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EDITED BY
Sehrish Iftikhar,
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Prince Mohammad Bin Fahd University,
Saudi Arabia
Ghulam Mustafa,
Zhejiang Normal University, China

*CORRESPONDENCE
Sana Saleem,
sanasaleem@umt.edu.pk
Andleeb Ibrar,
andleebibrar78@gmail.com
Musawa Yahya Almusawa,
malmusawi@jazanu.edu.sa
Sayed M. Eldin,
sayed.eldin22@fue.edu.eg
Muhammad Imran Asjad,
imran.asjad@umt.edu.pk

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Collapsing cylindrically symmetric filamentary stellar object

Sana Saleem[1]*, Andleeb Ibrar[1]*, Musawa Yahya Almusawa[2]*, Sayed M. Eldin[3]* and Muhammad Imran Asjad[1]*

[1]Department of Mathematics, University of Management and Technology, Lahore, Pakistan, [2]Department of Mathematics, Faculty of Science, Jazan University, Jazan, Saudi Arabia, [3]Center of Research, Faculty of Engineering and Technology, Future University in Egypt, New Cairo, Egypt

This work investigates the collapsing behavior of filamentary objects under the influence of dark matter. For this purpose, we use $f(R,T)$ gravity as a candidate for dark matter. The collapse equation is obtained by imposing the Darmois junction condition at the collapsing boundary. At the collapsing boundary, it is observed that the radial pressure is non-zero and is proportional to the field time-dependent component. Finally, we check the relationship between gravitational waves and dark source terms. It is concluded that the dark source terms disrupt the propagation of gravitational waves.

KEYWORDS

gravitational collapse, gravitational waves, dark matter, compact stellar filament, f(R,T) gravity

1 Introduction

Cosmological discoveries such as cosmic-accelerated expansion and the galaxy rotation curves reveal the existence of mysterious terms such as dark energy (DE) and dark matter (DM). The DE is the mysterious kind of energy that causes the cosmic expansion. The DM is a kind of non-baryonic matter that is neither an emitter nor an absorber of electromagnetic radiation. Its gravitational effects on baryonic matter indicate its existence. Observational studies concerning galaxy rotational curve issues and mass differences in galactic clusters provided evidence for the presence and significance of DM in the formation of stars (Oort, 1932; Zwicky, 1933). Planck statistics show that 68% of the universe is in the form of DE, 27% is in the form of DM, and the remaining 5% is in the form of baryonic matter (Planck collaboration and P.A.R. Ade, 2014).

The number of researchers provides a variety of models to explore the properties of DM and DE. The ΛCDM model based on the cosmological constant (Λ) is used to describe vacuum energy in the context of general relativity (GR). Unfortunately, this attempt fails due to "fine-tuning" (Sirivastava, 2008) and "cosmic coincidence" (Steinhardt et al., 1997) issues. In this context, the modified theory attains much attention. The $f(R)$ gravity (Capozziello, 2002) is one of the simplest extensions of GR constructed by placing the generic function $f(R)$ instead of R in the Einstein–Hilbert action. Numerous cosmological phenomena, including the inflationary phase (Barrow and Hervik, 2006), late-time cosmic evolution (Brookfield et al., 2006), validity with solar-system testing (Zhang, 2007), and astrophysical problems (R Manzoor, 2021), have been examined for various $f(R)$ models. In the same context, Sharif and Saleem (2020a) discussed the stability of the universe against anisotropic homogeneous perturbation for different models of the $f(R)$ theory.

Harko et al. (2011) presented the modified version of $f(R)$ gravity by inserting the generic function T (trace of the energy–momentum tensor), which is known as $f(R, T)$ gravity. This theory has attracted much attention as a viable gravity to discuss the mysteries of the universe. Harko et al. (2011) also discussed the stability analysis in the framework of the $f(R, T)$ theory. In the same context, Horvat et al. (2011) determined the stability of a matter structure in the presence of anisotropy pressure with radial perturbations. Sharif and Zubair (2012a), Sharif and Zubair (2012b), Sharif and Zubair (2013a), Sharif and Zubair (2013b), and Sharif and Zubair (2014) also investigated energy conditions and thermodynamical laws and presented the anisotropic universe models in the background of $f(R, T)$ gravity. In the case of the expansion-free cylindrically symmetric anisotropic fluid, Sharif and Yousaf (2012) found that certain solutions fulfill the Darmois junction conditions (DJCs) and certain solutions demonstrate the presence of a thin shell on the surface boundary. Sharif and Saleem (2020b) also discussed the stability of the universe by using homogenous anisotropic perturbations in the same context. Mumtaz at el. (2022) discussed the dissipative collapse of a cluster of stellar objects in $f(R, T)$ gravity.

The development of stellar objects (such as stars, planets, and galaxy clusters), gravitational waves (GWs), and radiations is the result of the gravitational collapse. As a result of this collapsing process, a stable cosmic stellar structure becomes unstable under the influence of its own gravity. Many scientists have investigated how gravitational collapse influences evolutionary stellar distributions. Ostriker (1964) examined the behavior of compressible cylindrically symmetric filamentary objects. Breysse (2014) examined the stability of filamentary objects with cylindrically symmetric structures of the self-gravitating fluid. Many researchers (Milgrom, 1997; Kneb, 2003; Bessho and Tsuribe, 2012; Freundlich, 2014) used cylindrical symmetry to explore filamentary objects analytically and numerically. Sharif and Manzoor (2016) examined the stability of filamentary objects in the context of the Brans–Dicke theory. Zubair et al. (2017) examined the stability of a cylindrically symmetric collapsing object under the influence of an anisotropic fluid in $f(R, T)$ gravity. In the same framework, Sharif and Waseem (2019) studied the stability of the Einstein universe under the influence of inhomogeneous perturbations and found the sable solutions. Guhaa and Ghosh (2021) discussed the dynamics of gravitational collapse instability in the context of $f(R, T)$ gravity. Hoemann et al. (2022) studied the collapse of filamentary objects in a two-phase process and found the collapse timescale.

The study of stellar objects influenced by mysterious components (DE and DM) may expose the mysterious change hidden in the universe's structure formation (Sharif and Manzoor, 2014). Herrera et al. (2005) studied the matching conditions for an anisotropic cylindrical fluid that is collapsing and demonstrated that the pressure at the surface of the cylinder is non-zero. Sharif and Fatima (2017) examined the dynamics of stellar filamentary objects in the context of the $f(G)$ theory and concluded that the existence of dark source terms influences both the universe's collapse and its rate of expansion. Manzoor et al. (2020) discussed the collapse of stellar filamentary objects in the background of $f(R)$ and Palatini $f(R)$ (Manzoor et al., 2019) gravity under the influence of exotic matter. Motivated by the previous work, we studied the dynamics of stellar filaments under the influence of exotic terms in the background of the $f(R, T)$ theory. This work is organized in the following pattern: **Section 2** examines the $f(R, T)$ theory. **Section 3** describes the use of DJCs to demonstrate the collapse of filamentary objects. The conclusions of the junction conditions are covered in **Section 4** to discuss the dynamics of the collapse. In order to evaluate the relationship between DM, and GWs, **Section 5** adds the specific model of the $f(R, T)$ theory to the collapsing mechanism. **Section 6** concludes the results.

2 The formalism of the $f(R, T)$ theory

Harko et al. (2011) introduced the $f(R, T)$ gravity, which has resulted in a number of exciting discoveries in the areas of cosmology and astrophysics. This theory is generated by inserting the generic trace term into the Einstein–Hilbert action. The action of $f(R, T)$ gravity is manifested as follows:

$$I = \int \sqrt{-g}\left[S_m + \frac{f(R,T)}{2\kappa}\right] d^4x, \tag{1}$$

where S_m stands for the matter Lagrangian, g exhibits the metric tensor's determinant, κ denotes the coupling parameter, R represents the Ricci scalar, and T manifests the trace of the stress energy tensor. The energy–momentum tensor for matter configuration is given by

$$T_{\zeta\eta} = -\left(\frac{2}{\sqrt{-g}}\right)\frac{\delta S_m}{\delta g^{\zeta\eta}}. \tag{2}$$

The modified field equations corresponding to Eq. 1 are determined as follows:

$$-\left(\Theta_{\zeta\eta} + T_{\zeta\eta}\right) f_T(R,T) + \kappa^2 T_{\zeta\eta} = -\frac{1}{2} f(R) g_{\zeta\eta} + f_R(R) R_{\zeta\eta} - \left(\nabla_\zeta \nabla_\eta - g_{\zeta\eta}\Box\right). \tag{3}$$

Here, the terms $f_R(R, T)$ and $f_T(R, T)$ are

$$f_R(R,T) = \frac{\partial f(R,T)}{\partial R}; \quad f_T(R,T) = \frac{\partial f(R,T)}{\partial T},$$

where $\Box = g^{\zeta\eta}\nabla_\zeta\nabla_\eta$ and ∇_ζ exhibit the covariant derivative, and $\Theta_{\zeta\eta}$ is given as follows:

$$\Theta_{\zeta\eta} = -2g^{\alpha\beta}\frac{\partial^2 S_m}{\partial g^{\alpha\beta}\partial g^{\zeta\eta}} + g_{\zeta\eta} S_m - 2T_{\zeta\eta}. \tag{4}$$

For matter configuration, we consider $S_m = \rho$ (Planck collaboration and P.A.R. Ade, 2014), which reduces Eq. 4 as follows:

$$\Theta_{\zeta\eta} = \rho g_{\zeta\eta} - 2T_{\zeta\eta}.$$

In the standard GR format, Eq. 3 can be rewritten as follows:

$$G_{\zeta\eta} = R_{\zeta\eta} - \frac{1}{2} R g_{\zeta\eta} = T^{eff}_{\zeta\eta}, \tag{5}$$

where $T^{eff}_{\zeta\eta}$ denotes the effective stress–energy tensor given by

$$T^{eff}_{\zeta\eta} = \frac{1}{f_R(R,T)}\Big[\left(1 + f_T(R,T)\right) T^{(m)}_{\zeta\eta} - \rho g_{\zeta\eta} f_T(R,T) - \frac{1}{2} g_{\zeta\eta}\left(R f_R(R,T) - f(R,T)\right) + f_R(R,T)\left(\nabla_\zeta\nabla_\eta - g_{\zeta\eta}\Box\right)\Big]. \tag{6}$$

The divergence of Eq. 6 is presented as follows:

$$\nabla^{\zeta}T^{eff}_{\zeta\eta}=-\frac{f_T(R,T)}{f_T(R,T)-1}\Big[\left(T_{\zeta\eta}+\Theta_{\zeta\eta}\right)\nabla^{\zeta}\{\ln f_T(R,T)\}$$
$$=-\frac{1}{2}g_{\zeta\eta}\nabla^{\zeta}T+\nabla^{\zeta}\Theta_{\zeta\eta}\Big]. \quad (7)$$

3 Filament structure

In this study, we consider a collapsing filament which is enclosed by a cylindrical surface and characterized by spacetime (Herrera et al., 2005).

$$ds^2=A^2(t,r)\,dr^2-A^2(t,r)\,dt^2+B^2(t,r)\,dz^2+C^2(t,r)\,d\phi^2. \quad (8)$$

In order to maintain cylindrical symmetry, the following limits $0\le\phi\le 2\Pi, -\infty\le t\le\infty, \text{and} -\infty<z<\infty$ are applied. We assign the coordinates as $x^1=r,\ x^0=t,\ x^2=z,$ and $x^3=\phi$. Baryonic matter composed of anisotropic and dissipative fluids is represented by the energy–momentum tensor given as follows:

$$T_{\zeta\eta}=P_r g_{\zeta\eta}+(\rho+P_r)V_\zeta V_\eta+(P_z-P_r)S_\zeta S_\eta$$
$$+\left(P_\phi-P_r\right)K_\zeta K_\eta+q\left(l_\zeta V_\eta+l_\eta V_\zeta\right). \quad (9)$$

Here, ρ manifests the energy density, whereas the terms P_r, P_ϕ, and P_z denote matter stresses, and $V_\zeta, K_\zeta, S_\zeta$, and l_ζ are four vectors. The vector V_ζ is a four-velocity vector, whereas the vectors K_ζ andS_ζ represent space-like vectors, and the vector l_ζ is a null-like vector perpendicular to V_ζ. These vectors satisfy the following identities:

$$V_\zeta=-A\delta^0_\zeta,\quad S_\zeta=B\delta^2_\zeta,\quad K_\zeta=C\delta^3_\zeta,\quad l_\zeta=A\delta^1_\zeta. \quad (10)$$

By using Eqs. 8–10, field Eq. 5 is reduced to five non-zero components; however, in this case, we will only focus on the respective two cases:

$$-\frac{B_{,tt}}{B}+\frac{B_{,t}}{B}\frac{A_{,t}}{A}-\frac{C_{,tt}}{C}+\frac{C_{,t}}{C}\frac{A_{,t}}{A}+\frac{B_{,r}}{B}\frac{A_{,r}}{A}+\frac{B_{,r}}{B}\frac{C_{,r}}{C}-\frac{C_{,t}}{C}\frac{B_{,t}}{B}+\frac{C_{,r}}{C}\frac{A_{,r}}{A}$$
$$=\frac{1}{f_R(R,T)}\Big[A^2p_r\left(f_T(R,T)+1\right)-A^2\rho f_T(R,T)$$
$$+\frac{1}{2}A^2\left(f(R,T)-Rf_R(R,T)\right)+f_R(R,T)_{,tt}$$
$$+\left(\frac{C_{,t}}{C}-\frac{A_{,t}}{A}+\frac{B_{,t}}{B}\right)f_R(R,T)_{,t}$$
$$+f_R(R,T)_{,r}\left(\frac{B_{,r}}{B}+\frac{A_{,r}}{A}+\frac{C_{,r}}{C}\right)\Big], \quad (11)$$

$$-\frac{B_{,r}}{B}\frac{A_{,t}}{A}+\frac{B_{,tr}}{B}-\frac{B_{,t}}{B}\frac{A_{,r}}{A}-\frac{A_{,t}}{A}\frac{C_{,r}}{C}+\frac{C_{,tr}}{C}-\frac{C_{,t}}{C}\frac{A_{,r}}{A}$$
$$=\frac{1}{f_R(R,T)}\Big[\left(f_T(R,T)+1\right)q+f_R(R,T)_{,tr}$$
$$-\frac{A_{,r}}{A}f_R(R,T)_{,t}-\frac{A_{,t}}{A}f_R(R,T)_{,r}\Big]. \quad (12)$$

Here, partial derivatives are given as follows:

$$f_R(R,T)_{,t}=\frac{\partial f_R(R,T)}{\partial t},\quad f_R(R,T)_{,tt}=\frac{\partial^2 f_R(R,T)}{\partial t^2},$$
$$f_R(R,T)_{,r}=\frac{\partial f_R(R,T)}{\partial r},\quad f_R(R,T)_{,rr}=\frac{\partial^2 f_R(R,T)}{\partial r^2},$$
$$f_R(R,T)_{,tr}=\frac{\partial^2 f_R(R,T)}{\partial t\partial r}. \quad (13)$$

3.1 Collapsing stellar filament boundary

Every interior structure that collapses has an exterior vacuum or non-vacuum distribution enclosing it. In this scenario, we use the cylindrical hypersurface $(\hat{\Sigma})$ related to the exterior vacuum configuration in Einstein and Rosen (1937). The spacetime that manifests the exterior case is presented as follows:

$$ds^2=-e^{2(\epsilon-\delta)}d\bar{T}^2+e^{2(\epsilon-\delta)}d\bar{R}^2+e^{2\delta}dZ^2+e^{-2\delta}\bar{R}^2d\phi^2, \quad (14)$$

where both coefficients ϵ and δ depend on temporal $\bar{T}$ and radial $\bar{R}$ components, respectively. The gravitational wave equation for the field equations $R_{\zeta\eta}=0$ is given by

$$\frac{\delta_{,\bar{R}}}{\bar{R}}+\delta_{,\bar{R}\bar{R}}-\delta_{,\bar{T}\bar{T}}=0, \quad (15)$$

$$\bar{R}\left(\delta^2_{,\bar{T}}+\delta^2_{,\bar{R}}\right)=\delta_{,\bar{R}},\quad 2\bar{R}\delta_{,\bar{R}}\delta_{,\bar{T}}=\epsilon_{,\bar{T}}. \quad (16)$$

Now, we use the DJCs (Darmois, 1927) to smoothly match the interior collapsing surface to the exterior surface. We take into account the DJC, which states that across the boundary $\hat{\Sigma}_-=\hat{\Sigma}_+=\hat{\Sigma}$, the interior spacetime (10) connected with the interior hypersurface $(\hat{\Sigma}_-)$ must be similar to the exterior spacetime (8) connected to the exterior hypersurface $(\hat{\Sigma}_+)$. Accordingly, we examine the continuity of the second basic form at $\hat{\Sigma}_-=\hat{\Sigma}=\hat{\Sigma}_+$. Using interior and exterior coordinates, the equation for the collapsed surface boundary $(\hat{\Sigma})=\hat{\Sigma}_-=\hat{\Sigma}_+$ is expressed as follows:

$$L_-=r-r_{\hat{\Sigma}}=0, \quad (17)$$

$$L_+=\bar{R}-\bar{R}_{\hat{\Sigma}}(\bar{T})=0. \quad (18)$$

where L_+ denotes the exterior spacetime and L_- denotes the interior spacetime. The hypersurface $(\hat{\Sigma})$ represents the co-moving boundary of collapsing surface, so the quantity $r_{\hat{\Sigma}}$ is taken as a constant. In order to use the junction conditions, we must make sure that the surface boundary uses the same parametrization whether it is supposed to be embedded in L_+ or L_-.

We will define the inner and exterior geometries on $\hat{\Sigma}$ in order to ensure the continuity of the first fundamental form. In order to achieve this, we use Eq. 17 on Eq. 8, which produces an interior metric on the boundary surface represented by

$$ds^2_{\hat{\Sigma}_-}=-d\tau^2+B^2dz^2+C^2d\phi^2. \quad (19)$$

Here, the time coordinate τ on $\hat{\Sigma}$ is specified as

$$d\tau=Adt. \quad (20)$$

We will consider $\Omega^0 = \tau$, $\Omega^2 = z$, *and* $\Omega^3 = \phi$ as parameters on the boundary surface $\hat{\Sigma}$. By utilizing (18), the exterior spacetime (14) reduces $\hat{\Sigma}$ to

$$ds^2_{\hat{\Sigma}_+} = -e^{-2(\epsilon-\delta)}\left[1-\left(\frac{d\bar{R}}{d\bar{T}}\right)^2\right]d\bar{T}^2 + e^{2\delta}dz^2 + e^{-2\delta}\bar{R}^2 d\phi^2. \quad (21)$$

According to the continuity of the first fundamental form, interior metric (19) is similar to Eq. 21 at $\hat{\Sigma}$ if the following requirements are fulfilled:

$$d\tau = e^{(\epsilon-\delta)}\left[1-\left(\frac{d\bar{R}}{d\bar{T}}\right)^2\right]^{\frac{1}{2}} d\bar{T}, \quad (22)$$

$$B = e^{\delta}, \quad (23)$$

$$C = e^{-\delta}\bar{R}, \quad (24)$$

with

$$1-\left(\frac{d\bar{R}}{d\bar{T}}\right)^2 > 0. \quad (25)$$

Depending on the extrinsic curvature ($K_{\theta\psi}$), the second fundamental form of the boundary condition is defined as follows:

$$K_{\theta\psi} d\Omega^\theta d\Omega^\psi, \qquad \theta,\psi = 0,2,3, \quad (26)$$

where

$$K^{\pm}_{\theta\psi} = -n^{\pm}_a\left(\frac{\partial^2 x^a}{\partial\Omega^\theta \partial\Omega^\psi} + \Gamma^{(a)}_{\rho\sigma}\frac{\partial x^\rho}{\partial\Omega^\theta}\frac{\partial x^\sigma}{\partial\Omega^\psi}\right). \quad (27)$$

Here, the term $n^{\pm}_a$ denotes the outward unit normals to the surface $\hat{\Sigma}$ associated with interior and exterior spacetimes, while x^ϵ represents the equation of $\hat{\Sigma}$ connected with L_- or L_+. The Christoffel symbols should be computed using the respective exterior Eq. 8 or interior Eq. 14 metrics. From Eqs 17, 18, the derived outward unit normals of $\hat{\Sigma}$ corresponding to L_- or L_+ are as follows:

$$n^-_\zeta = (0, A, 0, 0), \quad (28)$$

$$n^+_\zeta = \left[1-\left(\frac{d\bar{R}}{d\bar{T}}\right)^2\right]^{-\frac{1}{2}}\left(-\frac{d\bar{R}}{d\bar{T}}, 1, 0, 0\right)e^{(\epsilon-\delta)},$$
$$= \left(-\dot{\bar{R}}, \dot{\bar{T}}, 0, 0\right)e^{-2(\epsilon-\delta)}, \quad (29)$$

where dot represents the differential with respect to τ (the time coordinate discussed in Eq. 22). If Eq. 25 is satisfied, then unit vectors Eqs 28 and 29 are both space-like. The following are the non-zero components of the extrinsic curvature $K^{\pm}_{\theta\psi}$:

$$K^-_{00} = -\frac{1}{A^2}A_{,r}, \quad (30)$$

$$K^+_{00} = \left(\left(\ddot{\bar{R}}^2 - \dot{\bar{T}}^2\right)\left[\dot{\bar{R}}\left(\epsilon_{,\bar{T}} - \delta_{,\bar{T}}\right) + \dot{\bar{R}}\left(\epsilon_{,\bar{R}} - \delta_{,\bar{R}}\right)\right] - \dot{\bar{R}}\ddot{\bar{T}} + e^{2(\delta-\epsilon)}\dot{\bar{T}}\ddot{\bar{R}}\right), \quad (31)$$

$$K^-_{22} = -\frac{B}{A}B_{,r}, \quad (32)$$

$$K^+_{22} = e^{2\delta}\left(\dot{\bar{R}}\delta_{,\bar{T}} + \dot{\bar{T}}\delta_{,\bar{R}}\right), \quad (33)$$

$$K^-_{33} = -\frac{C}{A}C_{,r}, \quad (34)$$

$$K^+_{33} = e^{-2\delta}\left(\frac{\dot{\bar{T}}}{\bar{R}} - \dot{\bar{T}}\delta_{,\bar{R}} - \dot{\bar{R}}\delta_{,\bar{T}}\right)\bar{R}^2. \quad (35)$$

Eqs 20, 22–24 produce the entire Darmois conditions along with the continuity of $K_{\theta\psi}$ over the surface $\hat{\Sigma}$.

4 Results of boundary conditions

This section derives the important results of boundary conditions in the framework of collapsing galactic filaments. In this regard, we use the field equations related to interior and exterior geometries to concise the boundary conditions and formulate certain helpful formulas (Herrera et al., 2005; Herrera et al., 2007).

Equation 22 provides the following result:

$$\left(\dot{\bar{T}}^2 - \dot{\bar{R}}^2\right)e^{2(\epsilon-\delta)} = 1, \quad (36)$$

and by using Eqs 23, 24, we obtain

$$\bar{R} = BC. \quad (37)$$

Using Eq. 30, the previous equation can be differentiated as follows:

$$\dot{\bar{R}} = \frac{(BC)_{,t}}{A}. \quad (38)$$

Also, by using Eqs 23, 24, the continuity of curvatures K_{22} and K_{33} lead to the following outcome:

$$\dot{\bar{T}} = \frac{(BC)_{,r}}{A}. \quad (39)$$

Differentiating (11), (12), and (30) along with Eqs 38, 39 yields

$$\begin{aligned}\dot{\bar{R}}\ddot{\bar{T}} - \dot{\bar{T}}\ddot{\bar{R}} = \frac{1}{A^4}\Bigg[&(BC)_{,t}\Bigg[C_{,t}(AB)_{,r} + B_{,t}(AC)_{,r}\\ &-\left(\frac{1}{f_R(R,T)}\left[\left(f_T(R,T)+1\right)q - f_R(R,T)_{,r}\frac{A_{,t}}{A}\right.\right.\\ &\left.\left.+f_R(R,T)_{,tr} - \frac{A_{,r}}{A}f_R(R,T)_{,t}\right]\right)ABC\\ &+(BC)_{,r}\Bigg[\Bigg(\frac{1}{f_R(R,T)}\Big[A^2p_r\left(1+f_T(R,T)\right)\\ &-\rho A^2 f_T(R,T) + \frac{1}{2}A^2\left(f(R,T) - Rf_R(R,T)\right)\\ &+f_R(R,T)_{,tt} + \left(\frac{B_{,t}}{B} + \frac{C_{,t}}{C} - \frac{A_{,t}}{A}\right)f_R(R,T)_{,t}\\ &+\left(\frac{C_{,r}}{C} + \frac{B_{,r}}{B} + \frac{A_{,r}}{A}\right)f_R(R,T)_{,r}\Big]\Bigg)ABC\\ &-\ AB_{,t}C_{,t} - AB_{,r}C_{,r} - A_{,r}(BC)_{,r}\Bigg]\Bigg].\end{aligned} \quad (40)$$

The curvatures K_{00} and K_{22} and Eqs 16, 23, 24, 36, 37, 39, and 40 give the following expression:

$$\begin{aligned}&\frac{1}{A^4}\Bigg[\left(C_{,r}B_{,t} - B_{,r}C_{,t}\right)^2 + \Bigg(\frac{1}{f_R(R,T)}\Big[A^2p_r\left(f_T(R,T)+1\right)\\ &-A^2\rho f_T(R,T) + \frac{1}{2}A^2\left(f(R,T) - Rf_R(R,T)\right)\\ &+f_R(R,T)_{,tt} + \left(\frac{C_{,t}}{C} + \frac{B_{,t}}{B} - \frac{A_{,t}}{A}\right)f_R(R,T)_{,t}\\ &+\left(\frac{B_{,r}}{B} + \frac{A_{,r}}{A} + \frac{C_{,r}}{C}\right)f_R(R,T)_{,r}\Big]\Bigg)(BC)^2_{,r}\\ &-\Bigg(\frac{1}{f_R(R,T)}\Big[\left(1+f_T(R,T)\right)q + f_R(R,T)_{,tr}\\ &-\frac{A_{,t}}{A}f_R(R,T)_{,r} - \frac{A_{,r}}{A}f_R(R,T)_{,t}\Big]\Bigg)\Bigg]\\ &= \left(\dot{\bar{T}}^2 - \dot{\bar{R}}^2\right)^2\delta^2_{,\bar{T}}.\end{aligned} \quad (41)$$

Differentiating Eqs 23, 24 with Eqs 20, 22 results in

$$\frac{B_{,t}}{A} = e^{2\delta-\epsilon}\left[1-\left(\frac{\dot{\bar{R}}}{\dot{\bar{T}}}\right)^2\right]^{\frac{-1}{2}}\delta_{,\bar{T}}, \tag{42}$$

$$\frac{C_{,t}}{A} = e^{-\epsilon}\left[1-\left(\frac{\dot{\bar{R}}}{\dot{\bar{T}}}\right)^2\right]^{\frac{-1}{2}}\left(\frac{\dot{\bar{R}}}{\dot{\bar{T}}}-\bar{R}\delta_{,\bar{T}}\right). \tag{43}$$

Now, the connection between the extrinsic curvatures K_{22} and K_{33} and Eqs 42, 43 yields the following result:

$$\left(C_{,r}B_{,t}-B_{,r}C_{,r}\right)\frac{1}{A^2} = -\dot{\bar{R}}\dot{\bar{T}}\delta_{,\bar{R}}-\left(\dot{\bar{R}}^2-\dot{\bar{T}}^2\right)\delta_{,\bar{T}}. \tag{44}$$

Finally, by utilizing Eqs 16, 39 and inserting Eqs 44 into 41, we obtain

$$\begin{aligned}
&\frac{1}{f_R(R,T)}\Big[A^2p_r\left(f_T(R,T)+1\right)-\rho A^2 f_T(R,T)\\
&-\frac{1}{2}A^2\left(Rf_R(R,T)-f(R,T)\right)+f_R(R,T)_{,tt}\\
&+\left(\frac{B_{,t}}{B}-\frac{A_{,t}}{A}+\frac{C_{,t}}{C}\right)f_R(R,T)_{,t}\\
&+\left(\frac{A_{,r}}{A}+\frac{C_{,r}}{C}+\frac{B_{,r}}{B}\right)f_R(R,T)_{,r}\Big]\\
&=A^2e^{2(\delta-\epsilon)}\delta_{\bar{R}}^2\left(2\frac{\delta_{,\bar{T}}}{\delta_{,\bar{R}}}v-\frac{v^2}{1-v^2}\right)\\
&+\Big(\frac{1}{f_R(R,T)}\Big[\left(1+f_T(R,T)\right)q+f_R(R,T)_{,tr}\\
&-\frac{A_{,r}}{A}f_R(R,T)_{,t}-\frac{A_{,t}}{A}f_R(R,T)_{,r}\Big]\Big)\left((BC)_{,r}^{-2}\right).
\end{aligned} \tag{45}$$

Here, the radial velocity of the collapsed boundary surface $\hat{\Sigma}$ is represented by $v=\frac{d\bar{R}}{d\bar{T}}$. Eq. 45 represents a collapse equation for the stellar filamentary object under the influence of DM. Because of dissipation and exotic matter, it is clear from the previous equation that the radial pressure on the boundary surface $\hat{\Sigma}$ remains non-zero.

5 Stellar filaments and matter–curvature coupling

According to cosmological observations, it is observed that galaxies across the entire universe are linked by the system of filaments *via* DM. For several years, it has been assumed that there are DM filaments linking galaxies, acting as a sort of superstructure or web. In the present work, we will discuss the specific viable model of the $f(R,T)$ theory to examine the effect of DM.

5.1 Minimally coupled model

To examine the role of DM on the stellar collapsing filament, we take a specific minimally coupled model of the $f(R,T)$ theory, which is given as follows:

$$f(R,T) = R-\nu(-T)^m, \tag{46}$$

where ν denotes the coupling parameter and the parameter m represents the matter effects. For $m=0$, the aforementioned model reduces to GR, and for $0<m<1$, this model serves as a dark matter candidate at galactic scales (Zaregonbadi et al., 2016). By using Eq. 46, we obtained the following differentials:

$$\begin{aligned}
&f_T(R,T)=-\nu m(-T)^m,\quad f_R(R,T)=1,\quad f_R(R,T)_{,t}=0,\\
&f_R(R,T)_{,tt}=0,\quad f_R(R,T)_{,r}=0,\quad f_R(R,T)_{,rr}=0,\quad f_R(R,T)_{,rt}=0.
\end{aligned}$$

According to a specific model Eq. 46 of $f(R,T)$ gravity, collapsing Eq. 45 reduces to the following expression:

$$\begin{aligned}
&A^2P_r-A^2\nu m(-T)^{m-1}\left(P_r-\rho\right)+\frac{A^2}{2}\left(-\nu(-T)^m\right)\\
&=A^2e^{2(\delta-\epsilon)}\delta_{\bar{R}}^2\left(2\frac{\delta_{,T}}{\delta_{,\bar{R}}}v-\frac{v^2}{1-v^2}\right)\\
&+\left(1-\nu m(-T)^m\right)q\left((BC)_{,r}^{-2}\right).
\end{aligned} \tag{47}$$

It demonstrates that because of dissipation, the velocity of the collapsing boundary, components of the dark source, and the radial fluid pressure are non-zero on the collapsing boundary. Ordinary matter is believed to condense into galaxies and cosmic clusters in filament formations under the gravitational influence of DM. According to our collapsing model, the gravitational pull of mysterious matter on the collapsed filamentary object is similar to the effects of exotic terms.

The body's natural tendency to gravitate toward itself throughout the collapse process causes its gravitational force to dominate its internal pressure. A GW momentum flux occurs as a result of the collapsing body dissipating energy. In the collapsing scenario, Eq. 47 may be used to explain the connections between DM and various phenomena.

5.2 Gravitational waves and exotic material

A new and exciting area of research into gravitational theories has begun with the first detection of GWs by the LIGO and Virgo Collaboration in 2015 (Abbott, 2016a). The GWs are the result of different phenomena such as the Big Bang event and the gravitational collapse of star clusters. According to recent research, DM may have an influence on GW propagation in the same way that various propagation mediums affect the propagation of the light wave. However, this influence is so slight that it would be much below the sensitivity of the present detector.

The study of GWs has become more important. Gravitational waves generated by different phenomena can be observed and examined to discover more about the kinematics of the cosmos and how cosmic structures develop. While investigating these waves, polarization modes will indicate their geometrical direction. A GW has two polarized modes in GR; however, it can have additional modes in modified theories. For instance, it has been demonstrated that a GW has two more modes than GR in the $f(R)$ and $f(R,T)$ theories (Alves, 2016).

The collapse in Eq. 47 may be used to examine the relationships between GWs and dark source terms of filamentary collapse. Assume a cylindrical source that is static for a while before it begins to contract and produces a strong radiation pulse emitting from the axis. In this regard, the corresponding function may be expressed as follows (Ostriker, 1964):

$$\delta = \frac{1}{2\pi}\int_{-\infty}^{\bar{T}-\bar{R}} \delta_{st} + \frac{g(\bar{T}')}{((\bar{T}-\bar{T}')-\bar{R}^2)^{1/2}} d\bar{T}', \tag{48}$$

where the time-dependent function is $g(\bar{T}) = g_0\delta(\bar{T})$ that depicts the intensity of the wave path and δ_{st} represents the static Levi–Civita solution. Here, in this context, g_0 stands for a constant and $\delta(\bar{T})$ represents the delta function. Eq. 48 and wave Eq. 15 provide

$$\delta = \delta_{st}, \quad \bar{R} > \bar{T},$$
$$\delta = \frac{g_0}{2\pi(v^2-\bar{R}^2)^{1/2}} + \delta_{st}, \quad \bar{R} < \bar{T}.$$

Eqs 15, 48 demonstrate the relationship between GW and DM in the presence of pressure and visible matter dissipation at the surface. Assuming that the pressure and dissipation of baryonic matter are insignificant at $\hat{\Sigma}$ $(q \approx 0, p_r \approx 0)$, we can derive a relationship between DM and GW from Eq. 46, which is as follows:

$$A^2 vm(-T)^{m-1}\rho + \frac{A^2}{2}(-v(-T)^m) = A^2 e^{2(\delta-\epsilon)}\delta_{\bar{R}}^2\left(2\frac{\delta_{,T}}{\delta_{,\bar{R}}}v - \frac{v^2}{1-v^2}\right). \tag{49}$$

Eqs. 47 – 49 indicate the presence of exotic matter can interrupt the propagation medium of GWs in the collapsed mechanism.

6 Conclusion

The significance of DM at cosmic levels is predicted by observational investigations. On galactic and supergalactic levels, it is regarded as an essential component of the filamentary structure. The structure of the cosmos is characterized by stellar filaments. At various scales, these formations can be found everywhere in the cosmos. In this work, we explored the collapsing behavior of stellar filamentary objects under the consideration of exotic materials. In this regard, we used the higher-order minimally coupled gravity ($f(R, T)$ theory) to include the dark source terms in consideration. To define the collapsing structure, the DJC has been used on the collapsed surface boundary. By constructing a collapsing equation at the collapsing surface $\hat{\Sigma}$, it is discovered that the existence of dark components together with dissipation causes the radial pressure on the collapsing boundary to remain non-zero.

Observations demonstrated that a significant amount of DM is required to form galaxies and massive clusters. For collapsing context, we used the specific term of the $f(R, T)$ theory as the dark source. The existence of exotic material as a source of GW transmission can expose the mysteries of dark terms (Flauger and Weinberg, 2018; Tamfal et al., 2018). The main concluding remarks are as follows:

- It is discovered that the relationship between radial pressure (associated with baryonic contribution) and dark terms determines the stability scenario for collapsing structures.
- We have examined the connection of GWs emitting from collapsed filamentary objects in exotic terms. From the collapsing equation, a relation between dark terms and GWs has been obtained, which indicates that the presence of dark source terms can alter the medium of GWs.
- In GR, all collapsing variables are directly dependent on matter contribution, but in our case, exotic terms also play an essential role. It is worthwhile to mention that this work reduces to $f(R)$ gravity (Manzoor et al., 2020) when $T = 0$ and GR (Herrera et al., 2005) by substituting $f(R) = R$.

Data availability statement

The original contributions presented in the study are included in the article/Supplementary Material; further inquiries can be directed to the corresponding authors.

Author contributions

All authors listed have made a substantial, direct, and intellectual contribution to the work and approved it for publication.

Conflict of interest

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

Publisher's note

References

Abbott, B. P. (2016). Virgo and LIGO scientific collaborations). *Phys. Rev. Lett.* 116, 061102.

Abbott, B. P. (2016). Virgo and LIGO scientific collaborations). *Phys. Rev. Lett.* 116, 241103.

Alves, M. E. S. (2016). *Phys. Rev. D.* 94, 024032.

Barrow, J. D., and Hervik, S. (2006). Anisotropically inflating universes. *Phys. Rev. D.* 73, 023007. doi:10.1103/physrevd.73.023007

Bessho, S., and Tsuribe, T. (2012). Fragmentation of primordial filamentary clouds under far-ultraviolet radiation. *Publ. Astron. Soc. Jpn.* 64, 104. doi:10.1093/pasj/64.5.104

Bonnor, W. B., and Vickers, P. A. (1981). Junction conditions in general relativity. *Gen. Rel. Grav.* 13, 29–36. doi:10.1007/bf00766295

Breysse, P. C. (2014). *Mon. Not. R. Astron. Soc.* 437, 2675.

Brookfield, A. W., Bruck, C., and Hall, L. M. H. (2006). Viability off(R)theories with additional powers of curvature. *Phys. Rev. D.* 74, 064028. doi:10.1103/physrevd.74.064028

Capozziello, S. (2002). *Int. J. Mod. Phys. D.* 483, 11.

Darmois, G. (1927). *Memorial des Sciences Mathematiques, GauthierVillars*. Paris, Fasc, 25.

Einstein, A., and Rosen, N. (1937). On gravitational waves. *J. Frankl. Inst.* 223, 43–54. doi:10.1016/s0016-0032(37)90583-0

Flauger, R., and Weinberg, S. (2018). Gravitational waves in cold dark matter. *Phys. Rev. D.* 97, 123506. doi:10.1103/physrevd.97.123506

Freundlich, J. (2014). *Astron. Astrophys.* 564, A7.

Guhaa, S., and Ghosh, U. (2021). *Eur. Phy. J. Plus* 136, 460.

Harko, T., Lobo, F. S., Nojiri, S. I., and Odintsov, S. D. (2011). $f(R, T)$ gravity. *Phys. Rev. D.* 84, 024020. doi:10.1103/physrevd.84.024020

Herrera, L., Santos, N. O., and Mac Callum, M. A. H. (2007). Cylindrical collapse and gravitational waves. *Cl. Quantum Grav.* 24, 1033. doi:10.1088/0264-9381/24/4/c01

Herrera, L., Santos, N. O., and MacCallum, M. A. H. (2005). Cylindrical collapse and gravitational waves. *Cl. Quantum Grav.* 24, 1033. doi:10.1088/0264-9381/24/4/c01

Hoemann, E., Heigl, S., and Burkert, A. (2022). Filament collapse: A two phase process. arXiv preprint arXiv:2203.07002.

Horvat, D., Ilijic, S., and Marunovic, A. (2011). *Cl. Quantum Grav.* 28, 25009.

Kneb, A. (2003). *Mon. Not. R. Astron. Soc.* 345, 1285.

Manzoor, R., Adeel, M., and Saeed, M. (2020). Dynamics of collapsing stellar filament and exotic matter. *Int. J. Mod. Phys. D.* 29 (5), 2050036.

Manzoor, R., Jawad, A., Adeel, M., Saeed, M., and Rani, S. (2019). Collapsing stellar filament and exotic matter in Palatini $f(R)$ gravity. *Eur. Phy. J. C* 79 (10), 831.

Manzoor, R., Mumtaz, S., and Intizar, D. (2022). Dynamics of evolving cavity in cluster of stars. *Eur. Phys. J. C* 82 (8), 739.

Manzoor, R., and Shahid, W. (2021). Evolution of cluster of stars in $f(R)$ gravity. *Phys. Dark Universe* 33, 100844. doi:10.1016/j.dark.2021.100844

Milgrom, M. (1997). *Astrophys. J.* 478, 7.

Mumtaz, S., Manzoor, R., Saqlain, M., and Ikram, A. (2022). Dissipative charged homologous model for cluster of stars in $f(R, T)$ gravity. *Phys. Dark Universe* 37, 101096.

Oort, J. H. (1932). *Astron. Inst. neth.* 6, 249.

Ostriker, J. (1964). *Astrophys. J.* 140, 1029.

Planck collaboration, P.A.R. Ade (2014). Planck 2013 results. I. Overview of products and scientific results. *Astron. Astrophys.* 571, A1.

Sharif, M., and Fatima, H. I. (2017). Dynamics of stellar filaments in f(G) gravity, *Eur. Phy. J. Plus* 132, 127, doi:10.1140/epjp/i2017-11421-1

Sharif, M., and Manzoor, R. (2014). Mod. Phys. Lett. A 29, 1450192. *Eur. Phys. J. C* 76(2016)276; 330; *Ann. Phys.* 376(2017)1.

Sharif, M., and Manzoor, R. (2016). Stellar filaments in self-interacting Brans–Dicke gravity. *Eur. Phys. J. C* 76, 276. doi:10.1140/epjc/s10052-016-4113-x

Sharif, M., and Saleem, S. (2020). Stability of anisotropic perturbed Einstein universe in f(R) gravity. *Mod. Phys. Lett. A* 35, 2050152. doi:10.1142/s021773232050 1527

Sharif, M., and Saleem, S. (2020). Stability of anisotropic perturbed Einstein universe in f(R,T) theory. *Mod. Phys. Lett. A* 35, 2050222. doi:10.1142/s021773232050 2223

Sharif, M., and Waseem, A. (2019). Inhomogeneous perturbations and stability analysis of the Einstein static universe in $f(R, T)$ gravity. *Astrophys. Space Sci.* 364, 221. doi:10.1007/s10509-019-3711-0

Sharif, M., and Yousaf, M. (2012). Expansion-free cylindrically symmetric models. *Can. J. Phys.* 9, 865–870. doi:10.1139/p2012-070

Sharif, M., and Zubair, M. (2014). *Astrophys. Space Sci.* 349, 457465.

Sharif, M., and Zubair, M. (2012). Anisotropic universe models with perfect fluid and scalar field in $f(R, T)$ gravity. *J. Phys. Soc. Jpn.* 81, 114005. doi:10.1143/jpsj.81.114005

Sharif, M., and Zubair, M. (2013). Cosmological reconstruction and stability in $f(R, T)$ gravity. *Gen. Relativ. Gravit.* 46, 1723. doi:10.1007/s10714-014-1723-1

Sharif, M., and Zubair, M. (2013). Energy conditions constraints and stability of power law solutions in $f(R, T)$ gravity. *J. Phys. Soc. Jpn.* 82, 014002. doi:10.7566/jpsj.82.014002

Sharif, M., and Zubair, M. (2012). Thermodynamics in $f(R, T)$ theory of gravity. *J. Cosmol. Astropart. Phys.* 03, 028. doi:10.1088/1475-7516/2012/03/028

Sirivastava, S. K. (2008). *General relativity and cosmology*. Prentice Hall of India.

Steinhardt, P. J., Fitch, V. L., and Marlow, R. (1997). *Critical problems in Physics*. Princeton University Press.

Tamfal, T., Capelo, P. R., Kazantzidis, S., Mayer, L., Potter, D., Stadel, J., et al. (2018). Formation of LISA black hole binaries in merging dwarf galaxies: The imprint of dark matter. *Astrophys. J. Lett.* 864 (1), L19.

Zaregonbadi, R., Farhoudi, M., and Riazi, N. (2016). Dark matter from $f(R, T)$ gravity. *Phys. Rev. D.* 94 (8), 084052.

Zhang, P. (2007). Behavior of $f(R)$ gravity in the solar system, galaxies, and clusters. *Phys. Rev. D.* 76 (2), 024007.

Zlatev, I., Wang, L., and Steinhardt, P. J. (1999). Quintessence, cosmic coincidence, and the cosmological constant. *Phys. Rev. Lett.* 82, 896–899. doi:10.1103/physrevlett.82.896

Zubair, M., Azmat, H., and Noureen, I. (2017). Dynamical analysis of cylindrically symmetric anisotropic sources in $f(R, T)$ gravity. *Eur. Phy. J. C* 77, 1–10.

Zwicky, F. (1933). Die rotverschiebung von extragalaktischen nebeln. *Helv. Phys. Acta* 6, 110–127.