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A cosmological model with time dependent Λ , G and viscous fluid in general relativity

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In this paper, we investigate Bianchi type – V cosmological models with bulk viscous fluid and time varying cosmological Λ and Newtonian G parameters. The Einstein's field equations have been transformed into a coupling non-linear, first-order differential equations, and the fourth-order Runge-Kutta method of numerical integration has been used to integrate the differential equations with appropriate initial conditions consistent with current cosmological observations. We show that the model describes a universe that starts off with a negative cosmological term, as well as a matter-dominated and decelerated early epoch that, eventually becomes Λ -dominated and expanding with acceleration, in concordance with current observations.

KEYWORDS

time dependent G and Λ , bianchi metric, viscous fluid, anisotropy, cosmological parameter

1 Introduction

Bianchi cosmological models are homogeneous and anisotropic models that can be viewed as a generalization of the homogeneous and isotropic Friedman-Lemaître-Robertson-Walker (FLRW) space-times on which the concordance cosmology is based. These models are interesting because, although the universe is almost isotropic on the largest possible scales, small-scale anisotropies are a feature of the observed universe. Current cosmological observations (Perlmutter et al., 1997; Perlmutter et al., 1998; Riess et al., 1998; Perlmutter et al., 1999) point out that the universe is expand with acceleration that was previously thought to be decelerating. Dark energy (DE), which in the standard Λ -Cold Dark Matter (ACDM) paradigm is represented by the cosmological constant in Einstein Field Equations (EFEs) of general relativity (GR), is thought to be responsible for the late-time accelerated expansion. Whereas DE is estimated to account for about 70% of the total matter-energy budget of the universe, the significant other proportion of 25% is thought to exist in the form of dark matter (DM), a non-luminous,

and yet-to-be discovered, form of matter whose presence can only be detected through its gravitational effects.

Several proposals have been put forward to address the issues of DE and DM, such as additions of exotic matter forms (such as Λ , the Chaplygin gas, scalar fields, etc.), or modifications of GR itself [in the form of $f(R)$, $f(T)$, $f(R, T)$, $f(Q)$, etc. (Nojiri and Odintsov, 2004; Bergliaffa, 2006; Bertolami et al., 2007; Bertolami and Sequeira, 2009; Sotiriou and Faraoni, 2010; Liu and Reboucas, 2012; Atazadeh, Darabi; Wang and Liao, 2012; Sharif and Ikram, 2016; Bamba et al., 2017; Moraes et al., 2019; Mandal et al., 2020; Arora et al., 2021)] or the non-relativistic limit of Newtonian theory (MOND) in the case of DM. Another consideration in the literature has been one based on Dirac's hypothesis on the evolution of the fundamental "constants". P. Dirac hypothesized that Λ must be a time-dependent function (Dirac, 1937) because the theoretical prediction from quantum field theory (QFT) differs significantly from observations in the value of Λ (Chen and Wu, 1990; Sahni and Starobinsky, 2000). Since then, various scientists have indicated an interest in investigating cosmological models in the context of GR with time-dependent cosmological constant Λ . Many authors have investigated different forms of Λ with standard and non-standard cosmological models based on the same assumption (Vishwakarma and Abdussattar, 1996a; Vishwakarma and Abdussattar, 1996b; Vishwakarma et al., 1999; Vishwakarma, 2000; Vishwakarma, 2001; Vishwakarma, 2005; Bali et al., 2012; Alfedeel et al., 2018; Alfedeel and Abebe, 2020). Bulk viscosity is important in cosmology because it plays a role in the universe's accelerated expansion, also known as the inflationary phase. Over the course of the universe's history, bulk viscosity could manifest in a variety of ways (Ellis, 1971). It is believed that viscosity emerges when neutrinos disengage from the cosmic fluid (Misner, 1968), at the time of galaxies formation and particle synthesis at the early stages of our cosmos (Hu et al., 1983). For all these reasons, there have been many attempts to study non-standard cosmological models involving viscous fluids. Several researchers have recently studied various Bianchi-type cosmological models with varying cosmological constant (Λ) and bulk viscous fluid, including (Huang, 1990; Arbab, 1997; Arbab, 1998; Bali and Pradhan, 2007; Bali and Kumawat, 2008; Tiwari et al., 2016; Tiwari et al., 2017a; Tiwari et al., 2017b; Tiwari et al., 2018a; Tiwari et al., 2018b). For example, the influence of bulk viscosity on cosmic evolution has been studied in (Huang, 1990; Bali and Pradhan, 2007; Bali and Kumawat, 2008). Singh et al. (Singh et al., 2016) investigated the Bianchi type – V cosmological models for a viscous fluid, assuming that the Hubble parameter H is a linear hyperbolic function of cosmic time t . They discovered that using the proposed functional form for the Hubble parameter results in cosmological models that are compatible with current observations. Bali et al. (Bali et al., 2012) studied the Bianchi type – V cosmological model for viscous fluid distribution with variable cosmological term Λ . They analyzed a cosmic scenario

after assuming the rule of variation for the Hubble parameter H , i.e., $H = a(R^{-n} + 1)$, where a, n are constants and R is the average scale factor. They discovered that the model isotropizes asymptotically, and that the existence of shear viscosity speeds up the isotropization. Singh and Baghel (Singh and Baghel, 2010) have investigated Bianchi type – V cosmological models in the presence of bulk viscosity. They derived an accurate solution for the EFEs by assuming that the shear scalar σ is proportional to the volume expansion θ , and that the coefficient of bulk viscosity is a power function of energy density ρ or volume expansion θ . They discovered that Λ should be negative, and the models derived are expanding, shearing, and non-rotating, with no approach to isotropy at late periods. The same authors analyzed spatially homogenous and anisotropic Bianchi type – V space-times with a bulk viscous fluid source and a time-dependent cosmological term (Singh and Baghel, 2009). They arrived to cosmological models by assuming a law of variation for the Hubble parameter, which results in a constant deceleration parameter $q = m - 1$, where m is a constant. They came to the conclusion that the model reflected the universe's accelerating phase for particular values of m . Padmanabhan and Chitre (Padmanabhan and Chitre, 1987) looked at the influence of bulk viscosity on the development of the cosmos as a whole. They demonstrate that the bulk viscosity can result in inflation-like solutions.

Motivated by the above discussion, in this paper we will investigate the Bianchi type – V cosmological model for bulk viscous universe with time-dependent cosmological parameter Λ and Newtonian gravitational parameter G , which is inspired by previous works as mentioned above. We will not assume any coupling relation between the metric variables in this study when solving the gravitational field equations for model physical parameters or imposing any extra constraints, as others do. Instead, we will recast the governing equations for the Bianchi type – V model as a dimensionless, non-linear, first order, coupling differential equation for cosmological observations $h(z)$, $\Omega_m(z)$, Ω_Λ , Ω_χ , and Ω_σ , then integrate them in parallel to estimate the other model characterized parameters. The following is how the rest of this paper is organized: Section 2 introduces the Bianchi type – V metric and the field equations that go with it. The solution to the field equation is presented in Section 3. Section 4 will offer several cosmological models based on the selection of time-varying shear and bulk viscosity. Finally, we bring the article to a close with our conclusion in Section 5.

2 Metric and field equations

The Bianchi type – V line-element in orthogonal space and time coordinates is represented by the following formula:

$$ds^2 = dt^2 - A^2 dx^2 - e^{2mx} [B^2 dy^2 + C^2 dz^2]. \quad (1)$$

where $A = A(t)$, $B = B(t)$ and $C = C(t)$ are the metric potential and m is constant. We assume that the universe is filled by a viscous fluid whose distribution in space is represented by the following energy-momentum tensor:

$$T_{ij} = (\rho + \bar{p})v_i v_j + \bar{p}g_{ij} - 2\eta\sigma_{ij}, \tag{2}$$

where ρ is matter energy density, p is the isotropic pressure, η and ξ are coefficient of shear and bulk viscosity respectively, $v^i = (v^1, v^2, v^3, v^4) = (0, 0, 0, 1)$ is 4-velocity vector of the cosmic fluid and it is time-like quantity that satisfying $v_i v^i = -1$, σ_{ij} is the shear and \bar{p} is the effective pressure which is given by

$$\bar{p} = p - \xi v_{;i} = p - (3\xi - 2\eta)H. \tag{3}$$

Note that the bulk and shear viscosities, ξ and η , are both positive, i.e., $\eta > 0$, $\xi > 0$. We will assume them as either constant or function of time or energy, such as $\eta \sim H$ and $\xi \sim \rho^n$ and n is a numerical constant. Here, the cosmic fluid is assumed to satisfy a linear equation of state

$$p = w\rho, \quad -1 \leq w \leq 1,$$

where w is the equation of state parameter (EoS) which relates p to the energy density. The shear tensor is given by

$$\sigma_{ij} = (v_{i;k} h_j^k + v_{j;k} h_i^k) - \frac{1}{3}\theta h_{ij}, \tag{4}$$

where $h_{ij} = g_{ij} + v_i v_j$ is the projection tensor. The Einstein field equations (EFEs) of the gravitation with time-varying cosmological constant (Λ) in geometrical units where $c = 1$ are given by

$$R_{ij} - \frac{1}{2}g_{ij}R = -\kappa G T_{ij} + \Lambda g_{ij}. \tag{5}$$

Here $\kappa \equiv 8\pi$ and R_{ij} is Ricci tensor, R is Ricci scalar and g_{ij} is the symmetric second-rank metric tensor. Using Eqs 1–4, the EFEs in Eq. 5 for a viscous fluid distribution reduce to the following set of partial differential Eq. 1:

$$\frac{m^2}{A^2} - \frac{\ddot{B}}{B} - \frac{\ddot{C}}{C} - \frac{\dot{B}\dot{C}}{BC} + 2\eta\frac{\dot{A}}{A} = \kappa G \left[p - \left(\xi - \frac{2}{3}\eta \right) \theta \right] - \Lambda, \tag{6}$$

$$\frac{m^2}{A^2} - \frac{\ddot{A}}{A} - \frac{\ddot{C}}{C} - \frac{\dot{A}\dot{C}}{AC} + 2\eta\frac{\dot{B}}{B} = \kappa G \left[p - \left(\xi - \frac{2}{3}\eta \right) \theta \right] - \Lambda, \tag{7}$$

$$\frac{m^2}{A^2} - \frac{\ddot{A}}{A} - \frac{\ddot{B}}{B} - \frac{\dot{A}\dot{B}}{AB} + 2\eta\frac{\dot{C}}{C} = \kappa G \left[p - \left(\xi - \frac{2}{3}\eta \right) \theta \right] - \Lambda, \tag{8}$$

$$-\frac{3m^2}{A^2} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{B}\dot{C}}{BC} = \kappa G\rho + \Lambda, \tag{9}$$

$$\frac{\dot{B}}{B} + \frac{\dot{C}}{C} - 2\frac{\dot{A}}{A} = 0. \tag{10}$$

Generally, one can consider that the covariant derivative of the energy-momentum tensor T_{ij} is proportional to the time

1 Overdots represent partial differentiation with respect to cosmic time t .

variation of the cosmological “constant” and the gravitational “constant”, thus:

$$\kappa G \left[\dot{\rho} + (\bar{p} + \rho) \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) \right] + \kappa \rho \dot{G} + \dot{\Lambda} - 4\kappa G \eta \sigma^2 = 0. \tag{11}$$

$$\dot{\rho} + 3H[p + \rho - (3\xi - 2\eta)H] - 4\eta\sigma^2 = 0, \tag{12}$$

$$\kappa \rho \dot{G} + \dot{\Lambda} = 0. \tag{13}$$

Using $\bar{p} = p - (3\xi - 2\eta)H$, if the total matter content of the universe is conserved, Eq. 11 can be split into two independent equations:

According to Eq. 13, G turns out to be constant for non-zero energy density ρ when Λ is constant or $\Lambda = 0$. Note that we have used $H \equiv 1/3 \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right)$ as we will show later, and σ is the scalar shear tensor is given by

$$\sigma^2 = \frac{1}{2} \sigma_{ij} \sigma^{ij} = \frac{\sigma_0^2}{a^6}, \tag{14}$$

where σ_0 is a constant that is related to the universe anisotropy. The spatial volume V for Bianchi type - V space-time given by

$$V = a^3 = \sqrt{|-g_{ij}|} = ABC, \tag{15}$$

where (a) is the average scale factor of universe. In addition to that, the generalized Hubble parameter H , and the deceleration parameter q are defined as

$$H \equiv \frac{\dot{a}}{a} = \frac{1}{3} (H_x + H_y + H_z), \quad q \equiv -\frac{a\ddot{a}}{\dot{a}^2} = -\frac{\dot{H}}{H^2} - 1, \tag{16}$$

where H_x , H_y and H_z are the directional Hubble parameters along x , y and z directions respectively. The components of the shear tensor σ_{ij} for the metric in Eq. 1 are calculated as

$$\sigma_{11} = H_x - H, \quad \sigma_{22} = H_y - H, \quad \sigma_{33} = H_z - H, \quad \sigma_{44} = 0, \tag{17}$$

and the shear scalar σ now gives

$$\sigma^2 = \frac{1}{6} \left[\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B} \right)^2 + \left(\frac{\dot{B}}{B} - \frac{\dot{C}}{C} \right)^2 + \left(\frac{\dot{C}}{C} - \frac{\dot{A}}{A} \right)^2 \right]. \tag{18}$$

The average anisotropy parameter A_p is defined as

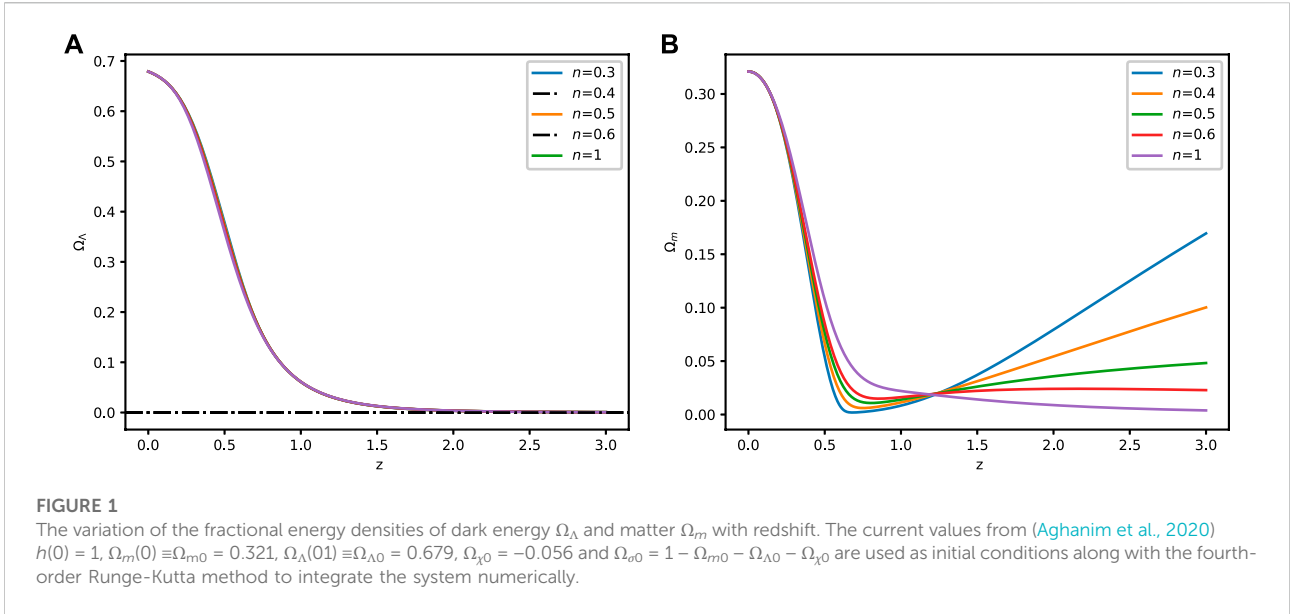
$$A_p = \frac{1}{3} \sum_{i=1}^3 \left(\frac{H_i - H}{H} \right)^2. \tag{19}$$

Subtracting the field Eqs 7, 8 gives

$$\frac{\ddot{B}}{B} - \frac{\ddot{C}}{C} + \left(\frac{\dot{B}}{B} - \frac{\dot{C}}{C} \right) \left\{ \frac{1}{2} \left(\frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) + 2\eta \right\} = 0, \tag{20}$$

which can be integrated to give

$$\frac{\dot{B}}{B} - \frac{\dot{C}}{C} = \frac{k_1}{a^3} e^{-2 \int \eta dt}, \tag{21}$$



Similarly, the rest of the field Eqs 6–10 can also be solved to give a coupled first order differential equation for the metric variables A , B and C as

$$\frac{\dot{A}}{A} = \frac{\dot{a}}{a}, \tag{22}$$

$$\frac{\dot{C}}{C} - \frac{\dot{A}}{A} = \frac{k_2}{a^3} e^{-2} \int \eta dt. \tag{23}$$

Integrating Eq. 22 and absorbing the constant of integration into A or B yields

$$A = a. \tag{24}$$

Thus, plugging Eq. 24 into Eqs 21, 23 produces

$$\frac{\dot{B}}{B} = \frac{\dot{a}}{a} + \frac{k_1}{a^3} e^{-2} \int \eta dt, \tag{25}$$

$$\frac{\dot{C}}{C} = \frac{\dot{a}}{a} + \frac{k_2}{a^3} e^{-2} \int \eta dt. \tag{26}$$

Integrating these equations one more time gives an expression for the metric function B and C as

$$B = d_1 a \exp \left[\int \left(\frac{k_1}{a^3} e^{-2} \int \eta dt \right) dt \right], \tag{27}$$

$$C = d_2 a \exp \left[\int \left(\frac{k_2}{a^3} e^{-2} \int \eta dt \right) dt \right], \tag{28}$$

where k_1 , k_2 , d_1 and d_2 are constants of integration. Eqs 6–10 can be written in terms of H , σ and q as

$$\kappa G \bar{p} - \Lambda = H^2 (2q - 1) - \sigma^2 + \frac{m^2}{A^2}, \tag{29}$$

$$\kappa G \rho + \Lambda = 3H^2 - \sigma^2 - \frac{3m^2}{A^2}. \tag{30}$$

Eq. 29 and Eq. 30 are the generalized Friedmann equations for Bianchi type-V spacetimes endowed with the viscous-fluid model under consideration. The generalized Raychaudhuri equation reads:

$$\dot{H} + 3H^2 - \frac{2m^2}{a^2} - \Lambda + \frac{\kappa G}{2} (p - \rho) - \kappa G \left(\frac{3\xi}{2} - \eta \right) H = 0. \tag{31}$$

This equation cannot be solved as it stands because of the unknown variables η , ξ , a , G , Λ , p and ρ . In order to facilitate the solution process by providing extra information in the form of initial conditions and a constraint, we divide the re-arranged form of the Friedmann Eq. 30 by $3H^2$ and write

$$1 = \Omega_m + \Omega_\Lambda + \Omega_\sigma + \Omega_\chi \tag{32}$$

such that

$$\Omega_m \equiv \frac{\kappa G \rho_m}{3H^2}, \quad \Omega_\Lambda \equiv \frac{\kappa G \rho_\Lambda}{3H^2}, \quad \Omega_\sigma \equiv \frac{\sigma^2}{3H^2}, \quad \Omega_\chi \equiv \frac{3m^2}{3H^2 a^2}. \tag{33}$$

The present-day values of the above dimensionless quantities are given by

$$\Omega_{m0} = \frac{\kappa G_0 \rho_{m0}}{3H_0^2}, \quad \Omega_{\Lambda0} = \frac{\kappa G_0 \rho_{\Lambda0}}{3H_0^2}, \quad \Omega_{\sigma0} = \frac{\sigma_0^2}{3H_0^2}, \tag{34}$$

$$\Omega_{\chi0} = \frac{3m^2}{3H_0^2 a_0^2},$$

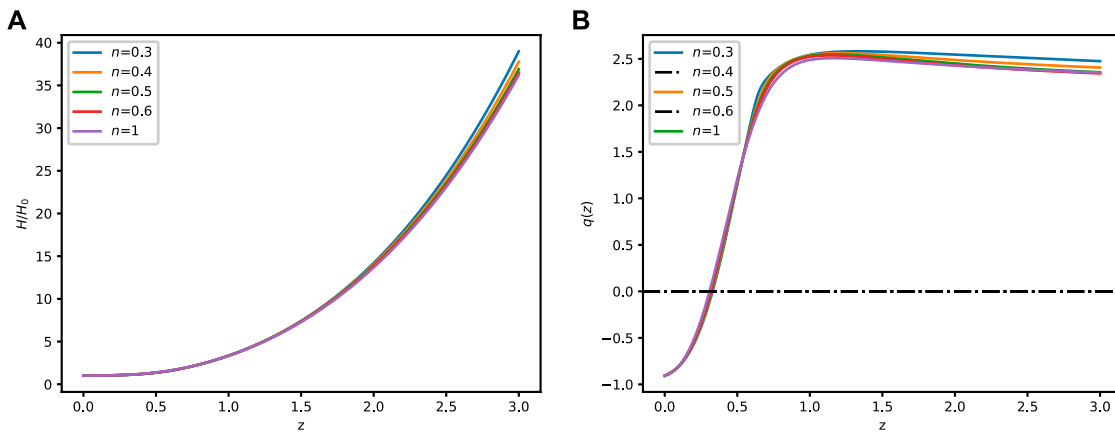


FIGURE 2 The variations of the normalized expansion rate h and the deceleration parameter q with redshift. The current values from (Aghanim et al., 2020) $h(0) = 1$, $\Omega_m(0) \equiv \Omega_{m0} = 0.321$, $\Omega_\Lambda(0) \equiv \Omega_{\Lambda0} = 0.679$, $\Omega_\chi(0) = -0.056$ and $\Omega_\sigma(0) = 1 - \Omega_{m0} - \Omega_{\Lambda0} - \Omega_{\chi0}$ are used as initial conditions along with the fourth-order Runge-Kutta method to integrate the system numerically.

In terms of the dimensionless parameters defined here, Eqs. 12, 13 can be rewritten as:

$$\dot{\Omega}_m + \left(2\frac{\dot{H}}{H} - \frac{\dot{G}}{G} \right) \Omega_m + 3H \left[(1 + w_m)\Omega_m - \frac{\kappa G}{3H} (3\xi - 2\eta) \right] - 4\kappa G\eta\Omega_\sigma = 0, \tag{35}$$

$$\dot{\Omega}_\Lambda + 2\frac{\dot{H}}{H}\Omega_\Lambda + \frac{\dot{G}}{G}\Omega_m = 0. \tag{36}$$

These evolution equations together with the constraint (Eq. 32) need one extra equation to solve for the different Ω_i 's. Thus we give the following additional evolution equations for the fractional energy density of the:

$$\dot{\Omega}_\chi + 2\left(H + \frac{\dot{H}}{H} \right) \Omega_\chi = 0, \tag{37}$$

$$\dot{\Omega}_\sigma + \left(6H + 2\frac{\dot{H}}{H} \right) \Omega_\sigma = 0. \tag{38}$$

Our next step is to numerically integrate these equations and see if/how the results compare with those of the Λ CDM model.

3 Numerical integration

We observe that a viscous fluid Bianchi type-V model with time varying G and Λ is characterized by $A, B, C, h, q, \Omega_m, \Omega_\Lambda$ and G , but the system of equations Eqs 6–10, 12, 13 only provides five differential equations. To complete the solutions processes an extra equation or assumption is required. According to the Dirac (Dirac, 1937) ansatz, the gravitational constant must decrease with time, and based on this we assume that

$$G(t) = G_0 a^\delta \Rightarrow \dot{G} = G\delta H, \tag{39}$$

where $\delta = -1/60$ is a constant obtained from observational constraints (Williams et al., 2009). In order to transform the governing Bianchi type-V evolution equations in redshift space, we use

$$\dot{Q} = \frac{dQ}{dt} = \frac{dQ}{dz} \frac{dz}{da} \frac{da}{dt} = -(1+z)HQ' \tag{40}$$

for any time-dependent quantity Q , and with the dimensionless parameters

$$h \equiv \frac{H}{H_0}, \quad a = \frac{1}{(1+z)},$$

$$\xi = \alpha H_0 (\rho_m / \rho_{m0})^n, \quad \text{and} \quad \eta = \beta H.$$

Here α and β are dimensionless constants and $0 \leq n \leq 1$. We can thus rewrite our previous Eqs 31, 35–37 in fully dimensionless forms as follows:

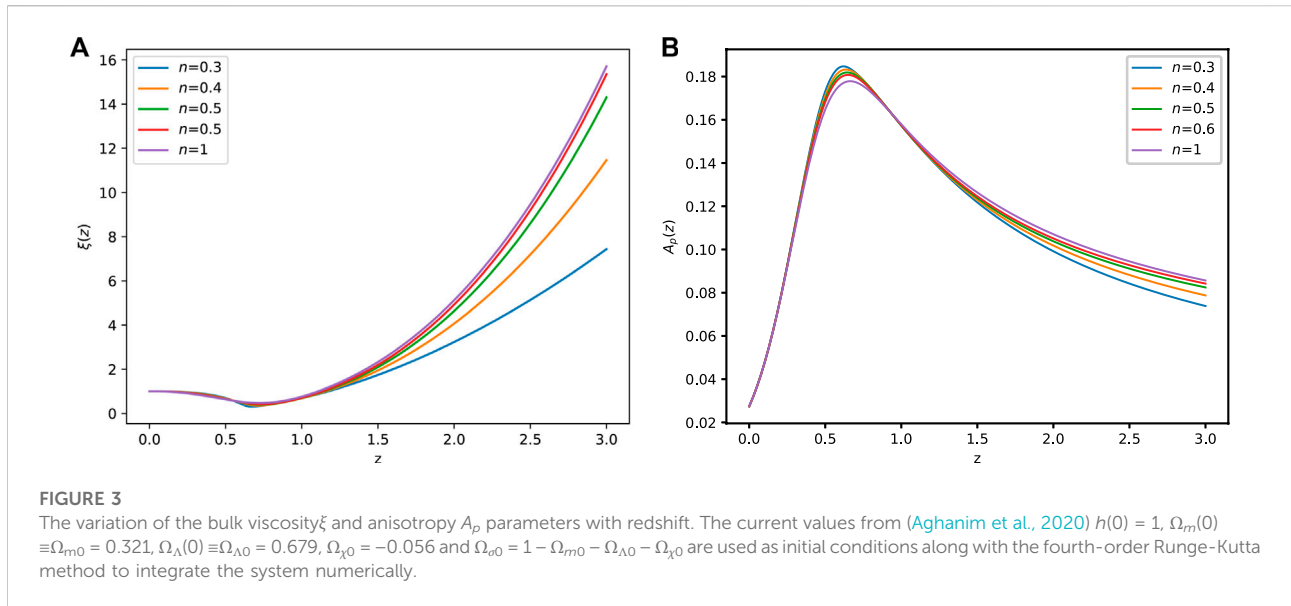
$$h' = \frac{h}{(1+z)} \left[3 - 2\Omega_\chi - 3\Omega_\Lambda - \frac{3}{2}(1 - w_m)\Omega_m \right] - \left[\frac{3\alpha\kappa G_0}{2} \left(\frac{h^2 \Omega_m}{\Omega_{m0}} \right)^n - \kappa G_0 \beta h \right] \frac{1}{(1+z)^{1+\delta}} \tag{41}$$

$$\Omega_m' = -\frac{2h'}{h}\Omega_m + \frac{1}{1+z} (\Omega + 3 + 3w_m)\Omega_m - \frac{3\alpha\kappa G_0}{h(1+z)^{1+\delta}} \left(\frac{h^2 \Omega_m}{\Omega_{m0}} \right)^n + \frac{2\kappa G_0 \beta}{(1+z)^{1+\delta}} - \frac{4\beta\kappa G_0 \Omega_\sigma}{(1+z)^{1+\delta}}, \tag{42}$$

$$\Omega_\Lambda' = -\frac{2h'}{h}\Omega_\Lambda - \frac{\delta}{1+z}\Omega_m, \tag{43}$$

$$\Omega_\chi' = -\frac{2h'}{h}\Omega_\chi + \frac{2\Omega_\chi}{1+z}, \tag{44}$$

$$\Omega_\sigma' = -\frac{2h'}{h}\Omega_\sigma + \frac{6\Omega_\sigma}{1+z}, \tag{45}$$



Eqs 41–45 are first-order coupled differential equations that describe the evolution of h , Ω_m and Ω_Λ with respect to the redshift z . The deceleration parameter q , the metric variables A, B and the volume expansion V are given by:

$$q = 2 - 2\Omega_\chi - 3\Omega_\Lambda - \frac{3}{2}(1 - w_m)\Omega_m - \frac{3\alpha\kappa G_0}{2h(1+z)^\delta} \left(\frac{h^2\Omega_m}{\Omega_{m0}} \right)^n + \frac{\beta\kappa G_0}{(1+z)^\delta}, \tag{46}$$

$$B = \frac{d_1}{(1+z)} \exp \left\{ \frac{\kappa_1}{H_0} \int \frac{(1+z)^{2+2\alpha}}{h} dz \right\}, \tag{47}$$

$$C = \frac{d_2}{(1+z)} \exp \left\{ \frac{\kappa_2}{H_0} \int \frac{(1+z)^{2+2\alpha}}{h} dz \right\}, \tag{48}$$

$$V = ABC = \frac{d_3}{(1+z)^3} \exp \left\{ \frac{\kappa_3}{H_0} \int \frac{(1+z)^{2+2\alpha}}{h} dz \right\}, \tag{49}$$

where $d_3 = d_1 d_2$ and $\kappa_3 = \kappa_1 + \kappa_2$ are numerical constants.

4 Results and discussion

The model governing system of Eqs 41–43 is numerically solved for $h(z)$, Ω_m and Ω_Λ along with the normalized initial conditions $h(0) = 1$, $\Omega_m(0) \equiv \Omega_{m0} = 0.321$, $\Omega_\Lambda(0) \equiv \Omega_{\Lambda0} = 0.679$, $\Omega_{\chi0} = -0.056$ and $\Omega_{\sigma0} = 1 - \Omega_{m0} - \Omega_{\Lambda0} - \Omega_{\chi0}$ using the fourth-order Runge-Kutta method. The numerical results were obtained for several values of the constant n in the range $0 \leq n \leq 1$ and $\alpha = \beta = \kappa G_0 = 1$. The behaviors of Ω_m , Ω_Λ , h , q , ξ and A_p are graphically represented in Figures 1–3.

From Figure 1 we see that Ω_m starts evolving with redshift from having large value at an earlier stage of cosmic evolution gradually decreasing to its minimum value around $z \sim 1$, then reaching its current value of Ω_{m0} at $z = 0$, whereas Ω_Λ grew from a small value at the early times to its current positive value at $z = 0$. This result is in agreement with results from the Λ CDM model.

As seen in Figures 2, 3, the normalized Hubble parameter h , the bulk viscosity ξ and the anisotropy parameter A_p have become smaller today compared to their values at larger redshifts, for all values of n considered. It appears from our analysis, however, that the anisotropy term at about $z \sim 1$ (when the fractional energy density was at its minimum) reaches a maximum value before it decreases to its minimum value today.

The right panel of Figure 2 shows demonstrates that the deceleration parameter changes sign at small redshift values, from negative $q > 0$ at the early times to $q < 0$ at the present time for all different values of n considered. The change in q indicates that the universe expansion in this model has gone through a phase transition from slowing (decelerating) early epoch on to a speeding up (accelerating) universe now, with the transition from deceleration to acceleration happening at $z \sim 0.5$, as predicted by observations as well.

5 Conclusion

The major goal of this paper was to investigate the homogeneous and anisotropic Bianchi type – V cosmological model in the presence of shear η and bulk ξ

viscosities in the cosmic fluids for time-varying gravitational G and cosmological Λ parameters. The governing background EFEs were simplified to second-order differential equations for the metric variables A , B and C , as well as generalized Friedman equations. In this research we have transformed the basic governing equations into non-linear first-order differential equations for h , Ω_m , Ω_Λ , Ω_σ , Ω_χ in the redshift space, which may be solved by numerically integrating in parallel using the fourth-order Runge-Kutta method. Unlike previous studies that required a relationship between the model's characteristic parameter to describe the model in time domain, the current method of integration is significant because it allows us to determine the behaviour of the model directly from redshift-dependent measurable quantities and to compare it to current and future data. Our results showed that the model describes a universe that starts off with a negative cosmological term, dominated by non-relativistic matter and decelerated, that eventually becomes dark energy-dominated and hence expanding with acceleration, in concordance with current observations. Our future endeavour in this direction will involve a more rigorous data analysis to observationally constrain the different assumed parameters of the model.

Data availability statement

The original contributions presented in the study are included in the article/Supplementary Material, further inquiries can be directed to the corresponding author.

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Conflict of interest

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