



OPEN ACCESS

EDITED BY

Sayantana Choudhury,
Shree Guru Gobind Singh Tricentenary (SGT)
University, India

REVIEWED BY

Grigorios Panotopoulos,
University of La Frontera, Chile
Anzhong Wang,
Baylor University, United States

*CORRESPONDENCE

Jae-Weon Lee,
✉ scikid@gmail.com

RECEIVED 30 November 2023

ACCEPTED 22 January 2024

PUBLISHED 12 February 2024

CITATION

Matos T, Ureña-López LA and Lee J-W (2024),
Short review of the main achievements of the
scalar field, fuzzy, ultralight, wave, BEC dark
matter model.
Front. Astron. Space Sci. 11:1347518.
doi: 10.3389/fspas.2024.1347518

COPYRIGHT

© 2024 Matos, Ureña-López and Lee. This is
an open-access article distributed under the
terms of the [Creative Commons Attribution
License \(CC BY\)](https://creativecommons.org/licenses/by/4.0/). The use, distribution or
reproduction in other forums is permitted,
provided the original author(s) and the
copyright owner(s) are credited and that the
original publication in this journal is cited, in
accordance with accepted academic practice.
No use, distribution or reproduction is
permitted which does not comply with
these terms.

Short review of the main achievements of the scalar field, fuzzy, ultralight, wave, BEC dark matter model

Tonatiuh Matos¹, Luis A. Ureña-López² and Jae-Weon Lee^{3*}

¹Departamento de Física, Centro de Investigación y de Estudios Avanzados del IPN, Mexico City, Mexico, ²Departamento de Física, DCI, Campus León, Universidad de Guanajuato, León, Mexico, ³Department of Electrical and Electronic Engineering, Jungwon University, Koesan, Republic of Korea

The Scalar Field Dark Matter model has been known in various ways throughout its history; Fuzzy, BEC, Wave, Ultralight, Axion-like Dark Matter, etc. All of them consist in proposing that dark matter of the universe is a spinless field Φ that follows the Klein-Gordon (KG) equation of motion $\square\Phi - dV/d\Phi = 0$, for a given scalar field potential V . The difference between different models is sometimes the choice of the scalar field potential V . In the literature we find that people usually work in the non-relativistic, weak-field limit of the Klein-Gordon equation, where it transforms into the Schrödinger equation and the Einstein equations into the Poisson equation, reducing the KG-Einstein system, to the Schrödinger-Poisson system. In this paper, we review some of the most interesting achievements of this model from the historical point of view and its comparison with observations, showing that this model could be the last answer to the question about the nature of dark matter in the universe.

KEYWORDS

dark matter, scalar field, fuzzy dark matter, galaxy, black hole

1 Introduction

The Scalar Field Dark Matter (SFD) model proposes that the dark matter of the universe is a spinless particle Φ fulfilling the Klein-Gordon equation

$$\square\Phi - \frac{dV}{d\Phi} = 0 \quad (1)$$

where \square is the D'Alembertian operator in a curved spacetime, whose source of Einstein's equations is the Scalar Field (SF), where the SF can be neutral (real) or charged (complex). Therefore, strictly speaking, we have to solve the Einstein-KG system. However, in galaxies and in general in the universe, systems are non-relativistic, and gravitation is weak enough so that the Einstein-KG system can be reduced to the Schrödinger-Poisson system, which in general is much easier to solve.

The origin of SF is unknown since we do not have a definitive unifying theory of all interactions. The easiest way to understand its origin is to add an SF term to the Standard Model (SM) of particles as we do with the rest of the terms of this model. There are other proposals, such as that the SF comes from superstring theory or that it has a QCD-type origin but with ultralight mass, etc. We will treat it here in a generic way, and we will focus only on the results as a dark matter candidate, leaving its origin for future work.

Dark matter behaves in a way similar to dust, as in the case of Cold Dark Matter (CDM), and SF mimics dust when it oscillates at the minimum of a potential (Turner, 1983). Therefore, the most popular SF potentials are those in which the SF potential V has a minimum. It is convenient to expand the SF potential into an even series of Φ as

$$V = V_0 + \frac{1}{2}m^2\Phi^2 + \frac{\lambda}{4}\Phi^4 + \dots \quad (2)$$

where we can interpret V_0 as the expectation value of the SF vacuum, m is the mass and λ the self-interaction parameter, where for complex SF the Φ^2 changes by $|\Phi|^2$. This expansion is generic as long as we do not know the correct form of V . However, there are some proposals for V coming from some theories, such as QCD axions, with the SF potential $V = f\sin(b\Phi)$ or $V = A(\cosh(b\Phi) - 1)$ (Sahni and Wang, 2000; Matos and Urena-Lopez, 2001), derived from superstring theory.

To the best of our knowledge, the idea that an SF could be dark matter of the universe began in 1983 by Baldeschi et al. (1983), where they fit the rotation curves of galaxies using boson and fermion particles. In 1990 the authors in (Press et al., 1990) presented a model in which a quartic potential $V = V_0(1 - \lambda|\Phi|^2)^2$ can explain the large-scale structure of the universe; this SF covers the missing mass of the universe. The first comparison of the model with real galaxies was made in 1992 (Sin, 1994; Ji and Sin, 1994), where the authors proposed a Bose gas to fit the rotation curves of galaxies, with a mass of the SFDM of the order of $10^{-23}eV$. The first numerical simulations with SF to form galaxies were presented in 1993 in (Widrow and Kaiser, 1993), where they obtained the shape of a galaxy using the Schödiger–Poisson system.

In 1995 (Lee et al., 1996) suggested that repulsive quartic self-interaction increases the length scale to $O(\sqrt{\lambda}m_p/m^2)$ even for a tiny λ , where m_p is the Planck mass. An approximate analytic solution for the ground state was obtained for the Thomas–Fermi limit. Based on the theory of boson stars, the maximum stable central density and the maximum halo mass for the $\lambda = 0$ case yielded a bound of $10^{-28} \leq m \leq 10^{-22}eV$.

Independently, in 1998, it was proposed in (Matos and Guzman, 2000) that SF can solve the problem of dark matter and that this field can explain the rotation curves of galaxies, initiating the first systematic study of this paradigm. Subsequently, the idea of SFDM has been rediscovered many times, such as Fuzzy DM (Hu et al., 2000), Quintessential DM (Arb et al., 2001; Arbey et al., 2002); Ultralight DM (Amendola and Barbieri, 2006; Lundgren et al., 2010); Bose-Einstein condensate DM (Boehmer and Harko, 2007; Rindler-Daller and Shapiro, 2010; Chavanis, 2011); Wave DM ((Bray, 2010; Schive et al., 2014)); Super Fluid DM (Berezhiani et al., 2023a), etc. In 2017 this idea became fashionable and one of the favorite candidates to explain DM after the publication of the article (Hui et al., 2017).

The SFDM model has two parameters, the mass $m = \widehat{m}c/\hbar$, where \widehat{m} is the mass in grams, and the self-interaction parameter λ . With these two parameters, it is possible to fit a large number of DM observations into the universe. The objective of this work is to historically list some of its most important achievements, and we will focus on the following.

- Good agreement of the rotation curves of stars and dust around galaxies.

- Good agreement with cosmological constrains and Big Bang Nucleosynthesis
- An alternative solution to the cusp-core problem.
- An alternative solution to the problem of satellites.
- The central black holes in galaxies.
- This is in excellent agreement with the CMB and MPS cosmological observations.
- The agreement with the cosmological numerical simulation.
- A natural explanation of the anomalous trajectories of satellites around galaxies.

Some predictions.

- The existence of a soliton at the center of galaxies.
- The structure formation of galaxies according to cosmological numerical simulations
- An alternative explanation to the Fermi Bubbles

In what follows, we want to touch on each of these issues by giving a historical development of the paradigm showing how the SFDM model solves or reduces the problem in each of these issues.

2 Rotation curves

Completely independently, in 1998 it was proposed by Tonatiuh Matos and Francisco S. Guzmán the idea that dark matter (DM) could be a scalar field as a doctoral topic. In this work (*Scalar fields as dark matter in spiral galaxies*, Matos and Guzman, arXiv:gr-qc/9810028 (Matos and Guzman, 2000)), was shown that this hypothesis could explain the observed rotation curves of stars and gas around galaxies. As in (Sin, 1994), the authors can fit well the rotation curves of specific galaxies. The fit of the rotation curves has been carried out through approximate solutions of the Schrödinger–Poisson system or through numerical simulations. For example, if the SF is real, in (Lee et al., 1996) and (Harko, 2011a) the authors used a Thomas–Fermi approximation to fit the rotation curves. They found that the density profile ρ of the SFDM is

$$\rho = \rho_0 \frac{\sin(x)}{x} \quad (3)$$

where ρ_0 is a constant and x is a unitless distance parameter. In (Robles and Matos, 2013a) the authors find that if the SF is complex, using some approximations, they find

$$\rho = \rho_0 \frac{\sin^2(x)}{x^2} \quad (4)$$

Or, using numerical simulations, in (Schive et al., 2014) they find that the density profile is

$$\rho = \frac{\rho_0}{(1+x^2)^8} \quad (5)$$

in the core of the galaxy, but the NFW profile $\rho = \frac{\rho_0}{x(1+x^2)}$ for x is larger than this core. All of these profiles fit well the rotation curves of galaxies.

However, in 2002 it was noted that there were two problems with the model that had to be dealt with. The first is that galaxies were found to be unstable according to this model. The second is that

supermassive black holes in galaxies could swallow the SFDM halo. The second problem will be described in Section 6. Next, we describe the first problem.

In (Guzman and Urena-Lopez, 2003; Guzman and Urena-Lopez, 2004; Guzman and Urena-Lopez, 2006) the authors found that the numerical simulations of the collapse of the SFDM do not stabilize, and the collapse continues until it forms a too compact object; the authors call it gravitational cooling. This collapse depends on the mass of the object; for large objects, this gravitational cooling is long enough to explain the existence of galaxy clusters, but for smaller masses it is too short to explain the shape of galaxies. There are at least two ways out of this problem. The first is to take into account the rotation of the SFDM. If the SFDM has a spin, the spin prevents the galaxy from collapsing. The second way out is to take into account the quantum character of the SFDM and invoke the excited states of the system.

It is possible to derive a Tully-Fisher relation from SFDM as in MOND (Bray and Goetz, 2014; Lee et al., 2019).

Another line of research is that when the SFDM is in a thermal bath and its finite temperature is taken into account, the Schrödinger equation predicts that the SF can have fundamental and excited states. The first attempts at this line of research are (Matos and Suarez, 2011), using a one-loop SF potential,

$$V = -m_{\Phi}^2 \Phi \Phi^* + \frac{\lambda}{2} (\Phi \Phi^*)^2 + \frac{\lambda}{4} \Phi \Phi^* T^2 + \frac{\pi^2}{90} T^4 \quad (6)$$

and (Harko and Madarassy, 2012) using the SF expansion. More approaches to this are given in (Robles and Matos, 2013a; Matos and Suárez, 2014), where the rotation curves are fitted using the ground and excited states of the SFDM. But the most interesting success of this line of research is the fact that the excited states are figure 8-shaped and are capable of explaining the Vast Polar Orbits (VPO), which we will see in a later section.

3 The cosmological constraints

Since the model fits the rotation curves of galaxies well, it must also be checked whether this model complies with cosmological observations, for example, Big Bang Nucleosynthesis (BBN), Mass and Power Spectrum (MPS), Cosmic Microwave Background (CMB), etc. This was done for the first time in (Matos and Urena-Lopez, 2001), where the authors used a modified code CMBFAST to obtain the MPS and CMB of the model, and they found more or less agreement with the data found up to this point. At that time, the data were not that good, but it was enough to say that the agreement between the data and the model is good enough for the model to be viable. This analysis was then performed several times as the data were improved, showing that the model agrees excellently with the cosmological data; see, for example, (Harko, 2011b; Hlozek et al., 2015; Ureña López and Gonzalez-Morales, 2016; Cedeño et al., 2017).

It is usual to separate the background part from a perturbed fluctuation, $\Phi = \Phi_0 + \delta\Phi$, then the fluctuation $\delta\Phi$ fulfills the following equations:

$$\begin{aligned} \nabla^2 \delta\Phi - \delta\ddot{\Phi} - 2H\delta\dot{\Phi} + V_{,\Phi_0\Phi_0} a^2 \delta\Phi + V_{,\Phi_0^*\Phi_0^*} a^2 \delta\Phi^* - 2V_{,\Phi_0\Phi_0^*} a^2 \delta\Phi + 4\dot{\Phi}_0 \delta\dot{\Phi} &= 0, \end{aligned} \quad (7)$$

$$2\nabla^2 \phi - 6H(\dot{\phi} + H\phi) = \kappa^2 \left[(\Phi_0 \delta\Phi^* + \dot{\Phi}_0^* \delta\Phi) - 2\phi \dot{\Phi}_0 \dot{\Phi}_0^* + a^2 \delta V \right], \quad (8)$$

where ϕ is the Newtonian potential, and δV is defined as

$$\delta V := V_{,\Phi_0} \delta\Phi + V_{,\Phi_0^*} \delta\Phi^*. \quad (9)$$

(7) is the KG equation for the perturbation $\delta\Phi$, and (8) is the Poisson equation. These two are part of the fluctuation equations used to study perturbations in an SFDM system. The rest of the equations are the same as in the Λ CDM model.

The next observation to be taken into account is the BBN. In (Li et al., 2014) it has been shown that to comply with the BBN constraint, it is necessary for the SF to have a self-interaction $\lambda\Phi^4$, with $\lambda \sim 10^{-90}$ ultra small but not zero. This result is very important because it implies that the SF must have a small self-interaction to satisfy all cosmological constraints, although this very small self-interaction seems to have no observational repercussions at the galactic level.

Furthermore, gravitational waves (GW) impose some restrictions on the SFDM model. In (Li et al., 2017) it is shown that the GWs produced during inflation can interact with the SFDM and produce a signal detectable by LIGO/Virgo and LISA. This possible interaction imposes some limitations on the reheating temperature of the universe, excluding a range of reheating-temperature parameters for a certain range of mass and self-interaction parameters.

4 The cusp-core problem

This problem has been observed since the 1990s, when people realize that CDM numerical simulations predict a huge concentration of DM at the center of galaxies and observations in dwarf galaxies seem to indicate that there is not as much concentration there, that is, the density profile of the DM in the center appears to be constant. There is a lot of discussion on this topic, but observations at the center of galaxies are not compatible with CDM predictions. To solve this problem, CDM needs more physical assumptions to flatten the density profile. There is a great deal of literature on this. Furthermore, in the satellite galaxies of our neighborhood, the center of these galaxies shows that they all have the same mass of $10^7 M_{\odot}$ within the first 300 pc, for at least six orders of magnitude in luminosity of the satellite galaxies (Strigari et al., 2008). Although it has been confronted with LCDM, its solution is not satisfactory.

The first paper proposing that SFDM could provide a natural solution to this problem was in (Hu et al., 2000), where the authors solve the SFDM equations

$$i \left(\partial_t + \frac{3}{2} \frac{\dot{a}}{a} \right) \psi = \left(-\frac{1}{2m} \nabla^2 + m\phi \right) \psi \quad (10)$$

where $\Phi = \psi e^{imt} + \psi^* e^{-imt}$, in one dimension to show that SFDM does not have such concentrations. The idea they put forward is that the SFDM model, which they called Fuzzy Dark Matter, has a quantum character and it is the uncertainty principle that prevents the DM from concentrating on a point, because in that case we are able to locate it. Due to the uncertainty principle, the location of the

DM causes the angular momentum to grow, and this prevents the concentration of matter in the center. The first analytical result on this topic was published in (Bernal et al., 2008), where the authors use approximations to find the density profile of dwarf galaxies and find a fit between observations of real dwarf galaxies and the central density profile predicted by SFDM, showing that this problem is solved naturally for the SFDM model without further assumptions. See also (Harko, 2011a; Su and Chen, 2011) for an alternative approach. This result was corroborated in (Schive et al., 2014) using 3D numerical simulations, and the authors show that SFDM does indeed have a core density profile as previously predicted by (Hu et al., 2000) and (Bernal et al., 2008) and opened an important line of research for 3D numerical simulations followed by (Du et al., 2017), (Mocz, 2019), among others. Numerical simulations of SFDM today have the same level of accuracy as CDM, which shows that SFDM is a viable model for the DM of the universe.

The observation that all satellite galaxies have the same amount of DM inside of the first 300 pc has also been faced using SFDM in (Lee and Lim, 2010) using numerical simulations, here the authors find that the visible matter density of a dwarf galaxy has a universal core size $r_c = 4,000$, corresponding to the physical size $r_c \sim 300$ pc and the total mass within this size $M_{tot} \sim 0.00019$, corresponding to $4.75 \times 10^7 M_\odot$. However, the SFDM is a field and satisfies the Schrödinger-Poisson field equations that contain scale invariance. In (Ureña López et al., 2017) using analytical methods, it was shown that this scale invariance implies that all galaxies must have the same mass within ~ 350 pc, giving a completely natural explanation to the observations of (Strigari et al., 2008).

5 The satellites problem

In the year 2000, Tonatiuh Matos and Luis A. Ureña-López, studied for the first time the SFDM hypothesis from a cosmological point of view. In (Matos and Urena-Lopez, 2001) the results were spectacular, finding for the first time that all cosmological observations up to that time were explained within the error bars by SFDM. They showed that the CMB and MPS were in agreement with those observed at the time and began the systematic cosmological study of this paradigm. The main result in (Matos and Urena-Lopez, 2001) was that, using the mass of the scalar field as a free parameter, it was shown that the SF has a natural cutoff of the mass power spectrum, which implies that the theoretical number of satellite galaxies is of the order of magnitude of those observed, provided that the mass of the SF is 10^{-22} eV, coinciding with the mass necessary to explain the rotation curves of galaxies. This result was corroborated many years later by numerical simulations (Schive et al., 2014) and semi-analytical analysis (Bozek et al., 2015).

Simply, the main idea was that perturbations of the scalar field $\delta\Phi$ from the decomposition $\Phi = \Phi_0 + \delta\Phi$ form the large-scale fabric of space-time. Here Φ_0 is the SF for the background and depends only on t . The point is that this perturbation $\delta\Phi$ follows a damped harmonic Equation (7) with the potential $V = m^2\Phi^2$.

$$\ddot{\Phi}_k + 2H\delta\dot{\Phi}_k - (k^2 + m^2 a^2) \delta\Phi_k = -2m^2\Phi_0 a^2 \phi - 4\dot{\phi}\dot{\Phi}_0, \quad (11)$$

where Φ_k is the Fourier transform of Φ and k is its corresponding wave number, driven by a force $-2m^2\Phi_0 a^2 \phi - 4\dot{\phi}\dot{\Phi}_0$ that oscillates

with a frequency very similar to the mass of the SF. The damped term essentially depends on the mass of the SF, the scale factor a that determines the redshift of the perturbation, and the wave number k of the Fourier transform of the perturbation, that is, the size of the perturbation. When the damping term is in resonance with the oscillating force term, we will have an increasing perturbation. But when they are not in resonance, the damping term will cause the perturbation to decrease and disappear. With this, a relationship was found between the size and redshift of the perturbation and the mass of the SF versus the frequency of the driving force. The free parameter m was set using the hitherto known number of galaxies in our neighborhood given the magic number of $m \sim 10^{-22}$ eV, according to the same mass value found to fit rotation curves in galaxies. In fact, the disturbance equation was solved with numerical methods, giving a relationship between all the quantities involved.

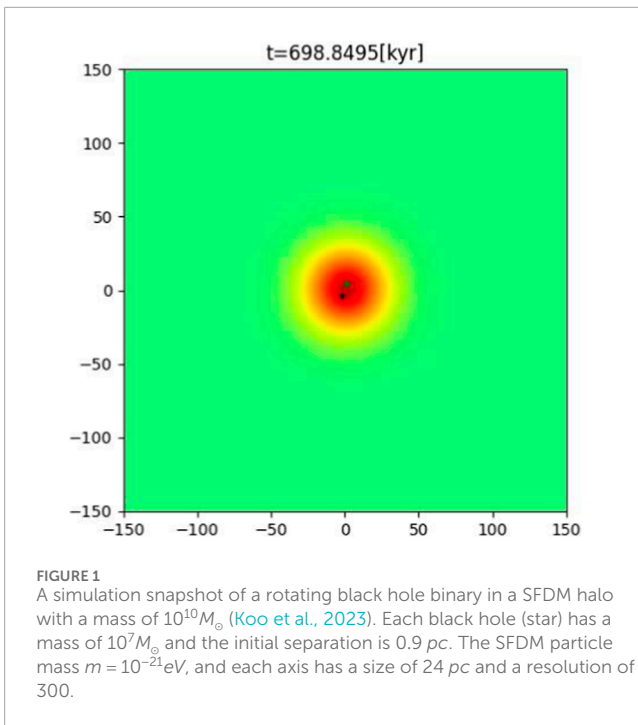
The first work that proved that this result is right was in (Guzman and Urena-Lopez, 2004), where the author found that the gravitational collapse of a SF contains, in fact, this cutoff point of the mass power spectrum. Further in (Schive et al., 2014), using numerical 3D simulations, the author finally showed that indeed the number of satellite galaxies was on the order of the observed one in our galaxy. See also (Du et al., 2017; Mocz, 2019).

It is also suggested that the SFDM model exhibits a characteristic size and mass scale consistent with dwarf galaxies and can account for the observed size evolution of very massive compact galaxies in the early universe (Le and e, 2009; Lee, 2016).

6 The central black holes

Galaxies contain a Supermassive Black Hole (SMBH) at their center. However, it is difficult to understand how some SMBHs formed, especially those at high redshifts. If the growth of these SMBH were by accretion, we have to explain how this accretion of matter grows the SMBH from $10^2 M_\odot$ to, say, $10^9 M_\odot$ with an accretion of one M_\odot per year, at high redshifts. There are a few proposals for this, ranging from galaxy collisions to primordial formation of SMBH during the Big Bang. All of them contain pros and cons, and the SFDM offers an alternative explanation of this problem that seems very natural but so far incomplete. The proposal is based on numerical simulations carried out in (Seidel and Suen, 1991), where the authors found that the critical mass of collapse of a real scalar field is $M_{crit} = 0.6m_{pl}^2/m$, where m_{pl} is the Planck mass and m is the SFDM mass. If we plug $m \sim 10^{-22}$ eV into this formula, we find $M_{crit} \sim 10^{13} M_\odot$, which is very large. For a complex scalar field, the result is similar (Balakrishna et al., 1998), the authors find that the critical mass of collapse for a complex SF is $M_{crit} = 0.1m_{pl}^2/m$. This gives two results; the first is that SFDM galaxy halos have a given natural mass limit, so that beyond this critical mass the SF collapses to form BH. However, at the same time, this large collapsing mass could explain the formation of SMBH at the center of galaxies, provided that a mechanism can be provided to reduce this mass. This hypothesis was proposed in (Torres et al., 2000; Urena-Lopez and Liddle, 2002) and further discussed in (Avilez et al., 2018; Lee et al., 2020; Padilla et al., 2021), giving some optimistic results.

However, there is an interesting problem that we must face. Supermassive black holes at the center of galaxies could swallow all the SF. This problem was addressed in (Cruz-Osorio et al., 2011) and



years later in (Hui et al., 2019; Kiczek and Rogatko, 2020). The results show that the SF can be accreted by the central SMBH, but at a very small rate, so that the SF can coexist with the SMBH for longer than the lifetime of the universe.

In 2023 (Koo et al., 2023) it was proposed that halos surrounding rotating SMBH binaries with SFDM spikes (Shen et al., 2023) can emit dark matter waves (Bromley et al., 2023). These waves could carry away orbital energy from the black holes, causing them to merge rapidly and potentially providing a solution to the final parsec problem (See Figure 1).

7 Cosmological numerical simulations

The first simulations of SFDM were using the Schrödinger-Poisson system approximation

$$\begin{aligned} \partial_t \psi &= \left(-\frac{1}{2m} \nabla^2 + m\phi \right) \psi \\ \nabla^2 \phi &= 4\pi G \delta\rho, \end{aligned} \quad (12)$$

where $\delta\rho = \frac{m^2}{2} \delta\psi$, were performed at (Alcubierre et al., 2002a; Alcubierre et al., 2002b; Alcubierre et al., 2003), where it was shown that SF collapse forms stable objects and can be compared to the halo of a galaxy, generating a central nucleus and with a natural cut in the mass power spectrum. However, the first 3D numerical simulations were performed at (Schive et al., 2014), where previous results, such as the central nucleus in the center of the galaxy and the cutoff in the mass spectrum, were corroborated beyond doubt. This article started a series of works on 3D numerical simulations that were very successful; see, for example, (Du et al., 2017; Mocz et al., 2017; Du et al., 2018; Church et al., 2019; Mocz, 2019; Davies and Mocz, 2020; Mocz et al., 2020; Veltmaat et al., 2020), by reproducing the shape of the universe, we observe with fewer satellite galaxies and

a central nucleus in galaxies without additional physics. Today, these numerical simulations can be compared with the same ones made with Λ CDM, showing only some essential differences. The most important thing is that, while Λ CDM predicts a central cusp density profile, SFDM predicts a galactic center, with a flat region called the soliton, discovered in 3D simulations in (Schive et al., 2014). This feature can make the difference between these two models (Dave and Goswami, 2023a). The other is the number of satellite galaxies around their hosts. While Λ CDM predicts thousands of satellite dark halos that need to be detected somehow, SFDM predicts a moderate number of them, which agree well with what is observed. Another feature that should be the difference is that the SFDM predicts that this soliton at the center of galaxies moves with time (Chowdhury et al., 2021), this feature is a footprint to be observed in galaxies if the SFDM is the DM in the universe.

The first numerical simulations with spherical symmetry also showed another problem; after the formation of the SFDM the object continues to collapse forming an object too dense to simulate a galaxy halo. This collapse is scale dependent; for large objects, such as galaxy clusters, this collapse takes so long that the formation of galaxy clusters can be explained very well with this model, but the formation of galaxies and dwarf galaxies collapse too soon (Guzman and Urena-Lopez, 2003; Guzman and Urena-Lopez, 2004; Guzman and Urena-Lopez, 2006). This problem was addressed for the first time in the literature considering the quantum characteristics of the scalar field using the excited states of the system in (Urena-Lopez and Bernal, 2010; Matos and Suárez, 2014). The idea is that since the SFDM is a quantum-mechanical system, it should contain excited states that could give the system some additional stability. The main result here is that galaxy halos, even for small galaxies such as LSB or dwarf, remain stable (Urena-Lopez and Bernal, 2010; Guzmán and Ureña López, 2020). Therefore, it is necessary for the halo of galaxies to have at least two cohabiting states to adapt to the rotation curves of the galaxies. With this in mind, in (Robles and Matos, 2013a) some galaxy rotation curves were fitted using excited states of the SFDM, giving a good result. More recently, this idea has been used to explain other phenomena such as VPOs and Fermi bubbles, which we will discuss in the following. This idea started a new paradigm in the literature called ℓ -boson stars (Alcubierre et al., 2018).

8 The vast polar orbits (VPO) problem

For some time, astronomers have observed that the satellite galaxies of the Milky Way are not distributed homogeneously but that there are anomalous trajectories of the satellites of this galaxy, called VPO (Pawlowski et al., 2013; Pawlowski and Kroupa, 2020), establishing that among the 50 satellites in the Local Group, 43 are contained in four different planes, but this is inconsistent with simulations based on CDM because it predicts that this distribution must be isotropic. Furthermore, this alignment has also been observed in M31 (Conn et al., 2013; Ibata et al., 2013). Numerical CDM simulations predict that satellite galaxies around their host should be homogeneously distributed (Shaya and Tully, 2013; Pawlowski, 2018). More recently, this same alignment of the paths of satellite galaxies has been observed in Centaurus, where 31 satellite galaxies in the constellation of Centaurus interact gravitationally with the elliptical galaxy Centaurus A and display a

similar anisotropic alignment (Müller et al., 2018). In a recent work (Solís-López et al., 2021), the authors took into account these excited states of the galaxy's halo, treating the galaxy's halo as an atom

$$\Phi = R(r) Y_l^m(\varphi, \theta) T(t) \quad (13)$$

where Y_l^m are the spherical harmonics functions, to explain these observations and show that this behavior of the satellites is very natural in SFDM. This is so because the quantum character of the SFDM (Matos, 2022) allows us to put the states of the SF as in an atom. The ground state is spherically symmetric, but the first excited state has a figure-8 shape, aligning surrounding objects with the figure-8 shape. This is an important feature of the SFDM, the probability of finding this alignment is too low to be able to be explained as a simple chance, but if the SFDM is the real nature of the DM, we must find this alignment in more galaxies. These observations may be made in the near future. In (Park et al., 2022) another mechanism for the formation of the satellite plane based on the gravitational cooling of the SFDM was proposed.

9 Detection

It is actually very difficult to detect or design an experiment to detect a spinless particle with an ultralight mass. There are several hypotheses and proposals that attempt to do that, but so far these attempts have had limited results. We want to mention only some of them here.

In (Bozek et al., 2015) the authors use the MPS cutoff for the SFDM to construct a UV luminosity function and compare it to the Hubble ultra-deep field UV luminosity function, giving a constraint on the SFDM mass $m \geq 10^{-22}$ eV. Observables of 21 cm and fluctuations in CMB are proposed in (Kadota et al., 2014), including measurements of the CMB lens that can provide information on the existence of SFDM for different masses $m < 10^{-26}$ eV.

Other possible ways to detect SFDM are using atomic methods such as hyperfine frequencies (Hees et al., 2016), or atomic clocks (Kouvaris et al., 2020; Filzinger et al., 2023), interferometers (Aiello et al., 2022; Zhao et al., 2022; Kim, 2023), atom multigradiometry (Badurina et al., 2023; Badurina et al., 2023), or neutrino interactions (Cordero et al., 2023).

Gravitational waves produced by Black Hole (BH) mergers are one of the most anticipated ways to detect any sign of the existence of SFDM, e.g., gravitational waves emitted by BH (D'Antonio, 2018; Isi et al., 2019; Morisaki and Suyama, 2019; Palomba et al., 2019; Sun et al., 2020; Ng et al., 2020; Ng et al., 2021a; Banerjee et al., 2023; Liu et al., 2021; Manita et al., 2023; Yu et al., 2023; Miller and Mendes, 2023; Tsutsui and Nishizawa, 2023), or gravitational wave resonance (Speeney et al., 2022; Delgado, 2023), using superradiance from BH (Hannuksela et al., 2019), BH mergers (Ng et al., 2021b; Chung et al., 2021; Chan and Hannuksela, 2022), or the deflection angle of black holes (Pantig and Övgün, 2022). Another possible way to detect SFDM is observations in galaxies and their surroundings, e.g., with short-range gravity experiments (Qin et al., 2022), gravitational lensing (Powell et al., 2022; Cabrera-Rosas and Matos, 2023) or dynamical friction (Traykova et al., 2021; Boudon et al., 2022; Vicente and Cardoso, 2022; Wang and Easther, 2022; Berezhiani et al., 2023b), binary

pulsars (Blas et al., 2020) compact eccentric binaries (Su et al., 2021), extreme mass-ratio inspirals (Barsanti et al., 2023), direct BH observations using the Event Horizon Telescope (Davoudiasl and Denton, 2019; Saha et al., 2022; De Luca and Houry, 2023), or by asteroid date (Chakrabarti et al., 2022; Tsai et al., 2023a; Tsai et al., 2023b), quadruply-imaged quasars (Laroche et al., 2022) frame dragging effect (Poddar, 2022), kinetic Sunyaev-Zel'dovich effect (Farren et al., 2022), Shapiro delay (Poddar, 2021), or in the center of our galaxy by motion of the S2 star around Sgr A (Yuan et al., 2022; Della Monica and de Martino, 2023).

An alternative is to use the Pulsar Timing Array (PTA) to detect SFDM particles (Porayko et al., 2018; Afzal et al., 2023; Antoniadis et al., 2023; Hwang et al., 2023; Smarra et al., 2023; Xia et al., 2023).

The existence of a central soliton is another distinguishing feature of SFDM (De Martino et al., 2020). Another interesting method is by tidal effects and tunneling out of satellite galaxies (Dave and Goswami, 2023b). The number of proposals is huge, there are many possible ways to find a signal of the existence of SFDM (for a recent one see (Kousha et al., 2023)), and it is possible that the detection of this type of DM is close.

10 Conclusions and challenges

The SFDM model has proven to be a paradigm that is very capable of explaining DM in the universe. This model has surpassed 25 years of most new observations of the universe and every time the observations go further with better resolution, the model fits the observations with better clarity, especially cosmological observations which are now very high resolution. This model is today a true competitor to the cold dark matter model, as it offers alternative explanations for the phenomena we see in the universe, with fewer hypotheses and less additional physics.

We believe that there are at least two observations that other DM models cannot explain, the first is that all satellite galaxies in our neighborhood contain the same amount of matter within the first 300pc. As we saw, the SFDM model is capable of explaining this fact in a very natural way. The other observation that can be explained using the SFDM in a very natural way is the VPOs. One of the main characteristics of the SFDM is its quantum character (Matos, 2022), even though the SF is considered classical in this model, the SF complies with the Schrödinger equation that contains excited states. This is a characteristic unique to this model, therefore, if we see more galaxies with VPO or with Femi bubbles, this will be a strong corroboration that SFDM or some very similar candidate is the final answer for the DM nature in the universe. We do not know of any other model that can explain these observations in a way as natural as the SFDM model does.

Nevertheless, the model still faces some challenges, some of which we list here. Perhaps one of the strongest constraints placed on the SFDM is that of the Lyman- α forest observations. In (Armengaud et al., 2017; Iršič et al., 2017; Nori et al., 2019) it was found that the SFDM masses $m < 2.3 \times 10^{-21}$ eV are discarded. Of course, there is a possibility that the observations of the Lyman- α forest are not so fine that we cannot say with enough certainty that the model is incorrect; we have to expect new observations of this in the near future, e.g., DESI observations, which will come

to light shortly. On the other hand, there are ways to solve this problem; one is provided in (Schive and Chiueh, 2018) with an extension to the SFDM model or it has been argued in (Robles and Matos, 2013b) that the SFDM mass that we read in galaxies is the effective mass with the finite-temperature contribution, but the SFDM mass could be $m_\phi \sim 10^{-21}$ eV, and each galaxy could show an apparently different mass because we read the effective mass due to the finite temperature of the SFDM in each galaxy. The temperature of the SFDM alters the scale l for each galaxy as $l^2 = \omega^2 - \frac{1}{2}(T_c^2 - T^2)a^2l^2 = \omega^2 - m^2a^2$, explaining why we see that the mass of the SFDM at cosmological scales is $m_\phi \sim 10^{-21}$ eV and in galaxies could be $m \sim 10^{-22} - 10^{-24}$ eV. Incorporating the self-interaction of SFDM could be another potential solution to this problem.

The SFDM is today one of the most studied DM models in the community and it is not far from this model proving to be the last answer to the DM problem in the universe.

Author contributions

TM: Writing—original draft. LU-L: Writing—original draft. J-WL: Writing—original draft.

Funding

The author(s) declare financial support was received for the research, authorship, and/or publication of this article. This work

References

- Afzal, A., Agazie, G., Anumrapudi, A., Archibald, A. M., Baker, P. T., Becsy, B., et al. (2023). The NANOGrav 15 yr data set: search for signals from new physics. *Astrophys. J. Lett.* 951, L11. 2306.16219.
- Aiello, L., Richardson, J. W., Vermeulen, S. M., Grote, H., Hogan, C., Kwon, O., et al. (2022). Constraints on scalar field dark matter from colocated michelson interferometers. *Phys. Rev. Lett.* 128, 121101. doi:10.1103/physrevlett.128.121101
- Alcubierre, M., Barranco, J., Bernal, A., Degollado, J. C., Diez-Tejedor, A., Megevand, M., et al. (2018). ℓ -boson stars. *Cl. Quant. Grav.* 35, 19LT01. 1805-11488. doi:10.1088/1361-6382/aadcb6
- Alcubierre, M., Becerril, R., Guzman, S. F., Matos, T., Nunez, D., and Urena-Lopez, L. A. (2003). Numerical studies of 2-oscillatons. *Cl. Quant. Grav.* 20, 2883–2903. gr-qc/0301105. doi:10.1088/0264-9381/20/13/332
- Alcubierre, M., Guzman, F. S., Matos, T., Nunez, D., Urena-Lopez, L. A., and Wiederhold, P. (2002a). Galactic collapse of scalar field dark matter. *Cl. Quant. Grav.* 19, 5017–5024. gr-qc/0111012. doi:10.1088/0264-9381/19/19/314
- Alcubierre, M., Guzman, F. S., Matos, T., Nunez, D., Urena-Lopez, L. A., and Wiederhold, P. (2002b). 4th international heidelberg conference on DM, 356–364. astro-ph/0204307.
- Amendola, L., and Barbieri, R. (2006). Dark matter from an ultra-light pseudo-Goldstone-boson. *Phys. Lett. B* 642, 192–196. hep-ph/0509257. doi:10.1016/j.physletb.2006.08.069
- Antoniadis, J., Arumugam, P., Arumugam, S., Babak, S., Bagchi, M., Bassa, C. G., et al. (2023). The second data release from the European Pulsar Timing Array III. Search for gravitational wave signals. arXiv 2306.16227.
- Arbey, A., Lesgourgues, J., and Salati, P. (2001). Quintessential halos around galaxies. *Phys. Rev. D.* 64, 123528. astro-ph/0105564. doi:10.1103/physrevd.64.123528
- Arbey, A., Lesgourgues, J., and Salati, P. (2002). Cosmological constraints on quintessential halos. *Phys. Rev. D.* 65, 083514. astro-ph/0112324. doi:10.1103/physrevd.65.083514
- Armengaud, E., Palanque-Delabrouille, N., Yèche, C., Marsh, D. J. E., and Baur, J. (2017). Constraining the mass of light bosonic dark matter using SDSS Lyman- α forest. *Mon. Not. Roy. Astron. Soc.* 471, 4606. 1703-4614. doi:10.1093/mnras/stx1870
- Avilez, A. A., Padilla, L. E., Bernal-Marin, T., and Matos, T. (2018). On the possibility that ultra-light boson haloes host and form supermassive black holes. *Mon. Not. Roy. Astron. Soc.* 477, 3257. doi:10.1093/mnras/sty572
- Badurina, L., Gibson, V., McCabe, C., and Mitchell, J. (2023). Ultralight dark matter searches at the sub-Hz frontier with atom multigradiometry. *Phys. Rev. D.* 107, 055002. doi:10.1103/physrevd.107.055002
- Balakrishna, J., Seidel, E., and Suen, W.-M. (1998). Dynamical evolution of boson stars. II. Excited states and self-interacting fields. *Phys. Rev. D.* 58, 104004. gr-qc/9712064. doi:10.1103/physrevd.58.104004
- Baldeschi, M. R., Ruffini, R., and Gelmini, G. B. (1983). On massive fermions and bosons in galactic halos. *Phys. Lett. B* 122, 221–224. doi:10.1016/0370-2693(83)90688-3
- Banerjee, S., Bera, S., and Mota, D. F. (2023). Prospects of probing dark matter condensates with gravitational waves. *JCAP* 03, 041. 2211.13988.
- Barsanti, S., Maselli, A., Sotiriou, T. P., and Gualtieri, L. (2023). Detecting massive scalar fields with extreme mass-ratio inspirals. *Phys. Rev. Lett.* 131, 051401. doi:10.1103/physrevlett.131.051401
- Berezhiani, L., Cintia, G., De Luca, V., and Khoury, J. (2023b). *Dynamical friction in dark matter superfluids: the evolution of black hole binaries*. arXiv 2311.07672.
- Berezhiani, L., Cintia, G., and Khoury, J. (2023a). Thermalization, fragmentation, and tidal disruption: the complex galactic dynamics of dark matter superfluidity. *Phys. Rev. D.* 107, 123010. 2212-10577. doi:10.1103/physrevd.107.123010
- Bernal, A., Matos, T., and Núñez, D. (2008). Flat central density profiles from scalar field dark matter halos. *Rev. Mex. Astron. Astrofísica* 44, 149.
- Blas, D., López Nacir, D., and Sibiryakov, S. (2020). Secular effects of ultralight dark matter on binary pulsars. *Phys. Rev. D.* 101, 063016. doi:10.1103/physrevd.101.063016
- Boehmer, C. G., and Harko, T. (2007). Can dark matter be a Bose-Einstein condensate? *JCAP* 06, 025. 0705.4158.
- Boudon, A., Brax, P., and Valageas, P. (2022). Subsonic accretion and dynamical friction for a black hole moving through a self-interacting scalar dark matter cloud. *Phys. Rev. D.* 106, 043507. doi:10.1103/physrevd.106.043507

was partially supported by Programa para el Desarrollo Profesional Docente; Dirección de Apoyo a la Investigación y al Posgrado, Universidad de Guanajuato; CONACyT México under Grants No. A1-S-17899, A1-S-8742, 304001, 376127, 240512, FORDECYT-PRONACES grant No. 490769 and I0101/131/07C-234/07 of the Instituto Avanzado de Cosmología (IAC) collaboration (<http://www.iac.edu.mx/>).

Conflict of interest

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

The author(s) declared that they were an editorial board member of Frontiers, at the time of submission. This had no impact on the peer review process and the final decision.

Publisher's note

All claims expressed in this article are solely those of the authors and do not necessarily represent those of their affiliated organizations, or those of the publisher, the editors and the reviewers. Any product that may be evaluated in this article, or claim that may be made by its manufacturer, is not guaranteed or endorsed by the publisher.

- Bozek, B., Marsh, D. J. E., Silk, J., and Wyse, R. F. G. (2015). Galaxy UV-luminosity function and reionization constraints on axion dark matter. *Mon. Not. Roy. Astron. Soc.* 450, 209. doi:10.1093/mnras/stv624
- Bray, H. L., and Goetz, A. S. (2014). *Wave dark matter and the tully-Fisher relation*. arXiv 1409.7347.
- Bray, H. L. (2010). *Scalar field dark matter as an alternative explanation for the polar orbits of satellite galaxies*. arXiv 1004.4016.
- Bromley, B. C., Sandick, P., and Shams, B. (2023). arXiv:2311.18013.
- Cabrera-Rosas, O. D. J., and Matos, T. (2023). arXiv:2305.16523.
- Cedeño, F. X. L., González-Morales, A. X., and Ureña López, L. A. (2017). One-parametric description for scalar field dark matter potentials. *Phys. Rev. D* 96, 061301. 1703.10180.
- Chakrabarti, S., Dave, B., Dutta, K., and Goswami, G. (2022). Constraints on the mass and self-coupling of Ultra-Light Scalar Field Dark Matter using observational limits on galactic central mass. *JCAP* 09, 074. 2202.11081.
- Chan, K. H. M., and Hannuksela, O. A. (2022). *Extracting ultralight boson properties from boson clouds around post-merger remnants*. arXiv 2209.03536.
- Chavanis, P.-H. (2011). BEC dark matter, Zeldovich approximation, and generalized Burgers equation. *Phys. Rev. D* 84, 063518. doi:10.1103/physrevd.84.063518
- Chowdhury, D. D., van den Bosch, F. C., Robles, V. H., van Dokkum, P., Schive, H.-Y., Chiueh, T., et al. (2021). On the random motion of nuclear objects in a fuzzy dark matter halo. *Astrophys. J.* 916. 2105.05268.
- Chung, A. K.-W., Gais, J., Cheung, M. H.-Y., and Li, T. G. F. (2021). Searching for ultralight bosons with supermassive black hole ringdown. *Phys. Rev. D* 104, 084028. doi:10.1103/physrevd.104.084028
- Church, B. V., Ostriker, J. P., and Mocz, P. (2019). Heating of Milky Way disc stars by dark matter fluctuations in cold dark matter and fuzzy dark matter paradigms. *Mon. Not. Roy. Astron. Soc.* 485, 2861. 1809-2876. doi:10.1093/mnras/stz534
- Conn, A. R., Lewis, G. F., Ibata, R. A., Parket, Q. A., Zucker, D. B., Martin, N. F., et al. (2013). The three-dimensional structure of the M31 satellite system; strong evidence for an inhomogeneous distribution of satellites. *Astrophys. J.* 766, 120. 1301.7131.
- Cordero, R., Delgado, L. A., and Miranda, O. G. (2023). European Spallation Source as a searching tool for an ultralight scalar field. *Phys. Rev. D* 107, 075023. 2207-11308. doi:10.1103/physrevd.107.075023
- Cruz-Osorio, A., Guzman, F. S., and Lora-Clavijo, F. D. (2011). Scalar field dark matter: behavior around black holes. *JCAP* 06, 1008.0027. doi:10.1088/1475-7516/2011/06/029
- D'Antonio, S. (2018). *Phys. Rev. D* 98, 103017. arXiv:1809.07202.
- Dave, B., and Goswami, G. (2023a). *Self-interactions of ULDM to the rescue?* arXiv 2304.04463.
- Dave, B., and Goswami, G. (2023b). *ULDM self-interactions, tidal effects and tunnelling out of satellite galaxies*. arXiv 2310.19664.
- Davies, E. Y., and Mocz, P. (2020). Fuzzy dark matter soliton cores around supermassive black holes. *Mon. Not. Roy. Astron. Soc.* 492, 5721. 1908-5729. doi:10.1093/mnras/staa202
- Davoudiasl, H., and Denton, P. B. (2019). Ultralight boson dark matter and event horizon telescope observations of M87*. *Phys. Rev. Lett.* 123, 021102. doi:10.1103/physrevlett.123.021102
- Delgado, P. C. M. (2023). arXiv:2309.09946.
- Della Monica, R., and de Martino, I. (2023). Bounding the mass of ultralight bosonic dark matter particles with the motion of the S2 star around Sgr A*. *Phys. Rev. D* 108, L101303. 2305.10242. doi:10.1103/physrevd.108.L101303
- De Luca, V., and Khoury, J. (2023). Superfluid dark matter around black holes. *JCAP* 04, 048. 2302.10286. doi:10.1088/1475-7516/2023/04/048
- De Martino, I., Broadhurst, T., Tye, S. H. H., Chiueh, T., and Schive, H.-Y. (2020). Dynamical evidence of a dark solitonic core of 109M in the milky way. *Phys. Dark Univ.* 28, 100503. doi:10.1016/j.dark.2020.100503
- Du, X., Behrens, C., and Niemeyer, J. C. (2017). Substructure of fuzzy dark matter haloes. *Mon. Not. Roy. Astron. Soc.* 465, 941. doi:10.1093/mnras/stw2724
- Du, X., Schwabe, B., Niemeyer, J. C., and Bürger, D. (2018). Tidal disruption of fuzzy dark matter subhalo cores. *Phys. Rev. D* 97, 063507. doi:10.1103/physrevd.97.063507
- Farren, G. S., Grin, D., Jaffe, A. H., Hložek, R., and Marsh, D. J. E. (2022). Ultralight axions and the kinetic Sunyaev-Zeldovich effect. *Phys. Rev. D* 105, 063513. doi:10.1103/physrevd.105.063513
- Filzinger, M., Dörscher, S., Lange, R., Klose, J., Steinel, M., Benkler, E., et al. (2023). Improved limits on the coupling of ultralight bosonic dark matter to photons from optical atomic clock comparisons. *Phys. Rev. Lett.* 130, 253001. doi:10.1103/physrevlett.130.253001
- Guzman, F. S., and Urena-Lopez, L. A. (2003). Newtonian collapse of scalar field dark matter. *Phys. Rev. D* 68, 024023. astro-ph/0303440.
- Guzman, F. S., and Urena-Lopez, L. A. (2004). Evolution of the Schrödinger-Newton system for a self-gravitating scalar field. *Phys. Rev. D* 69, 124033. gr-qc/0404014.
- Guzman, F. S., and Urena-Lopez, L. A. (2006). Gravitational cooling of self-gravitating Bose condensates. *Astrophys. J.* 645, 814-819. astro-ph/0603613. doi:10.1086/504508
- Guzmán, F. S., and Ureña López, L. A. (2020). Self-interacting multistate boson stars. *Phys. Rev. D* 101, 081302. 1912.10585.
- Hannuksela, O. A., Wong, K. W. K., Brito, R., Berti, E., and Li, T. G. F. (2019). Probing the existence of ultralight bosons with a single gravitational-wave measurement. *Nat. Astron.* 3, 447. 1804-09659. doi:10.1038/s41550-019-0712-4
- Harko, T. (2011a). *Bose-Einstein condensation of dark matter solves the core/cusp problem*. arXiv 1105.2996.
- Harko, T. (2011b). Evolution of cosmological perturbations in Bose-Einstein condensate dark matter: cosmological perturbations in BEC. *Mon. Not. Roy. Astron. Soc.* 413, 3095-3104. doi:10.1111/j.1365-2966.2011.18386.x
- Harko, T., and Madarassy, E. J. M. (2012). Finite temperature effects in Bose-Einstein condensed dark matter halos. *JCAP* 01, 020. 1110.2829. doi:10.1088/1475-7516/2012/01/020
- Hees, A., Guéna, J., Abgrall, M., Bize, S., and Wolf, P. (2016). Searching for an oscillating massive scalar field as a dark matter candidate using atomic hyperfine frequency comparisons. *Phys. Rev. Lett.* 117, 061301.
- Hložek, R., Grin, D., Marsh, D. J. E., and Ferreira, P. G. (2015). A search for ultralight axions using precision cosmological data. *Phys. Rev. D* 91, 103512. 1410-2896. doi:10.1103/physrevd.91.103512
- Hu, W., Barkana, R., and Gruzinov, A. (2000). Fuzzy cold dark matter: the wave properties of ultralight particles. *Phys. Rev. Lett.* 85, 1158-1161. astro-ph/0003365. doi:10.1103/physrevlett.85.1158
- Hui, L., Kabat, D., Li, X., Santoni, L., and Wong, S. S. C. (2019). Black hole hair from scalar dark matter. *JCAP* 06, 1904.12803.
- Hui, L., Ostriker, J. P., Tremaine, S., and Witten, E. (2017). Ultralight scalars as cosmological dark matter. *Phys. Rev. D* 95, 043541. doi:10.1103/physrevd.95.043541
- Hwang, J.-C., Jeong, D., Noh, H., and Smarra, C. (2023). *Pulsar timing Array signature from oscillating metric perturbations due to ultra-light axion*. arXiv 2311.00234.
- Ibata, R. A., Lewis, G. F., Conn, A. R., Irwin, M. J., McConnachie, A. W., Chapman, S. C., et al. (2013). A vast, thin plane of corotating dwarf galaxies orbiting the Andromeda galaxy. *Nature* 493, 62. doi:10.1038/nature11717
- Iršič, V., Viel, M., Haehnelt, M. G., Bolton, J. S., and Becker, G. D. (2017). First constraints on fuzzy dark matter from Lyman- α forest data and hydrodynamical simulations. *Phys. Rev. Lett.* 119, 031302. doi:10.1103/physrevlett.119.031302
- Isi, M., Sun, L., Brito, R., and Melatos, A. (2019). Directed searches for gravitational waves from ultralight bosons. *Phys. Rev. D* 99, 084042. 049901 (2020)], 1810.03812. doi:10.1103/physrevd.99.084042
- Ji, S. U., and Sin, S. J. (1994). Late-time phase transition and the galactic halo as a Bose liquid. II. The effect of visible matter. *Phys. Rev. D* 50, 3655-3659. hep-ph/9409267. doi:10.1103/physrevd.50.3655
- Kadota, K., Mao, Y., Ichiki, K., and Silk, J. (2014). Cosmologically probing ultra-light particle dark matter using 21 cm signals. *JCAP* 06, 011. 1312.1898.
- Kiczek, B., and Rogatko, M. (2020). Influence of dark matter on black hole scalar hair. *Phys. Rev. D* 101, 084035. doi:10.1103/physrevd.101.084035
- Kim, H. (2023). *Gravitational interaction of ultralight dark matter with interferometers*. arXiv 2306.13348.
- Koo, H., Bak, D., Park, I., Hong, S. E., and Lee, J.-W. (2023). arXiv e-prints arXiv:2311.03412. arXiv:2311.03412.
- Kousha, H. M., Ansarifard, M., and Abolhasani, J. (2023). arXiv:2312.10745.
- Kouvaris, C., Papantonopoulos, E., Street, L., and Wijewardhana, L. C. R. (2020). Probing bosonic stars with atomic clocks. *Phys. Rev. D* 102, 063014.
- Laroche, A., Gilman, D., Li, L., Bovy, J., and Du, X. (2022). arXiv:2206.11269.
- Lee, J.-W. (2009). Are galaxies extending? *Phys. Lett. B* 681, 118. doi:10.1016/j.physletb.2009.10.005
- Lee, J.-W. (2016). Characteristic size and mass of galaxies in the Bose-Einstein condensate dark matter model. *Phys. Lett. B* 756, 166. 1511-169. doi:10.1016/j.physletb.2016.03.016
- Lee, J.-W., Kim, H.-C., and Lee, J. (2019). Radial acceleration relation from ultra-light scalar dark matter. *Phys. Lett. B* 795, 206-210. doi:10.1016/j.physletb.2019.06.008
- Lee, J.-W., and Koh, I.-G. (1996). Galactic halos as boson stars. *Phys. Rev. D* 53, 2236-2239. hep-ph/9507385. doi:10.1103/physrevd.53.2236
- Lee, J.-W., Lee, J., and Kim, H.-C. (2020). The M-sigma relation of supermassive black holes from the scalar field dark matter. *Mod. Phys. Lett. A* 35, 2050155. doi:10.1142/s0217732320501552
- Lee, J.-W., and Lim, S. (2010). Minimum mass of galaxies from BEC or scalar field dark matter. *JCAP* 01, 0812.1342.
- Li, B., Rindler-Daller, T., and Shapiro, P. R. (2014). Cosmological constraints on Bose-Einstein-condensed scalar field dark matter. *Phys. Rev. D* 89, 083536. doi:10.1103/physrevd.89.083536

- Li, B., Shapiro, P. R., and Rindler-Daller, T. (2017). Bose-Einstein-condensed scalar field dark matter and the gravitational wave background from inflation: new cosmological constraints and its detectability by LIGO. *Phys. Rev. D* 96, 063505. doi:10.1103/physrevd.96.063505
- Liu, D., Yang, Y., Wu, S., Xing, Y., Xu, Z., and Long, Z.-W. (2021). Ringing of a black hole in a dark matter halo. *Phys. Rev. D* 104, 104042. doi:10.1103/physrevd.104.104042
- Lundgren, A. P., Bondarescu, M., Bondarescu, R., and Balakrishna, J. (2010). Lukewarm dark matter: Bose condensation of ultralight particles. *Astrophys. J. Lett.* 715, L35–L39. doi:10.1088/2041-8205/715/1/L35
- Manita, Y., Takeda, H., Aoki, K., Fujita, T., and Mukohyama, S. (2023). arXiv:2310.10646.
- Matos, T. (2022). The quantum character of the scalar field dark matter. *Mon. Not. Roy. Astron. Soc.* 517, 5247.
- Matos, T., and Guzman, F. S. (2000). Scalar fields as dark matter in spiral galaxies. *Cl. Quant. Grav.* 17, L9–L16. gr-qc/9810028. doi:10.1088/0264-9381/17/1/L02
- Matos, T., and Suarez, A. (2011). Finite temperature and dissipative corrections to the Gross-Pitaevskii equation from $\lambda\Phi^4$ one-loop contributions. *EPL* 96, 56005. 1110-3114. doi:10.1209/0295-5075/96/56005
- Matos, T., and Suárez, A. (2014). Bose-Einstein condensate dark matter phase transition from finite temperature symmetry breaking of Klein-Gordon fields. *Cl. Quant. Grav.* 31, 045015. doi:10.1088/0264-9381/31/4/045015
- Matos, T., and Urena-Lopez, L. A. (2001). Further analysis of a cosmological model with quintessence and scalar dark matter. *Phys. Rev. D* 63, 063506. astro-ph/0006024. doi:10.1103/physrevd.63.063506
- Miller, A. L., and Mendes, L. (2023). First search for ultralight dark matter with a space-based gravitational-wave antenna: LISA Pathfinder. *Phys. Rev. D* 107, 063015. 08736. doi:10.1103/physrevd.107.063015
- Mocz, P. (2019). First star-forming structures in fuzzy cosmic filaments. *Phys. Rev. Lett.* 123, 141301. 1910.01653.
- Mocz, P., Fialkov, A., Vogelsberger, M., Becerra, F., Shen, X., Robbers, V. H., et al. (2020). Galaxy formation with BECDM – II. Cosmic filaments and first galaxies. *Mon. Not. Roy. Astron. Soc.* 494, 2027. 1911.05746. doi:10.1093/mnras/staa738
- Mocz, P., Vogelsberger, M., Robles, V. H., Zavala, J., Boylan-Kolchin, M., Fialkov, A., et al. (2017). Galaxy formation with BECDM – I. Turbulence and relaxation of idealized haloes. *Mon. Not. Roy. Astron. Soc.* 471, 4559.
- Morisaki, S., and Suyama, T. (2019). Detectability of ultralight scalar field dark matter with gravitational-wave detectors. *Phys. Rev. D* 100, 123512. 05003. doi:10.1103/physrevd.100.123512
- Müller, O., Pawłowski, M. S., Jerjen, H., and Lelli, F. (2018). A whirling plane of satellite galaxies around Centaurus A challenges cold dark matter cosmology. *Science* 359, 534. 1802-537. doi:10.1126/science.aa01858
- Ng, K. K. Y., Hannuksela, O. A., Vitale, S., and Li, T. G. F. (2021b). Searching for ultralight bosons within spin measurements of a population of binary black hole mergers. *Phys. Rev. D* 103, 063010. 02312. doi:10.1103/physrevd.103.063010
- Ng, K. K. Y., Isi, M., Haster, C.-J., and Vitale, S. (2020). Multiband gravitational-wave searches for ultralight bosons. *Phys. Rev. D* 102, 083020. 2007.12793. doi:10.1103/physrevd.102.083020
- Ng, K. K. Y., Vitale, S., Hannuksela, O. A., and Li, T. G. F. (2021a). Constraints on ultralight scalar bosons within black hole spin measurements from the LIGO-virgo GWTC-2. *Phys. Rev. Lett.* 126, 151102. doi:10.1103/physrevlett.126.151102
- Nori, M., Murgia, R., Iršič, V., Baldi, M., and Viel, M. (2019). Lyman α forest and non-linear structure characterization in Fuzzy Dark Matter cosmologies. *Mon. Not. Roy. Astron. Soc.* 482, 3227. 1809-3243. doi:10.1093/mnras/sty2888
- Padilla, L. E., Rindler-Daller, T., Shapiro, P. R., Matos, T., and Vázquez, J. A. (2021). Core-halo mass relation in scalar field dark matter models and its consequences for the formation of supermassive black holes. *Phys. Rev. D* 103, 063012. doi:10.1103/physrevd.103.063012
- Palomba, C., D'Antonio, S., Astone, P., Frasca, S., Intini, G., Rosa, I., et al. (2019). Direct constraints on the ultralight boson mass from searches of continuous gravitational waves. *Phys. Rev. Lett.* 123, 171101. 1909.08854.
- Pantig, R. C., and Övgün, A. (2022). Dark matter effect on the weak deflection angle by black holes at the center of Milky Way and M87 galaxies. *Eur. Phys. J. C* 82, 391. doi:10.1140/epjc/s10052-022-10319-8
- Park, S., Bak, D., Lee, J.-W., and Park, I. (2022). Analyzing planar galactic halo distributions with fuzzy/cold dark matter models. *JCAP* 12, 2207.07192.
- Pawłowski, M. S. (2018). The planes of satellite galaxies problem, suggested solutions, and open questions. *Mod. Phys. Lett. A* 33, 1830004.
- Pawłowski, M. S., and Kroupa, P. (2020). The Milky Way's disc of classical satellite galaxies in light of Gaia DR2. *Mon. Not. Roy. Astron. Soc.* 491, 3042–3059. doi:10.1093/mnras/stz3163
- Pawłowski, M. S., Kroupa, P., and Jerjen, H. (2013). Dwarf galaxy planes: the discovery of symmetric structures in the Local Group. *Mon. Not. Roy. Astron. Soc.* 435, 1928–1957. doi:10.1093/mnras/stt1384
- Poddar, T. K. (2021). Ultralight dark matter: constraints from gravitational waves and other astrophysical observations. *JCAP* 9, 041. 2104.09772.
- Poddar, T. K. (2022). Constraints on ultralight axions, vector gauge bosons, and unparticles from geodetic and frame-dragging measurements. *Eur. Phys. J. C* 82, 982. doi:10.1140/epjc/s10052-022-10956-z
- Porayko, N. K., Zhu, X., Levin, Y., Hobbs, G., Postnov, K., Bailes, M., et al. (2018). Parkes Pulsar Timing Array constraints on ultralight scalar-field dark matter. *Phys. Rev. D* 98, 102002. 1810.03227.
- Powell, D. M., Vegetti, S., McKean, J. P., Spingola, C., Stacey, H. R., and Fassnacht, C. D. (2022). A lensed radio jet at milliarcsecond resolution I: bayesian comparison of parametric lens models. *Mon. Not. Roy. Astron. Soc.* 516, 1808. doi:10.1093/mnras/stac2350
- Press, W. H., Ryden, B. S., and Spergel, D. N. (1990). Single mechanism for generating large-scale structure and providing dark missing matter. *Phys. Rev. Lett.* 64, 1084–1087. doi:10.1103/physrevlett.64.1084
- Qin, C., Zhao, B., Du, A., Ke, J., Luo, J., Tan, Y., et al. (2022). arXiv:2212.06032.
- Rindler-Daller, T., and Shapiro, P. R. (2010). Complex scalar field dark matter on galactic scales. *Asp. Conf. Ser.* 432, 244. 0912.2897.
- Robles, V. H., and Matos, T. (2013a). Strong lensing with finite temperature scalar field dark matter. *Phys. Rev. D* 88, 083008. 1302.5944. doi:10.1103/physrevd.88.083008
- Robles, V. H., and Matos, T. (2013b). Exact solution to finite temperature sfdm: natural cores without feedback. *Astrophys. J.* 763, 19. 1207-5858. doi:10.1088/0004-637x/763/1/19
- Saha, A. K., Parashari, P., Maity, T. N., Dubey, A., Bouri, S., and Laha, R. (2022). arXiv:2208.03530.
- Sahni, V., and Wang, L.-M. (2000). New cosmological model of quintessence and dark matter. *Phys. Rev. D* 62, 103517. astro-ph/9910097. doi:10.1103/physrevd.62.103517
- Shive, H.-Y., and Chiueh, T. (2018). Halo abundance and assembly history with extreme-axion wave dark matter at $z \geq 4$. *Mon. Not. Roy. Astron. Soc.* 473, L36–L40. doi:10.1093/mnras/slx159
- Shive, H.-Y., Chiueh, T., and Broadhurst, T. (2014). Cosmic structure as the quantum interference of a coherent dark wave. *Nat. Phys.* 10, 496–499. 1406.6586. doi:10.1038/nphys2996
- Seidel, E., and Suen, W. M. (1991). Oscillating soliton stars. *Phys. Rev. Lett.* 66, 1659–1662. doi:10.1103/physrevlett.66.1659
- Shaya, E. J., and Tully, R. B. (2013). The formation of Local Group planes of galaxies. *Mon. Not. Roy. Astron. Soc.* 436, 2096–2119. 1307.4297. doi:10.1093/mnras/stt1714
- Shen, Z., Wang, A., Gong, Y., and Yin, S. (2023). arXiv:2311.12259.
- Sin, S.-J. (1994). Late-time phase transition and the galactic halo as a Bose liquid. *Phys. Rev. D* 50, 3650–3654. hep-ph/9205208. doi:10.1103/physrevd.50.3650
- Smarra, C., Goncharov, B., Barausse, E., Antoniadis, J., Babak, S., Nielsen, A. S. B., et al. (2023). Second data release from the European pulsar timing array: challenging the ultralight dark matter paradigm. *Phys. Rev. Lett.* 131, 171001. 2306-16228. doi:10.1103/PhysRevLett.131.171001
- Solis-López, J., Guzmán, F. S., Matos, T., Robles, V. H., and Ureña López, L. A. (2021). *Phys. Rev. D* 103, 083535. arXiv:1912.09660.
- Sweeney, N., Antonelli, A., Baibhav, V., and Berti, E. (2022). Impact of relativistic corrections on the detectability of dark-matter spikes with gravitational waves. *Phys. Rev. D* 106, 044027. 2204-12508. doi:10.1103/physrevd.106.044027
- Strigari, L. E., Bullock, J. S., Kaplinghat, M., Simon, J. D., Geha, M., Willman, B., et al. (2008). A common mass scale for satellite galaxies of the Milky Way. *Nature* 454, 1096–1097. doi:10.1038/nature07222
- Su, B., Xianyu, Z.-Z., and Zhang, X. (2021). Probing ultralight bosons with compact eccentric binaries. *Astrophys. J.* 923, 114. doi:10.3847/1538-4357/ac2d91
- Su, K.-Y., and Chen, P. (2011). Solving the cusp-core problem with a novel scalar field dark matter. *JCAP* 08, 016. 1008.3717. doi:10.1088/1475-7516/2011/08/016
- Sun, L., Brito, R., and Isi, M. (2020). Erratum: search for ultralight bosons in Cygnus X-1 with advanced LIGO [phys. Rev. D 101, 063020 (2020)]. *Phys. Rev. D* 102, 089902. 1909.11267. doi:10.1103/physrevd.102.089902
- Torres, D. F., Capozziello, S., and Lambiase, G. (2000). A supermassive boson star at the galactic center? *Phys. Rev. D* 62, 104012. astro-ph/0004064. doi:10.1103/physrevd.62.104012
- Traykova, D., Clough, K., Helfer, T., Berti, E., Ferreira, P. G., and Hui, L. (2021). Dynamical friction from scalar dark matter in the relativistic regime. *Phys. Rev. D* 104, 103014. doi:10.1103/physrevd.104.103014
- Tsai, Y. D., Farnocchia, D., Micheli, M., Vagnozzi, S., and Visinelli, L. (2023b). arXiv:2309.13106.
- Tsai, Y.-D., Wu, Y., Vagnozzi, S., and Visinelli, L. (2023a). Constraints on fifth forces and ultralight dark matter from OSIRIS-REX target asteroid Bennu. *JCAP* 04, 2107.04038.

- Tsutsui, T., and Nishizawa, A. (2023). Observational constraint on axion dark matter in a realistic halo profile with gravitational waves. *Phys. Rev. D.* 107, 103516. doi:10.1103/physrevd.107.103516
- Turner, M. S. (1983). Coherent scalar-field oscillations in an expanding universe. *Phys. Rev. D.* 28, 1243–1247. doi:10.1103/physrevd.28.1243
- Urena-Lopez, L. A., and Bernal, A. (2010). Bosonic gas as a galactic dark matter halo. *Phys. Rev. D.* 82, 123535. doi:10.1103/physrevd.82.123535
- Ureña López, L. A., and Gonzalez-Morales, A. X. (2016). Dark matter with ultralight bosons and its imprint on cosmological structure. *JCAP* 07, 048. 1511.08195.
- Urena-Lopez, L. A., and Liddle, A. R. (2002). Supermassive black holes in scalar field galaxy halos. *Phys. Rev. D.* 66, 083005. astro-ph/0207493. doi:10.1103/physrevd.66.083005
- Ureña López, L. A., Robles, V. H., and Matos, T. (2017). Mass discrepancy-acceleration relation: a universal maximum dark matter acceleration and implications for the ultralight scalar dark matter model. *Phys. Rev. D.* 96, 043005. doi:10.1103/physrevd.96.043005
- Veltmaat, J., Schwabe, B., and Niemeyer, J. C. (2020). Baryon-driven growth of solitonic cores in fuzzy dark matter halos. *Phys. Rev. D.* 101, 083518. doi:10.1103/physrevd.101.083518
- Vicente, R., and Cardoso, V. (2022). Dynamical friction of black holes in ultralight dark matter. *Phys. Rev. D.* 105, 083008. doi:10.1103/physrevd.105.083008
- Wang, Y., and Easther, R. (2022). Dynamical friction from ultralight dark matter. *Phys. Rev. D.* 105, 063523. doi:10.1103/physrevd.105.063523
- Widrow, L. M., and Kaiser, N. (1993). Using the schroedinger equation to simulate collisionless matter. *Astrophys. J. Lett.* 416, L71. doi:10.1086/187073
- Xia, Z.-Q., Tang, T.-P., Huang, X., Yuan, Q., and Fan, Y.-Z. (2023). Constraining ultralight dark matter using the Fermi-LAT pulsar timing array. *Phys. Rev. D.* 107, L121302. 2303-17545. doi:10.1103/physrevd.107.121302
- Yu, J.-C., Yao, Y.-H., Tang, Y., and Wu, Y.-L. (2023). Sensitivity of space-based gravitational-wave interferometers to ultralight bosonic fields and dark matter. *Phys. Rev. D.* 108, 083007. doi:10.1103/physrevd.108.083007
- Yuan, G.-W., Shen, Z.-Q., Tsai, Y.-L. S., Yuan, Q., and Fan, Y.-Z. (2022). Constraining ultralight bosonic dark matter with Keck observations of S2's orbit and kinematics. *Phys. Rev. D.* 106, 103024. doi:10.1103/physrevd.106.103024
- Zhao, W., Gao, D., Wang, J., and Zhan, M. (2022). Investigating the environmental dependence of ultralight scalar dark matter with atom interferometers. *Gen. Rel. Grav.* 54, 41. doi:10.1007/s10714-022-02925-4