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Generalized uncertainty principle and burning stars

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Gamow's theory of the implications of quantum tunneling for star burning has two cornerstones: quantum mechanics and the equipartition theorem. It has been proposed that both of these foundations are affected by the existence of a non-zero minimum length, which usually appears in quantum gravity scenarios and leads to the generalized uncertainty principle (GUP). Mathematically, in the framework of quantum mechanics, the effects of the GUP are considered as perturbation terms. Here, generalizing the de Broglie wavelength relation in the presence of a minimal length, GUP corrections to the Gamow temperature are calculated, and in parallel, an upper bound for the GUP parameter is estimated.

KEYWORDS
quantum gravity, minimal length, generalized uncertainty principle, Gamow theory, stellar formation

Introduction

In the first step of star burning, its constituents must overcome the Coulomb barrier to participate in nuclear fusion (NF). This means that when the primary gas ingredients have mass m and velocity v, then using the equipartition theorem, one gets

$$\frac{1}{2}mv^2 = \frac{3}{2}K_BT \geq U_c(r_0), \tag{1}$$

where K_B denotes the Boltzmann constant, the subscript c in $U_c(r_0)$ indicates the Coulomb potential, and correspondingly, $U_c(r_0) = \frac{Z_iZ_je^2}{r_0}$ denotes the maximum of the Coulomb potential between the ith and jth particles located at a distance r_0 from each other (Prialnik, 2000). In this article, Kelvin (K) is the temperature unit. Finally, we reach

$$T \geq \frac{2Z_iZ_je^2}{3K_Br_0} \simeq 1 \cdot 1 \times 10^{10}\frac{Z_iZ_j}{r_0}, \tag{2}$$

for the temperature required to overcome the Coulomb barrier. Therefore, NF happens whenever the temperature of the primary gas is comparable to Eq. 2, which clearly shows that, for the heavier nuclei, NF happens at higher temperatures. On the contrary, for the temperature of gas with mass M and radius R, we have (Prialnik, 2000)

$$\mathcal{T} \approx 4 \times 10^6\left(\frac{M}{M_\odot}\right)\left(\frac{R_\odot}{R}\right), \tag{3}$$

TABLE 1 Some bounds on the GUP parameter β_0.

Measurement/experiment	β_0	Refs.
Modified mass-temperature relation	10^{78}	Scardigli and Casadio (2015)
Light deflection	10^{78}	Scardigli and Casadio (2015)
Pulsar PRS B 1913 + 16 data	10^{71}	Scardigli and Casadio (2015)
Solar system data	10^{69}	Scardigli and Casadio (2015)
GW150914	10^{60}	Feng et al. (2017)
Dresselhaus interaction	10^{51}	Aghababaei et al. (2021)
Landau levels	10^{50}	Das and Vagenas (2008)
Sagnac effect	10^{49}	Feleppa et al. (2021)
Rashba effect	10^{46}	Aghababaei et al. (2021)

where $M_\odot$ and $R_\odot$ are the Sun mass and radius, respectively. Clearly, $\mathcal{T}$ and T are far from each other, meaning that NF cannot cause star burning (Prialnik, 2000). Therefore, NF occurs if a process reduces the required temperature (2). In fact, we need a process that decreases Eq. 2 to the values comparable to Eq. 3. Quantum tunneling lets particles pass through the Coulomb barrier, which finally triggers star burning, meaning that quantum tunneling allows NF to occur at temperatures lower than T (Prialnik, 2000). Indeed, if the distance between particles (r_0) becomes of the order of their de Broglie wavelength ($r_0 \simeq \frac{\hbar}{p} \equiv \lambda_Q$ where Q implies that we are in the purely quantum mechanical regime), then quantum tunneling happens and simple calculations lead to (Prialnik, 2000)

$$T \geq \frac{2Z_iZ_je^2}{3K_B\lambda_Q} \simeq 9\cdot 6\times 10^6 Z_i^2 Z_j^2 \left(\frac{m}{\frac{1}{2}}\right) \equiv \mathrm{T}, \tag{4}$$

instead of Eq. 2 for the temperature required to launch star burning. λ_Q can also be obtained by solving $\frac{p^2}{2m} = U_c(r_0)|_{r_0=\lambda_Q}$ which gives (Prialnik, 2000)

$$\lambda_Q = \frac{\hbar^2}{2mZ_iZ_je^2}, \tag{5}$$

meaning that quantum tunneling provides a platform for NF in stars (Prialnik, 2000). As an example, for hydrogen atoms, one can see that quantum tunneling leads to $\mathrm{T} \simeq 9\cdot 6\times 10^6\ K$ (comparable to (3)) as the Gamow temperature at which NF is underway. Based on the above argument, it is expected that any change in p affects λ_Q and, thus, these results.

It is also useful to mention here that the quantum tunneling theory allows the above process because the tunneling probability is not zero. Indeed, quantum tunneling is also the backbone of Gamow's theory of the α decay process (Gamow, 1928). Relying on the inversion of the Gamow formula for α decay, which gives the transmission coefficient, a method has also been proposed for studying the inverse problem of Hawking radiation (Völkel et al., 2019).

The backbone of quantum mechanics is the Heisenberg uncertainty principle (HUP),

$$\Delta x \Delta p \geq \frac{\hbar}{2}, \tag{6}$$

where x and p are ordinary canonical coordinates satisfying $[x_i, p_j] = i\hbar\delta_{ij}$. It has been proposed that, in quantum gravity scenarios, the HUP is modified such that (Kempf et al., 1995; Kempf, 1996)

$$\Delta X \Delta P \geq \frac{\hbar}{2}\left(1 + \frac{\beta_0 l_p^2}{\hbar^2}(\Delta P)^2\right), \tag{7}$$

called the GUP, where l_p denotes the Planck length and β_0 is the dimensionless GUP parameter. X and P are called generalized coordinates, and we work in a framework in which $X_i = x_i$, and up to the first order, we have $P_i = p_i(1 + \frac{\beta_0 l_p^2}{3\hbar^2}p^2)$ and $[X_i, P_j] = i\hbar(1 + \frac{\beta_0 l_p^2}{\hbar^2}P^2)\delta_{ij}$ (Das and Vagenas, 2008; Motlaq and Pedram, 2014). Moreover, the GUP implies that there is a non-zero minimum length $(\Delta X)_{\min} = \sqrt{\beta_0}\, l_p$. Indeed, the existence of a non-zero minimum length also emerges even when the gravitational regime is Newtonian (Mead, 1964), a common result with quantum gravity scenarios (Hossenfelder, 2013). More studies on quantum gravity can be traced to earlier studies (Lake et al., 2019; Lake et al., 2020; Lake, 2021; Lake, 2022). There have been various attempts to estimate the maximum possible upper bound on β_0 (Zhu et al., 2009; Chemissany et al., 2011; Das and Mann, 2011; Sprenger et al., 2011; Pikovski et al., 2012; Husain et al., 2013; Ghosh, 2014; Jalalzadeh et al., 2014; Scardigli and Casadio, 2015; Bosso et al., 2017; Feng et al., 2017; Gecim and Sucu, 2017; Bushev et al., 2019; Luciano and Petruzziello, 2019; Park, 2020; Aghababaei et al., 2021; Feleppa et al., 2021; Mohammadi Sabet et al., 2021), and among them, it seems that the maximum estimation for the upper bound is of the order of 10^{78} (Scardigli and Casadio, 2015). The implications of GUP on stellar evolution (Moradpour et al., 2019; Shababi and Ourabah, 2020) and the thermodynamics of various gases (Chang et al., 2002; Fityo, 2008; Wang et al., 2010; Hossenfelder, 2013; Motlaq and Pedram, 2014; Moradpour et al., 2021) have also been studied.

Indeed, the existence of a minimal length leads to the emergence of the GUP (Hossenfelder, 2013), and it affects thermodynamics (Chang et al., 2002; Fityo, 2008; Wang et al., 2010; Hossenfelder, 2013; Motlaq and Pedram, 2014; Moradpour et al., 2021) and quantum mechanics (Kempf et al., 1995; Kempf, 1996), as P can be expanded as a function of p. This letter deals with the GUP effects on star burning facilitated by quantum tunneling. Loosely speaking, we investigate the effects of a minimal length on T (the Gamow temperature).

GUP corrections to the tunneling temperature

To proceed further and in the presence of the quantum features of gravity, we introduce the generalized de Broglie wavelength as

$$\lambda_{GUP} \equiv \frac{\hbar}{P}. \tag{8}$$

It is obvious that, as $\beta_0 \to 0$, one obtains $P \to p$ and thus $\lambda_{GUP} \to \lambda_Q$, which is the quantum mechanical result. Indeed, up to first order in β_0, we have $\lambda_{GUP} = \lambda_Q (1 - \frac{\beta_0 l_p^2}{3\lambda_Q^2})$, and the thermal energy per particle with temperature T is (Motlaq and Pedram, 2014)

$$\langle K\rangle = \langle\frac{P^2}{2m}\rangle = \frac{3}{2}K_B T - 3\frac{\beta_0 l_p^2}{\hbar^2} m K_B^2 T^2. \tag{9}$$

Mathematically, one should find the corresponding de Broglie wavelength by solving the following equation:

$$\frac{P^2}{2m} = U_c(r_0)\bigg|_{r_0=\lambda_{GUP}}. \tag{10}$$

Inserting the result into

$$\frac{3}{2}K_B T - 3\frac{\beta_0 l_p^2}{\hbar^2} m K_B^2 T^2 \geq U_c(r_0)\bigg|_{r_0=\lambda_{GUP}}, \tag{11}$$

one can finally find the GUP corrected version of Eq. 4.

Now, inserting λ_{GUP} into Eq. 10 and then combining the results with Eq. 11, we find

$$T^{\pm}_{GUP} = \frac{\hbar^2\left(1 \pm \sqrt{1 - 8\beta_0 l_p^2 m K_B \mathrm{T}/\hbar^2}\right)}{4\beta_0 K_B l_p^2 m}. \tag{12}$$

in which Eq. 4 has been used for simplification. To estimate the magnitude of $l_p^2 m K_B \mathrm{T}/\hbar^2$, we consider the hydrogen atom for which $m \sim 10^{-27}\, kg$. Now, since $l_p \sim 10^{-35}\, m$, $K_B \sim 10^{-23}\, \frac{m^2 kg}{s^2 K}$, $\hbar \sim 10^{-34}\, \frac{m^2 kg}{s}$, and $\mathrm{T} \sim 10^6\, K$, one easily finds $l_p^2 m K_B \mathrm{T}/\hbar^2 \sim 10^{-46}$. Moreover, because the effects of the GUP in the quantum mechanical regime are small (Hossenfelder, 2013), a reasonable basic assumption could be that $\beta_0 l_p^2 m K_B \mathrm{T}/\hbar^2 \ll 1$. Indeed, if $\beta_0 \ll 10^{46}$, then we always have $\beta_0 l_p^2 m K_B \mathrm{T}/\hbar^2 \ll 1$ meaning that *i)* we can Taylor expand our results and *ii)* 10^{46} is an upper bound for β_0, which is comparable to those found in previous works (Das and Vagenas, 2008; Scardigli and Casadio, 2015; Feng et al., 2017; Aghababaei et al., 2021; Feleppa et al., 2021) summarized in Table 1.

Expanding the above solutions (12) and bearing in mind that the true solution should recover T at $\beta = 0$, one can easily find that T^-_{GUP} is the proper solution leading to

$$T^-_{GUP} = \mathrm{T}\left(1 + 2\beta_0\left(\frac{l_p^2 m K_B \mathrm{T}}{\hbar^2}\right)\right). \tag{13}$$

up to first order in β_0. Hence, because it seems that β_0 is positive (Das and Vagenas, 2008; Scardigli and Casadio, 2015; Feng et al., 2017; Aghababaei et al., 2021; Feleppa et al., 2021), one can conclude that $\mathrm{T} < T^-_{GUP}$.

Conclusion

Motivated by the GUP proposal and the vital role of the HUP in quantum mechanics and, thus, the quantum tunneling process that facilitates star burning, we studied the effects of the GUP on the Gamow temperature. In order to determine this, the GUP modification to the de Broglie wavelength was addressed, which finally helped us to find the GUP correction to the Gamow temperature and also estimate an upper bound for β_0 (10^{46}), which agrees well with those found in previous works (Das and Vagenas, 2008; Scardigli and Casadio, 2015; Feng et al., 2017; Aghababaei et al., 2021; Feleppa et al., 2021).

Finally, based on the obtained results, it may be expected that the GUP also affects the transmission coefficients (Gamow's formula) (Gamow, 1928; Hossenfelder, 2013; Völkel et al., 2019), meaning that the method of Völkel et al. (2019) will also be affected. This is an interesting topic for future study because Hawking radiation is a fascinating issue in black hole physics (Wald, 2001).

Data availability statement

The original contributions presented in the study are included in the article/Supplementary material. Further inquiries can be directed to the corresponding author.

Author contributions

All authors listed have made a substantial, direct, and intellectual contribution to the work and approved it for publication.

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Conflict of interest

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

Publisher's note

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