known to disbelieve φ . This shows that constitutive rules cannot be understood as semantic meaning components of reported assertions such that ‘*A told B that φ .*’ would mean that *there is an event e which is an utterance event with speaker A and addressee B and propositional content φ for which rule (1) holds.* Nevertheless, the constituting rules must play some role in reported utterance events.

With Searle, we assume that constitutive and regulative rules define speech acts as social institutions. They are a form of conventional linguistic behavior. We postulate that this behavior is defined for a core of perfect communicative situations in which interlocutors can entertain only true beliefs and are assumed to be fully cooperative. The constitutive rules only apply here. From this core, communicative acts are extended to more complex and possibly non-cooperative utterance situations via epistemic paths that involve changing perspectives between interlocutors. For example, in (3) the speaker S who reports Leo’s utterance believes that from Leo’s perspective constitutive rule (1) is satisfied, and, hence, that the utterance can be called an assertion from Leo’s perspective. We assume that the path from S ’s to Leo’s perspective allows S to call

Leo’s utterance an ‘*assertion*’, although the constitutive rule (1) is violated from S ’s own perspective. In (4), the constitutive rule (1) is violated from both the speaker S ’s perspective and from Leo’s perspective. However, Leo must think that from S ’s perspective it is satisfied. Hence, it is the path from S to Leo to S that allows S , or us as readers of (4), to classify Leo’s utterance as assertion. However, paths can be more convoluted than suggested by (3) and (4) alone. Suspicions may introduce circular paths and mutual mistrust in the validity of constitutive rules. We show also for these situations how epistemic paths can justify the classification of utterances as assertions.

We present a theory that explains how epistemic paths can give rise to felicitous joint communicative acts that extend beyond the epistemic core of perfect utterance situations. In contrast to Searle, we take an interactional perspective on speech acts (see Clark, 1996) where speaker and hearer have each to perform their own required act: the speaker performs an utterance act that is followed by an appropriate response of the hearer. We introduce two epistemic felicity constraints that decide whether a joint communicative act is consistent with the interlocutor’s beliefs: a *licensing* constraint and a *uniqueness* constraint. *Licensing* requires that the joint act is possible from the interlocutor’s perspectives, and *uniqueness* that the hearer’s response is uniquely determined by the speaker’s utterance act. We will see how the constraints eliminate infelicitous communicative acts when joint acts are extended to new epistemic situations.

In the next section, we present a general format for the representation of constitutive rules for speech acts. We then consider referential uses of definite descriptions in more detail and demonstrate how epistemic paths allow extended uses outside the communicative core situations. In particular, we consider the examples discussed by Clark and Marshall (1981) that are supposed to show that felicitous references to an object r with *the φ* require that $\varphi(r)$ is common knowledge. We show that this has only to be true for communicative core situations. In **Section 5**, we introduce the formal model. We represent utterance situations and epistemic states in a possible worlds framework of knowledge and belief (e.g., Hintikka, 1962; Barwise, 1989; Fagin et al., 1995; Gerbrandy, 1998; Baltag et al., 2008), building up, in particular, on (Benz, 2008, 2012). We construct the class of situations in which referential uses of definite descriptions are felicitous, first for the core situations that satisfy common knowledge of true beliefs and cooperativity, then for situations that show an internal hierarchical structure in which the utterance situation is connected to a core situations only via epistemic paths. Throughout, we discuss examples of assertions and referential uses of definite descriptions in parallel. Finally, in **Section 6**, we return to the introductory examples, and discuss wider ramifications of the proposed account for speech act theory.

2 REPRESENTING CONSTITUTIVE RULES AS JOINT PROJECTS

Searle (1969, Sec. 2.5) illustrates the difference between constitutive and regulative rules with the rules of Chess. The

rules of Chess are a paradigmatic example of constitutive rules, the main purpose of which it is to define what counts as a move of the game. In addition to constitutive rules there may also be regulative rules, for example, that the players should not smoke and abstain from distracting behavior. However, these rules do not define chess. As an example of a constitutive rule, Searle (1969, p. 34) cites the rules for *checkmate*. In general, these rules take into account only the position of pieces on the chess board. Some rules may also take into account the game history, for example, the rule of *castling*. For example, moving the White King from his start position¹ two squares to the right and the Rook from its start position to the left of the King counts as legal chess move called *castling kingside* if King and Rook had not moved before, none of the squares between them are occupied, and the King does not move out of, through, or into check. If this rule that defines the legal move of kingside castling in chess were given to a program that checks the moves of players, then any violation of its conditions would mean that the program would reject the move as a move of chess. Nevertheless, we can, without contradiction, make statements as in (5) and (6), which are analogous to (3) and (4).

- 5) Leo castled kingside, but I knew that the King had moved before.
- 6) Leo castled kingside, but I knew that she is cheating.

As in the case of speech acts, exploiting the different perspectives of people involved can explain why one can call a move ‘*castling*’ although it violates its defining rules. In (5), the move may seem legal from Leo’s perspective, or from the perspective of an observer who does not know the history of the game. In (6), the move may seem legal from an outside observer’s perspective, or the violation may go unnoticed from the opponent’s perspective. Also in (7), the speaker can describe what he did as *castling kingside*.

- 7) I castled kingside. Luckily, my opponent didn’t remember that the King had moved before.

The speaker could not say ‘*I moved the pieces as if I castled kingside*,’ or ‘*I pretended to castle kingside*.’ He has to say that he castled kingside, although one could say that he pretended to perform a legal move.

There seem to be the same pragmatic mechanisms at work that widen the meaning of ‘*castling*’ and the meaning of ‘*asserting*’. However, playing chess differs in important respects from conversation. Chess is a game with strictly opposed players, whereas we assume with Grice (1975) that an unmarked conversational situation is one where speakers and hearers are cooperative. Chess is a game without private information, i.e., whatever happens in the game as well as the positions of the pieces on the board are shared knowledge between players. In

a typical dialogue situation the knowledge of speaker and hearer differ. And performing a certain speech act, for example, *asserting*, requires the speaker to have more knowledge than the hearer. A further difference is that chess games can be described as sequences of moves by the White and Black players. It has been argued forcibly by, for example, Clark (1996) that conversation is a sequence of joint coordinated actions, i.e., that each communicative act performed by the speaker needs a corresponding communicative act on the hearer side to be completed. These pairs of communicative acts have been called ‘*joint projects*’ Clark (1996).² We follow this line of research and represent communicative acts as triples consisting of a model m , a communicative act a performed by the speaker, and a response act r by the hearer. Hence, each joint project is a set of triples $\langle m, a, r \rangle$. We call the triples joint communicative acts. For assertions, we assume that the speaker’s act is an utterance of a sentence s with some propositional content φ , and that the hearer reacts with a grounding act that updates the common ground with the fact that the speaker asserted φ . The constitutive rule (1) for assertions then translates into the following representation (8).

- 8) **Assertion.** Let \mathcal{M} be a set of models, \mathcal{L} a set of sentences of a given language, and Φ a set of logical forms for sentences of \mathcal{L} . Asserting sentence s with propositional content φ in situation m is a legal communicative act if φ is true in m . We identify the joint project of asserting s with meaning φ with $p_{ass}(s, \varphi) = \{\langle m, s, \varphi \rangle \mid m \in \mathcal{M} \text{ and } m \models \varphi\}$.

Note that each sentence s defines its own joint project. Hence, the classification into joint projects is more fine-grained than the classification into speech acts. This is also true of the following representation of referential uses of definite descriptions. We assume that each pair consisting of a description *the* φ and a referent r define their own joint project.

- 9) **Referential definite description.** Let \mathcal{M} be a set of models, and Φ a set of logical forms. The utterance of a definite description *the* φ with intended referent r in a situation m is a legal communicative act if $\varphi(r)$ is true in m . We identify the joint project of referring to object r with definite description *the* φ with $p_{def}(\text{the } \varphi, r) = \{\langle m, \text{the } \varphi, r \rangle \mid m \in \mathcal{M} \text{ and } m \models \varphi(r)\}$.

The constitutive rules have to be accompanied by regulative rules. Together they define the felicity conditions of a speech act. Here, we only consider felicity conditions that pertain to the epistemic perspectives of speaker and hearer. As mentioned before, we consider two constraints called *licensing* and *uniqueness*.

- 10) **Licensing.** Let p be a given joint project. An utterance act a is epistemically licensed for the speaker, if from the speaker’s perspective doing a can initiate the joint project p in all

¹The chess board is an 8×8 -square with columns named a to h , and rows numbered 1 to 8. The White King’s start position is e1, and the Kingside Rook’s start position h1.

²‘*Joint projects*’ can be seen as a generalization of the notion of *adjacency pairs* (Schegloff and Sacks, 1973).

possible state of affairs m and for all possible belief states of the hearer. An utterance act a is epistemically licensed for the hearer, if from the hearer's perspective doing a can initiate the joint project p in at least one possible state of affairs m and for at least one belief states of the speaker.

- 11) **Uniqueness.** Let p be a given joint project. An utterance act a satisfies the uniqueness condition for p from the speaker's or hearer's perspective, if it holds for all their possible states of affairs m for which doing a can initiate any joint project that a leads to the same hearer response r such that the joint communicative act $\langle m, a, r \rangle$ belongs to p .

If we ask for a deeper reason for these constraints, then the answer is the requirement that interlocutors should not gamble. If *licensing* is violated, then the speaker believes that the attempted speech act may fail. For the hearer, a violation of *licensing* means that the speaker's utterance act is inconsistent with the hearer's beliefs. If *uniqueness* is violated, then it is unclear how to respond to the utterance act. This complete ban on gambling may be too strict a requirement for realistic utterance situations, but, as a logical idealization, it is appropriate for our purposes.³

3 THE REFERENTIAL USE OF DEFINITE DESCRIPTIONS

In this section we consider referential uses of definite descriptions. As mentioned before, there are two closely related problems about the interpretation of communicative acts: the classification problem and the meaning problem. The classification problem stems from the fact that utterances and uses of definite descriptions can be classified as assertions or referential uses although their constitutive conditions are not satisfied. Referential uses of definite descriptions provide examples that are particularly suitable for studying the role of epistemic paths in the classification problem.

In their influential study, Clark and Marshall (1981, C&M) discuss a series of examples that show that reference to an object r with definite description *the* φ can fail although any finite sequence of conditions *the speaker believes that* $\varphi(r)$, *the hearer believes that* $\varphi(r)$, *the speaker believes that the hearer believes that* $\varphi(r)$, *the hearer believes that the speaker believes that* $\varphi(r)$, etc are true. C&M concluded that successful referential uses of definite description require *common knowledge* of $\varphi(r)$. The relevant examples consist of short stories about two protagonists who read the early edition of a newspaper together and discuss the fact that it says that *A Day at the Races*, a movie with the Marx Brothers, is showing that night at the local cinema Roxy. Then,

one of the protagonists, or both learn individually that the movie has been changed to *Monkey Business*. The stories always end with one of the protagonists asking 'Have you ever seen the movie showing at the Roxy tonight?' The question is then whether the definite description 'the movie showing at the Roxy tonight' refers to *Monkey Business*. Version 4 of their examples reads as follows:

On Wednesday morning Ann and Bob read the early edition of the newspaper and discuss the fact that it says that *A Day at the Races* is playing that night at the Roxy. Later, Ann sees the late edition, notes that the movie has been corrected to *Monkey Business*, and marks it with her blue pencil. Still later, as Ann watches without Bob knowing it, he picks up the late edition and sees Ann's pencil mark. That afternoon, Ann sees Bob and asks, "Have you ever seen the movie showing at the Roxy tonight?" (Clark and Marshall, 1981, p. 13, Version 4)

Here, Bob must reason as follows: Ann knows that *Monkey Business* is playing tonight. But she thinks I believe that we both are mutually convinced that *A Day at the Races* is showing. So she must think that I think she refers to *A Day at the Races*. Hence, knowing that *Monkey Business* is showing, and knowing that the speaker knows that *Monkey Business* is showing is not enough to ensure successful reference to *Monkey Business*.

More and more complicated examples can be constructed that show that any finite sequence of sentences 'Ann knows that Bob knows that . . . that *Monkey Business* is showing that night' is not enough to ensure reference to *Monkey Business*. Clark & Marshall arrive at the conclusion that both participants need to know that all sentences of the form (12) have to be true in order to secure reference to *Monkey Business*.

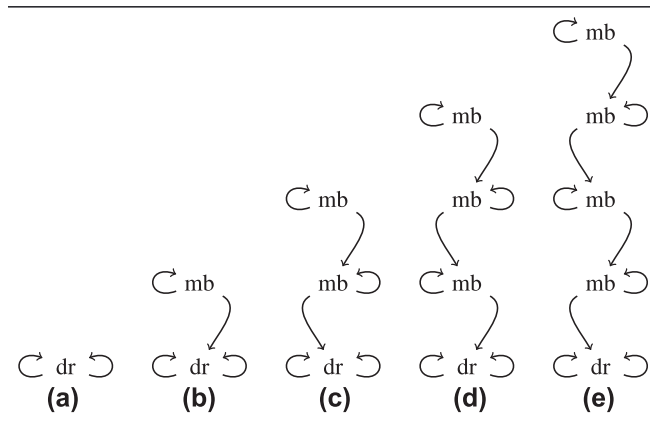
- 12) X_1 knows that X_2 knows that X_3 knows that . . . that X_n knows that *Monkey Business* is showing tonight.

Here, the X_i 's are dialogue participants, and n is any natural number. This condition is equivalent to: It must be *common knowledge* that *Monkey Business* is showing. Common knowledge of $\varphi(r)$ entails all sentences of (12). **Table 1** shows graphical representations of the different epistemic states considered by C&M. Underlying is a possible worlds representation of beliefs, which will be defined in **Section 5**.

We are interested in the question: What does the definite description *the* φ = 'The movie showing at the Roxy tonight?' actually refer to? Each of C&M's scenarios starts with Ann and Bob reading together that *A Day at the Races* is showing. This initial epistemic state is represented by a) in **Table 1**. We can distinguish a reading that is based on *public* information, and one that is based on *private* information. In the a) and the b) situation, where Bob thinks to be in situation a), *the* φ obviously refers to *A Day at the Races*, which is based on shared *public* belief. In situation c), the answer is not as clear cut. Bob may answer 'No, I've never seen *A Day at the Races*. But, you know, the program has been corrected. *Monkey Business* is showing.', because he thinks that Ann thinks that it is public knowledge that *A Day at the Races* is showing. Bob

³This does not mean that we consider ambiguity and lack of understanding to be minor dialogue problems. The *uniqueness* constraint is a regulatory, i.e., normative constraint. Normative rules can be violated with or without intend. Crucial for us is that their logical consequences can be studied without considering repair strategies that apply in case of violations.

TABLE 1 | Different information states considered by Clark and Marshall (1981, C&M, pp. 11–14). Abbreviations: dr: *A Day at the Races* is showing, mb: *Monkey Business* is showing. Each node represents a possible world. Arrows $w \rightarrow v$ say that v is an epistemic possibility in world w ; arrows to the left point to possible worlds of the speaker's information state ('Ann'), arrows to the right to possible worlds of the hearer's information state ('Bob'). b) is Version 2 of C&M; c) is C&M's Version 3; d) is their Version 4; and e) is their Version 5.



may also answer 'Yes, I have. You know, the program has been corrected and *Monkey Business* is showing. I saw the movie last year on TV.' This interpretation of the definite description is based on Bob's *private* beliefs about which movie is showing. This reading involves a repair, as Bob must think that Ann will, at first, interpret the Yes-answer as a confirmation of the proposition that Bob has seen *A Day at the Races*. We are only interested in the interpretation based on public information, that does not involve a repair. In C&M's more complex scenarios, the two readings seem both to be available. We, therefore, modify C&M's examples in a way that favors the *public* reading. The modified examples show that the public reading is available although the conditions about beliefs in (12) may be violated for arbitrarily large n .

In the following scenarios, the question is always what is the referent of the $\varphi = \text{the movie showing at the Roxy tonight?}$ Version 1 in (13) is the basic scenario in which $\varphi(mb)$ is common knowledge between Ann and Bob. The epistemic states are represented graphically in **Table 2**.

13) **Version 1.** On Wednesday morning Ann and Bob read the newsletter on Ann's computer and discuss the fact that it says that *Monkey Business* is playing at the Roxy that night. Later Ann decides that she wants to stay at home. She calls Bob and asks, 'Do you want to watch the movie showing at the Roxy tonight on Netflix with me?'

In the next version, the beliefs of Ann and Bob have not changed, but the truth of $\varphi(mb)$ is not given.

14) **Version 2.** On Wednesday morning Ann and Bob read the newsletter on Ann's computer and discuss the fact that it says that *Monkey Business* is playing at the Roxy that night. Later, a correction was sent saying that, in fact, *A Day at the Races* is playing. Neither Ann nor Bob

notice the correction. Later Ann decides that she wants to stay at home. She calls Bob and asks, 'Do you want to watch the movie showing at the Roxy tonight on Netflix with me?'

In Version 3, Ann learns that $\varphi(dr)$, but Bob's beliefs are unchanged. This does not block the reference to *Monkey Business*.

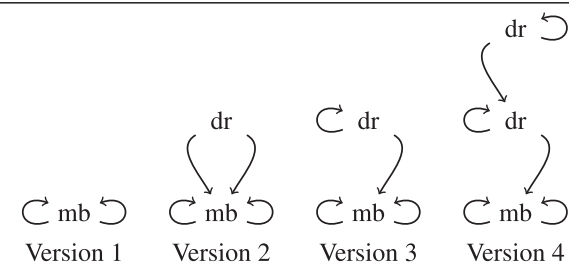
15) **Version 3.** On Wednesday morning Ann and Bob read the newsletter on Ann's computer and discuss the fact that it says that *Monkey Business* is playing at the Roxy that night. Later, a correction was sent saying that, in fact, *A Day at the Races* is playing. Only Ann notices the correction. She doesn't like *A Day at the Races*. She knows that Bob would love to see it, but that he couldn't have noticed the correction. She calls Bob and asks, 'Do you want to watch the movie showing at the Roxy tonight on Netflix with me?'

In Version 4, both Ann and Bob learn that $\varphi(dr)$, and Bob learns that Ann learns it. Again, this does not block reference to *Monkey Business*.

16) **Version 4.** On Wednesday morning Ann and Bob read the newsletter on Ann's computer and discuss the fact that it says that *Monkey Business* is playing at the Roxy that night. Later, a correction was sent saying that, in fact, *A Day at the Races* is playing. Ann notices the correction. Later, Bob reads her email and notices the correction, and notices also that Ann has read it. Bob would love to see *A Day at the Races* but he knows that Ann doesn't like it at all. He wants to please her, but doesn't want her to know that he reads her mail without her knowing it, he calls Ann and asks, 'Do you want to watch the movie showing at the Roxy tonight on Netflix with me?'

In this manner, more and more complex epistemic states can be created in which it holds that X_1 knows that X_2 knows that X_3 knows that . . . that X_n knows that *A Day at the Races* is showing, and, hence, in which X_1 knows that X_2 knows that . . . that X_n knows that *Monkey Business* is not showing.

TABLE 2 | Different information states considered in (13)–(16). Abbreviations: dr: *A Day at the Races* is showing, mb: *Monkey Business* is showing. Each node represents a possible world. Arrows $w \rightarrow v$ say that v is an epistemic possibility in world w ; arrows to the left point to possible worlds of the speaker's information state, arrows to the right to possible worlds of the hearer's information state.



Nevertheless, reference to *Monkey Business* is possible. This leads to the following paradox: Clark and Marshall (1981) showed that successful reference to a referent r requires that it is common knowledge that $\varphi(r)$ holds; common knowledge of $\varphi(r)$ entails all conditions of the form (12); however, the procedure for constructing (13)–(16) shows that all these sentences can be false, and, still, the referential act can be successful. How is this possible?

The graphs in **Tables 1** and **2** point to a solution. The complex states constructed by C&M and by us embed a basic situation in which common knowledge of $\varphi(r)$ is satisfied. In this basic situation, *the* φ refers to r on the basis of public information. The interpretation of *the* φ as r then travels upwards along epistemic paths to the real situation, and licenses this interpretation although the constituting rules are not satisfied. What if we ask about the deeper reason for the interpretation's ability to travel along epistemic paths? If we consider Version 2, then we see that the situation is *indistinguishable* from Version 1 from the perspective of both interlocutors. Hence, the interlocutors should behave identically in both situation. In Version 3, the situation is distinguishable from the core situation in Version 1, but it is still *indistinguishable* for the hearer. This allows the speaker to behave identically in both situations. In Version 4, both interlocutors can see that the situation is different from that of Version 1. However, for the speaker it is *indistinguishable* from that of Version 3, which immediately explains why she behaves identically in both situations, and why the hearer can behave identically in Version 3 and 4. *Behaving identically* means here that the speaker chooses the same utterance act and the hearer the same interpretation. This solution will be further worked out in the following, and transferred to similar problems with assertions.

4 ASSERTIONS

In this section, we consider assertions in situations that show epistemic relations between speaker and hearer similar to those seen with the modified Clark & Marshall scenarios. The examples are taken from (Benz, 2008) and slightly modified for the present discussion. There are two possible states of affairs, either it snows in the Alps (φ), or it does not snow in the Alps ($\bar{\varphi}$). In each example, the classification of the speech act of uttering '*It is snowing in the mountains.*' is of interest. Version 1 in (17) shows the basic situation equivalent to Version 1 for definite descriptions in (13). Version 2 is a case in which the constitutive truth-condition is violated due to a false belief of the speaker. In Version 3 the speaker is lying outright, and Version 4 is a case in which the lie is detected by the hearer. Although the utterance of φ in versions 2–4 violates the constitutive condition of assertions as formulated in (1) and (8), we could, in each case, classify it as an instance of an assertion.

17) **Version 1.** Helga calls up her son Stephan who lives in a small town in the Alps and asks him whether he wants to visit her in Munich. Stephan answers: '*It is snowing in the mountains. So I don't want to drive now.*'

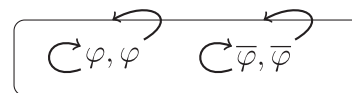
Version 2. Helga calls up her son Stephan and asks him whether he wants to visit her in Munich. Stephan answers: '*It is snowing in the mountains. So I don't want to drive now.*' But he has not checked the weather for some time, and it is now raining and the streets are clear.

Version 3. Helga calls up her son Stephan and asks him whether he wants to visit her in Munich. Stephan has a new girl-friend and prefers to stay at home this weekend. He answers: '*It is snowing in the mountains. So I don't want to drive now.*' However, he knows that it is not snowing and that the streets are clear.

Version 4. Helga calls up her son Stephan and asks him whether he wants to visit her in Munich. Stephan answers: '*It is snowing in the mountains.*' Helga has just talked to her daughter, who lives next to Stephan, and learned from her that it is not snowing and that the streets are clear. She also learned that Stephan has a new girl-friend and prefers to stay at home this weekend.

The epistemic relations in the basic situation is shown in (18). There are two states of the world: one in which it snows in the Alps (φ), and in which the speaker knows that it snows and says so, and another in which it does not snow ($\bar{\varphi}$), and in which the speaker knows it and says so. The hearer does not know which world is the real one, and all this is common knowledge. The state of the world and the possible utterance are coded as a pair of two formulas. In the basic Version 1, the two formulas are identical.

18) The epistemic relations in Version 1 of (17):



In (8), the joint project of asserting a sentence s with meaning φ has been defined as the set of all triples $\langle m, s, \varphi \rangle$ where m is a model that represents the state of the world and makes φ true. The idea behind this representation is that the state of affairs not only represents what is true about the world (the model m), but also what the possible future utterance events are. In (18), if φ is true in m , then the speaker can utter, according to constitutive rule (1), a sentence s with meaning φ , and thereby initiate the corresponding joint project. In the graph in (18), the state of affairs is represented by a pair of formulas, e.g., $\langle \varphi, \varphi \rangle$. The pair φ, φ represents an instance $\langle m, s, \varphi \rangle$ of the joint project of asserting s with meaning φ . The graphs in **Table 3** show the epistemic relations for Versions 2–4 of (17). They are all instances in which the proposition supported by the outer state of affairs and the meaning of the sentence uttered by the speaker are different from each other.

A comparison between **Tables 2** and **3** shows that the graphs are structurally identical except for their respective basic versions. The same reasoning that explains why the use of a definite description *the* φ can count as a referential act with target *Monkey Business* in the situations represented by the top nodes of Versions 2–4 in **Table 2** explains why the utterance of '*It is snowing in the mountains.*' can count as an assertoric act with propositional content φ in the situations represented by the top nodes of Versions 2–4 in **Table 3**. The classification as assertoric act travels along the epistemic path leading from the top node down to the basic situation that properly licenses the assertion.

TABLE 3 | Information states in versions 2 to 4 of (17). The nodes of the graph are pairs of formulas, where the first formula says whether φ or $\bar{\varphi}$ is true in the world, and the second formula represents an utterance by the speaker.

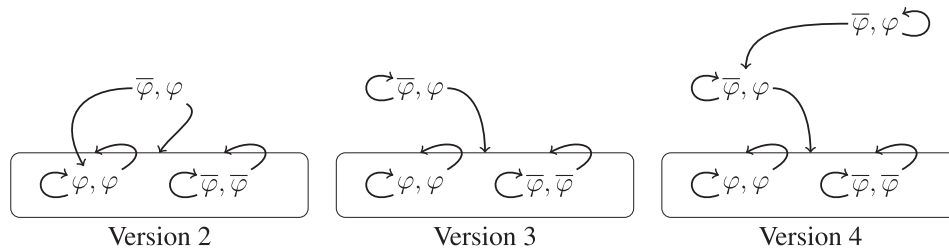
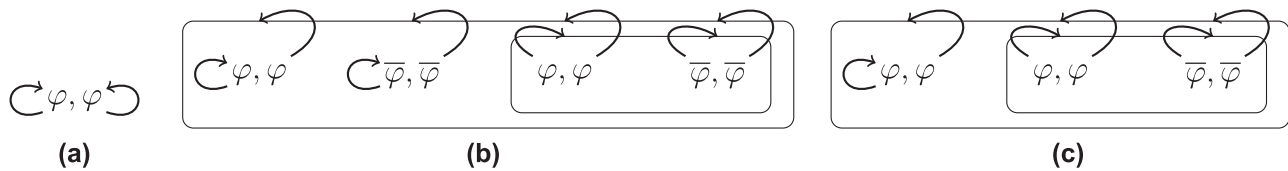


TABLE 4 | Different basic situations for Version 1 of (17).



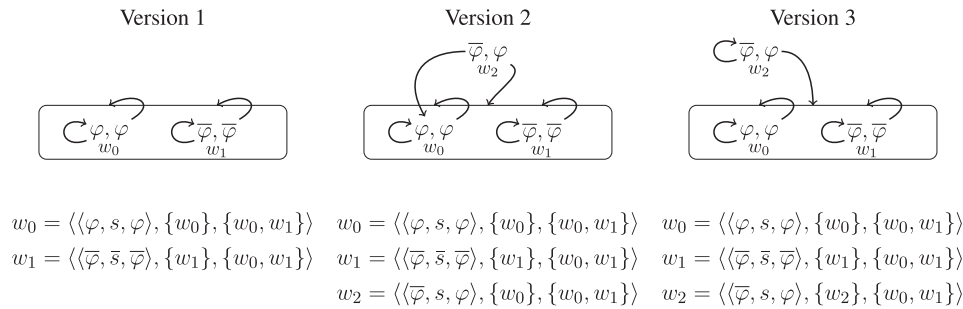
We will introduce the mathematical framework which allows us to handle these examples precisely in **Section 5**. Before we turn to formal representations, we have a closer look at the structure of the epistemic graphs. They can be divided into a base and a hierarchical structure building up on it. The hierarchical part shows descending paths. The bases can differ in their internal structure. As we have seen, the bases for assertions in (18) and that for referential uses of definite descriptions shown in **Table 2** have different structure. **Table 4** shows three further possibilities for the base of the assertoric speech act. The first a) is a copy of the base for the referential use of definite descriptions. As a plausible base for Version 1 of (17) it is ruled out by an additional pragmatic constraint that says that the speaker should not say what is already common belief. However, we do not formalize this constraint so that a) remains a theoretical possibility. In setting up the epistemic graph in (18), we made the assumption that it is known that the speaker knows whether it is snowing, or not. This assumption does not follow, however, from Version 1 of (17). **Table 4b and c** show two possibilities where the hearer thinks it possible that the speaker does not know whether it snows. There are even more possibilities. For example, by bending the hearer's edge going out from the rightmost $\bar{\varphi}, \bar{\varphi}$ -world back to this world, we would have a licit epistemic graph that allows for an assertion of φ in the leftmost φ, φ -world. We will discuss more examples once we have introduced formal representations.

Our task is to explain why a certain utterance can be classified as an assertion in a given node in an epistemic graph. We adopt the following strategy: once it is explained why this classification is justified in a base situation, the classification can travel upward through the hierarchical part of the graph. This means, we can divide our considerations into that of the basic level and that of the higher hierarchical levels. Once the classification problem is solved for the base, the solution for the hierarchical part follows. One characteristic of the bases is the absence of descending paths. This means that all nodes in the bases are connected with each other. This leads to circular structures. We therefore consider circular structures separated from hierarchical ones.

5 THE MODEL

As explained in **Section 2**, we adopt a Clark (1996) perspective and represent *communicative acts* not as isolated acts but as coordinated joint projects consisting of a linguistic act by the speaker and a response by the hearer. A joint project consists of triples $\langle m, a, r \rangle$, where m represents the outer facts of the world, a the speaker's act, and r the hearer's response. This representation is, in general, more fine-grained than the traditional classification of speech acts. For example, we defined the joint project of referring to an object r with definite description *the φ* as the set of all triples $\langle m, \text{the } \varphi, r \rangle$ for which m makes $\varphi(r)$ true ($m \models \varphi(r)$). The *referential use of definite descriptions* then consists of many such joint projects. It consists of all joint projects p for which there is a one-place predicate $\varphi(\cdot)$ and an object r such that $p = \{ \langle m, \text{the } \varphi, r \rangle \mid m \models \varphi(r) \}$. Similarly, we defined the joint project of asserting a sentence s with meaning φ as the set of all triples $\langle m, s, \varphi \rangle$ for which $m \models \varphi$. The phenomenon of *assertive utterances* is then represented by the set of all joint projects p for which there is a sentence s with reading φ such that $p = \{ \langle m, s, \varphi \rangle \mid m \models \varphi \}$.⁴ In the previous section, we simplified the notation. For example, in Version 1 of (17) there are two state of affairs, one in which it is snowing and one in which it is not snowing. We identified them with two formulae, φ and $\bar{\varphi}$. There are two sentences $s = \text{'It is snowing in the mountains'}$ and $\bar{s} = \text{'It is not snowing in the mountains'}$, which were again identified with φ and $\bar{\varphi}$ respectively. Hence, there were two joint projects involved: $p = \{ \langle \varphi, s, \varphi \rangle \}$ and $\bar{p} = \{ \langle \bar{\varphi}, \bar{s}, \bar{\varphi} \rangle \}$.

⁴The representation of the hearer's response by a formula φ is not essential here. We could have represented the same joint project as $p = \{ \langle m, s, \llbracket s \rrbracket \rangle \mid m \in \llbracket s \rrbracket \}$, which would have made the connection to formal semantics even clearer.

TABLE 5 | Worlds and their epistemic graphs for scenarios in (17).

5.1 Possible Worlds and Epistemic Relations

The joint projects do not represent epistemic relations between interlocutors and interlocutors and the world. We adopt a possible worlds representation in which beliefs are modeled as sets of epistemically possible worlds. A world has the form:⁵

- 19) **Possible world:** A *possible world* w is a triple $w = \langle \langle m, a, r \rangle, S, H \rangle$, where $\langle m, a, r \rangle$ is an element of some joint project p , and S and H are sets of possible worlds representing, respectively, the speaker's and hearer's beliefs.

We write S^w for the speaker's information state, and H^w for the hearer's information state in world w . Furthermore, we write $\langle m^w, a^w, r^w \rangle$ or d^w for the joint communicative act represented by w . Example definitions of worlds and their epistemic graphs are shown in **Table 5**.

In standard set theory, there is no w_0 that could satisfy equation $w_0 = \langle \langle \varphi, s, \varphi \rangle, \{w_0\}, \{w_0, w_1\} \rangle$ due to the Axiom of Foundation. We therefore turn to a variant of set theory with Anti-Foundation Axiom (AFA) developed by Aczel (1988). This theory has been used extensively for modeling circular structures (Barwise, 1989; Barwise and Etchemendy, 1989; Barwise and Moss, 1996; Gerbrandy and Groeneveld, 1997; Gerbrandy, 1998; Benz, 2008). We do not go into the intricacies of this theory. We need one important property: in AFA-set theory every system of equations has a unique solution. For example, the equations for the different graphs shown in **Table 5** are systems of equations. We can consider the names of worlds w_0, w_1, \dots as *variables* for which we seek a solution. A solution is a function that maps the variables to ordinary (non-well-founded) sets that satisfy the equations. As we have said, every such system of equations has a unique solution in AFA-set theory. Hence, the worlds shown in **Table 5** are well-defined set-theoretic entities. The property also allows for simple representations of *belief updates*. Propositions can be identified with sets of possible worlds. If an interlocutor X learns that a proposition φ holds, then this can be represented by intersecting the set of worlds that represent X 's beliefs with the set of worlds representing the meaning of φ . We say then that X 's beliefs

have been *updated* with φ . If the proposition is mutually learned, then each interlocutor has to update not only his/her own belief set, but also the belief sets representing the beliefs of others, and this update has to be iteratively applied to each other's beliefs. In terms of systems of equations, this can be modeled by first writing down the original system of equations, and then intersecting all belief sets occurring in the system with the set representing φ . Finally, the modified system of equations has to be solved again. The solution then represents the updated system of beliefs. The results of updating the worlds in **Table 5** with φ , i.e., with $\{w_0\}$, are shown in **Table 6**. The results for Version 1 and 2 follow immediately from the definition. However, Version 3 is not yet accounted for. If we update with $\{w_0\}$, then w_2 should be eliminated from the speaker's belief state, and, therefore, we should expect the empty, i.e., contradictory, belief state after updating w_2 . We will see later how to account for the result shown in **Table 6**.

The update that we just described can be represented by a formal update operator $*$. It models the effect of *mutual learning* some information Y .⁶ In **Eq. 1**, $w * Y$ denotes the update of beliefs in a world w with Y , and in **Eq. 2**, $X * Y$ the update of a belief set X with Y .

$$w * Y := \langle \langle m, a, r \rangle, S * Y, H * Y \rangle \quad (1)$$

$$X * Y := \{v * Y \mid v \in X \cap Y\} \quad (2)$$

The graph of Version 1 of **Table 6** represents $w_0 * \{w_0\}$ with w_0 as in Version 1 of **Table 5**, and the graph of Version 2 of **Table 6** represents $w_2 * \{w_0\}$ with w_2, w_0 defined as in Version 2 of **Table 5**. World w_2 survives in Version 2 as only worlds in belief states are eliminated. If a system of equations represents a belief state, i.e., a set of possible worlds, then updating the system of equations with information Y is equivalent to removing all variables w_i from *both sides* of the system for which the solution $s(w_i)$ is not an element of Y .

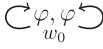
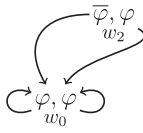
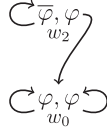
We are now in a position to explain an important modeling decision. Why do possible worlds represent joint communicative acts $\langle m, a, r \rangle$, and not only the state of affairs m ? Let us consider Version 3 in **Table 5**, and let us change the definition of worlds such that only the outer state of affairs is represented. Then Version 3 is represented by the following system of equations:

$$20) \quad w_0 = \langle \varphi, \{w_0\}, \{w_0, w_1\} \rangle, \quad w_1 = \langle \bar{\varphi}, \{w_1\}, \{w_0, w_1\} \rangle, \\ w_2 = \langle \bar{\varphi}, \{w_2\}, \{w_0, w_1\} \rangle$$

⁵For the relation between possible worlds as defined in (19) and *Kripke*-frames for modal logics see **Appendix A.2**.

⁶The notation with $*$ follows (Barwise and Moss, 1996).

TABLE 6 | Worlds and their epistemic graphs for scenarios in (17) after updating with $\varphi = \{w_0\}$.

Version 1	Version 2	Version 3
		
$w_0 = \langle \langle \varphi, s, \varphi \rangle, \{w_0\}, \{w_0\} \rangle$	$w_0 = \langle \langle \varphi, s, \varphi \rangle, \{w_0\}, \{w_0\} \rangle$ $w_2 = \langle \langle \bar{\varphi}, s, \varphi \rangle, \{w_0\}, \{w_0\} \rangle$	$w_0 = \langle \langle \varphi, s, \varphi \rangle, \{w_0\}, \{w_0\} \rangle$ $w_2 = \langle \langle \bar{\varphi}, s, \varphi \rangle, \{w_2\}, \{w_0\} \rangle$

If we replace w_2 by w_1 , then (20) turns into (21):

$$\bar{T}(w) := \bigcup_n T^n. \quad (4)$$

$$21) \ w_0 = \langle \varphi, \{w_0\}, \{w_0, w_1\} \rangle, \ w_1 = \langle \bar{\varphi}, \{w_1\}, \{w_0, w_1\} \rangle$$

Every solution that solves (21) also provides a solution for (20). As every system of equations has only one solution, it follows that the solutions for w_1 in (21) and for w_2 in (20) must be identical. A graphical representation corresponding to that of Version 3 in **Table 5** accompanied by the equation in (20) can easily create the illusion of w_1 being distinct from w_2 . If we represent not only the state of affairs but also possible interactions in worlds, then w_1 and w_2 become distinct. In (Benz, 2008) the distinction between w_1 and w_2 was achieved by including the speaker's goal in the structure of possible worlds. By including sequences $\langle m, a, r \rangle$ we achieve the same effect: we represent the speaker's intention, the intention to evoke response r by doing a in situation m . Without intentions, we could not distinguish lies from honest assertions.

Possible worlds defined by systems of equations can represent utterance situations one at a time. It would be desirable to have definitions of whole classes of utterance situations that share certain characteristics. To avoid the necessary apparatus, we continue on a case by case basis.⁷

We need some additional concepts. First, we introduce the notion of an *epistemic path*. An epistemic path from w_1 to w_{n+1} is a sequence $\langle w_0, X_0, \dots, w_n, X_n, w_{n+1} \rangle$ with the property: for all i , X_i is either S^{w_i} or H^{w_i} , and $w_{i+1} \in X_i$.

The *transitive hull* of a world w is the set of worlds that includes w itself and all worlds that can be reached from w via a connecting epistemic path. Let w be a possible world. We first construct sets of worlds that are reachable in $0, \dots, n$ steps:

$$T^0 = \{w\}, \ T^{n+1} = T^n \cup \bigcup \{X^v \mid v \in T^n \wedge X = S, H\}. \quad (3)$$

The transitive hull of w is then defined as the union of all T^n s:

⁷For example, the class of all possible worlds could be introduced as the maximal fixed-point of the set continuous operator $\Gamma X := \{ \langle d, x, y \rangle \mid d \in D, x, y \subseteq X \}$, where D is some set of instances of joint projects.

It can be verified that $\bar{T}(w)$ is the set of all worlds that are reachable via an epistemic path from w . For example, in Version 1 of **Table 5**, $\bar{T}(w_0) = \{w_0, w_1\} = \bar{T}(w_1)$, and in Versions 2 and 3, we find again $\bar{T}(w_0) = \{w_0, w_1\} = \bar{T}(w_1)$, and $\bar{T}(w_2) = \{w_0, w_1, w_2\}$. Hence, $\bar{T}(w_0) = \bar{T}(w_1) \subsetneq \bar{T}(w_2)$. This shows the hierarchical structure of w_2 , and helps distinguishing worlds in the base of a graph where it holds for all v, w that $\bar{T}(v) = \bar{T}(w)$, and the worlds w which are higher up in the graph, for which it holds that there is a $v \in \bar{T}(w)$ such that $\bar{T}(v) \subsetneq \bar{T}(w)$.

Finally, we introduce two formal properties of possible worlds w :

$$\forall v \in \bar{T}(w) \forall X = S, H : X^v \neq \emptyset \wedge \forall u \in X^v X^u = X^v \quad \text{introspection (5)}$$

$$\forall v \in \bar{T}(w) : v \in S^v \cap H^v \quad \text{truthfulness (6)}$$

The first property entails that interlocutors know what they know, and know what they do not know. This is sometimes considered too strong an assumption about beliefs. We assume it here for convenience. The other property says that it is common knowledge that interlocutors have only true beliefs. If *truthfulness* holds for w , then every path in $\bar{T}(w)$ can be reversed, i.e., if some world v can be reached from another world u , then u can also be reached from v . In particular, *truthfulness* entails that for all $v \in \bar{T}(w) : \bar{T}(v) = \bar{T}(w)$.

We always assume *introspection*, and for elements of the base of an epistemic graph, we also assume *truthfulness*.

5.2 The Base Level of an Epistemic Graph

In **Table 4** we have seen various examples of basic epistemic graphs. They have in common that all worlds are connected with each other. This is entailed by the *truthfulness* condition that we assume to hold for all well-behaved communicative situations. The idea is that we can first solve the simpler task of classifying communicative acts in well-behaved situations, and then generalize the classification to the ill-behaved ones.

The epistemic relations in an utterance situation is represented by an epistemic graph. The goal of this section is to show how a sub-graph can be constructed that satisfies all epistemic felicity conditions. This construction will be a fixed-point construction. We first introduce formal variants of the *licensing* and *uniqueness* conditions.

Let us consider *licensing* from the speaker's perspective. If the speaker wants to start a joint project p he has to be sure that it can be performed in all epistemically possible worlds. The speaker can only perform a single act. Hence, there must be an act a such that for all epistemically possible states of affairs m there is a response r and a world $v \in S^w$ such that $\langle m^v, a^v, r^v \rangle = \langle m, a, r \rangle \in p$. For example, if the speaker wants to assert s with meaning φ , then φ has to be true in all epistemically possible states of affairs. As information states are sets of possible worlds, not sets of state of affairs, the actual definition that follows in (9) has to be slightly more roundabout. Assume that there are several sentences s_0, s_1, s_2, \dots with different and non-exclusive meanings φ_i that the speaker knows to be true. Then, for each joint communicative act $\langle m, s_i, \varphi_i \rangle$ there is a world w_i in the speaker's belief state in which the joint act is performed. For this world w_i it would not be clear what it should mean that another joint act $\langle m, s_j, \varphi_j \rangle$ can be performed. So, the requirement that it must be possible to perform a joint communicative act in all the speaker's epistemically possible worlds has to be re-worded: For all possible worlds w there must exist a world v that represents the joint act and agrees with w in the state of affairs m and the speaker's and hearer's belief states. Hence, we say that two worlds w and v are *similar*, if $\langle m^v, S^w, H^w \rangle = \langle m^v, S^v, H^v \rangle$. For the following it is convenient to introduce notation for the set of worlds out of a given set X that are similar to a given world w :

$$[w]_X := \{v \in X \mid m^v = m^w \wedge S^v = S^w \wedge H^v = H^w\}. \quad (7)$$

For convenience, we also introduce notation for the set X_a of all worlds that share the same utterance act a , and the set X^p of all worlds with a joint communicative act that belongs to a given project p :

$$X_a := \{v \in X \mid a^v = a\}, \quad X^p := \{v \in X \mid \langle m^v, a^v, r^v \rangle \in p\}. \quad (8)$$

With these preparations, we can introduce the formal constraints for *licensing* and *uniqueness*. They are formulated as conditions on information states, i.e., sets of possible worlds X , that depend on a project p and an act a :

$$L_{p,a}X := \Leftrightarrow \forall w \in X \exists v \in [w]_X : a^v = a \wedge \langle m^v, a^v, r^v \rangle \in p \quad \text{licensing} \quad (9)$$

$$U_{p,a}X := \Leftrightarrow \forall w, v \in X_a : \langle m^v, a^v, r^v \rangle \in p \rightarrow r^v = r^w \quad \text{uniqueness} \quad (10)$$

The *uniqueness* condition says that for every state of affairs in which act a can initiate a joint communicative act it will lead to the same response. *Uniqueness* is downward entailing, i.e., $X \subseteq Y$ entails $U_{p,a}Y \rightarrow U_{p,a}X$, and depends only on the joint communicative acts represented in X .

We can now show how to construct a maximal sub-set of a given set X in which the epistemic felicity conditions *licensing* and *uniqueness* are mutually guaranteed to hold. Let there be a given set \mathcal{P} of joint projects. Let X be a set of possible worlds such that for each $v \in \bar{T}(w)$ it holds that its joint communicative act $\langle m^v, a^v, r^v \rangle$ belongs to some project $p \in \mathcal{P}$. If X is the

speaker's belief state, then she knows that the epistemic felicity conditions for initiating a certain project p with a certain act a are satisfied in the following sub-set of X :

$$F_{p,a}^S X = \{v \in X_a^p \mid L_{p,a}S^v \wedge U_{p,a}S^v\}. \quad (11)$$

This is the set of all $v \in X$ with joint communicative act $\langle m^v, a^v, r^v \rangle \in p$ and utterance act $a^v = a$ that satisfy *licensing* and *uniqueness*.

The hearer, in contrast to the speaker, does not need to believe that act a initiates project p in all possible worlds. It suffices that he believes that it is consistent with his information. Hence, *licensing* can be restricted to a non-empty sub-set of his belief state:

$$F_{p,a}^H X = \{v \in X_a^p \mid \exists Y \subseteq H^v (\emptyset \neq Y \wedge L_{p,a}Y) \wedge U_{p,a}H^v\}. \quad (12)$$

We can construct the set of possible worlds in which the epistemic felicity conditions are mutual knowledge by an iterative process of eliminating worlds that do not satisfy them. The construction proceeds in parallel for all acts a and joint projects $p \in \mathcal{P}$. We start with a set X of possible worlds for which *truthfulness* holds and *transitivity* holds, i.e., for each $w \in X$ it holds that $\bar{T}(w) \subseteq X$. We set $F_*^0 = F^0 = X$. In the first step, we collect all worlds v in X which satisfy the speaker's epistemic felicity conditions, and *update* X with the information that they are satisfied. We do this for all joint projects $p \in \mathcal{P}$ and acts a :

$$F^1 = \bigcup_{p,a} F_{p,a}^S F_*^0 \quad (\text{selects the worlds in which the speaker's epistemic conditions are satisfied}) \quad (13)$$

$$F_*^1 = F_*^0 * F^1 \quad (\text{updates with this information}) \quad (14)$$

In the next step, this is repeated for the hearer's epistemic felicity conditions:

$$F^2 = \bigcup_{p,a} F_{p,a}^H F_*^1 \quad (\text{selects the worlds in which the hearer's epistemic conditions are satisfied}); \quad (15)$$

$$F_*^2 = F_*^1 * F^2 \quad (\text{updates with this information}) \quad (16)$$

This construction continues such that in each odd step the speaker's epistemic felicity conditions are checked, and in the even steps the hearer's:

$$F^{2n+1} = \bigcup_{p,a} F_{p,a}^S F_*^{2n} \quad F_*^{2n+1} = F_*^{2n} * F^{2n+1} \quad (17)$$

$$F^{2n+2} = \bigcup_{p,a} F_{p,a}^H F_*^{2n+1} \quad F_*^{2n+2} = F_*^{2n+1} * F^{2n+2} \quad (18)$$

Fortunately, it is not necessary to repeat this infinitely often. We can show that:

$$\forall n \geq 3 : F_*^n = F_*^3. \quad (19)$$

Why should the construction stabilize after three steps? After the first step, it is common knowledge that licensing and uniqueness hold from the speaker's perspective. As belief states can only become smaller by updating, the speaker's uniqueness condition is guaranteed to hold for all following construction steps. As for each remaining world w , it holds that $w \in H^w$ due to *truthfulness*, the hearer's licensing condition is automatically satisfied. Some worlds may be removed in step two due to the hearer's uniqueness

condition. After step two, the hearer's uniqueness condition is guaranteed to hold in all subsequent construction steps. Updating in step two may introduce violations of the speaker's licensing condition. In step three, worlds that violate speaker's licensing are again removed. As both the speaker and the hearer's uniqueness conditions must hold, only the licensing conditions could remove further worlds. However, as *truthfulness* holds, hearer's licensing is entailed by the speaker's licensing condition. Hence, in step four, none of the remaining worlds can be removed.

22) **Fixed-point.** Given a set of joint projects \mathcal{P} and a set X of possible worlds for \mathcal{P} where *truthfulness* and *transitivity* hold, then the maximal sub-set of X in which it is common knowledge that the epistemic felicity conditions of speaker and hearer are satisfied is $\nabla_{\mathcal{P}}X := F_{\mathcal{P}}^3$.

We next consider some examples. The first (23) demonstrates several points: first, basic cases can become more complex than the ones considered before; second, there are additional modeling assumptions that have to be made; third, for visualization there is a different type of graph that is better suited for base situations; and fourth, it shows how the construction is applied for finding fixed-points for epistemically felicitous referential uses of definite descriptions.

23) **Scenario.** The following is common knowledge. Either (*mb*) *Monkey Business* or (*dr*) *A Day at the Races* is showing at the Roxy. Ann has read the program, and knows which one it is. The newsfeed that Bob uses would only announce the program if *Monkey Business* is showing. Hence, if the state of affairs (m_0) is such that *A Day at the Races* is on the program, Bob will be uncertain. If *Monkey Business* is showing (m_1, m_2, m_3), he might have read the announcement (m_2, m_3), or not (m_1). If he has read it, Ann may know that (m_3), or not (m_1, m_2).

In which situation is it mutually felicitous to refer to *Monkey Business* with *the φ* = 'The movie showing at the Roxy'? The answer is only in m_3 . We will see how this comes out. Two graphical representations and a system of equations are shown in **Table 7**. The graph in c) shows the joint projects more clearly. Vertical lines in the center

column shows situations that are indiscernible for the hearer after the speaker's action (*the φ*), and vertical lines in the first column shows situations that are indiscernible for the speaker before acting. The graphs in a) and c) are equivalent.

There are two competing joint projects starting with *the φ* : The project $q = \{\langle m_0, \textit{the } \varphi, \textit{dr} \rangle\}$ where reference to *A Day at the Races*, and a project $p = \{\langle m_i, \textit{the } \varphi, \textit{mb} \rangle \mid i = 1, 2, 3\}$ where reference to *Monkey Business* is intended. We also assume, for reasons that will soon become clear, that there is a *do-nothing* project $l = \{\langle m_i, \varepsilon, \varepsilon \rangle \mid i = 1, \dots, 4\}$ where no action is performed. We construct $\nabla_{\mathcal{P}}X$ for $\mathcal{P} = \{p, q, l\}$ and $X = \{w_0, w_1, w_2, w_3\}$. For now, we ignore project l . In the first construction step, we test for each project whether the speaker's felicity conditions are satisfied. It can be verified that for w_0 the conditions for q hold, and that for w_1, w_2 , and w_3 the conditions for p hold. Hence, none of the worlds is eliminated. We turn to the hearer and the second construction step. The hearer's licensing condition is automatically satisfied as in each case $\{w_i\} \subseteq H^{w_i}$. However, *uniqueness* is violated for $H^{w_0} = H^{w_1}$. For w_2 and w_3 uniqueness is satisfied. Hence, the system has to be updated with $\{w_2, w_3\}$. This would lead to (24).

$$24) \quad w_2 = \langle \langle m_2, \textit{the } \varphi, \textit{mb} \rangle, \{w_2\}, \{w_2, w_3\} \rangle \\ w_3 = \langle \langle m_3, \textit{the } \varphi, \textit{mb} \rangle, \{w_3\}, \{w_2, w_3\} \rangle$$

Clearly, *licensing* is satisfied for the speaker's information state in both w_2 and w_3 . The construction stabilizes, and we arrive at the prediction that the referential act is mutually felicitous in both w_2 and w_3 . This is obviously not correct. In the original w_2 the speaker Ann did not know whether Bob has read the announcement, and, hence, she thought that he may be ignorant about the movie playing. This cannot have changed by just reasoning about felicity conditions. What went wrong? When updating with $\{w_2, w_3\}$, we eliminated w_1 . This means that Ann, in a situation in which she does not know whether Bob read the program $S = \{w_1, w_2\}$, would reason that Bob must have read the program (w_2) because, otherwise, he would not know to what she is referring to with *the φ* . This wishful reasoning is blocked by the *do-nothing* project l . It has the effect that none of the possible state of affairs m_0, \dots, m_3 are eliminated. (25) shows the system of equations for the epistemic relations with project l .

TABLE 7 | Representations of scenario (23).

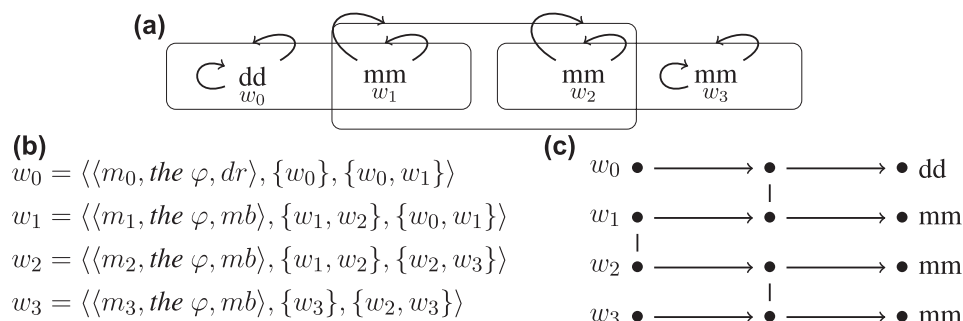
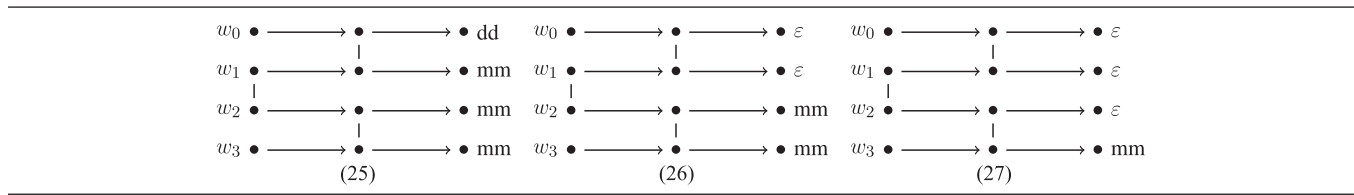


TABLE 8 | The graphs for construction steps 1, 2, and 3 for Example 23 with *do-nothing* project *l*.

- 25) $w_0 = \langle \langle m_0, \text{the } \varphi, \text{dr} \rangle, \{w_0, w_{0,\varepsilon}\}, \{w_0, w_1, w_{0,\varepsilon}, w_{1,\varepsilon}\} \rangle$
 $w_1 = \langle \langle m_1, \text{the } \varphi, \text{mb} \rangle, \{w_1, w_2, w_{1,\varepsilon}, w_{2,\varepsilon}\}, \{w_0, w_1, w_{0,\varepsilon}, w_{1,\varepsilon}\} \rangle$
 $w_2 = \langle \langle m_2, \text{the } \varphi, \text{mb} \rangle, \{w_1, w_2, w_{1,\varepsilon}, w_{2,\varepsilon}\}, \{w_2, w_3, w_{2,\varepsilon}, w_{3,\varepsilon}\} \rangle$
 $w_3 = \langle \langle m_3, \text{the } \varphi, \text{mb} \rangle, \{w_3, w_{3,\varepsilon}\}, \{w_2, w_3, w_{2,\varepsilon}, w_{3,\varepsilon}\} \rangle$
 $w_{0,\varepsilon} = \langle \langle m_0, \varepsilon, \varepsilon \rangle, \{w_0, w_{0,\varepsilon}\}, \{w_0, w_1, w_{0,\varepsilon}, w_{1,\varepsilon}\} \rangle$
 $w_{1,\varepsilon} = \langle \langle m_1, \varepsilon, \varepsilon \rangle, \{w_1, w_2, w_{1,\varepsilon}, w_{2,\varepsilon}\}, \{w_0, w_1, w_{0,\varepsilon}, w_{1,\varepsilon}\} \rangle$
 $w_{2,\varepsilon} = \langle \langle m_2, \varepsilon, \varepsilon \rangle, \{w_1, w_2, w_{1,\varepsilon}, w_{2,\varepsilon}\}, \{w_2, w_3, w_{2,\varepsilon}, w_{3,\varepsilon}\} \rangle$
 $w_{3,\varepsilon} = \langle \langle m_3, \varepsilon, \varepsilon \rangle, \{w_3, w_{3,\varepsilon}\}, \{w_2, w_3, w_{2,\varepsilon}, w_{3,\varepsilon}\} \rangle$

The update in Step 2 again eliminates $\{w_0, w_1\}$, which leads to (26). Note that *licensing* and *uniqueness* are trivially satisfied for project *l*: licensing says that the project can be initiated for all state of affairs, and uniqueness says that, once initiated, it can only be completed in one way. Hence, no update can remove any of the $w_{i\varepsilon}$ -worlds.

- 26) $w_2 = \langle \langle m_2, \text{the } \varphi, \text{mb} \rangle, \{w_2, w_{1,\varepsilon}, w_{2,\varepsilon}\}, \{w_2, w_3, w_{2,\varepsilon}, w_{3,\varepsilon}\} \rangle$
 $w_3 = \langle \langle m_3, \text{the } \varphi, \text{mb} \rangle, \{w_3, w_{3,\varepsilon}\}, \{w_2, w_3, w_{2,\varepsilon}, w_{3,\varepsilon}\} \rangle$
 $w_{0,\varepsilon} = \langle \langle m_0, \varepsilon, \varepsilon \rangle, \{w_{0,\varepsilon}\}, \{w_{0,\varepsilon}, w_{1,\varepsilon}\} \rangle$
 $w_{1,\varepsilon} = \langle \langle m_1, \varepsilon, \varepsilon \rangle, \{w_1, w_{1,\varepsilon}, w_{2,\varepsilon}\}, \{w_{0,\varepsilon}, w_{1,\varepsilon}\} \rangle$
 $w_{2,\varepsilon} = \langle \langle m_2, \varepsilon, \varepsilon \rangle, \{w_2, w_{1,\varepsilon}, w_{2,\varepsilon}\}, \{w_2, w_3, w_{2,\varepsilon}, w_{3,\varepsilon}\} \rangle$
 $w_{3,\varepsilon} = \langle \langle m_3, \varepsilon, \varepsilon \rangle, \{w_3, w_{3,\varepsilon}\}, \{w_2, w_3, w_{2,\varepsilon}, w_{3,\varepsilon}\} \rangle$

Now, if we consider w_2 , we see that licensing is not satisfied for *p* and S^{w_2} as there is a possible world ($w_{1,\varepsilon}$) in S^{w_2} with state of affairs m_1 , for which the speaker knows that it is not possible to initiate *p*. Hence, w_2 is eliminated. As w_3 satisfies licensing and uniqueness, it survives. The final system of equations is shown in (27).

- 27) $w_3 = \langle \langle m_3, \text{the } \varphi, \text{mb} \rangle, \{w_3, w_{3,\varepsilon}\}, \{w_3, w_{2,\varepsilon}, w_{3,\varepsilon}\} \rangle$
 $w_{0,\varepsilon} = \langle \langle m_0, \varepsilon, \varepsilon \rangle, \{w_{0,\varepsilon}\}, \{w_{0,\varepsilon}, w_{1,\varepsilon}\} \rangle$
 $w_{1,\varepsilon} = \langle \langle m_1, \varepsilon, \varepsilon \rangle, \{w_{1,\varepsilon}, w_{2,\varepsilon}\}, \{w_{0,\varepsilon}, w_{1,\varepsilon}\} \rangle$
 $w_{2,\varepsilon} = \langle \langle m_2, \varepsilon, \varepsilon \rangle, \{w_{1,\varepsilon}, w_{2,\varepsilon}\}, \{w_3, w_{2,\varepsilon}, w_{3,\varepsilon}\} \rangle$
 $w_{3,\varepsilon} = \langle \langle m_3, \varepsilon, \varepsilon \rangle, \{w_3, w_{3,\varepsilon}\}, \{w_3, w_{2,\varepsilon}, w_{3,\varepsilon}\} \rangle$

Now, the prediction is that Ann can use *the* φ for referring to *Monkey Business* only in situation w_3 . If she uses it, then the system in (27) is updated with the set of all worlds that instantiate a project starting with *the* φ ; i.e. it has to be updated with $\{w \mid a^w = \text{the } \varphi\} = \{w_3\}$. This update leads to $w_3 = \langle \langle m_3, \text{the } \varphi, \text{mb} \rangle, \{w_3\}, \{w_3\} \rangle$. This implies that the definite description not only tells Bob to pick out *Monkey Business* as the referent, it also tells him that Ann knows that he has read the program.

The definition of possible worlds becomes more complicated when adding project *l*. As project *l* is defined for all state of affairs, it does not need to be shown in graphs, except some state of affairs would otherwise be eliminated. The simplified graphs in **Table 8**

represent the possible worlds defined in construction steps (25)–(27) with joint project *l* only showing when necessary.

5.3 Hierarchical Epistemic Graphs

The base level of a graph consist of worlds that satisfy *truthfulness* and *introspection*. Now, we turn to examples where the *truthfulness* condition is violated. All the examples that we have seen are represented by graphs that have a base in $\nabla\mathcal{P}$ over which a hierarchical structure is erected. All worlds in the upper structure are rooted in the base by epistemic paths reaching down to it. This section will be less technical. We will concentrate on showing different types of epistemic graphs that can be found on higher levels. We first clarify in which sense the worlds have a hierarchical structure. It is possible to distinguish different levels in this hierarchical structure, depending on how deeply the base is embedded in a world. Each level is characterized by a unique *order type* which is shared by all worlds at this level. As we have seen, the *truthfulness* condition implies that each world in the transitive hull $\overline{T}(w)$ of a world w is connected to every other world by an epistemic path, in particular, it holds that the transitive hulls of all worlds in $\overline{T}(w)$ are identical. We give these worlds the order type 0. We define the order type *otp* of other worlds recursively using the transitive hull.

$$\text{otp}(w) = 0 \Leftrightarrow \forall v \in \overline{T}(w) w \in \overline{T}(v). \quad (20)$$

$$\text{otp}(w) = \sup\{\text{otp}(v) + 1 \mid v \in \overline{T}(w) \wedge w \notin \overline{T}(v)\} \quad (21)$$

$$\text{otp}(X) = \sup\{\text{otp}(v) \mid v \in X\} \quad (22)$$

The first condition says that worlds at the base have order type 0. The second, that for other worlds w the order type is the smallest ordinal that is larger than all order types of worlds from which w cannot be reached by an epistemic path.⁸ The last condition introduces the order type of a set of possible worlds which is the smallest ordinal that is at least as large as the order types of all the worlds in the set. For example, in **Table 7**, the worlds w_0 and w_1 have order type 0, and w_2 has order type 1. In **Table 3**, the top worlds in Versions 1 and 2 have order type 1, and that of Version 3 order type 2, and in **Table 1** we see examples with order types increasing from 0 in a) to 4 in e).

Let us first consider Versions 2 and 3 in **Table 5** with the corresponding examples in (17). The joint project

⁸Set theoretically the *supremum* of a set of ordinals is just the union of these ordinals. The definition is maximally general and extends into the transfinite. However, in this article, we only consider worlds with finite order type.

of *asserting* sentence s with meaning φ was defined as the set of all triples $\langle m, s, \varphi \rangle$ consisting of a model m that makes φ true, the speaker's utterance s and the hearer's interpretation φ . In epistemic graphs, as in **Table 5**, such a triple was represented by the pair φ, φ , the first φ saying that m is such that φ is true, and the second φ representing the hearer's interpretation of s . Hence, in the base level of an epistemic graph, the formulae appearing in the pairs must always be identical. As Version 2 and 3 in **Table 5** show, this may no longer be the case in higher levels. To account for this possibility, we have to make the joint projects independent of the state of affairs. Let $p \in \mathcal{P}$ be a joint project, then the *extended* joint project \tilde{p} is defined as

$$\tilde{p} = \{ \langle m, a, r \rangle \mid \exists m' \langle m', a, r \rangle \in p \}. \quad (23)$$

This means, the joint communicative acts $\bar{\varphi}, \varphi$ that we see in **Table 5** are elements of the extended joint project of asserting φ . We allow extended projects to occur only on higher levels of the hierarchy. We are going to show that the *licensing* and *uniqueness* conditions can be re-used at higher levels to determine the worlds where asserting s is epistemically felicitous. For extended projects, the conditions are shown in **Eqs. (24)** and **(25)**.

$$L_{\tilde{p},a}X \Leftrightarrow \forall w \in X \exists v \in [w]_X : a^v = a \wedge \langle m^v, a^v, r^v \rangle \in \tilde{p} \quad \text{licensing} \quad (24)$$

$$U_{\tilde{p},a}X \Leftrightarrow \forall w, v \in X_a : \langle m^v, a^v, r^v \rangle \in \tilde{p} \rightarrow r^v = r^w \quad \text{uniqueness} \quad (25)$$

The conditions are unchanged, except that basic projects have been replaced by extended projects. The *licensing* condition says that the joint communicative act can be performed in all epistemically possible state of affairs, and *uniqueness* that performing it leads to a unique response for each state. The operators selecting worlds satisfying the epistemic felicity constraints stay the same, except that the basic projects are replaced by extended projects. For convenience, they are shown in **Eq. 26** and **Eq. 27**.

$$F_{\tilde{p},a}^S X = \{ v \in X_a^{\tilde{p}} \mid L_{\tilde{p},a} S^v \wedge U_{\tilde{p},a} S^v \}. \quad (26)$$

$$F_{\tilde{p},a}^H X = \{ v \in X_a^{\tilde{p}} \mid \exists Y \subseteq H^v (\emptyset \neq Y \wedge L_{\tilde{p},a} Y) \wedge U_{\tilde{p},a} H^v \}. \quad (27)$$

Apart from checking whether *licensing* and *uniqueness* hold for the speaker and hearer's perspective, the operators check whether the joint communicative act represented by a possible world is an instantiation of a given extended joint project \tilde{p} performed with a special act a .

With these operators, a fixed-point can be constructed as in 17, the only difference being that the construction is applied bottom up, level by level. We eschew the technical details and demonstrate their workings with some examples. Let us consider the graphs in **Table 9**. In graphs a), c), and d), the belief states of participants are subsets of the base level. In w_0 and w_1 the epistemic felicity conditions for assertions are satisfied in a) and b), and for definite references in c) and d). Graphically, it should be easy to check that the felicity conditions of *licensing* and *uniqueness* are satisfied for w_2 and w_3 in a), b), and d), and violated in c). Checking the formal definitions needs more effort. First, we note that for all graphs the fixed-point of the base level $\nabla_{\mathcal{P}}\{w_0, w_1\}$ is equal to the base level itself, and that in a) and b) assertions of φ are licensed in w_0 , and of $\bar{\varphi}$ in w_1 . We consider first world w_2 in a). The abbreviation $\bar{\varphi}, \varphi$ stands for the joint communicative act $\langle m, s, \varphi \rangle$ with a model m that supports $\bar{\varphi}$ and an assertion of a sentence s that expresses semantically that φ . Hence, asserting s in m violates the constituting rules of assertions. However, $\langle m, s, \varphi \rangle$ is an element of the extended joint project of asserting φ . We have to check the felicity conditions of *uniqueness* and *licensing* for w_2 . As mentioned before, *uniqueness* is trivially satisfied for assertions, as we assumed that semantic meaning is not ambiguous. Only licensing has to be checked. This is identical to checking licensing for w_0 in graph (18), as the belief states of speaker and hearer in w_2 and w_0 are identical. As the felicity conditions are satisfied in w_0 , it only remains to check the condition ' $v \in X_a^{\tilde{p}}$ ' in Definition (26). As fixed-points are calculated level by level, X must be the restriction of $\bar{T}(w_2)$ to Level 1, i.e., $X = \{w_2\}$. As $\langle \bar{\varphi}, \varphi \rangle$ is an instance of the extended project of asserting φ , the condition is satisfied. Hence, applications of $F_{\tilde{p},a}^S$ and $F_{\tilde{p},a}^H$ to $\{w_2\}$ return again $\{w_2\}$. Clearly, further applications of these operators cannot change the result, so that $\{w_2\}$ must be a fixed-point of these operators. This shows that asserting s with interpretation φ satisfies the joint epistemic felicity conditions, and, hence, it is the case that both interlocutors agree on the interpretation of s , and that they both believe that they can mutually figure this out. The case of w_3 is symmetrical, where φ and $\bar{\varphi}$ change places. In sum, it follows that asserting a sentence s with meaning φ is epistemically felicitous in w_2 , and asserting a sentence \bar{s} with meaning $\bar{\varphi}$ in w_3 is likewise epistemically felicitous.

We next turn to b) in **Table 9**. From the hearer's perspective, the situation is identical to that of a) or that of Version 1 in (17) with graph (18). Hence, we only need to consider the speaker's perspective in w_2

TABLE 9 | Epistemic graphs: (a), (b) for assertions in scenarios similar to (17); (a): false belief case, (b): lying; (c), (d) for reference in scenarios similar to (13)–(16); (c): failed joint reference on higher level, (d): successful joint reference on higher level. Abbreviations: φ a proposition, $\bar{\varphi}$ its negation; m : *Monkey Business*, d : *A Day at the Races*.

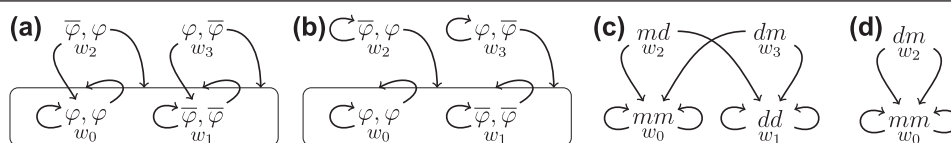
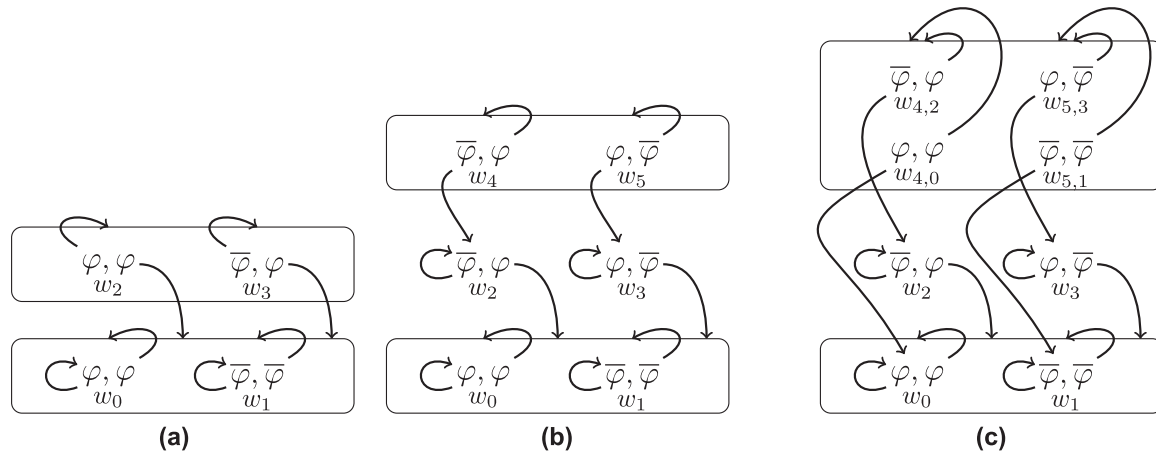


TABLE 10 | Epistemic graphs with uncertainty on higher levels. **(a)**, **(b)** for assertions in scenarios similar to (17); **(a)**: case of insufficient information, **(b)**: detected lying, **(c)**: hearer uncertainty: is speaker honest or lying. Abbreviations: φ a proposition, $\bar{\varphi}$ its negation.



and w_3 . Clearly, the speaker thinks that in all her epistemically possible worlds an assertion of a sentence s with meaning φ is possible (as an extended joint act of asserting), and also thinks that it leads to a unique response. As w_2 is itself an instance of the extended project \tilde{p}_{ass} of asserting φ , it follows that an application of $F_{\tilde{p},s}^S$ to $\{w_2\}$ just returns $\{w_2\}$. Further applications of $F_{\tilde{p},s}^S$ and $F_{\tilde{p},s}^H$ do not change the result, and, hence, w_2 is an element of the Level 1 fixed-point of extended project \tilde{p}_{ass} . Analogously, it follows that $\{w_3\}$ is a fixed-point of the extended project of asserting $\bar{\varphi}$.

With c) and d), we switch to referential uses of definite descriptions. Clearly, in c) the interpretation of *the* φ = ‘The movie showing at the Roxy tonight’ cannot agree between speaker and hearer, neither in w_2 , where *Monkey Business* is showing and a use of *the* φ has to result in a reference to *A Day at the Races*, nor in w_3 , where *A Day at the Races* is showing and a use of *the* φ has to result in a reference to *Monkey Business*. In d), however, where *A Day at the Races* is showing but both interlocutors think that *Monkey Business* is showing, *the* φ will from both interlocutors’ perspective felicitously refer to *Monkey Business*.

In all examples of Table 9, the belief states of interlocutors are subsets of the base level or singleton sets. We can also find natural situations with belief states with uncertainty at higher levels. Examples are shown in Table 10. In a) The speaker does not know whether φ or $\bar{\varphi}$ is true, but she is convinced that uttering s will lead in all her epistemic possibilities to joint interpretation φ . From the hearer’s perspective, the situation is indistinguishable from the base situation. In contrast to b) in Table 9, a) is a case of an assertion with insufficient information, hence, a violation of Grice (1975) maxim of quality.

In b) of Table 10, a case is shown in which the hearer knows that the speaker is lying but does not know whether φ or $\bar{\varphi}$ is the case. Furthermore, the hearer knows that the speaker thinks him to be gullible. Graph c) seems at first overly complicated, but it represents a natural situation: in it the hearer does not know whether the speaker is honest and says the truth (w_0 and w_1), or is dishonest and lies (w_2 and w_3). Furthermore, the hearer does not know himself whether φ is true, or not. He again knows that the

speaker knows the state of the world and that she thinks him to be unsuspecting. For all the worlds, our criterion predicts that the assertion of φ is mutually guaranteed to be successful in the worlds on the left side, and an assertion of $\bar{\varphi}$ in the worlds on the right side.

The examples that we have considered so far show strictly hierarchical belief states. This means, in every possible world that is not in the base level, there is one agent whose belief set has an order type that is smaller than the world’s order type. Graphically, this means that the belief set of one agent is a subset of the levels that are below the actual world. More precisely, they are defined as follows:

- 28) A possible world is *strictly hierarchical*, if for all v in the transitive hull $\bar{T}(w)$ of w it holds that $otp(v) > 0$ implies:

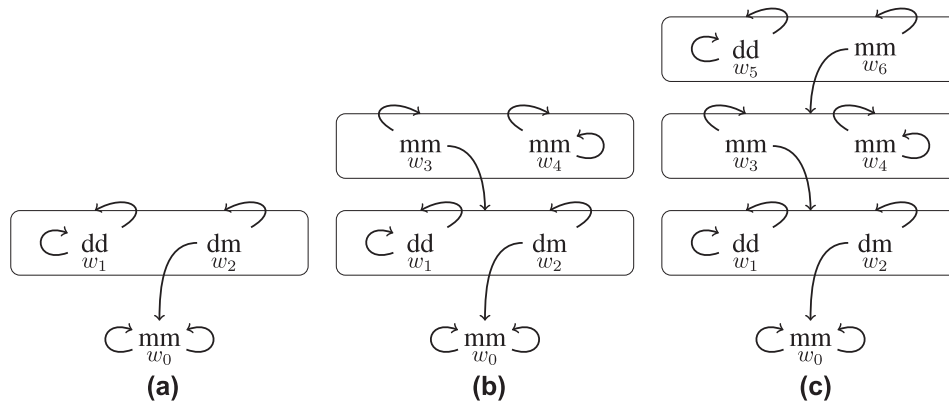
$$otp(S^v) < otp(v) \vee otp(H^v) < otp(v). \quad (28)$$

If belief states are not strictly hierarchical, they must show circular relations on higher levels. We consider some examples. Table 11 shows three epistemic graphs with possible worlds that can be reached from each other via epistemic paths.

We consider an example:

- 29) Ann and Bob attend a course on film studies. Together they listen as the lecturer tells the class that, this evening, the course will watch *Monkey Business* at the cinema. Later, in the library, Bob meets the lecturer as she talks to another film student. However, Bob cannot see who the student is. He thinks it is Clara, another student, or Ann. The lecturer notices him and says: “Oh, Bob! Good to see you. I made a mistake. The movie showing this evening is *A Day at the Races*, and not *Monkey Business*.” Bob leaves without asking who the other student is. He knows that Ann cannot have learned about the correction if she was not in the library. Later, he receives a mail from Ann telling him that she doesn’t like *the movie showing at the cinema tonight*.

What is Ann referring to? The situation is represented by Graph a) in Table 11. If Ann was not the other student in the

TABLE 11 | Graphs with circular belief states at higher levels.

library, then, clearly, she refers to *Monkey Business*. If she was there, then she knows that Bob knows that *A Day at the Races* is showing and that Bob knows that the other student knows it too. She also knows that he does not know that the other student was she herself. Hence, if she was the other student then she knows that Bob cannot know what *the φ = the movie showing at the cinema tonight* is referring to. There are two possibilities then: if she was not the other student, she thinks that *the φ* will successfully refer to *Monkey Business*, if she was the student, she should first tell Bob that she learned about the correction, and then refer to *A Day at the Races* with *the φ* . It follows that, if both of them assume that they are rational, that Bob can infer from an utterance of *the φ* that Ann was not the other student, and that she refers to *Monkey Business*. For Graph a) in **Table 11**, this means that the fixed-point construction on the first level should eliminate w_1 but not w_2 . Unfortunately, this is not the case. If we first apply the operator checking the speaker's epistemic felicity conditions from (26), then both worlds survive. If we then apply the operator for the hearer's epistemic felicity conditions, then both worlds are eliminated as the *uniqueness* condition is violated for $H^{w_1} = H^{w_2}$.

At this point, we should recall that the iterative application of the felicity operators corresponds to the iterative reasoning about each other and the ensuing step by step elimination of epistemic possibilities that are not consistent with *uniqueness* and *licensing*. The problem with world w_2 is that the speaker's belief state is a subset of the base level, hence, she is oblivious to the reasoning that goes on on the first level. The hearer cannot eliminate w_2 with the argument that the speaker will not make an attempt at referring to *Monkey Business* because she can see that doing this would be inconsistent with the hearer's *uniqueness* condition. The elimination step in the construction of the fixed-point cannot be applied to worlds with belief states in the lower levels. We say that a world w is *speaker* or *hearer rooted* in the lower level with respect to an act a and a project p , if the speaker's belief state S^w , or the hearer's H^w , are subsets of the lower levels and satisfy the felicity constraints there.

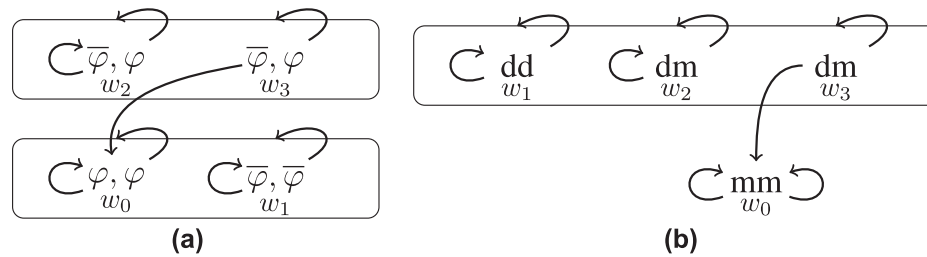
If a world w is rooted in the lower level with respect to an act a and a project p , and if $d^w \in \tilde{p}$ and $a^w = a$, then

it should be re-introduced when it is eliminated by a felicity operator during fixed-point construction. For a) in **Table 11** this means that after the elimination of w_2 due to the violation of the hearer's *uniqueness* condition, w_2 has to be re-introduced into the graph. This results into the graph consisting of two worlds, w_0 and w_1 , defined by the system of equations consisting of $w_0 = \langle \langle m, the \varphi, m \rangle, \{w_0\}, \{w_0\} \rangle$, $w_1 = \langle \langle d, the \varphi, m \rangle, \{w_0\}, \{w_1\} \rangle$. This graph also satisfies the two felicity constraints.

The next example is one that shows two levels with circular belief states. It uses the same type of communicative situation with uncertain bystander as Example (29).

30) Ann and Bob attend a course on film studies. Together they listen as the lecturer tells the class that, this evening, the course will watch *Monkey Business* at the cinema together. Later, in the library, Bob meets the lecturer as she talks to another film student. However, Bob cannot see who the student is. He thinks it is Clara, another student, or Ann. The lecturer notices him and says: "Oh, Bob! Good to see you. I made a mistake. The movie showing this evening is *A Day at the Races*, and not *Monkey Business*. Bob leaves without asking who the other student is. Still later, he meets the lecturer again in the cafeteria. She tells him that the program has changed again. Then Ann comes in. The lecturer tells her: "Hallo Ann, I have just told this student here that the program changed again. It is *Monkey Business* that is showing tonight." Bob noticed that Ann could not see him, that she must think that it could be him but that she could not be certain. He also knew that she must think that he could not learn about the change of program if he was not the student in the cafeteria. Bob also noticed that Ann must have been the other student in the library. Later, he receives a mail from Ann, telling him that she doesn't like *the movie showing at the cinema tonight*.

The situation is represented by b) in **Table 11**. It can be easily checked that the fixed-point on the second level is identical to the level consisting of w_3 and w_4 . The fixed-point of the first level again consists of only w_2 .

TABLE 12 | Graphs with circular belief states at higher levels.

In principle, we can add more and more levels with circular structure. Graph c) in **Table 11** shows an example with four levels. As world w_6 on Level 3 is rooted in level 2, it is not eliminated when the fixed-point on Level 3 is constructed. It is then predicted that a reference to *Monkey Business* in w_6 with *the movie showing at the cinema tonight* is felicitous, whereas a reference to *A Day at the Races* in w_5 is not felicitous.

As final example in this section, we present a situation that resembles (29) but is not about reference but about assertions.

- 31) Helga calls up her son Stephan and asks him whether he wants to visit her in Munich. Stephan tells her that he will watch the weather forecast this evening and call her in the morning. Helga knows the channel where Stephan watches the late news and learns that it is snowing in the Alps the next day. Next morning a mutual friend video calls her and mentions that the forecast has changed and that the streets are free of snow. In the background, Helga can see someone who resembles her son Stephan, but she cannot be sure. Shortly afterward, she receives a text message from Stephan saying that he cannot visit her because snow is forecasted and he doesn't want to drive then. He also wrote that he will not have his smartphone with him and cannot read text messages that day. She knows that Stephan has a new girl-friend and prefers to stay at home.

Is Stephan lying about snow in the Alps, or not? The situation is represented by the graph in **Table 12a**.

World w_3 is rooted in the base level, and anchored to a world in which the speaker is licensed to assert $\varphi = \text{'It is snowing in the Alps.'}$ As it is itself an instance of the extended project of asserting φ , it will be in the fixed-point on Level 1. World w_2 is also an instance of the extended project, the speaker is licensed to assert φ in all epistemic possibilities, and the hearer's belief state also satisfies *licensing* of asserting φ . As mentioned before, *uniqueness* is trivially satisfied for assertions. Hence, w_2 will also be in the fixed-point. The prediction is then that Helga cannot tell whether Stephan lied or said what he believed to be true.

What is the difference between the graphs in **Tables 11a** and **12a**? The answer is that we chose a minimal representation of (29) in **Table 11a**. We saw in (18) and **Table 4** that the basic utterance situation for assertions can come in different varieties. The textual description of the utterance situation in (17) leaves the exact epistemic relations between speaker and hearer underspecified. The same underspecification is encountered with Example (29). An alternative to the graph

in **Table 11a** is shown in **Table 12b**.⁹ Here, world w_2 corresponds to w_2 in **Table 12a**. Both survive the tests for *licensing* and *uniqueness* conditions and the subsequent updates. In the case of assertions, there cannot be a possibility corresponding to world w_1 in **Tables 11a** and **12b**, as there is no ambiguity about semantic interpretation equivalent to ambiguity about choice of referent.

6 COMPARISON AND OUTLOOK

We developed a theory of epistemic felicity conditions and speech acts that followed a path charted by the works of J. Searle, H.P. Grice, and H.H. Clark. For both assertions and definite descriptions there is a large body of literature, so large that we can only hint at how our model fits into the general landscape of semantic and pragmatic theories. For both referential uses of definite descriptions and assertions we make minimal assumption about dialogue context. In our model, *familiarity* (Heim, 1982) and *uniqueness* (Russell, 1905, 1919) of referents are not semantic properties of definite descriptions but follow from pragmatic felicity conditions that hold in very basic epistemic graphs only. If the felicity conditions are not met, then the referent remains undefined (see Strawson, 1950). Our model also accounts for situation in which the description of a definite does, or may not apply to the referent as in Donnellan's (1966) famous Martini-glass example (an example is shown in **Table 11**, Graph a)). For assertions, our constitutive rules only require that the asserted proposition is true (Weiner, 2005), from which the requirement that the speaker believes it (Williamson, 1996; Turri, 2016) follows as a felicity requirement of basic utterance situations, but it may be violated at higher order belief states. In particular, our model shows how the existence of non-cooperative language use and un-truthfulness can be reconciled with the constitutive requirement of truthfulness (see Pagin 2016 for an overview of the related philosophical discussion).

Our model is about epistemic felicity conditions of speech acts. Which speech acts can be performed in a dialogue situation is pragmatically dependent on the interlocutors' beliefs about the

⁹There are, in fact, an infinitude of alternatives. We leave the clarification of this issue to future research.

world and about each other. There are theories that try to predict possible speech acts without reference to private beliefs. Prominent examples are *commitment* theories, *discourse structural* approaches, or approaches based on the idea of common *scoreboards*. In a commitment approach, if a speaker asserts a sentence then s/he takes on the (social) obligation of defending its truth; s/he does, however, not necessarily express a belief in it.¹⁰ Discourse structural approaches explain the possible sequences of speech acts by discourse relations that must hold between dialogue moves. Example are the *Segmented Discourse Representation Theory* (Asher and Lascarides, 2003) and the *Rhetorical Structure Theory* (Mann and Thompson, 1988). Relevant is here, for example, the account of strategic conversation in non-cooperative discourse by Asher and Lascarides (2013). The idea that information update in dialogue can be modeled with public scoreboards can be traced back to Lewis (1979). The scoreboard represents the public information of interlocutors. Each communicative act updates the scoreboard in specific ways. In ideal cases, the update only depends on the old scoreboard and the sentence uttered. Hence, the update after an honest assertion and a lie would be the same. A comparison of our model to any one of these approaches would go beyond the scope of this article. A common motivation for all of them are the problems that epistemic accounts of speech acts face when confronted with non-cooperative discourse or utterance situations with higher-order belief states. Our model shows how these problems can be overcome.

In the previous sections we have seen how the interlocutors' limited perspectives can give rise to extended uses of communicative acts. On the base level, where interlocutors follow constitutive rules and have only truthful beliefs, the joint communicative acts that mutually satisfy the epistemic felicity conditions of *licensing* and *uniqueness* can be found by a fixed-point construction. The fixed-point construction depends only on a given set of joint projects, hence, it generalizes to any type of communicative act, the constitutive uses of which can be represented by joint communicative acts of the form $\langle m, a, r \rangle$, i.e., as a set of triples consisting of a state of affairs m , an utterance act a , and a response r .

The elements of $\langle m, a, r \rangle$ are abstract representations of the state of affairs, acts and responses. For example, the state of affairs m can represent a concrete situation in the world, but it can also represent a more abstract dialogue scoreboard. As an example, we may consider Ginzburg (2012) KoS framework. In this framework a and r would each be the latest moves in a pair of dialogue states representing the precondition and effects of performing the respective speech act. If a and r belong to a joint project, then the effect state of a must be a subtype of the preconditions of r . By identifying the pre-state of a with m , we can see how adjacency pairs in the KoS-format can be translated into joint communicative acts of the form $\langle m, a, r \rangle$, and, thereby, plugged into our epistemic model. In this way, our model could benefit from the additional fine-structure that KoS has

to offer. It also shows how a scoreboard approach and an epistemic approach as the one proposed in this article can be reconciled. In contrast to chess, dialogue game boards are not physically given. They have to be maintained and coordinated by speaker and hearer, and so perspectives must have a role to play.¹¹

We said before that constitutive rules define a form of social institution consisting in a conventionalized regularity of linguistic behavior. In the following, we tied this behavior to a class of well-behaved utterance situations at the base of the epistemic graphs that we have seen. From there, the behavior is extended to a wider class of hierarchical epistemic states. We have seen that, in extending the behavior, indistinguishability between utterance situations play a crucial role. The extended use of a communicative act can travel up the hierarchy along epistemic paths because the situation on the higher level is for one, or sometimes both interlocutors indistinguishable from one at a lower level.

If a communicative act is defined for a constitutive core, then our theory also predicts that extended uses that violate the constitutive rules exist. Hence, if honest, truthful assertions exist, then also assertions based on false beliefs must exist, as well as outright lies. This also means that the definitions of speech acts can be simplified considerably, as only constitutive rules for uses in the well-behaved core have to be considered.¹² A non-trivial observation is that extended uses can still be classified with the same name as the uses in the constitutive core. In the introduction we mentioned the following examples in (32) and (33):

- 32) Leo told me that it is snowing in the Alps, but I knew that the snowing had stopped. (false belief).
 33) Leo told me that it is snowing in the Alps, but I knew that she is lying. (lying).

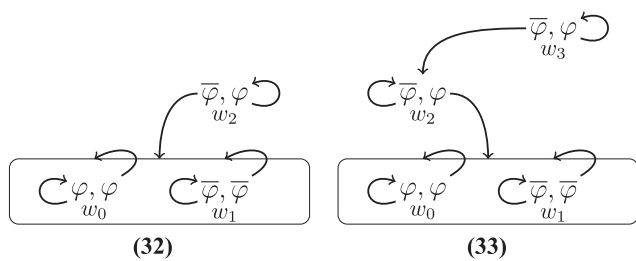
Assuming that *tell* reports an assertion event, then the examples show that classifying an utterance as *assertion* is consistent with false beliefs and lying. This raises a question about the semantics of *tell*. If the constitutive rules were part of the semantic meaning of assertions, then, given how we have defined the constitutive joint project of *asserting*, the sentence '*Leo told me that φ* ' should mean that Leo uttered a sentence with meaning φ and she uttered it in a situation in which this sentence is true. Clearly, the examples in (3') and (4') are not consistent with such a semantic rule. **Table 13** shows two graphs for the examples.

In our model, we distinguished between the project as defined by its constitutive rules and the extended project that is defined by the action-response pairs alone. This means, if p is a joint project, then the extended project \tilde{p} is $\{\langle m, a, r \rangle \mid \exists m' \langle m', a, r \rangle \in p\}$. We make two assumptions: a) '*tell*' semantically applies to joint communicative acts in the constituting joint project of assertions p_{ass} , but it can be extended to joint communicative acts of the extended joint project \tilde{p}_{ass} ; b) pragmatically an

¹⁰There are, however, various meanings that have been given to the term *commitment*. For an older overview, see (Brabanter and Dendale, 2008). For recent discussions, see e.g., (Krifka, 2012; Geurts, 2019; Krifka, 2019).

¹¹In line with H.H. Clark's propositions 3 and 6, (1996, p. 23/24).

¹²However, we have to concede that extending the account to cover intricate problems that motivate, for example, dynamic syntactic theories like DS-TTR (see Gregoromichelaki et al., 2011) needs further work (e.g., on the problem of *split turn taking*; see Gregoromichelaki and Kempson, 2016 for an overview; I thank the reviewers for bringing this important phenomenon to my attention).

TABLE 13 | Epistemic graphs for examples of assertions with false beliefs and lying.

application of ‘tell’ to a joint project is felicitous only if the denoted joint communicative acts is uttered in a world that belongs to a fixed-point of either the base, or one of the higher levels of an epistemic graph. These assumptions allow the felicitous use of ‘tell’ to travel up the paths of an epistemic graph as indicated in the introduction. They also explain how the reports in (32) and (33) can be felicitous. In **Table 13**, the theory predicts a felicitous use of φ in world w_2 for (32) and in world w_3 for (33). The two assumptions entail that ‘tell’ can be felicitously used for reporting the utterance events in these worlds.

This solution assumes that lexical meaning is flexible and allows for contextual adjustment taking the interlocutors perspectives and the resulting indistinguishability between utterance situations into

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account. There are other paths for seeking a solution that come to mind. For example, one could assume that the lexical meaning of *tell* has a meaning that is weak enough to be consistent with all epistemic graphs that we have considered in this article. *Commitment* approaches belong here. We must, however, leave the comparison and further pursuit of the semantic issues to future research.

DATA AVAILABILITY STATEMENT

The original contributions presented in the study are included in the article/Supplementary Material, further inquiries can be directed to the corresponding author.

AUTHOR CONTRIBUTIONS

AB is the sole author of this article.

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7 APPENDIX

7.1 How to read epistemic graphs

Table 14 shows how to read epistemic graphs. We assume that there is a speaker A and an addressee B . Arrows to the left of a world point to A 's information state, and arrows to the right to B 's information state. An information state is a set of possible worlds. If an arrow points to a single world w , then the respective information state is a set with w as single element. If an information state has more than one element, it is represented by a box encircling its elements. For convenience, basic building units of graphs are shown in the Table 14.

7.2 Epistemic Graphs and Kripke Frames

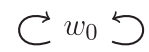
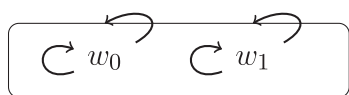
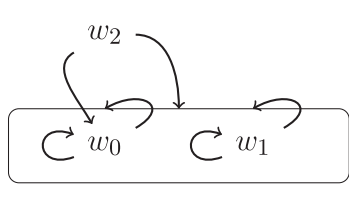
We address the question how epistemic graphs are related to *Kripke frames* for epistemic modal logic. Epistemic modal logic can be traced back to (Hintikka, 1962). For a newer introduction and an overview see e.g., (van Benthem, 2011) and (Baltag et al., 2008; Baltag and Renne, 2016). In epistemic modal logics, epistemic possibility is modeled by accessibility relations between worlds. For each agent i there is a relations R_i between possible worlds with the meaning that if $\langle v, w \rangle \in R_i$ then i believes in world v that w represents a possible state of affairs. If there are two agents, a speaker and a hearer, then the

beliefs of each one is represented by his/her own accessibility relation R_S or R_H . Given a set of possible worlds \mathcal{W} and accessibility relations R_S and R_H an equivalent epistemic graph is defined by the following system of equations:

$$v = \langle v, \{w \mid R_S(v, w)\}, \{w \mid R_H(v, w)\} \rangle, \quad v \in \mathcal{W}. \quad (29)$$

In reverse, if a system of equations is given that defines an epistemic graph, and \mathcal{W} is the set of solutions, i.e., all $v \in \mathcal{W}$ are of the form $v = \langle d^v, S^v, H^v \rangle$, then $R_S := \{\langle v, w \rangle \mid v, w \in \mathcal{W} \wedge w \in S^v\}$ and $R_H := \{\langle v, w \rangle \mid v, w \in \mathcal{W} \wedge w \in H^v\}$ are the accessibility relations of the corresponding Kripke frame. If a modal logic with belief operators for speaker and hearer is interpreted in the Kripke frame and the epistemic graph, then the two constructions are equivalent in the sense that corresponding worlds make the same modal logic formulas true. As we are not concerned with modal logics but directly reason with epistemic graphs, there is nothing to be gained by using Kripke frames. For our purposes, Kripke frames have disadvantages. For example, the graph defined by $w = \langle m, \{w\}, \{w\} \rangle$ could be represented in infinitely many ways by equivalent Kripke frames. Hierarchical and circular structures and the order types of worlds are not immediately definable. Their definition would have required normalization with respect to maximal *bisimulations* (see Barwise and Moss, 1996; Gerbrandy and Groeneveld, 1997; Gerbrandy, 1998). AFA-set theory allows one to avoid this step.

TABLE 14 | Reading epistemic graphs. The comments to the right explain new features of the respective graphs.

	<p>arrow to the left is pointing from w_0 back to w_0: A's information state in w_0 is $\{w_0\}$; arrow to the right is pointing from w_0 back to w_0: B's information state in w_0 is $\{w_0\}$;</p>
	<p>two worlds with arrows to the right that both point to the box encircling w_0 and w_1; they show that B's information states in w_0 and w_1 both equal $\{w_0, w_1\}$ (encircled worlds);</p>
	<p>arrow to the left of w_2 is pointing at w_0; it shows that A's information state in w_2 is $\{w_0\}$; arrow to the right of w_2 is pointing at the box encircling w_0 and w_1; it shows that B's information state in w_2 equals $\{w_0, w_1\}$ (encircled worlds);</p>