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Anisotropic cloud string cosmological model with five-dimensional kaluza-klein space-time

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Anisotropic cloud string cosmological models has been investigated in the context of five dimensional Kaluza- Klein space time. In this paper the energy momentum tensor is generated by rest energy density and tension density of the string with particle density attached to them. To obtained the exact solutions of the Einstein field equations we assumed a scale factor $a(t) = e^{\frac{1}{\beta} \sqrt{2\beta t + c}}$ where β and c are positive constant, which yields a variable deceleration parameter (DP) $q = -\frac{a\ddot{a}}{\dot{a}^2} = \beta H + \alpha$. The physical and geometrical behavior of the models is also discussed in detail.

KEYWORDS

kaluza-klein space time, string tension density, particle density, variable deceleration parameter, cloud string

Introduction

Now a day, string cosmology has attracted lots of attention, because of its significant role in the study of the origin and evaluation of the Universe before the creation of particles. It is a fascinating field for cosmologists to study and discover the mysterious phenomena that have yet to be observed and explore the unseen information of our Universe. As a result, cosmologists are extremely interested in learning more about the past, present, and future evolution of the Universe. But, as of now, we lack strong evidence to make a conclusive statement about its origin and evolution. So, further investigation is required to discover the mysterious phenomena of the entire universe. [Stachel \(1980\)](#) and [Letelier \(1983\)](#) was started the study of string in the context of general relativity. Because the string is extremely appropriate in describing the early phase of the evolution of our Universe. Many eminent authors are interested to work in the field of cosmic strings within the context of general relativity ([Kibble, 1976; 1983](#)), and it is thought that strings cause density perturbations that lead to the formation of massive scale structures in the Universe ([Zel'dovich et al., 1974; Zel'dovich, 1980](#)).

Strings are stable topological structures that formed during the early universe phase transition due to a drop in temperature below certain critical temperatures. Observations of our universe using contemporary technical tools also suggest that in the early stages of our Universe, there existed a massive scale network of strings. Geometric strings and massive strings are two types of strings that contain stress-energy. The presence of strings

is responsible for the universe is anisotropy; nevertheless, strings are no longer visible. These strings are not damaging the cosmological models, instead, they can lead to a variety of fascinating astrophysical results. Strings can also be used to describe the nature and essential arrangement of the early Universe. String theory describes the early stage of evolution of the Universe in terms of (vibrating) strings instead of particles and gives us a single theoretical structure in which all matter and forces are unified. Because strings play such an important role in describing the evolution of the early stages of our Universe. Several authors have recently focused their attention on string cosmological models. According to GUT (grand unified theories), after the big-bang explosion, there is a symmetry flouting during the phase transition of the early stages of the Universe, and these strings appear when the cosmic temperature descends below certain critical temperatures (Everett, 1981; A. Vilenkin, 1981a,b).

The study of Kaluza-Klein (KK) (Kaluza, 1921; Klein, 1926) theory is a model that sought to integrate Einstein's theory of gravity and Maxwell's electromagnetism theory, which revolves around the concept of the fifth dimension, beyond the four dimensions of space and time. The study of KK cosmology became popular because of its illustrious history and some interesting features to revolutionize the study of the universe. This allows the universe to expand early and study its evolution and behavior, adding extra dimensions to Einstein's field equations as seen nowadays. It is becoming very fascinating to study string cosmology in higher-dimensional space-time in the context of general relativity. Several researchers like Chodos and Detweiler (1980), Appelquist et al. (1987) have investigated a homogeneous higher dimensional cosmological model with massive string in general relativity. Naidu et al. (2013) and Reddy and Lakshmi (2014) have explored the possibility of higher dimensional space-time in the field of cosmology. Jain and Shyamsunder (2015), Khadekar and Patki (2008), Sharif and Khanum (2011), Venkateswarlu and Kumar (2006), Khadekar and Vaishali (2010), Samanta and Dhal (2013), Raut et al. (2015) have discussed five-dimensional KK cosmological models with different matters. Adhav et al. (2008) and Yilmaz (2006) have investigated KK cosmic solutions are examined in higher dimensions for quark matter along with string cloud and domain walls in the context of general relativity. Reddy et al. (2007) and Reddy and Naidu (2007) have investigated a higher-dimensional string cosmological model in different theories of gravitation. Khadekar et al. (2008) investigated string dust cosmological models with particles attached to them by considering three different forms of variable Λ in the context of five-dimensional KK space-time. Khadekar et al. (2007) studied a string cosmological model with bulk viscosity in higher dimensional space-time. Nimkar (2017) discussed String cosmological model with the electromagnetic field in general relativity. Pawar et al. (2018) discussed KK string cosmological model in f (R, T) theory of gravity. Krori et al.

(1994) have investigated a higher dimensional Bianchi type-I cosmological model with string and they found that matter and string coexist throughout the evolution of the universe. Mohanty et al. (2002), Sahoo et al. (2017) have investigated the anisotropic cosmological model universe in Bianchi type-I space-time. Venkateswarlu and Pavankuma (2005) have investigated a string cosmological model in higher dimensional space-time with scale covariant theory of gravitation. Rahaman et al. (2003) obtained the exact solutions of the field equations for the higher dimensional space time in the framework of Lyra manifold when the source of gravitational field is a massive string. Kandalkar et al. (2012) constructed Bianchi type-III string cosmological models in presence of magnetic field in the context of general relativity and obtained exact solution of the field equations by using the condition that the sum of the energy density and tension density is zero. Mohanty and Samanta (2009) have investigated a five dimensional axially symmetry string cosmological models in general theory of relativity in presence of bulk viscous fluid. Samanta and Debata (2011) constructed Bianchi type-I five dimensional string cosmological model in the framework of Lyra manifold. Choudhury (2017), Tripathi et al. (2021), Dubey et al. (2018), Tiwari et al. (2019), Ram and Verma (2019), Mollah et al. (2019) and Singh and Baro (2020) are some of the eminent authors who studied different string cosmological models in higher dimensional space time in the contexts of the general relativity. Recently Mollah and Singh (2021) and Baro et al. (2021) constructed higher dimensional Bianchi type-III string cosmological in the framework of general relativity.

In this article, we discuss anisotropic cloud string cosmological models with particles attached to them in the five-dimensional KK space-time. This article is prepared as follows: Sec.2 is devoted to the metric and Einstein's field equations. In Sec. 3 we presented the solutions of the field equations. The geometrical and physical interpretation of the results is given in sec. 4. In the last section, we give the conclusions.

Metric and field equations

The five-dimensional KK metric is given by

$$ds^2 = dt^2 - A^2(dx^2 + dy^2 + dz^2) - B^2d\phi^2 \quad (1)$$

where A and B are functions of cosmic time t only and the fifth coordinate ϕ is taken to be extended space like coordinate.

Einstein's field equation is given by

$$R_{ij} - \frac{1}{2}Rg_{ij} = -T_{ij} \quad (2)$$

where R_{ij} is the Ricci tensor R is the Ricci scalar g_{ij} is the metric tensor and T_{ij} is the energy-momentum tensor for a cloud string respectively.

Thus the energy-momentum tensor for a cloud string is given by

$$T_{ij} = \rho v_i v_j - \lambda x_i x_j \tag{3}$$

where v_i and x_i satisfy the conditions

$$v^i v_i = -x^i x_i = -1, v^i x_i = 0 \tag{4}$$

Here ρ is the rest energy density for a cloud of strings with particles attached to them. $\rho = \rho_p + \lambda$, ρ_p being the rest energy density of particles attached to the strings and λ the tension density of the strings. Here p and ρ are a function of cosmic time t only. x_i is a unit space-like vector instead of the direction of strings so that $x^2 = x^3 = x^4 = 0$ and $x^1 \neq 0$.

The energy-momentum tensor T_{ij} in co-moving coordinates for could string is given by

$$T_0^0 = \rho, T_1^1 = \lambda, T_2^2 = T_3^3 = T_4^4 = 0 \tag{5}$$

The field Eq 2 for the line-element (1) with the help of Eqs. 3–5 can be written explicitly as

$$3 \frac{\dot{A}^2}{A^2} + 3 \frac{\dot{A}\dot{B}}{AB} = \rho \tag{6}$$

$$2 \frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + 2 \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}^2}{A^2} = \lambda \tag{7}$$

$$2 \frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + 2 \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}^2}{A^2} = 0 \tag{8}$$

$$3 \frac{\ddot{A}}{A} + 3 \frac{\dot{A}^2}{A^2} = 0 \tag{9}$$

An over dot indicates a derivative with respect to cosmic time t .

The spatial volume for the model (1) is given by

$$V = a^4 = A^3 B \tag{10}$$

The generalized signify Hubble parameter for Kaluza-Klein space time is given by

$$H = \frac{1}{4} \left(3 \frac{\dot{A}}{A} + \frac{\dot{B}}{B} \right) \tag{11}$$

The directional Hubble parameters H_x, H_y, H_z and H_ϕ in the direction of x, y, z and ϕ respectively for the Kaluza-Klein metric are

$$H_x = H_y = H_z = \frac{\dot{A}}{A}$$

and

$$H_\phi = \frac{\dot{B}}{B}$$

The scalar expansion θ and shear scalar σ^2 are given by

$$\theta = 4H = \frac{3\dot{A}}{A} + \frac{\dot{B}}{B} \tag{12}$$

$$\sigma^2 = \frac{1}{2} \left[\sum_{i=1}^4 H_i^2 - 4H^2 \right] = \frac{4}{2} \Delta H^2 \tag{13}$$

The expansion of signify anisotropic parameter (Δ) is given by

$$\Delta = \frac{1}{4} \sum_{i=1}^4 \left(\frac{\Delta H_i}{H} \right)^2 \tag{14}$$

where $\Delta H_i = H_i - H$ and $H_i = 1, 2, 3, 4$ represent the directional Hubble parameters in H_x, H_y, H_z and H_ϕ directions respectively.

Solutions of the field equations

The set of linearly independent field Eqs 6–9 with five unknown A, B, ρ, λ and ρ_p .

To solve the system of equations we consider deceleration parameter (q) as a linear function of hubble parameter (Tiwari et al., 2015; Tiwari et al., 2018; Sharma et al., 2019):

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = \beta H + \alpha \tag{15}$$

Here α and β arbitrary constants.

For $\alpha = -1$ in Eq 15

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = -1 + \beta H$$

which yields the following differential equation

$$\frac{a\ddot{a}}{\dot{a}^2} + \beta \frac{\dot{a}}{a} - 1 = 0 \tag{16}$$

After integration Eq 16 we get

$$a(t) = e^{\frac{1}{\beta} \sqrt{2\beta t + c}} \tag{17}$$

where c is an integrating constant.

Collins et al. (1980) have exposed that for a spatially homogeneous metric, a large class of solutions that can satisfy the condition $\frac{\sigma}{\theta}$ is constant, where θ is the expansion in the model. So we assume the shear scalar σ is proportional to the expansion scalar θ . This gives the relation between scale factor A and B as,

$$A = B^n \tag{18}$$

where n is constant and $n \neq 1$.

From Eqs. 10, 17, 18 the metric component are

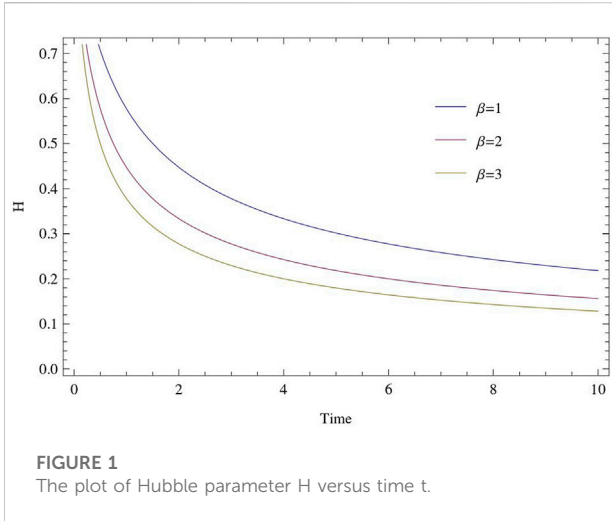
$$A(t) = e^{\frac{4n}{\beta(3n+1)} \sqrt{2\beta t + c}} \tag{19}$$

and

$$B(t) = e^{\frac{4}{\beta(3n+1)} \sqrt{2\beta t + c}} \tag{20}$$

Therefore the metric (1) reduce to

$$ds^2 = dt^2 - e^{\frac{4n}{\beta(3n+1)} \sqrt{2\beta t + c}} (dx^2 + dy^2 + dz^2) - B(t) e^{-\frac{4}{\beta(3n+1)} \sqrt{2\beta t + c}} d\phi^2 \tag{21}$$



Eq. 21 represents Five-Dimensional KK Cosmological Models with variable deceleration parameter.

Physical properties of the model

We have obtained the cosmological model (21), the directional Hubble parameters H_x, H_y, H_z and H_ϕ , the physical quantities such as Hubble parameter H, spatial volume V, signify anisotropy parameter Δ , expansion scalar θ , shear scalar σ^2 , energy density ρ , particles density ρ_p and tension density of the string λ are obtained as follows:

The directional Hubble parameters H_x, H_y, H_z and H_ϕ are

$$H_x = H_y = H_z = \frac{4n}{(3n+1)\sqrt{2\beta t+c}}$$

and

$$H_\phi = \frac{4}{(3n+1)\sqrt{2\beta t+c}}$$

For Kaluza-Klein space-time, the signify Hubble parameter(H) is given by

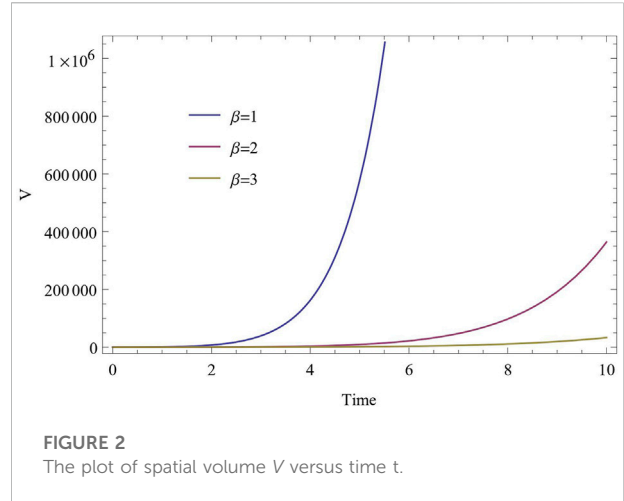
$$H = \frac{1}{\sqrt{2\beta t+c}} \tag{22}$$

The spatial volume(V) is given by

$$V = e^{\frac{4}{3}\sqrt{2\beta t+c}} \tag{23}$$

The expansion of signify anisotropic parameter (Δ) is given by

$$\therefore \Delta = \frac{3(n-1)^2}{(3n+1)^2} = \text{constant} (\neq 0 \text{ where } n \neq 1) \tag{24}$$



The expansion scalar (θ) is given by

$$\theta = \frac{4}{\sqrt{2\beta t+c}} \tag{25}$$

The shear scalar (σ^2) is given by

$$\sigma^2 = \frac{3(n-1)^2}{2(3n+1)^2(2\beta t+c)^2} \tag{26}$$

From Eqs 25, 26 we obtain

$$\lim_{t \rightarrow \infty} \frac{\sigma^2}{\theta^2} = \frac{3(n-1)^2}{8(3n+1)^2} = \text{constant} (\neq 0 \text{ where } n \neq 1) \tag{27}$$

The energy density ρ is given by

$$\rho = \frac{48n(n+1)}{(3n+1)^2(2\beta t+c)} \tag{28}$$

The tension density λ for the string is given by

$$\lambda = \frac{16(3n^2+2n+1)}{(3n+1)^2(2\beta t+c)} - \frac{4\beta(2n+1)}{(3n+1)(2\beta t+c)^{\frac{3}{2}}} \tag{29}$$

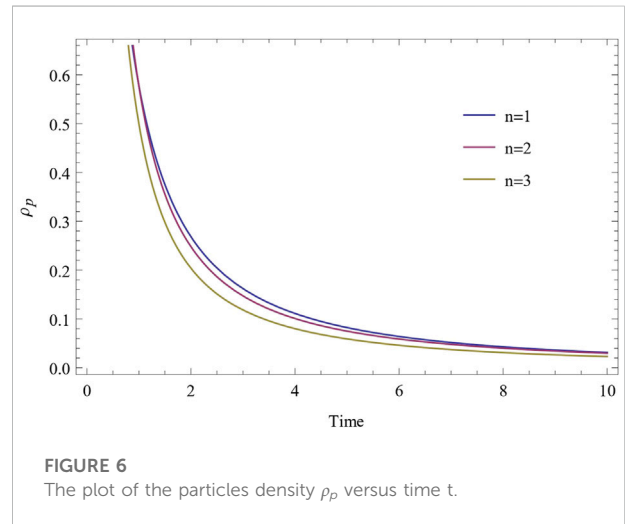
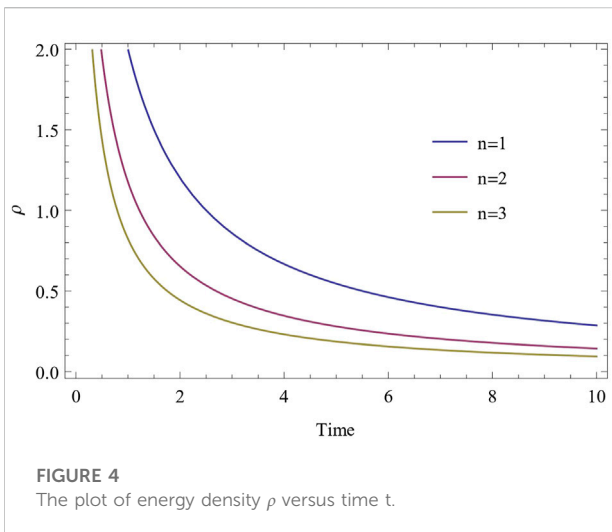
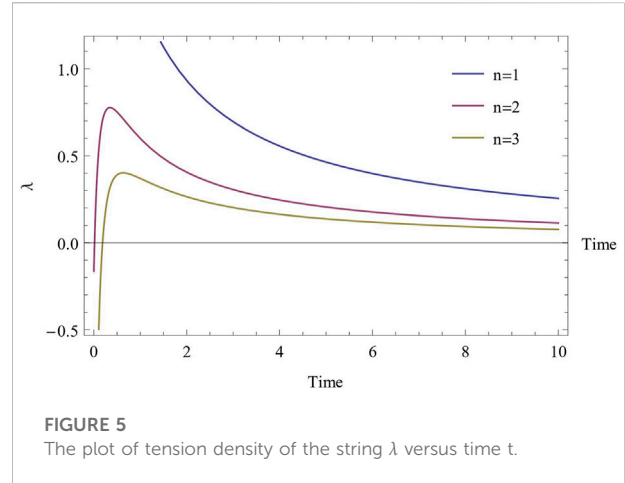
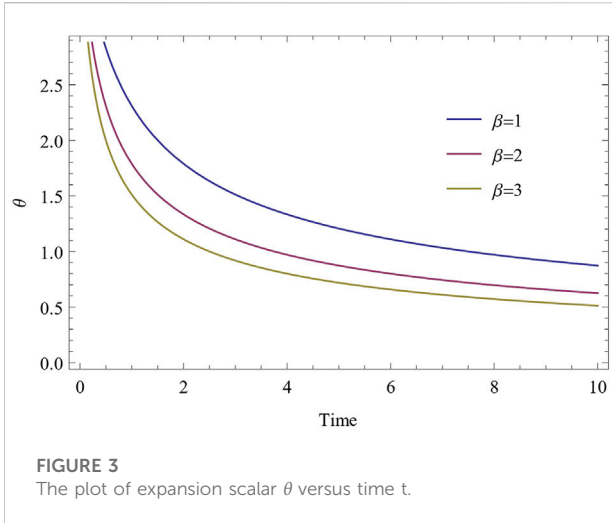
The particles density ρ_p is obtained by

$$\rho_p = \frac{16(n-1)}{(3n+1)^2(2\beta t+c)} + \frac{4\beta(2n+1)}{(3n+1)(2\beta t+c)^{\frac{3}{2}}} \tag{30}$$

The deceleration parameter (q) is given by

$$q = -1 + \frac{\beta}{\sqrt{2\beta t+c}} \tag{31}$$

It can be seen that from Eqs. 22, 25, both the Hubble parameter (H) and expansion scalar (θ) is a positive and decreasing function of cosmic time t. The Hubble parameter (H) and expansion scalar (θ) tend to infinity as $t \rightarrow 0$ and tend to a finite value as $t \rightarrow \infty$ are shown in Figures 1, 3 which are agrees with established theories. Figure 2



shows variation of spatial volume w.r.t. time. We have also noticed that $\frac{dH}{dt}$ is negative which indicates that our universe is expanding rapidly.

From Eq. 23 shows that at $t = 0$ the spatial volume is finite and thereafter increases continuously when cosmic time t is increasing. Figure 3 depicts the nature of variations of V versus t .

From Eq. 31 it is observed that the deceleration parameter $q > 0$ for $t < \frac{\beta^2 - c}{2\beta}$ which indicates that our model universe is a decelerating phase. It is also observed that the deceleration parameter $q < 0$ for $t > \frac{\beta^2 - c}{2\beta}$ which indicates that our model universe is a accelerating phase, which agrees with present day's observations (Riess et al., 1998; Perlmutter et al., 1999).

The expansion of signify anisotropic parameter $\Delta \neq 0$ (constant) for $n \neq 1$ and $\Delta = 0$ for $n = 1$. We also observed from Eq. 27 that $\lim_{t \rightarrow \infty} \frac{\sigma^2}{\theta^2} \neq 0$ (constant) for $n \neq 1$ and

$\lim_{t \rightarrow \infty} \frac{\sigma^2}{\theta^2} = 0$ for $n = 1$, which means that our model is anisotropic when $n \neq 1$ and it is isotropic when $n = 1$.

From Eq. 28 it is seen that the expansion for rest energy density ρ is a decreasing function of cosmic time t . This shows that the rest energy density is positive and satisfies the condition of energy $\rho \geq 0$ for all $n \geq -1$. Also from Figure 4, it is seen that the rest energy density ρ is decreasing when time t is increasing and initially $\rho \rightarrow \infty$ when $t \rightarrow 0$, thus has an initial singularity.

It is seen from Figures 5, 6 that both the string tension density λ and particle density ρ_p are positive, decreasing function of cosmic time t , and become zero as $t \rightarrow \infty$. Also, we observed that initially both the string tension density λ and particle density ρ_p tend to infinity when t tends to zero which suggests that the universe began with big bang and as time progresses, both the string tension density λ and particle density ρ_p decreases with the expansion of the universe.

Conclusion

In the present article, we have investigated the behavior of anisotropic cloud string cosmological models in five-dimensional KK space-time to describe the mysterious phenomena of the entire universe. To get the exact solutions of the Einstein field equations, we assumed a scale factor $a(t) = e^{\frac{1}{2}\sqrt{2\beta t + c}}$ where β and c are positive constant, which yields a variable deceleration parameter (DP) $q = -\frac{a\ddot{a}}{\dot{a}^2} = \beta H + \alpha$. Our model depicts to have an anisotropic phase for $n \neq 1$ throughout the evolution of the universe as it does not depend on the cosmic time t . According to present day's observations, there is a disparity in measuring microwave intensity from different directions of the sky. This motivated us to investigate the universe using the anisotropic five dimensional Kaluza-Klein space-time in order to better describe our universe. Several cosmological observations such as Cosmic Background Explorer (COBE) and the Wilkinson Microwave Anisotropic Probe (WMAP) are also evidence that we live in a globally anisotropic universe. In order to produce any significant amount of shear in recent periods, one must cause anisotropy in space-time and WMAP, where they found small anisotropy in microwave background radiation. Also, the models represents an exponentially expanding Universe that begins with the big bang at cosmic time $t = 0$ with finite volume and extends at an accelerating rate. The deceleration parameter "q" of the universe has certainly changed its sign from positive to negative (signature flipping), which indicates that the universe has decelerated expansion in the past and accelerated expansion at present day's observations (Amendola, 2003; Padmanabhan and Choudhury, 2003; Kandalkar and Samdurkar, 2015). Our model satisfies the condition of energy density $\rho \geq 0$ and $p_p \geq 0$. The particle density and string tension density are equivalent, but the string tension density vanishes faster than the particle density, so our model reflects a matter-dominated

universe that accords with current observational data in the late time period.

Data availability statement

The original contributions presented in the study are included in the article, further inquiries can be directed to the corresponding author.

Author contributions

All of the authors listed have contributed a significant, direct, and intellectual contribution to the work and have given their permission for it to be published.

Conflict of interest

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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