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The potential of Shannon entropy to find the large separation of δ *Scuti* stars: The entropy spectrum

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This study explores the use of Shannon entropy to find periodic patterns in the oscillation spectra of δ *Scuti* stars. We have developed a new diagnostic tool for detecting potential patterns that scans for minimal entropic states in the well-known échelle diagrams. Here, we describe the basic mathematical grounds of the Shannon entropy and how it can be applied to échelle diagrams through a new diagnostic diagram: the entropy (H) spectrum (HSpec). The method is first validated with the solar-like pulsator HD 49933, for which the large separation was found compatible with values published in the literature. Then we computed the entropy spectrum for two well-studied δ *Scuti* stars: HD 174936 and HD 174966, for which HSpec analysis was able to accurately determine their large separation (or some multiple or submultiple of it). Although these results are promising, the HSpec tool presents several limitations: it has a strong dependence on the probability distribution of the frequencies in the échelle diagram, and on the way it is calculated. We discuss possible solutions to this that rely on 2D probability distributions and conditional entropy.

KEYWORDS

asteroseismology, time series analysis, stellar structure and evolution, information theory, stellar pulsations, *delta Scuti* stars, solar-like oscillations

1 Introduction

The analysis of periodic patterns is crucial for asteroseismology. Up to date, the search for individual mode identification has been found to be an almost unreachable task for most of the variable stars. This has barely progressed even with the availability of ultra-precise data from space. However, some structures in the oscillation power spectrum of pulsating stars provide very useful information about the stellar interiors. This has been especially successful for the Sun and solar-like pulsators (e.g., red giants), whose periodic patterns can be observed without difficulty (see e.g., Aerts et al., 2010; Corsaro et al., 2012). In fact, from main-sequence to read giants solar-like pulsators, the automated mode identification (including mixed modes) is nowadays a reality (details in e.g., Kallinger, 2019; Corsaro et al., 2020; Nielsen et al., 2021) This has led to a revolution in the understanding of their internal structure and evolution with the use of scaling relations

(see e.g., Hekker and Christensen-Dalsgaard, 2016, and references therein), which have been used as a proxy of stellar masses, radii, mean densities, ages, etc. This has allowed us to perform more accurate stellar population studies of the galaxy (Miglio et al., 2016, 2013). On the other hand, for other stars with different pulsation mechanisms, such periodic patterns are not so easy to detect.

Thanks to a great observational effort, Breger et al. (1999) found periodic regularity in the oscillation spectrum of FG Vir, which was proposed as possible large separation-like pattern in the low-frequency regime (around the fundamental radial mode). Later on, in the era of space missions like *MOST* (Walker et al., 2003), *CoRoT* (Baglin et al., 2006), and *Kepler* (Koch et al., 2010), thanks to ultra-precise photometric time series, such patterns in the low-frequency regime of δ *Scuti* stars were again observed (see e.g., García Hernández et al., 2009; Handler, 2009; García Hernández et al., 2013). These patterns were also predicted theoretically (Reese et al., 2008; Ouazzani et al., 2015) and were found to be compatible with a large separation that scales with the mean density of the star ($\Delta\nu$, Suárez et al., 2014). This scaling was then empirically confirmed by García Hernández et al. (2015) using binary systems with a δ *Scuti* component. Then Hernández et al. (2017) used the scaling $\Delta\nu - \rho$ relation to demonstrate that it was possible to accurately determine the surface gravity of those stars when precise measurements of stellar parallaxes are available. This paved the way to perform multivariable correlations analyses in the observed seismic data (Moya et al., 2017) as well as to find the empiric relation between the frequency at maximum power of their oscillation spectra ($\nu_{\max}$) and the effective temperature (Barceló Forteza et al., 2018), or even with surface gravity through the study of the gravity darkening in fast rotators (Barceló Forteza et al., 2020). Interestingly, Mirouh et al. (2019) retrieved a similar $\Delta\nu - \rho$ scaling relation for fast rotating stars using island modes identified using a convolutional neural network. The most recent theoretical relation has been obtained by Rodríguez-Martín et al. (2020) using synthetic spectra of rotating models.

A confident knowledge of the large separation is also valuable because it allows to better search for other interesting patterns, such as the rotational splitting (see e.g., Barceló Forteza et al., 2017; Ramón-Ballesta et al., 2021), the identification of certain radial modes in young δ *Scuti* stars (see Bedding et al., 2020, for more details), or put asteroseismic constraints on the age of open clusters (Pamos Ortega et al., 2022).

Despite all this progress, the detection of periodicities remains critical. In the last decades, methods based on pattern recognition have been applied in the time domain (see e.g., Mosser and Appourchaux, 2009, and references therein) with significant success in solar-like pulsators, red giants, etc. For A–F stars, the discrete Fourier transform (DFT) of the frequency positions (García Hernández et al., 2009) is the most successful method to detect large separations in A–F stars. Some attempts to automatize its use have been developed (see e.g., Paparó et al., 2016); however, all of them still requires some supervision, preventing the implementation in automatic pipelines.

In this work, we tackled the problem of frequency pattern detection with a concept borrowed from Information Theory: the Shannon entropy. This concept has mainly been applied in the realm of information transmission, computer science, etc. In astronomy, Cincotta et al. (1995) and Cincotta et al. (1999) developed the first methods to detect patterns in astronomical time series.

In recent years, Shannon entropy has regained interest in the so-called big data and machine learning fields as a reliable measure of statistical dependence (see e.g., Malakar et al., 2012), as well as a criterion for feature selection in engineering (Tourassi et al., 2001). This work considers the frequency patterns as features in stellar power spectra, and thus we applied the concept of Shannon entropy to detect them with no hypothesis about their statistical distributions. In Section 2, we define the concept of Shannon entropy in the manner we calculated it and the new diagnostic tool based on it, the entropy spectrum (HSpec). The method is then validated against well-known frequency patterns of the solar-like star HD 49993 and applied to the two selected δ *Scuti* stars (Section 3). Finally, in Section 4, we interpret the results as compared with other methods and discuss the limitations of the tool as well as possible solutions to improve its robustness.

2 Methods

The methodology followed in this work relies entirely upon the use of the Shannon entropy, a concept which was introduced for the first time by Claude Shannon in the context of communication theory (see his famous study Shannon, 1948, for details). In the following, for the sake of simplicity the word "entropy" will be used as a shortcut of "Shannon Entropy". It is thus worthy defining it to better understand its importance for the method. The entropy H of a discrete random variable X is defined in terms of its probability distribution $P(X)$, as follows:

$$H(X) = -\sum_{x} P(x)\log P(x). \tag{1}$$

When there is no knowledge about the distribution of probabilities $P(x)$ we need to estimate it. This thus implies that instead of calculating the entropy we will estimate it $\hat{H}(X)$. The better the estimate of the distribution of probabilities is, the closer $\hat{H}(X)$ will be to the actual entropy. In the present work, we use the method of maximum-likelihood (ML) to estimate $\hat{P}(x)$ from the observed counts (see Hausser and Strimmer, 2009, for details). In practice, we computed the following:

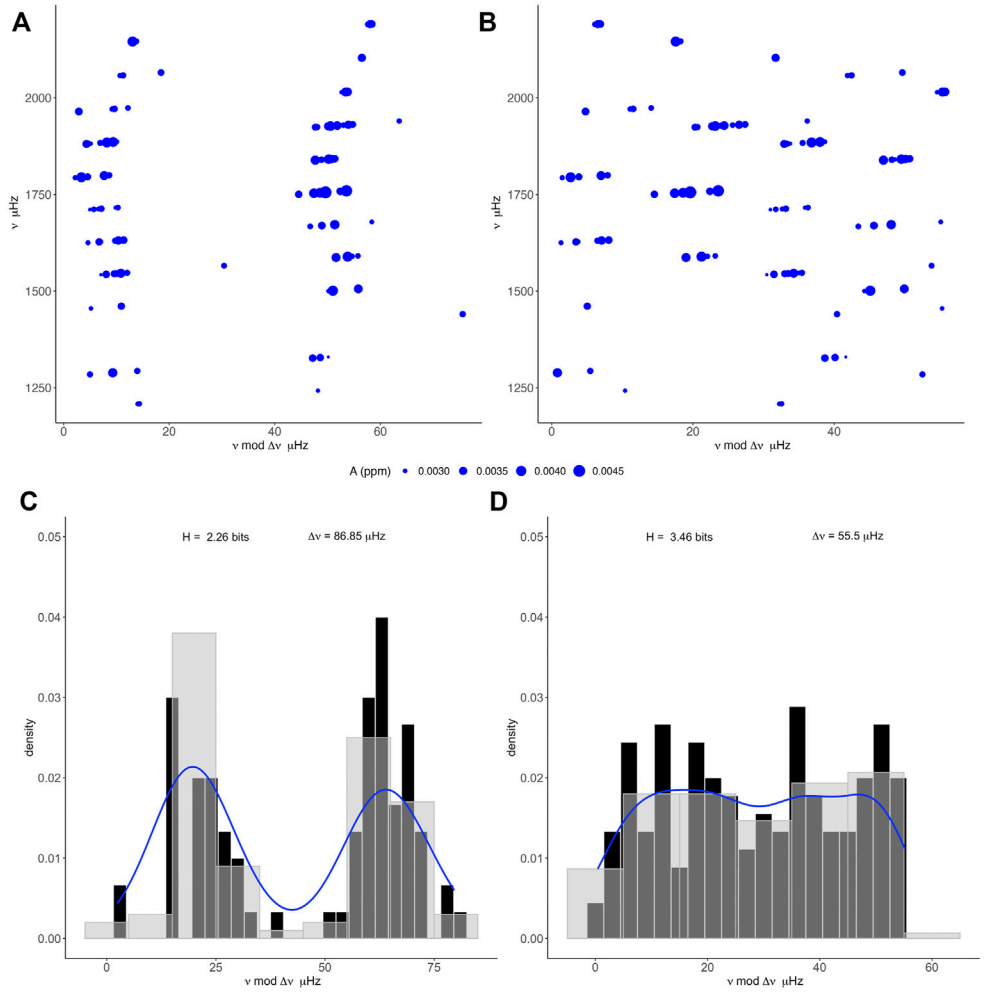

FIGURE 1
Upper panels show the échelle diagrams of the solar-like star HD 49933, the first solar-like star observed by *CoRoT*. These were built using its oscillation frequencies obtained from a pre-whitening process (see Section 3.1). The size of dots indicates the amplitude of the observed modes in ppm. **(A)** shows the classical vertical ridges corresponding to modes ordered by $\Delta\nu$ = 86.85 μHz. **(B)** shows the same diagram for a random value of $\Delta\nu$ = 55.5 μHz. Lower panels, **(C,D)**, are the histograms (density of counts) of the échelle diagrams corresponding to **(A,B)**, respectively. Grey and black represent histograms computed with bandwidths of 10 μHz and 3 μHz, respectively. For illustration the density curve for the latter is also depicted in blue color.

$$\hat{H}(X) = -\sum_{x} \hat{P}(x)\log\hat{P}(x), \tag{2}$$

where

$$\hat{P}(x) = \frac{y_k}{n} \tag{3}$$

is the ML estimate of probability distribution based on the observed counts y_k normalized by the total number of observations.

2.1 The échelle diagrams

We are interested in finding frequency patterns in the oscillation spectra, that is, the set of observed oscillation frequencies of a given pulsating star. For this purpose, we use the following statistical variables:

$$x = \left[\nu_i \bmod \Delta\nu_j\right], \tag{4}$$

$$y = [\nu_i], \tag{5}$$

where $[\nu_i]$ represents the set of observed frequencies and $\Delta\nu_j$ is a given frequency periodicity.

Such a relation between variables is the well-known échelle diagram, widely used in asteroseismology for the visual inspection and determination of large separations (mainly in solar-type stars). When $\Delta\nu_j$ corresponds to the large separation, the oscillation modes appear aligned, with a vertical distance between points of $\Delta\nu_j$ for the same angular degree. The different

ridges represent the modes with different mode degrees, although the identification of the modes is out of the scope of this study.

Figure 1 shows the échelle diagram of HD 49993 for two different spacing: its large separation and a random value. Notice that when vertical ridges come up, most of the modes are vertically aligned, which indicates certain regularity. This is indeed, the way of finding the correct $\Delta\nu$ using échelle diagrams. However, the clarity of the vertical ridges is progressively lost as one moves away from the optimal value, up to some $\Delta\nu_j$ at which points seem to be randomly distributed, and therefore indicates less regularity, or at least, less regularity than the previous case, which is the main point of the present method.

2.2 The entropy of échelle diagrams

The concept of order and disorder are somewhat ambiguous. Instead, let us use the notion of measure of information, as originally proposed by C. Shannon. This measure can also be understood as a kind of uncertainty about the information itself[1]. The measure of information conveys how hard it is to transmit information through a message. Hard means that you need more bits of information to transmit the same information. In this case, the message is carried by the distribution of points in the échelle diagram. In other words, an ordered-like distribution as the one seen in Figure 1A requires less bits of information to be transmitted, as compared with the random-like distribution of Figure 1B. Qualitatively this means that $H_A < H_B$.

In order to quantify the entropy, H, it is necessary to know the distribution of the points $P(X)$ in the échelle diagram (see Eq. 1). The simplest way of obtaining such information is to compute the histogram of the échelle diagram's abscissa, that is, how ν $|\Delta\nu_j|$, for a given $\Delta\nu_j$. This is illustrated in panels C and D of Figure 1, where the distribution of counts (bars) or density (shaded curve) is P(X). As expected, when points are distributed in two regions (lobes of panel A), the uncertainty about the location of points is low (or equivalently, the amount of bits necessary to transmit the information about the distribution of the points is low), $H_A = 1.96$ bits, as compared with a uniform P(X) of panel B, for which is $H_B = 3.46$ bits.

At this point it is important to emphasize that this is an interpretation of entropy in terms of measure of information, hence nothing can be said about the information it may conveys. That is, one can deduce or extract useful information from panels A or B of Figure 1, regardless their measure of information, which is just related with how the distribution of points in the échelle diagrams can be coded. It can also be interpreted as the degree of uncertainty or even as the average unlikelihood of occurrence of the outcomes.

2.3 The Hspec

The main objective of the method is to find the distribution of points in the échelle diagram that minimizes its entropy H. In practice, we proceed according to the following steps:

1. Selection of the interval of the periodicities $\Delta\nu_j$ to be explored and its resolution R. Depending on the previous knowledge about type of pulsation, estimates of periodic patterns, etc., we establish how wide the scan frequency will be. For solar-like pulsations this is quite straightforward since scaling relations can provide an initial guess on $\Delta\nu$, which may help to shorten the interval to explore. However, the scaling relations for δ *Scuti* stars relies the large separation with the mean density of the star (see Rodríguez-Martín et al., 2020, for the most updated relation), calibrated for eclipsing binaries, for which independent measurements of the mean density are available. The resolution, that is, the step in frequency $R = \Delta\nu_j - \Delta\nu_{j-1}$ can be arbitrarily small. However, this can increase the amount of numerical operations needed. The lower limit, in any case, should be twice the observational uncertainties of the oscillation frequencies: $2\ |\epsilon(\nu_j)|$.
2. Choosing the range of observed frequencies to be explored. Although, this step is not strictly required, it is recommended to restrict it (if possible) to the frequency domain where the targeted periodic patterns are expected to be. Complete exploration is also possible, although the - spectra might be noisier or less reliable.
3. Estimation of the number of bins n_b to sample the probability distributions. We selected a given number of bins for the scan of $\Delta\nu_j$.
4. Building the entropy spectrum, HSpec. Échelle diagrams are computed for each the scanned $\Delta\nu_j$ values explored within the selected ranges of frequency and periodicities $[\Delta\nu_{\min}, \Delta\nu\max]$. Then H_j is calculated for each one of those échelle diagrams. The complete set of $(H_j,\ \Delta\nu_j)$ is what I called entropy spectrum.

The absolute minimum of entropy in HSpec is considered the dominant periodicity, which can be interpreted as the large separation or a submultiple of it (see Section 4).

2.4 A first glance to HSpec on a synthetic oscillation spectrum

Notice that the Shannon entropy is sensitive not only to variations in the overall distribution of the points but also in the

1 Note that, here, H does not deal with the information itself, but with its measure.

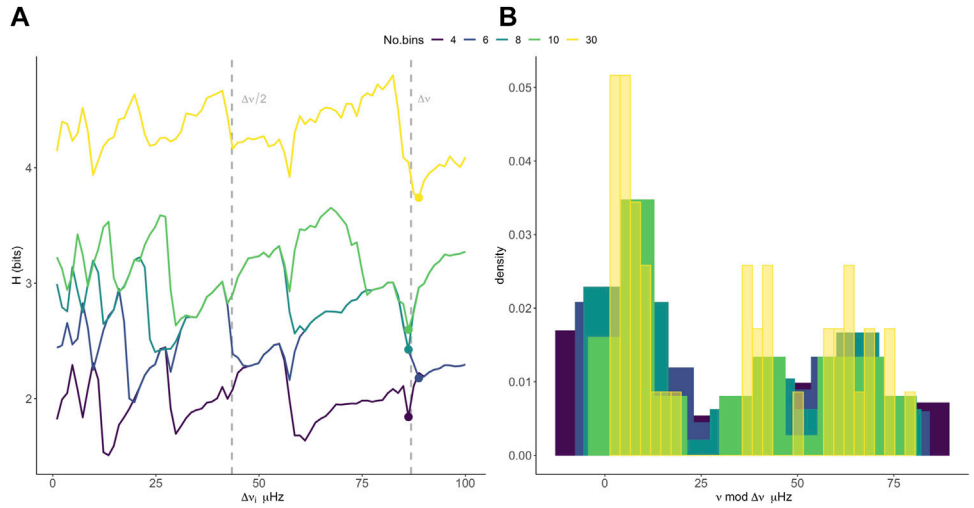

FIGURE 2
Panel **(A)** shows the variation of HSpec with the number of bins for an asteroseismic model of 1.52 $M_{\odot}$. For illustration, the large separation of the model, $\Delta\nu$ = 86.84 μHz, and its half, are shown as vertical dashed lines. Filled dots indicate the location of the local minima that spot the large separation value. **(B)** depicts the histograms from which the probability distributions were extracted to compute the entropy spectra shown in **(A)**.

way the probability distribution is computed, that is, the size and/or number of bins used in the histograms.

A detailed analysis of HSpec for different configuration of parameters lies beyond the scope of this study. Nonetheless, a first glance to synthetic oscillation spectra indicates that HSpec would be more sensitive to the number of bins than any other factor (see Figure 2). The entropy spectra showed in that figure were computed for an asteroseismic model of 1.52 $M_{\odot}$ using the evolutionary code MESA (Paxton et al., 2019) and the adiabatic pulsation code FILOU (Suárez and Goupil, 2008). Standard physics for δ *Scuti* stars were considered with solar metallicity and a moderately fast surface rotation (40% of the break-up frequency). Adiabatic frequencies were corrected for the effect of rotation including near-degeneracy effects (Suárez et al., 2006). The theoretical large separation of $\Delta\nu$ = 86.83 μHz was computed as the median of the large separations obtained for each ℓ within the range n = [2, 10].

The value of the actual large separation is always spotted whatever the number of bins considered. However, these local minima do not always correspond with the absolute minima. In some cases, it is hard to find out which of the peaks would be the right one. For lower $\Delta\nu_i$ values, the analysis is even more difficult since there are significant shifts of the peaks of about 5–20 μHz. Bear in mind that all the modes are considered (from ℓ = 0 to 2), that is, no visibility function was convoluted with the oscillation spectrum. This is therefore the best case, for which there is no lack of modes contributing to $\Delta\nu$. (see Section 4 for more details on the next steps foreseen to improve the method).

3 Results

As the main goal of the HSpec analysis is to find the large separation in δ *Scuti* stars, it is important to validate the method on stars for which the large separation is known and easily detectable, such as solar-type stars.

3.1 The HSpec of the solar-like pulsator HD 49933

For this star, we used the 60 days data from the *CoRoT* initial run (IR) processed with the MIARMA code to fill the gaps due to the satellite duty cycle (in particular the gaps caused by the passage of the satellite through the South Atlantic anomaly (see details in Pascual-Granado et al., 2015). We then obtained the pulsation frequencies of the star using the pre-whitening procedure described in Pascual-Granado et al. (2018), using the code SIGSPEC (Reegen, 2007).

The large separation of HD 49933 was estimated from the light curve by Appourchaux et al. (2008) and later on by Benomar et al. (2009) is $\Delta\nu$ = 85.9 ± 0.15 μHz. In the frequency domain,

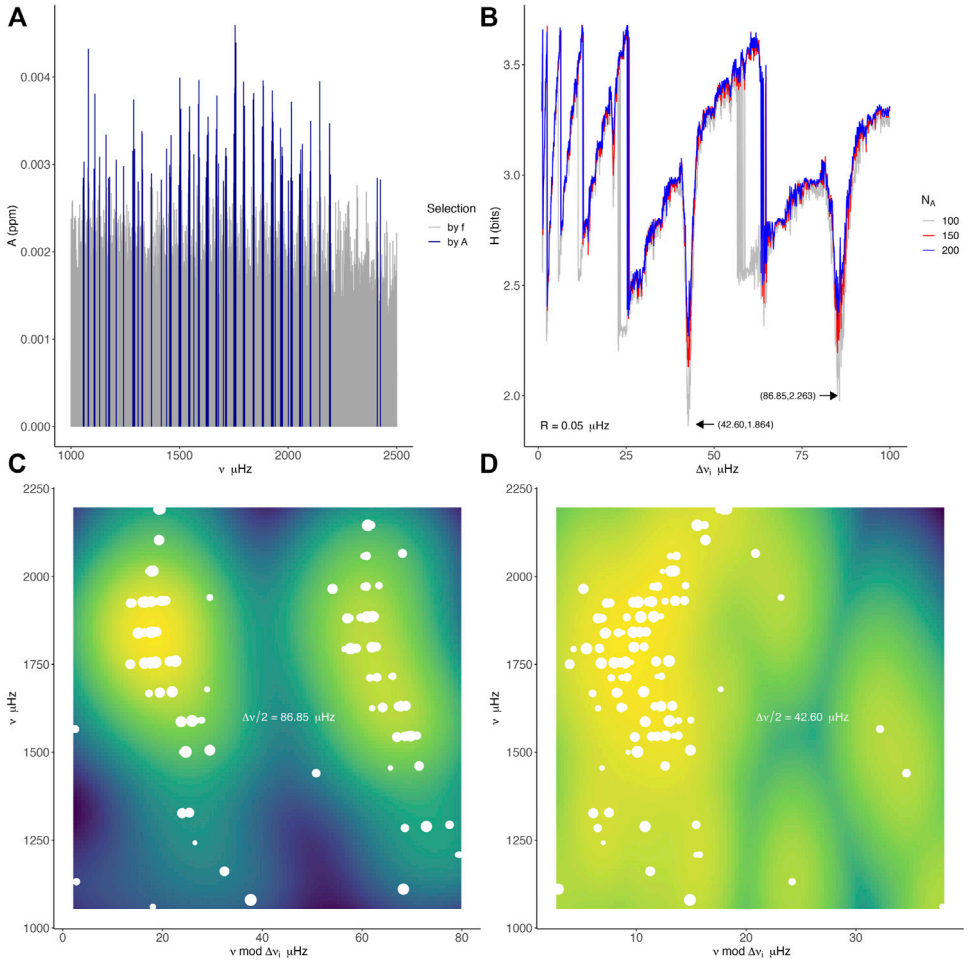

FIGURE 3
Panel **(A)** depicts the oscillation spectrum in the frequency range where p-modes are found (in grey), and a selection of N_A =100 frequencies from that range with the largest amplitude (an explanation of this arbitrary number is given in the discussion section). Panel **(B)** shows the entropy spectrum (HSpec) computed for three different numbers of large amplitude frequencies: N_A = 100, 150, and 200. Notice that the minimum H value is found for the half of the observed large separation whatever N_A. Panels **(C,D)** show the échelle diagram for both the estimated large separation and its half. As for panel 3, the size of white dots is proportional to the amplitude of the modes, and the background is the 2D density of the échelle diagrams..

using échelle diagrams, its value is confirmed (Figure 1A). In addition, in those studies a rotational splitting in the range of 3.5 − 6 μHz is also reported.

Following the procedure explained in the previous section, we built the entropy spectrum for HD 49933 by first selecting the frequency range, and second, by keeping only a set of frequencies with the largest amplitudes. This procedure, similar to the one followed in García Hernández et al. (2009), helps us to obtain better results, assuming that the modes with the highest amplitudes (in general low-degree modes when observed with photometry) are those that mostly contribute to $\Delta\nu$. Figure 3 shows the aforementioned selection (panel A) and the HSpec (panel B) for HD 49993.

As expected, HSpec shows high variability with different entropy minima corresponding to, say, local states of low H. When exploring low $\Delta\nu_i$ values peaks may correspond with échelle diagrams with a too small number of points. From the information theory interpretation, this is equivalent to the presence of a pattern; hence, the actual H value may be similar. Therefore, as happens for the DFT method, the peaks in that range will not be considered in this first study.

It can be seen that the absolute minimum (H = 1.864 bits) within the range of $\Delta\nu$ is reached for $\Delta\nu_i$ = 42.60 μHz, which is half of the large separation found by Benomar et al. (2009). The entropy spectrum was constructed with a resolution of R = 0.05 μHz. If we consider this resolution as the uncertainty of

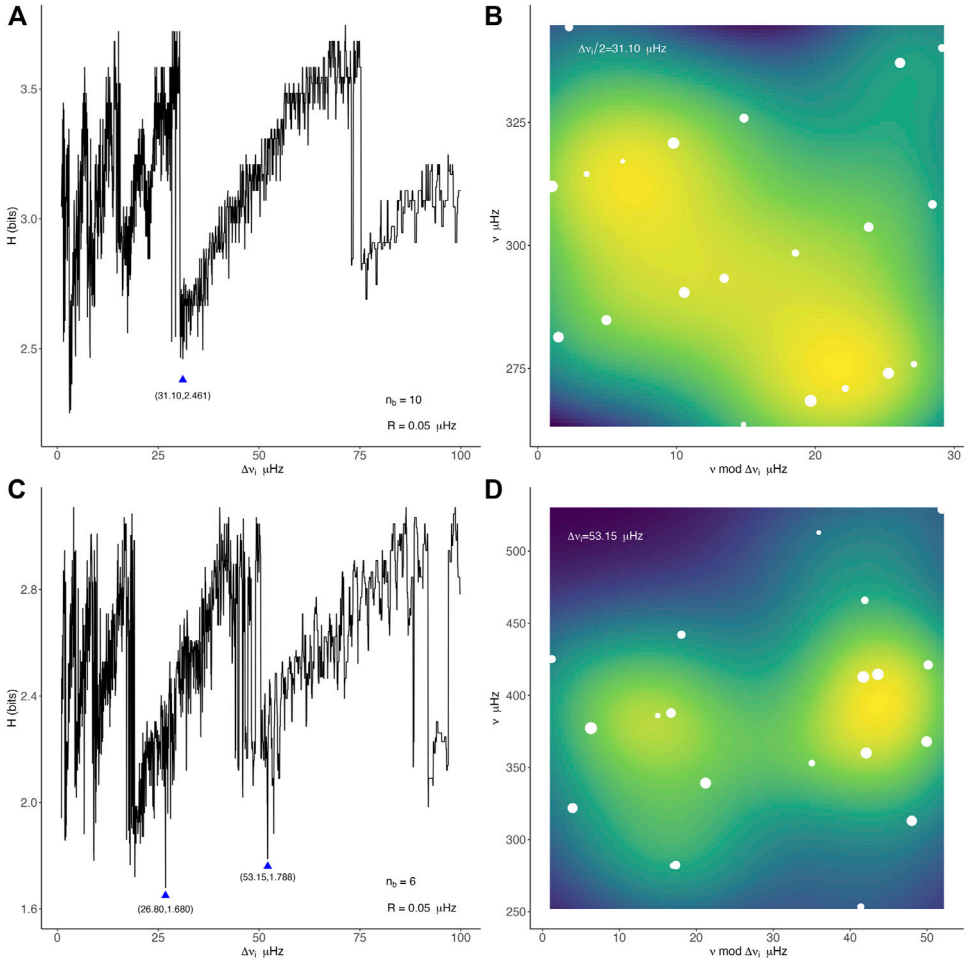

FIGURE 4
Entropy spectrum (HSpec) for HD 174966 **(A)** and HD 174936 **(C)**. Panels **(B,D)** show the resulting échelle diagram (white dots) for the large separation determined by the HSpec in panels **(A,B)**, respectively. The size of dots is proportional to the amplitude of the modes. Background corresponds with the 2D density of the échelle diagrams, which help to visually guess the probability distribution of the points in both vertical and horizontal axes. Notice that for HD 174966, the échelle diagram is built for the half of the large separation, corresponding to the absolute minimum in HSpec.

the estimate, the detected periodicity would thus be $\Delta\nu_i = 42.60 \pm 0.05\,\mu$Hz. However, this must be considered with caution, since a comprehensive analysis of the different sources of errors must be done in order to properly estimate the uncertainty of $\Delta\nu$. As an example, the HSpec shows different local minima and even some shifts in $\Delta\nu$ when using a different amount of frequencies of highest amplitude to construct the échelle diagrams for each $\Delta\nu_i$. The depth and width of the peaks and their relation might be good estimators for the uncertainty in the measure when all the error sources are included. They can be modified by, for example, the number of frequencies, number of bins used to construct the probability distribution, method used to estimate entropy, etc. (see Section 2). This however exceeds the scope of this exploratory work. As for the DFT and AC, the HSpec show both the large separation and its submultiples. The presence of an absolute minimum may be used to consider that periodicity as the one to which more modes are contributing due to their visibility.

3.2 The HSpec of the δ *Scuti* stars HD 174936 and HD 174966

These objects are two of the best-known δ *Scuti* stars, for which there is a reliable determination of the large separation using the discrete Fourier transform method. Any method based on statistical estimators has an intrinsic dependence upon the number of realizations (observations). This is the case of the discrete Fourier transform, autocorrelation (AC), and histogram of frequency differences (HFD),

currently used for determining the large separation of δ *Scuti* stars[2].

As for HD 49993, the light curves of both δ *Scuti* stars were observed by the *CoRoT* satellite (Seismo Field). Their oscillation spectra were obtained from the fractal analysis of the pre-whitening process (see De Franciscis et al., 2018; De Franciscis et al., 2019, for more details). That method allowed us to find the harmonic content of the light curve without imposing any statistical criterion about the significance, thereby minimizing the presence of spurious peaks, which is key to avoid any distortion in the échelle diagrams and hence on the computation of HSpec.

As it happens in other statistical methods (DFT, autocorrelation function, etc.), the detection of regularities in these stars is more difficult, due to the complexity of their oscillation spectrum (see Goupil et al., 2005, for an interesting review of the asteroseismology of δ *Scuti stars*). In contrast to the HSpec of HD 49993, the number of bins, resolution, and frequency domain play an important role in the determination of the large separation. This implies to perform several iterations to find the optimum HSpec from which the $\Delta\nu$ and its multiples (and submultiples) were determined.

Figure 4 depicts the HSpec for both stars together with the échelle diagram corresponding to the detected $\Delta\nu$. It is worth noting that the leftmost part of the entropy spectrum is quite similar to the one found for HD 49993, which reinforces the explanation of the peaks as a consequence of the lack points in the échelle diagram. When the number of frequencies is small this configuration may lead to the absolute minimum of entropy. This is the case of HD 174966, for which the second minimum is compatible with the half of the large separation determined by the DFT method $\Delta\nu = 65 \pm 1\ \mu$Hz (García Hernández et al., 2013). The next minima are above 75 μHz, far beyond twice the $\Delta\nu/2$. As discussed before, this may be caused by the way in which the distribution of probability is constructed.

Moreover, the number of bins used for HD 174966 is 10, while only six were sufficient for HD 174936. This is because although the total number of frequencies for both stars is somewhat similar (120 and 177 frequencies for HD 174966 and HD 174936, respectively), the frequency range in which the highest amplitude peaks are found is [250, 350] μHz for HD 174966 and [160, 650] μHz for HD 174936. This implies that échelle diagrams were built with 20 frequencies for HD 174966 and 127 frequencies for HD 174936.

For HD 174936, both the half of the large separation $\Delta\nu/2 = 28.80 \pm 0.05\ \mu$Hz and twice the value (i.e., the large separation) $\Delta\nu = 53.15 \pm 0.05\ \mu$Hz were found, which is compatible with the large separation found using the DFT technique, that is, $\Delta\nu = 52 \pm 10\ \mu$Hz (García Hernández et al., 2009). Notice that, once again the absolute minimum within the range of explored $\Delta\nu_i$ is identified with the half of the large separation, which might be interpreted in terms of the modes' visibility.

In addition, the échelle diagram of HD 174936 frozen at its large separation shows a glimpse of the classical échelle configuration with two vertical ridges composed by the modes with the highest amplitudes. In contrast, this configuration is not visible in the échelle diagram of HD 174966, but a series of inclined fringes with no clear pattern for the modes with the highest amplitudes. A similar distribution of points in the échelle diagram is found for twice $\Delta\nu/2$, even if this is not clearly a minimum in HSpec.

Regarding the values of H, it can be seen that the HSpec of HD 17966 is shifted by about 1 bit on average respect to the HSpec of HD 174936. Since there is no reference value for the entropy, except maybe its average over the HSpec, which allows us to calculate some statistics, there is no way to compare two HSpec by their absolute H values. The hypothesis that seems more plausible to explain such a shift is the amount of frequencies that are actually contributing to the large separation.

4 Discussion

The entropy spectrum analysis was found to be reliable and robust for the solar-like pulsator HD 49993. For this star $\Delta\nu/2$ and $\Delta\nu$ were found the two minima of the HSpec. As happens with other methods mentioned previously, additional information is required to spot $\Delta\nu$ multiples or submultiples, and thereby the actual value of $\Delta\nu$. This has implications on the visibility of the modes which are inherently related with the angle of inclination of the star respect to the observer.

For the two selected δ *Scuti* stars HD 174966 and HD 174936 HSpec was able to accurately determine the periodicities corresponding to either the large separation or its half. However, in contrast to HD 49993, the optimum HSpec was obtained by several iterations with different combinations of frequency ranges, number of bins, and number of frequencies with the highest amplitudes. Although this is an evident limitation of the method, the HSpec looks promising as a tool to determine the large separation of δ *Scuti* stars, even for oscillation spectra containing a few tens of modes.

In order to render the method here presented more robust, it is necessary to minimize both the human supervision and the additional information required to accurately determine the large separation of δ *Scuti* stars. The next steps will thus be focused on improving the treatment of the probability distribution of the échelle diagram points. I envisage to work on two main aspects: 1) the enhancement of the signal through the échelle diagrams, i.e., to implement a modified version of those diagrams that enhance the vertical alignment features in

2 Throughout the study, when referring to large separation in δ *Scuti* stars means the low-order (n = [2, 8–10]) large separation.

the probability distribution, and 2) the incorporation of joint probability distribution that accounts for the relation between *nu* ad ν modulo $\Delta\nu_j$. For 1) there is well-known method widely used for solar-like pulsators, the so-called collapsed échelle diagrams (see e.g., Corsaro et al., 2012, and references therein). These collapsed échelle diagrams take into account the entire observed power spectrum, which may render transitions between entropic states smoother. The background signal would also be present so it might be used somewhat as reference entropy for each star. Another interesting point is that it can be quite fast since no pre-whitening process is necessary, so it might be used in pipelines for asteroseismic analyses. Regarding the joint probability distributions 2), the inclusion of an additional dimension, the distribution of frequencies, might render HSpec method more sensitive to the presence of vertical patterns than any other patterns (for instance tilted ridges or even diagonal fringes).

Data availability statement

The original contributions presented in the study are included in the article further inquiries can be directed to the corresponding author.

Author contributions

JCS is the unique contributor to this work, from conception of the idea to development, test, and manuscript writing.

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Conflict of interest

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