



# Coupling Between Alfvén Wave and Kelvin–Helmholtz Waves in the Low Latitude Boundary Layer

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The Kelvin–Helmholtz (KH) instability of magnetohydrodynamic surface waves at the low latitude boundary layer is examined using both an eigenfrequency analysis and a time-dependent wave simulation. The analysis includes the effects of sheared flow and Alfvén velocity gradient. When the magnetosheath flows are perpendicular to the ambient magnetic field direction, unstable KH waves that propagate obliquely to the sheared flow direction occur at the sheared flow surface when the Alfvén Mach number is higher than an instability threshold. Including a shear transition layer between the magnetosphere and magnetosheath leads to secondary KH waves (driven by the sheared flow) that are coupled to the resonant surface Alfvén wave. There are remarkable differences between the primary and the secondary KH waves, including wave frequency, the growth rate, and the ratio between the transverse and compressional components. The secondary KH wave energy is concentrated near the shear Alfvén wave frequency at the magnetosheath with a lower frequency than the primary KH waves. Although the growth rate of the secondary KH waves is lower than the primary KH waves, the threshold condition is lower, so it is expected that these types of waves will dominate at a lower Mach number. Because the transverse component of the secondary KH waves is stronger than that of the primary KH waves, more efficient wave energy transfer from the boundary layer to the inner magnetosphere is also predicted.

**Keywords:** Kelvin–Helmholtz instability, Alfvén wave, boundary layer, magnetopause, mode conversion, wave coupling

## 1 INTRODUCTION

The Kelvin–Helmholtz (KH) instability has been widely investigated in the Earth’s magnetosphere (Johnson et al., 2014). Unstable KH waves generally occur at the interface between two fluids having different velocities and are fundamentally important for understanding dynamics within the boundary layer that develops between the flows. These waves can affect the exchange of mass, momentum, and energy across those boundaries (e.g., Miura, 1984; Thomas and Winske, 1993; Otto and Fairfield, 2000; Nykyri and Otto, 2001; Matsumoto and Hoshino, 2006; Cowee et al., 2010; Hwang et al., 2011; Nakamura et al., 2011; Moore et al., 2016; Nykyri et al., 2017; Johnson et al., 2021). Mass transport due to KH instability can result from diffusion through thin boundaries created by the instability (e.g., Nakamura et al., 2017) and/or as the result of secondary reconnection (e.g., Otto and Nykyri, 2003; Ma et al., 2017) which results in more effective transport (Ma et al., 2019). Cross-scale energy transport associated with the KH instability may result from the generation

of plasma waves leading to both ion and electron heating (Johnson and Cheng, 2001; Chaston et al., 2007; Moore et al., 2017; Nykyri et al., 2021a; Nykyri et al., 2021b; Delamere et al., 2021). The KH waves are also critical to the interaction between the solar wind and other planetary magnetospheres (McComas and Bagenal, 2008; Delamere and Bagenal, 2010; Delamere et al., 2021).

KH waves are surface waves because they are localized near the interface and exponentially decay away from the interface (e.g., Southwood, 1968; Pu and Kivelson, 1983). However, because the wave number is relatively small, the wave energy can still penetrate into the plasma sheet and/or the inner magnetosphere (e.g., Pu and Kivelson, 1983) and play a role in the generation of geomagnetic pulsations and mode conversion to the shear Alfvén waves (e.g., Chen and Hasegawa, 1974; Engebretson et al., 1998).

The magnetopause boundary is often assumed for simplicity to have zero thickness (Pu and Kivelson, 1983; Mills and Wright, 1999; Turkakin et al., 2013), and this assumption is valid for waves with wavelengths longer than the thickness of the boundary layer. When the shear velocity and the Alfvén speed jump at the zero-thickness interface, the linear dispersion relation of KH waves in a slab geometry for an incompressible plasma can be derived as follows (Chandrasekhar, 1961):

$$\omega = \frac{\mathbf{k} \cdot (\rho_{msh} \mathbf{V}_{msh} + \rho_{msp} \mathbf{V}_{msp})}{\rho_{msp} + \rho_{msh}} \pm i \sqrt{\frac{\rho^*}{\rho_{msh} + \rho_{msp}} \left( [\mathbf{k} \cdot (\mathbf{V}_{msh} - \mathbf{V}_{msp})]^2 - \frac{(\mathbf{k} \cdot \mathbf{B}_{msh})^2 + (\mathbf{k} \cdot \mathbf{B}_{msp})^2}{\mu_0 \rho^*} \right)}, \quad (1)$$

where  $\omega$  and  $\mathbf{k}$  are a wave frequency and vector, respectively,  $\mathbf{V}$  and  $\mathbf{B}$  are shear flow velocity and magnetic field,  $\rho$  and  $\rho^* = \rho_{msh}\rho_{msp}/(\rho_{msh} + \rho_{msp})$  are a mass density and a mean mass density, respectively,  $\mu_0$  is the magnetic permeability of free space, and msp(msh) denotes the magnetosphere (magnetosheath). When  $\mathbf{B}_{msp} = \mathbf{B}_{msh}$  and  $\rho_{msp} = \rho_{msh}$ , the KH wave frequency in Equation 1 is reduced to  $\omega = \omega_{KH0} = \frac{1}{2} \mathbf{k} \cdot \mathbf{V}_{msh}$ . In Equation 1, the KH waves become unstable when

$$[\mathbf{k} \cdot (\mathbf{V}_{msh} - \mathbf{V}_{msp})]^2 > \left[ (\mathbf{k} \cdot \mathbf{B}_{msh})^2 + (\mathbf{k} \cdot \mathbf{B}_{msp})^2 \right] / \mu_0 \rho^* \quad (2)$$

is satisfied; and the stability threshold condition (2) may be used to determine a critical Alfvén Mach number ( $M_{As}$ ) above which the KH wave is unstable.

In addition to the velocity transition at the magnetopause boundary, there is also a large gradient in the Alfvén velocity, which is typically wider in extent than the velocity shear layer (Paschmann et al., 1993). When an Alfvén velocity ( $V_A$ ) transition layer is included between the magnetosheath and magnetosphere, it can modify the KH wave properties. Strong coupling between the Alfvén surface wave and KH surface wave can result when the frequencies are comparable. This interaction between the two surface waves can lead to instability at a slower

flow velocity. This new instability has been referred to as the resonant flow instability (RFI) as it results when Doppler-shifted compressional waves originating at the velocity interface have approximately the same frequency as the Alfvén resonance frequency (Taroyan and Erdélyi, 2003). The RFI includes a negative absorption of the magnetosonic waves, and it has been investigated for the solar corona (Tirry et al., 1998; Andries et al., 2000; Andries and Goossens, 2001; Taroyan and Ruderman, 2011; Antolin and Van Doorselaere, 2019), magnetopause (Ruderman and Wright, 1998; Taroyan and Erdélyi, 2002, 2003), and magnetotail (Turkakin et al., 2014), respectively. While these works focused on shear in the velocity along the magnetic field direction, a similar instability can also result in velocity shear across the magnetic field or for discontinuous changes in the magnetic field direction at velocity interfaces. These modes can generally be referred to as secondary KH instabilities and are characterized by instability at a slower flow speed than the primary KH instability with growth occurring in a narrow range of propagation angle or Mach number (e.g., González and Gratton, 1994; Taroyan and Erdélyi, 2002; Turkakin et al., 2013). Turkakin et al. (2013) examined the primary and the secondary KH waves in the magnetopause and magnetotail when the magnetic fields in the magnetosheath and magnetosphere are perpendicular to each other. This mode may be particularly important during periods of low solar wind Alfvén Mach number (Lavraud and Borovsky, 2008; Lavraud et al., 2013; Génot and Lavraud, 2021) as it may be unstable even when the primary KH mode is stabilized. Although the (primary) KH wave is considered to be one source of the field-line resonances, the secondary KH instability is strongly coupled to the Alfvén waves. While it has been shown that the secondary KH instability is important in the solar corona, in this article, we show that the secondary KH waves also appear when the shear transition layer exists between the magnetosheath and magnetosphere. Using both eigenmode analysis and a newly developed time-dependent MHD wave model, detailed characteristics of the secondary waves are examined.

This article is structured as follows: in Section 2, the MHD wave equations are presented. Section 3 describes the dispersion relation of the KH waves when the zero-thickness interfaces are assumed. The eigenmode frequency, growth rate, and the KH wave amplitude ratio are also shown. In Section 4, we introduce a new time-dependent MHD wave simulation code. The simulation results are compared with the eigenfrequency analysis from Section 3. We also discuss the wave coupling between KH and Alfvén waves. The last section contains a brief discussion and conclusions.

## 2 MHD WAVE EQUATIONS IN COLD PLASMA

In a cold plasma, basic equations of an ideal MHD plasma are

$$\rho \left[ \frac{\partial}{\partial t} + \mathbf{V} \cdot \nabla \right] \mathbf{V} - \nu \mathbf{V} = \frac{1}{\mu_0} (\nabla \times \mathbf{B}) \times \mathbf{B}, \quad (3)$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{V} \times \mathbf{B}), \quad (4)$$

where  $\nu$  is a collisional frequency that is introduced to damp waves propagating outside the region of interest, which effectively imposes outgoing boundary conditions. It should be noted that collisional effects play no role in the stability of the primary or secondary KH instabilities that we analyze in the rest of this article.

We assume that a field variable consists of background equilibrium (0) and small perturbation (1) components ( $\mathbf{B} = \mathbf{B}_0 + \mathbf{B}_1$ ,  $\rho = \rho_0 + \rho_1$ , and  $\mathbf{V} = \mathbf{V}_0 + \mathbf{V}_1$ ), and a shear flow ( $\mathbf{V}_0(x) = V_0(x)\hat{y}$ ) and a uniform background magnetic field ( $\mathbf{B}_0 = B_0\hat{z}$ ) lie in the  $y$ - and  $z$ -directions, respectively. Then, the perturbed quantities can be Fourier analyzed in the  $y$ - and  $z$ -directions ( $\partial/\partial y \rightarrow ik_y$ , and  $\partial/\partial z \rightarrow ik_{\parallel}$ , where  $k_y$  and  $k_{\parallel}$  are wavenumbers in the  $y$  and field-aligned ( $z$ ) directions). Thus, the linearized MHD wave equations are

$$\rho_0 \left( \frac{\partial}{\partial t} + ik_y V_0 \right) V_{1x} + \nu V_{1x} = \frac{B_0}{\mu_0} \left( ik_{\parallel} B_{1x} - \frac{\partial B_{1z}}{\partial x} \right), \quad (5)$$

$$\rho_0 \left( \frac{\partial}{\partial t} + ik_y V_0 \right) V_{1y} + \nu V_{1y} = \frac{B_0}{\mu_0} \left( ik_{\parallel} B_{1y} - ik_y B_{1z} \right) - \rho_0 V_{1x} \frac{\partial V_0}{\partial x}, \quad (6)$$

$$\left( \frac{\partial}{\partial t} + ik_y V_0 \right) B_{1x} = ik_{\parallel} B_0 V_{1x}, \quad (7)$$

$$\left( \frac{\partial}{\partial t} + ik_y V_0 \right) B_{1y} = ik_{\parallel} B_0 V_{1y} + B_{1x} \frac{\partial V_0}{\partial x}, \quad (8)$$

$$\left( \frac{\partial}{\partial t} + ik_y V_0 \right) B_{1z} = -ik_y B_0 V_{1y} - B_0 \frac{\partial V_{1x}}{\partial x}. \quad (9)$$

In **Section 3**, we solve the spectrum of eigenmodes of these equations in slab geometry, while in **Section 4**, we solve these equations using a finite-difference time-domain method.

To proceed with the spectral analysis, we define an auxiliary set of variables including the fluid displacement ( $\xi$ )

$$\mathbf{V}_1 \equiv \left( \frac{\partial}{\partial t} + \mathbf{V}_0 \cdot \nabla \right) \xi, \quad (10)$$

the total pressure perturbation ( $p$ ), and compressibility ( $\psi$ ),

$$p \equiv B_0 B_{1z}, \quad (11)$$

$$\psi \equiv \nabla \cdot \mathbf{V}_1. \quad (12)$$

Taking the Fourier transform in time ( $\frac{\partial}{\partial t} \rightarrow -i\omega$ ) and ignoring the collision term ( $\nu \rightarrow 0$ ), **Equations 5–9** become

$$\mu_0 \rho_0 \tilde{\omega}^2 \xi_x = -iB_0 k_{\parallel} B_{1x} + \frac{\partial p}{\partial x}, \quad (13)$$

$$\mu_0 \rho_0 \tilde{\omega}^2 \xi_y = -i\tilde{\omega} \mu_0 \rho_0 \xi_x \frac{\partial V_0}{\partial x} - iB_0 k_{\parallel} B_{1y} + ik_y p, \quad (14)$$

$$\tilde{\omega} B_{1x} = i\tilde{\omega} B_0 k_{\parallel} \xi_x, \quad (15)$$

$$\tilde{\omega} B_{1y} = i\tilde{\omega} B_0 k_{\parallel} \xi_y + iB_{1x} \frac{\partial V_0}{\partial x}, \quad (16)$$

$$\tilde{\omega} B_{1z} = -iB_0 \psi, \quad (17)$$

where  $\tilde{\omega} = \omega - k_y V_0$ .

Then, **Equations 11–17** can be reduced to two coupled first-order differential equations,

$$\frac{dp}{dx} = \mu_0 \rho_0 (\tilde{\omega}^2 - k_{\parallel}^2 V_A^2) \xi_x, \quad (18)$$

$$\mu_0 \rho_0 \frac{d\xi_x}{dx} = - \left( \frac{\tilde{\omega}^2 - k_y^2 V_A^2 + k_{\parallel}^2 V_A^2}{\tilde{\omega}^2 - k_{\parallel}^2 V_A^2} \right) \frac{p}{B_0}. \quad (19)$$

We solve **Equations 18** and **19** to analyze the eigenmode frequency in **Section 3**.

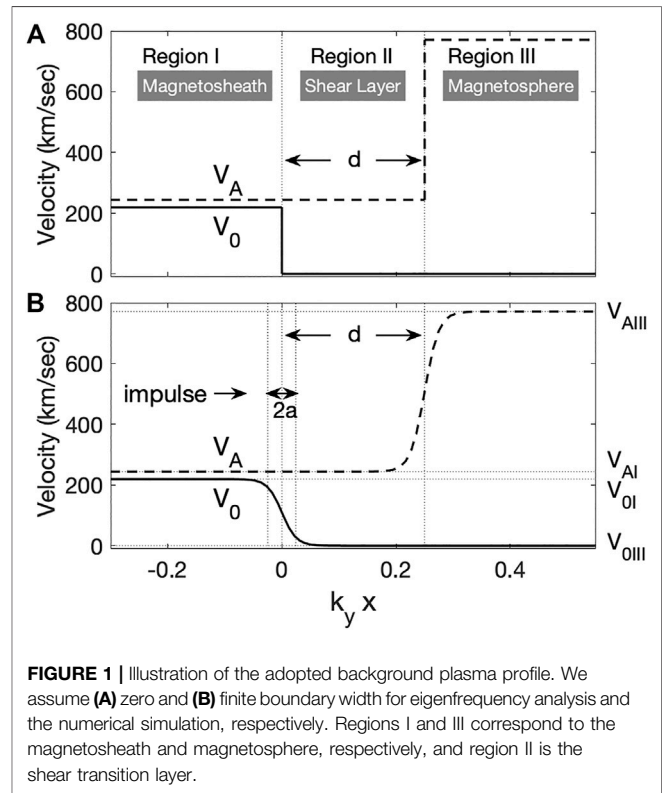
### 3 WAVE DISPERSION RELATION AT THE PLASMA INTERFACES

Eigenfrequency analysis is performed when the shear transition layer exists between magnetosheath and magnetosphere. For calculations,  $V_0$  and  $V_A$  are assumed to vary only in the direction of the  $x$ -axis, as shown in **Figure 1A**,

$$V_0(x) = V_{0I} \Theta(x), \quad (20)$$

$$V_A(x) = V_{AI} + (V_{AIII} - V_{AI}) \Theta(x-d), \quad (21)$$

where  $\Theta(x) = 0 (x < 0)$  or  $1 (x \geq 0)$  is a Heaviside step function. **Figure 1A** illustrates the transition from magnetosheath (I) to magnetosphere (III). The flow is sheared between regions I and II, while the Alfvén velocity increases between regions II



and III. Region II is the shear layer, which divides the plasma into two semi-infinite homogeneous regions (I and III) separated with a width  $d$ . It is generally expected that velocity shear between layers I and II can drive a KH instability that is localized at this interface, while the jump in Alfvén velocity between regions II and III supports surface Alfvén waves satisfying the Alfvén resonance condition. In the following analysis, we show how these modes couple when the transitions occur in close proximity.

The eigenmodes of these equations are localized, so they must satisfy exponentially decaying boundary conditions in regions I and III. Moreover, it is also expected that in region II that the solution decays away from either boundary. As such, the analytical forms of the solutions in each region  $J$  are as follows:

$$p_J(x) = p_J^- \exp(-\kappa_J x) + p_J^+ \exp(\kappa_J x), \quad (22)$$

$$\xi_{xJ}(x) = \xi_{xJ}^- \exp(-\kappa_J x) + \xi_{xJ}^+ \exp(\kappa_J x), \quad (23)$$

where  $\pm$  signs represent waves toward positive or negative directions in  $x$ .

For a surface wave, it is required that  $p_I^- = p_{III}^+ = \xi_{xI}^- = \xi_{xIII}^+ = 0$ , and  $\xi_x$  and  $p$  must be continuous at each interface; thus, at  $x = 0$

$$p_I^+ = p_{II}^- + p_{II}^+, \quad (24)$$

$$\xi_{xI}^+ = \xi_{xII}^- + \xi_{xII}^+, \quad (25)$$

and at  $x = d$ ,

$$p_{III}^- \exp(-\kappa_{III} x) = p_{II}^- \exp(-\kappa_{II} x) + p_{II}^+ \exp(\kappa_{II} x), \quad (26)$$

$$\xi_{xIII}^- \exp(-\kappa_{III} x) = \xi_{xII}^- \exp(-\kappa_{II} x) + \xi_{xII}^+ \exp(\kappa_{II} x). \quad (27)$$

The wave dispersion relation is obtained by inserting the solutions into **Equations 18** and **19** and noting that for solutions of the form  $\exp(\pm \kappa x)$  that

$$\kappa p = \mp \mu_0 \rho_0 (\tilde{\omega}^2 - k_{\parallel}^2 V_A^2) \xi_x, \quad (28)$$

$$\kappa \mu_0 \rho_0 \xi_x = \pm \left( \frac{\tilde{\omega}^2 - k_y^2 V_A^2 + k_{\parallel}^2 V_A^2}{\tilde{\omega}^2 - k_{\parallel}^2 V_A^2} \right) \frac{p}{B_0^2}, \quad (29)$$

and the relationship between  $p$  and  $\xi_x$  in each region  $J = I, II$ , and  $III$  in **Figure 1A** becomes

$$H_J \xi_{xJ} = p_J, \quad (30)$$

where  $H_J = \mu_0 \rho_{0J} (\tilde{\omega}^2 - k_{\parallel}^2 V_{AJ}^2) / \kappa_J$ .

From **Equations 24–27** and **30**, the wave dispersion can be derived as

$$D(\omega, k_y, k_{\parallel}, V_0, V_A) = (H_I + H_{II})(H_{II} + H_{III}) - (H_I - H_{II})(H_{II} H_{III}) \exp(-2\kappa_{II} d) = 0. \quad (31)$$

The amplitude ratio ( $A_p$ ) of the magnetic compressional component ( $p$ ) between the two interfaces ( $x = 0$  and  $d$ ) can also be determined:

$$A_p \equiv \frac{p_{x=d}}{p_{x=0}} = \frac{p_{II}^+ \exp(\kappa_{II} d) + p_{II}^- \exp(-\kappa_{II} d)}{p_{II}^+ + p_{II}^-} \quad (32)$$

$$= \frac{H_{III}}{H_I} \frac{H_I^2 - H_{II}^2}{H_{III}^2 - H_{II}^2}.$$

### 3.1 Primary and Secondary Kelvin–Helmholtz Waves

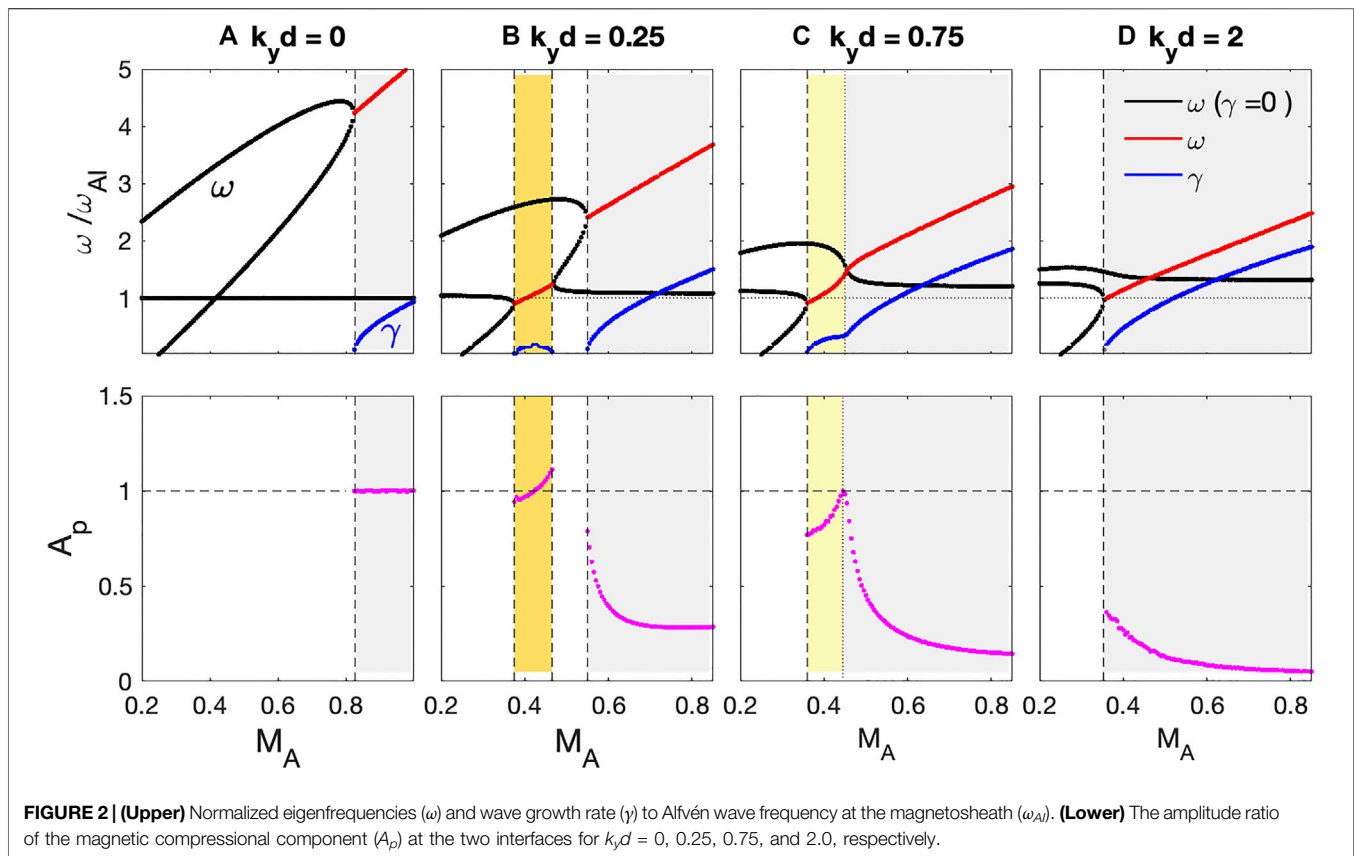
Using **Equations 31** and **32**, we calculate the eigenfrequency ( $\omega$ ), growth rate ( $\gamma$ ), and amplitude ratio between magnetic compressional component ( $A_p$ ) for various widths of the shear transition layer,  $k_y d = 0, 0.25, 0.75$ , and  $2.0$ , as shown in **Figure 2**. For these plots, the plasma densities in region I and region III are assumed to be  $N_{0I} = 5 \times 10^6/\text{m}^3$  and  $N_{0III} = 5 \times 10^5/\text{m}^3$ , and the background magnetic field strength is  $B_0 = 25\text{nT}$ . We also specify an angle of propagation ( $\phi$ ) with respect to the ambient magnetic field,  $\phi = \tan^{-1}(k_y/k_{\parallel}) = 80^\circ$  and  $\sqrt{k_y^2 + k_{\parallel}^2} = \pi/(2R_E)$ . For complete stability analysis, this angle would be varied to determine the maximum growth rate for a given Mach number. The upper panels of **Figure 2** are the calculated real (black and red) and imaginary (blue, growth rate  $\gamma$ ) frequencies as functions of the Alfvén Mach number ( $M_A \equiv V_{0I}/V_{AI}$ ), and the lower panels plot the amplitude ratio  $A_p$  of unstable wave modes.

In the absence of the shear transition layer ( $k_y d = 0$ ) as shown in **Figure 2A**, forward and backward propagating fast waves, which have positive and negative frequencies at  $M_A = 0$ , occur when  $M_A$  is small (Taroyan and Erdélyi, 2002). These waves are stable until  $M_A$  reaches the threshold of the KH instability,  $M_{As} = \tan^{-1}(\phi) \sqrt{2(1 + V_{AI}/V_{AIII})}$ . For  $M_A > M_{As}$  marked as a gray-shaded region in **Figure 2A**, the waves develop a complex frequency and become unstable. For the given range of  $M_A$ ,  $\omega$  and  $\gamma$  increase linearly with  $M_A$ . Because the characteristics of this wave mode are the same as the typical KH waves (e.g., Johnson et al., 2014), this wave corresponds to *primary* KH waves (hereafter PKHW). In this figure, we also found a shear Alfvén wave mode at  $\omega = \omega_{AI} = k_{\parallel} V_{AI}$ . The fast and shear Alfvén waves cross each other near  $M_A \sim 0.45$ , but the coupling of the two wave modes does not occur.

Introducing a finite width of the shear transition layer significantly changes the wave dispersion relations. In **Figure 2**, the PKHWs also occur for the cases of  $k_y d \neq 0$ . The  $M_A$  threshold decreases from  $0.83$  for  $k_y d = 0$  to  $0.35$  for  $k_y d = 2.0$ . Overall, the wave frequency  $\omega$  decreases, while the growth rate  $\gamma$  increases as  $k_y d$  increases. For example, for  $M_A = 0.85$ ,  $\omega/\omega_{AI} = (4.35, 3.68, 2.95, 2.48)$  and  $\gamma/\omega_{AI} = (0.35, 1.5, 1.85, 1.89)$  when  $k_y d = (0.0, 0.25, 0.75, 2.0)$ . Thus, when a shear transition layer is included, lower frequency PKHWs are excited with a stronger growth rate and lower  $M_A$  threshold.

For  $k_y d = 0.25$  in **Figure 2B**, coupling between the backward propagating fast and shear Alfvén waves occurs near  $\omega/\omega_{AI} \sim 1$ , and unstable waves also appear for  $0.355 \leq M_A \leq 0.47$  (shaded yellow in **Figure 2B**). These waves correspond to the *secondary* KH waves (hereafter SKHW) (Turkakin et al., 2013). In this case, the SKHWs are clearly separated from the PKHWs and have lower  $\omega$ , lower  $\gamma$ , and lower  $M_A$  threshold than the PKHWs. The





compressional amplitude ratio ( $A_p$ ) in the lower panel shows significant differences between PKHWs and SKHWs;  $A_p \ll 1$  for the PKHWs and  $A_p \sim 1$  for the SKHWs. Therefore, for SKHWs, the amplitude of the instability is similar at both the  $V_0$  and  $V_A$  interfaces, indicating a spreading of wave power over a more extended region, while the PKHWs are localized about the  $V_0$  interface. When the Mach number is low, it is expected that only the SKHWs would be excited.

When the  $V_A$  interface is further away from the  $V_0$  interface ( $k_y d = 0.75$ ), as shown in **Figure 2C**, the PKHW and SKHW modes merge near  $M_A \sim 0.47$ . Although  $\omega$  and  $\gamma$  monotonically increase as a function of  $M_A$ , the KH waves have similar behavior to the SKHW ( $A_p \sim 1$  and  $\omega \sim \omega_{AI}$ ) at smaller  $M_A$  and the PKHWs ( $A_p < 1$  and  $\omega \gg \omega_{AI}$ ) at larger  $M_A$ . Thus, the waves may still be divided into the semi-SKHW marked as a light yellow-shaded region and PKHW marked as a gray-shaded region in **Figure 2C**.

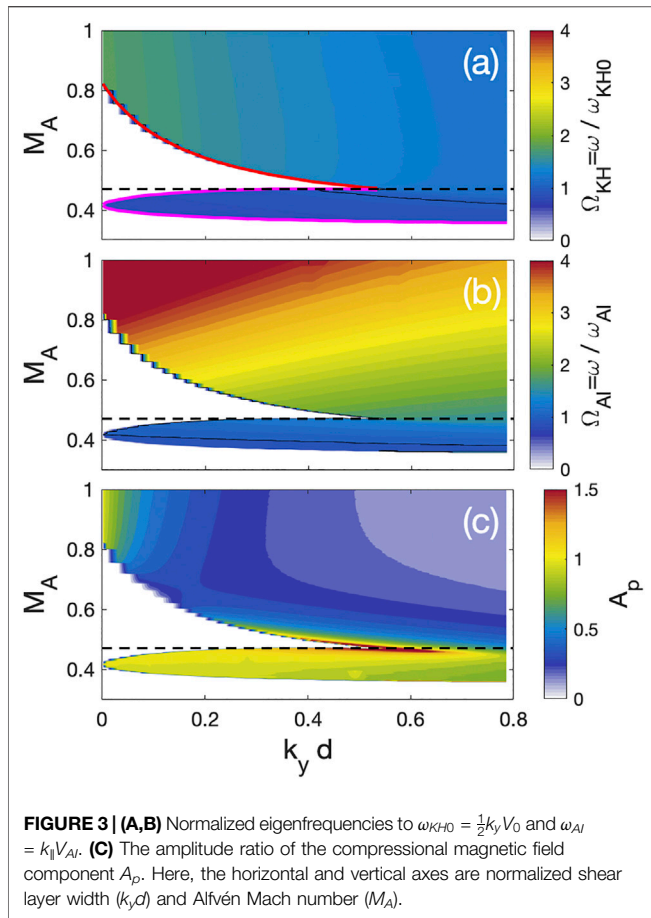
For  $k_y d = 2$ , as shown in **Figure 2D**, only a single unstable wave mode corresponding to the PKHWs occurs localized at the  $V_0$  interface. The  $V_A$  profile can be treated as a constant at the  $V_0$  interface and the  $M_A$  threshold becomes  $M_{As} \sim 2 \tan^{-1}(\phi) = 0.359$ . The threshold occurs near  $\omega / \omega_{AI} \sim 1$ ; thus, the wave frequencies are always higher than  $\omega_{AI}$ .

It is also useful to examine how  $\omega$  and  $A_p$  depend on  $M_A$  and  $k_y d$ . **Figure 3A,B** shows contour plots of  $\omega$  normalized to 1)  $\omega_{KHO}$  and 2)  $\omega_{AI}$ , respectively. In this figure, two wave modes are clearly organized by ranges of  $M_A$ ; the PKHW for  $M_A > 0.47$  and the SKHW for  $0.355 \leq M_A \leq 0.47$ . Red and magenta lines in

**Figure 3A** represent the  $M_A$  threshold for the PKHW and SKHW, respectively. The  $M_A$  threshold of the PKHWs decreases and the upper  $M_A$  limit of the SKHWs increases as  $k_y d$  increases. The thresholds merge near  $k_y d \sim 0.534$  and  $M_A \sim 0.47$ . Thus, for  $M_A > 0.47$ , a single wave mode appears (see **Figure 2C**); however, wave characteristics at lower and higher  $M_A$  are significantly different.

The PKHWs show that all parameters ( $\omega / \omega_{KHO}$ ,  $\omega / \omega_{AI}$ , and  $A_p$ ) have a strong dependence on  $k_y d$ , and they decrease as  $k_y d$  increases. For most  $M_A$ ,  $\omega / \omega_{KHO} \sim 1$  and  $1 < \omega / \omega_{KHO} < 2$ . Because both  $\omega$  and  $\omega_{KHO}$  increase proportionally to  $M_A$ ,  $\omega / \omega_{KHO}$  has less dependence on  $M_A$ . However, because  $\omega_{AI}$  does not depend on  $k_y$  and  $\omega$  increases as  $M_A$  increases,  $\omega / \omega_{AI}$  depends on both  $M_A$  and  $k_y d$ . For the given conditions,  $\omega / \omega_{AI}$  is maximized when  $k_y d$  is small and  $M_A$  is large. **Figure 3C** shows  $A_p < 1$  for  $M_A \geq 0.47$ , except  $k_y d \rightarrow 0$ . Thus, it shows that the PKHWs are almost always dominant at the  $V_0$  interface. For  $k_y d \rightarrow 0$ , a strong amplitude of the pressure term occurs at the secondary interface. However, this increase in  $A_p$  is not an indicator of a separate instability, but rather it simply indicates that the decay of the wave power from the  $V_0$  interface to the  $V_A$  interface reduced as the shear layer vanishes.

On the other hand, the eigenmode frequency of the SKHWs is comparable to  $\omega_{KHO}$  and  $\omega_{AI}$  ( $0.9 \leq \omega / \omega_{KHO(AI)} \leq 1.2$ ) in the entire range of  $k_y d$  and  $M_A$  because this wave mode appears due to the coupling between shear Alfvén mode and the fast compressional waves (thus,  $\omega_{KHO(AI)} \sim \omega_{AI}$ ). For the entire range of  $k_y d$ ,  $A_p$  is

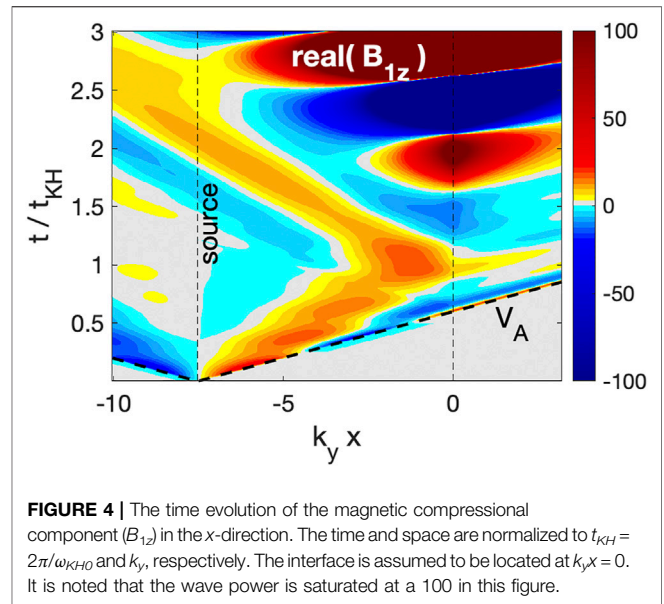


always close to or even higher than 1. These results suggest that the KH instability occurs at both the  $V_0$  and  $V_A$  interfaces with almost the same amplitude even though the interfaces are well separated.

The eigenmode calculations can be summarized as follows: the PKHWs are localized at the  $V_0$  interface having a higher frequency than  $\omega_A$  in the magnetosheath for faster shear flow velocity, while the SKHWs can be detected at both the  $V_0$  and  $V_A$  interfaces with similar wave frequency to  $\omega_{AI}$  in the magnetosheath for slower shear flow velocity.

### 4 MHD WAVE SIMULATIONS

In order to examine the PKHWs and SKHWs, we also developed an MHD wave simulation model. Similar to the previous fluid wave simulation codes (Kim and Lee, 2003; Kim et al., 2007), the finite-difference method is used in both time and space to solve the MHD Equations 5–9 as an initial-valued problem. We adopt a box model in which  $B_0$  is assumed to lie along the  $z$ -direction and inhomogeneity is introduced in the  $x$ -direction, while the boundary layer plasma flows in the  $y$ -direction with variation in the  $x$ -direction. Perfect reflecting boundaries are assumed and strong collisions are applied near the boundaries to describe semi-infinite space. Therefore, the total energy of traveling waves



decreases once the initial waves reach the boundary. Seed perturbations in the simulation domain result in linear growth of unstable modes, and the growth rate can be calculated once the unstable waves exceed the amplitudes of the initial perturbation.

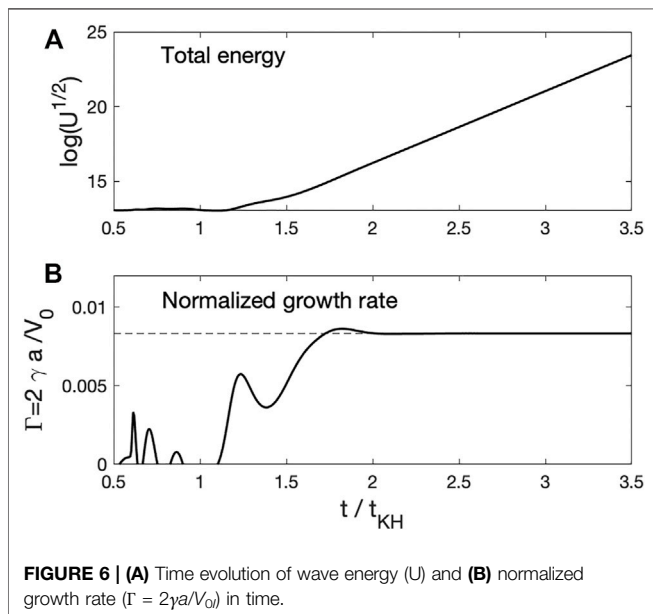
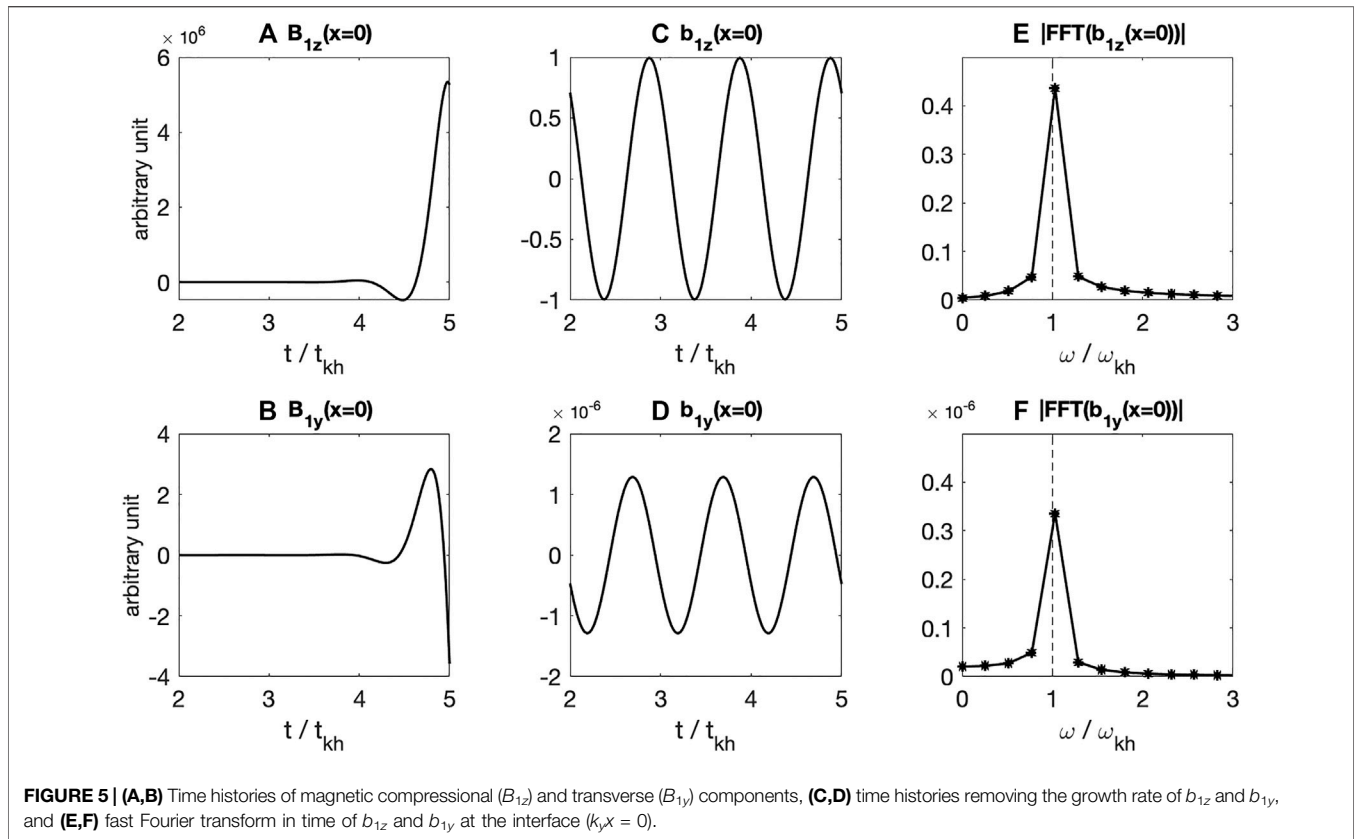
#### 4.1 KH Waves in Uniform $V_A$ Plasma

We first examine the KH waves in a plasma where  $V_A$  does not vary in space. In this simulation, a hyperbolic tangent  $V_0$  profile along with constant  $V_A$  was adopted in the wave code:

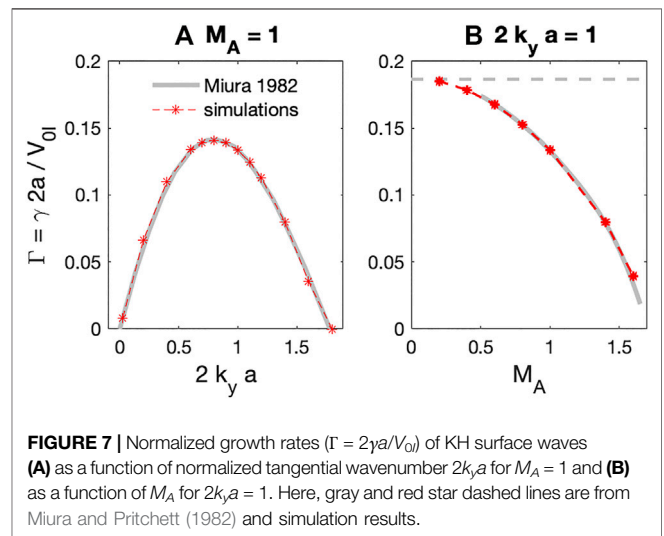
$$V_0(x) = \frac{V_{0I}}{2} \left[ 1 - \tanh\left(\frac{x}{a}\right) \right], \quad (33)$$

where  $V_{0I}$  is the flow velocity in region I, and this profile characterizes the  $V_0$  discontinuity in a scale length  $a$ , as shown in Figure 1B. One of the primary differences between the background profile used in the time-dependent analysis, and the previously discussed eigenmode analysis ( $a \rightarrow 0$ ) is the fact that the discontinuous profile has been smoothed.

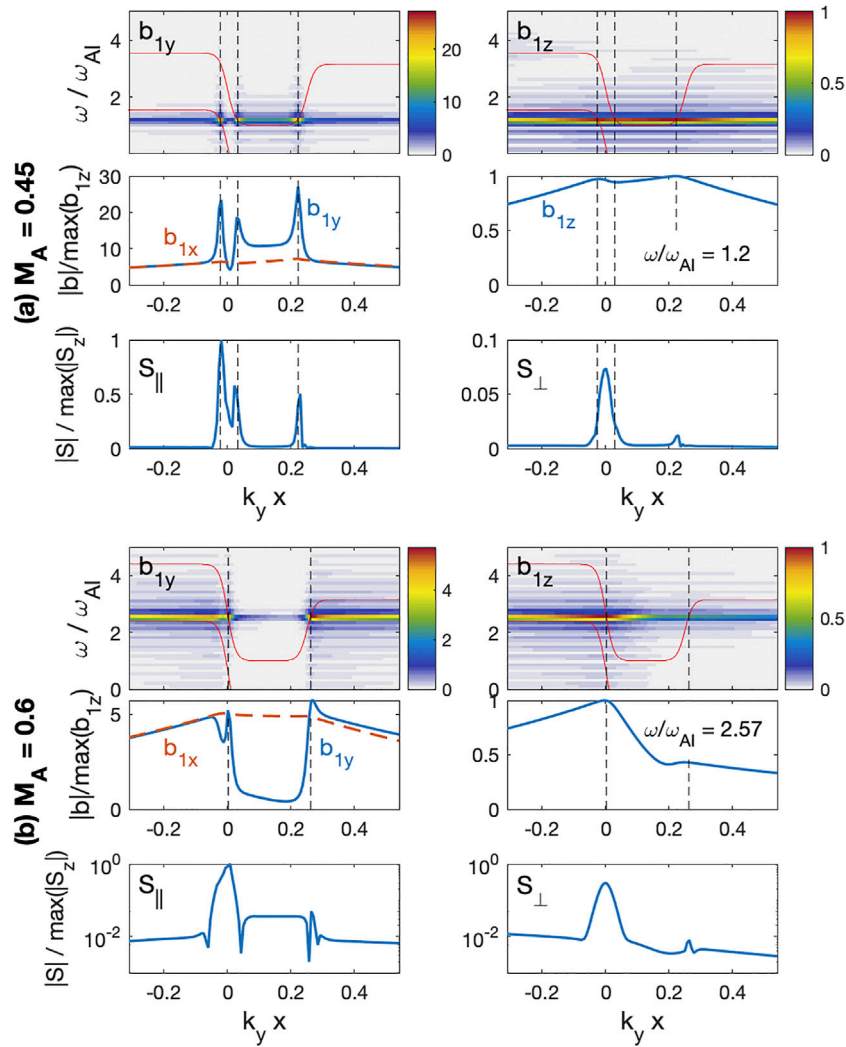
We assume that the length of the simulation box is  $L_x \sim 45/k_y$ . Since the KH surface wave is expected to not fully decay by the time it reaches the edge of the simulation domain in the  $x$ -direction, we add an absorption layer near the boundary ( $\sim 30/k_y$ ) in the simulation box to prevent reflection. An initial perturbation is launched as a compressional component of  $V_{1x}$  at the source location ( $k_y x_{source} \sim -7.5$ ) in region I (i.e., magnetosheath). This source is assumed to have a narrow spatial width ( $k_y \delta_{source} = 0.093$ ) and to include broadband frequencies,  $V_{1x}(x, t) = \exp(-1.5 \frac{t^2}{t_{KH}^2}) \exp(-\frac{(x-x_{source})^2}{\delta_{source}^2})$ , where  $k_{KH} = 2\pi/\omega_{KH0}$ . The simulation is run from  $t = 0$  to  $t = 5.6t_{KH}$ , and all components of  $B_1$  and  $V_1$  at each time step are stored during the simulation run time. The background densities in the magnetosheath (region I) and the magnetosphere (region III), the background magnetic field strength,  $k_y$ , and  $k_z$  are the same as in the eigenmode analysis of Section 3.



**Figure 4** shows the time evolution of the magnetic compressional component ( $B_{1z}$ ) in the  $x$ -direction for  $M_A = 1$  and  $k_y a = 0.025$ . Two vertical lines represent the source location ( $k_y a = -7.5$ ) and the  $V_0$  interface ( $k_y x = 0$ ), and thick dashed lines represent Alfvén speed ( $V_A$ ). Since the initial wave packet includes broadband frequencies, the wave packet



disperses in time and space. Leftward propagating waves reach a strong collisional layer near the boundary ( $k_y x < -10.5$ ) and are totally absorbed. Rightward propagating waves reach the  $V_0$  interface at  $k_y x = 0$  around  $t/t_{KH} = 0.7$ , and they partially reflect from the interface due to a steepened density gradient. The rest of the waves penetrates the  $V_0$  interface and reach the collisional layer ( $k_y x > 3.0$ ). Once the magnetic field and velocities are perturbed near the interface, an unstable



**FIGURE 8** | Wave spatial distribution for  $k_y d = 0.25$  **(A)**  $M_A = 0.45$  and **(B)**  $M_A = 0.6$ . Upper panels are perturbed magnetic field compressional ( $b_{1z}$ ) and transverse ( $b_{1y}$ ) components, middle panels are the spatial structure of the peak frequency, and lower panels are the Poynting flux parallel ( $S_{\parallel}$ ) and perpendicular ( $S_{\perp}$ ) to  $B_0$ .

wave mode begins to grow at around  $t/t_{KH} \sim 1.2$ . Unlike the initial perturbation, these waves decay in the  $x$ -direction rather than propagate. The wave amplitude in **Figure 4** saturates at  $\pm 100$ .

We focus on the surface waves at  $k_y x = 0$  and determine the growth rate, wave frequency, and polarization. Time histories of  $B_{1z}$  and  $B_{1y}$  at  $x = 0$  in **Figure 5A** rapidly grow in time; thus, the sinusoidal wave form is not clearly seen. However, the wave growth term can be removed from the time histories using the magnetic ( $U_B$ ), kinetic energy ( $U_V$ ), or total energy ( $U = U_B + U_V$ ). We plot  $U_{tot}(t) = \sum_x U(x, t)$  in the simulation box in **Figure 6A**. Early in the simulation period ( $t/t_{KH} < 1.2$ )  $U_{tot}$  is quasi-stable; however, once an unstable waves generated, it increases linearly. The wave magnetic field with a constant growth rate  $\gamma$  can be written as

$$B_1(x, t) \sim b_1(x, t) \exp[\gamma(x, t)t], \quad (34)$$

and because of the magnetic energy  $U_B \propto |B|^2$ , the wave growth rate  $\gamma$  in each grid point can be estimated from

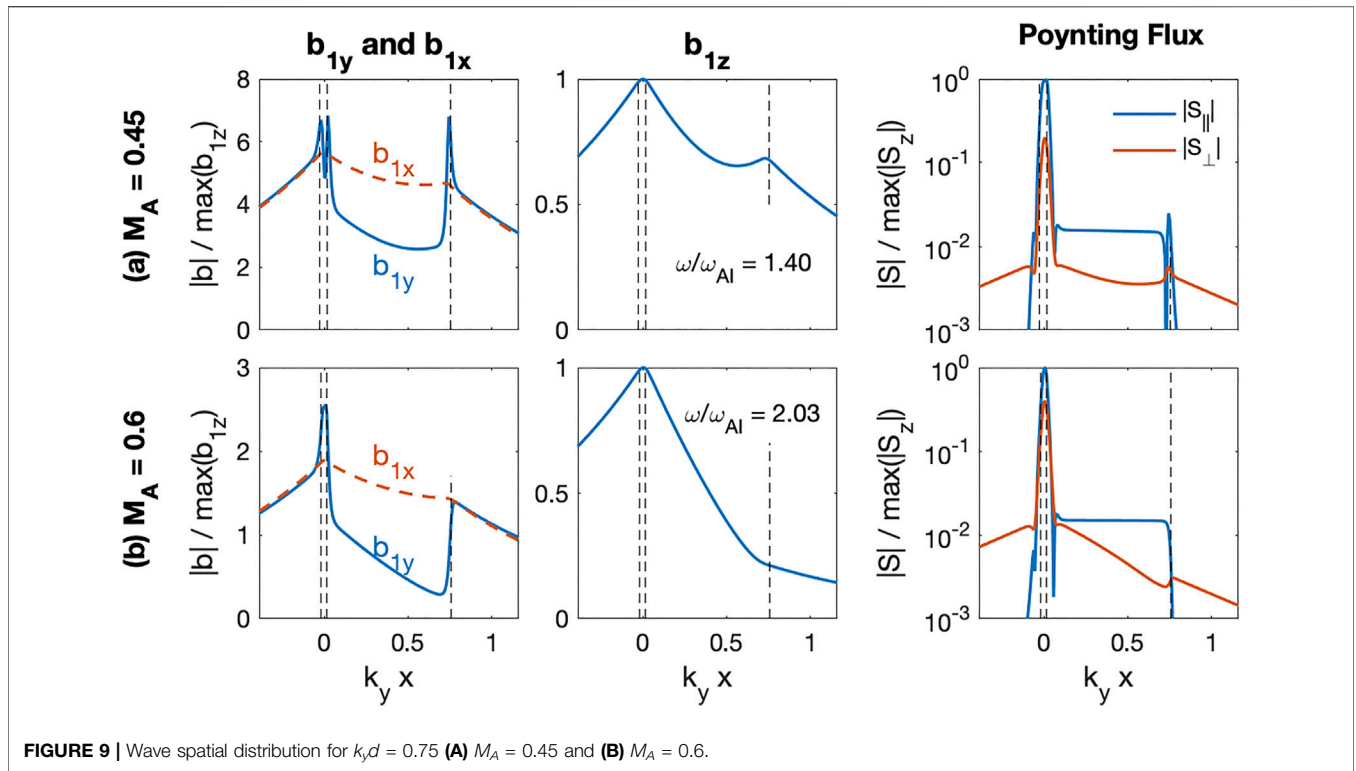
$$\gamma(x, t) \sim \frac{\partial}{\partial t} \ln(\sqrt{U_B(x, t)}). \quad (35)$$

We also confirmed that  $\gamma$  calculated using either the magnetic ( $U_B$ ) or kinetic energies ( $U_V$ ) are identical; thus,  $\gamma(x, t) = \frac{\partial}{\partial t} \ln(\sqrt{U(x, t)}) = \frac{\partial}{\partial t} \ln(\sqrt{U_B(x, t)}) = \frac{\partial}{\partial t} \ln(\sqrt{U_V(x, t)})$ . Furthermore, once the initial wave vanishes near the boundary, only the localized surface waves (such as KH waves) remain in the simulation domain; thus,  $\gamma$  also can be calculated using  $U_{tot}$ :

$$\gamma(t) = \frac{\partial}{\partial t} \ln(\sqrt{U_{tot}(t)}). \quad (36)$$

When a boundary has a finite thickness, the normalized growth rate ( $\Gamma \equiv 2a\gamma/V_{0l}$ ) becomes a function of normalized





boundary width ( $2k_y a$ ) (Miura and Pritchett, 1982). To illustrate, the time evolution of  $\Gamma(t)$  is plotted in **Figure 6B** for  $k_y d = 0.025$  and  $M_A = 1$ , and it converges to  $\sim 0.0083$ . Therefore, for these parameters, the normalized growth rate can be estimated as  $\Gamma = 0.0083$ .

Once the growth rate is determined, the wave components (and polarization) can be obtained from

$$b_1(t) \sim B_1(t) / \exp(\gamma(t - t_0)) \Big|_{t > t_0}, \quad (37)$$

where  $t_0$  is the time at which the wave growth begins. **Figures 5C,D** show that  $b_{1z}$  and  $b_{1y}$  have clear sinusoidal structures with a single frequency. The wave spectra of  $b_{1z}$  and  $b_{1y}$  in **Figures 5E,F** confirm that the single peak corresponds the KH wave frequency,  $\omega_{KH0} = \frac{1}{2} k_y V_0$ . In this manner, we can determine both the real and imaginary components of the frequency, which can be compared with the eigenmode analysis.

For code validation, we also compared the simulation results with prior analytical results in Miura and Pritchett (1982). **Figure 7** shows the growth rate,  $\Gamma$ , as a function of (a)  $2k_y a$  for  $M_A = 1$  and (b) as a function of  $M_A$  for  $2k_y a = 1$ . In this figure, the prior analytic results (gray lines) and our simulations (red stars) show excellent agreement with each other. For  $M_A = 1$  in **Figure 7A**, wave growth only occurs for a limited value of the normalized boundary width  $0 < 2k_y a < 1.8$  and maximizes near  $2k_y a = 0.8$ . For  $2k_y a = 1$ , the maximum  $\Gamma$  occurs for  $M_A \rightarrow 0$  and has a value of 0.144, as predicted from Miura and Pritchett (1982). The growth rate decreases as  $M_A$  increases and no KH wave arises for  $M_A > 1.6$ . Therefore, the new MHD wave code successfully demonstrates KH waves and benchmarking

comparisons of the simulations with previous analytical results validate the code accuracy.

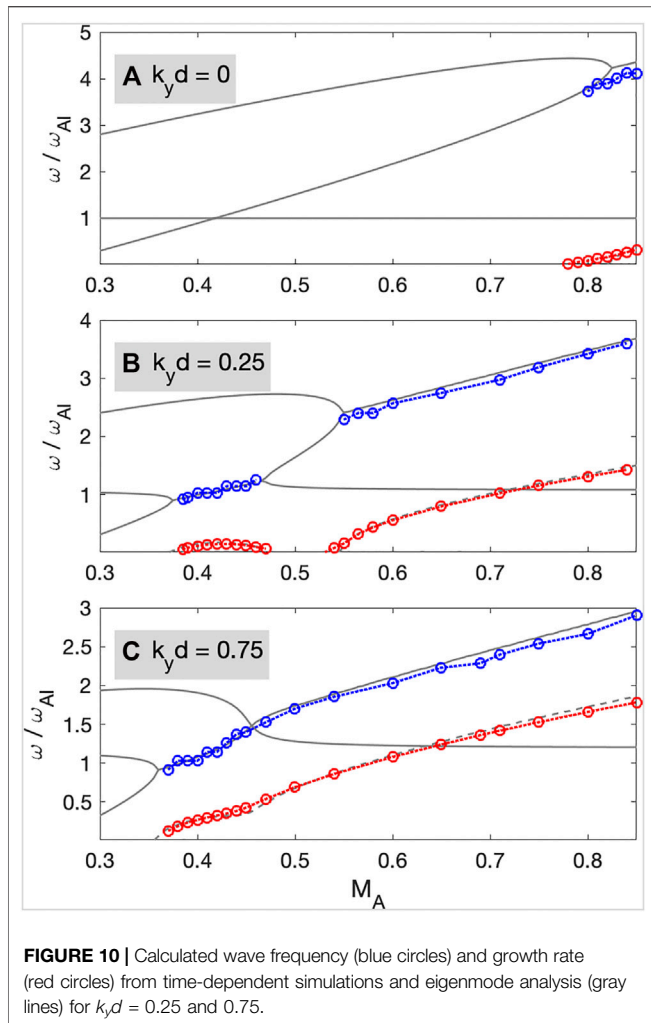
## 4.2 Coupling Between KH and Alfvén Resonant Waves

In this section, the simulation results include the shear transition layer between the magnetosphere and the magnetosheath, as shown in **Figure 1B**. In contrast to the results of **Section 3**, we consider a finite width of the boundary layer. Similar to the  $V_0$  profile in **Equation 33**,  $V_A$  is assumed to have a hyperbolic tangent profile:

$$V_A(x) = V_{AI} + \frac{V_{AI} + V_{AIII}}{2} \left[ \tanh\left(\frac{x-d}{a}\right) \right]. \quad (38)$$

The two interfaces are separated with width  $d$ , although each interface has its own width,  $a$ . From the eigenfrequency calculations in **Figure 2**, we showed that the inclusion of a shear transition layer effectively generates the SKHWs when the shear flow velocity is slow; thus, we ran the simulations for  $k_y d = 0.25$  and  $0.75$  and  $M_A < 0.85$  to compare with the eigenmode calculation.

We used the time histories of  $b_1$ , which does not include the exponential growth, in order to analyze the real frequency and relative strength of the field components. **Figure 8** presents wave spectra of perturbed magnetic field and the Poynting flux for  $k_y d = 0.25$ . For  $M_A = 0.45$  in **Figure 8A**, only the waves at  $\omega/\omega_{AI} \sim 1.2$  have strong amplitude. This frequency is close to the eigenmode frequency of  $\omega/\omega_{AI} = 1.17$  in **Figure 2**. The



estimated growth rate near the  $V_0$  and  $V_A$  interfaces are identical with  $\gamma/\omega_{AI} = 0.128$ . This growth rate is also in good agreement with the analytical results of  $\gamma/\omega_{AI} = 0.142$  in **Section 3**.

In order to examine the detailed wave properties, we plot spatial structures of the fluctuating magnetic field ( $b_{1x}$ ,  $b_{1y}$ , and  $b_{1z}$ ) at  $\omega/\omega_{AI} = 1.2$  in the middle row of **Figure 8A**. In this case, the compressional components ( $b_{1x}$  and  $b_{1z}$ ) maximize at the  $V_A$  and  $V_0$  interfaces and decay in the  $x$ -direction away from the interfaces. The  $b_{1z}$  and  $b_{1x}$  amplitudes at the two interfaces are comparable; thus,  $b_{1z}(x=d)/b_{1z}(x=0) = 1.025$ . This ratio is almost identical to the amplitude ratio of the pressure  $A_p = 1.06$  from **Figure 2B**.

On the other hand, the transverse component  $b_{1y}$  is enhanced at three different locations near  $V_0$  ( $k_y x = -0.022$  and  $0.033$ ) and  $V_A$  interfaces ( $k_y = 0.22$ ), where the wave frequency matches the Alfvén resonance condition ( $\omega_{AR}$ ):

$$\omega = \omega_{AR}^{\pm} \equiv k_y V_0 \pm k_{\parallel} V_A.$$

Due to the finite width of the  $V_0$  interface near  $x = 0$ ,  $\omega_{AR}^{\pm}$  can be positive at the  $V_0$  interface; thus, two separate regions of enhanced wave power can occur corresponding to Doppler-shifted resonance with both Alfvén resonances. In this case,  $b_{1y}$  is

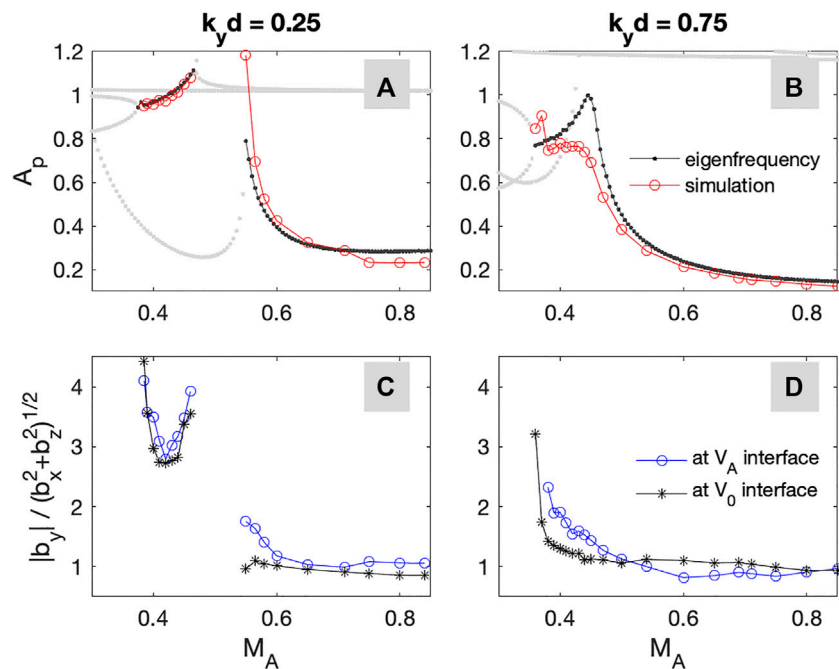
significantly stronger than  $b_{1z}$  or  $b_{1x}$ , and  $b_{1y}/\max(b_{1z}) \sim 28$  at the  $V_A$  interface. Furthermore, strong field-aligned Poynting flux occurs at the interfaces, as shown in the lower panels of **Figure 8A**. The Poynting flux parallel ( $S_{\parallel}$ ) and perpendicular ( $S_{\perp}$ ) to  $B_0$  show that the wave energy predominantly flows along the magnetic field line at both interfaces. Since we launch compressional waves (with  $V_{1x}$ ) in the magnetosheath, and the growing KH waves are compressional waves, the amplitude enhancement of the magnetic transverse component and intense field-aligned Poynting flux at the interfaces are clear evidences of the mode conversion from the surface KH waves to the surface Alfvén waves.

For the higher  $M_A$  case in **Figure 8B**, the amplitude is maximized at  $\omega = 2.57\omega_{AI}$ , which is similar to the analytical value of  $\omega = 2.6\omega_{AI}$  in **Section 3**. The  $b_{1z}$  component maximizes near  $x = 0$  and the secondary peak near  $k_y x = 0.25$  becomes weaker. The  $b_{1z}$  amplitude ratio between the two interfaces is 0.64, which is in good agreement with the analytical value of  $A_p = 0.4$ . The  $b_{1y}$  component shows strong amplitude near  $k_y x = 0$  and  $k_y x = 0.26$ . In this case, because the eigenmode frequency is higher than  $\omega_{AR}^{-}$ ,  $b_{1y}$  enhanced only at  $\omega = \omega_{AR}^{+}$ . The amplitude ratio between  $b_{1y}$  and  $b_{1z}$  is much lower than that for the case with  $M_A = 0.45$  in **Figure 8A**, having  $b_{1y}/\max(b_{1z}) \sim 6$  at  $x \sim d$ . Strong field-aligned flux  $S_{\parallel}$  appears at the  $V_0$  interface in the bottom panels, but  $S_{\perp}$  becomes stronger than the case of  $M_A = 0.45$ . The analytic eigenmode calculations predict that the PKHWs occur under  $(k_y d, M_A) = (0.25, 0.75)$ . The simulation results show that the mode conversion from the PKHWs to the surface Alfvén wave still occurs at each interface, but this process is less effective than that from the SKHWs.

For  $k_y d = 0.75$ , the waves have a strong amplitude peak near  $\omega/\omega_{AI} = 1.4$  and  $2.03$  for  $M_A = 0.45$  and  $0.6$ , respectively, and the spatial structures of these waves are presented in **Figure 9**. In this case, the PKHWs and SKHWs are not separated anymore (See **Figure 2**) and we define the KH waves in the lower  $M_A$  as semi-SKHWs in **Section 3**. For  $M_A = 0.45$  in **Figure 9A**,  $b_{1z}$  maximizes near  $x = 0$  and a weak secondary peak appears near  $x = d$ . On the other hand, three amplitude peaks near  $k_y x = -0.025$ ,  $4.018$ , and  $0.754$  appear in  $b_{1y}$ . The power ratio  $|b_{1y}/\max(b_{1z})|$  at the  $V_0$  interface is reduced to  $|b_{1y}/\max(b_{1z})| \sim 7$  from 23 for  $(k_y d, M_A) = (0.25, 0.45)$  in **Figure 8A**. The enhancement of  $S_{\parallel}$  is also seen at both interfaces and relatively strong  $S_{\perp}$  also appears. Near  $x = 0$  at the  $V_0$  interface,  $|S_{\perp}/S_{\parallel}|_{x=0}$  is about 0.39, which is almost twice as large as  $|S_{\perp}/S_{\parallel}|_{x=0} = 0.195$  for  $(k_y d, M_A) = (0.25, 0.45)$ . Therefore, even though the compressional wave behavior of the semi-SKHWs is similar to the SKHWs, the mode conversion from semi-SKHWs becomes much weaker than that from the SKHWs.

For  $M_A = 0.6$  in **Figure 9B**,  $b_{1z}$  decays along the  $x$ -direction from the  $V_0$  interface and no amplitude bump occurs at the  $V_A$  interface. The  $b_{1y}$  component shows a discontinuity at the  $V_A$  interface following the compressional Alfvén wave dispersion relation. Therefore,  $S_{\perp}$  becomes comparable to  $S_{\parallel}$  for  $k_y x > 0.75$ . In this case, the mode conversion at the  $V_A$  interface does not occur, but energy still flows along the magnetic field line at the  $V_0$  interface.

We also analyzed the cases for  $M_A$  for  $k_y d = 0, 0.25$ , and  $0.75$  at various  $M_A$ . **Figure 10** shows the extracted eigenmode frequency and growth rate from the simulations. In this figure, the red and



**FIGURE 11 | (A,B)**  $A_p$  from the simulations (red circles) eigenfrequency calculations (black dots) for  $k_y d = 0.25$  and  $0.75$ . **(C,D)** The amplitude ratio between the transverse and compressional components at the  $V_0$  interface (black stars) and the  $V_A$  interface (blue circles), respectively.

blue circled lines represent simulations, and the gray lines are taken from the eigenfrequency analysis from **Figure 2** in **Section 3**. Although the boundary thicknesses used in **Section 2** (slab) and **Section 3** (width  $a$ ) are different because the inhomogeneity scale length for the numerical simulation is much shorter than the wavelength, the eigenmode analytical and simulation results in  $\omega$  and  $\gamma$  show excellent agreement.

We also calculate the amplitude ratios between the compressional component at the two interfaces ( $A_p$ ) and between transverse ( $b_{1y}$ ) and compressional ( $\sqrt{b_{1x}^2 + b_{1z}^2}$ ) components at each interface. Due to the finite thickness of the boundary, two wave amplitude peaks can occur within the  $V_0$  interface as shown in **Figures 8, 9**, so we average the amplitude near  $x = 0$ , if  $\omega = \omega_{AR}^\pm = k_y V_0 \pm k_z V_A$ . The simulated  $A_p$  and eigenfrequency calculations show good agreement with each other in **Figure 11A,B**. In particular,  $A_p$  of the SKHWs in **Figure 11A** are almost identical to the analytic calculations. Thus, these results confirm that the SKHWs occur with nearly the same amplitude at both interfaces, while the PKHWs only happen at the  $V_0$  interface. The amplitude ratio between the transverse and compressional components in **Figure 11C,D** suggests that the transverse magnetic component of the SKHWs is dominant. In other words, the mode conversion to the shear Alfvén wave from the SKHWs effectively occurs at the interfaces. The PKHWs in **Figure 11C** and semi-SKHWs and PKHWs in **Figure 11D** show that the transverse mode amplitudes are comparable to the compressional mode amplitude; therefore, weaker or no

mode conversion occurs under given conditions at the  $V_A$  interface.

## 5 CONCLUSION AND DISCUSSION

This article investigates the coupling between KH and Alfvén waves when a shear transition layer exists between the magnetosheath and magnetosphere. Using the eigenfrequency analysis and time-dependent wave simulations, we showed that the SKHWs are generated when the shear velocity is slower than the typical threshold value for the onset of the KH instability.

The SKHWs occur with a frequency comparable to both KH wave frequency ( $\omega_{KH0} = \frac{1}{2}k_y V_0$ ) and the Alfvén frequency at the magnetosheath ( $\omega_{AI} = k_{\parallel} V_{AI}$ ), while the PKHWs have a much higher frequency than  $\omega_{AI}$ . These results suggest that PKHWs and SKHWs can be identified using the frequency ratio to  $\omega_{KH0}$  and  $\omega_{AI}$  from *in situ* observations. The SKHWs appear at both the  $V_0$  and  $V_A$  interfaces with nearly the same amplitude, while the PKHWs appear only at the  $V_0$  interface. Since  $V_0$  is uniform at the  $V_A$  interface, no KH waves can be generated at the  $V_A$  interface without coupling between the KH and Alfvén waves. For the given conditions of  $0 < k_y d \leq 0.5$ , where the SKHWs are well separated from the PKHWs in **Figure 3** and the shear transition layer width is  $0 < d \leq 0.3R_E$ ; thus, if the thickness of each boundary ( $a$ ) is much shorter than the width of the transition layer ( $d$ ), the SKHWs can be detected at the  $V_A$  interface.

The simulation results in **Figures 8, 9** show that the magnetic transverse component is dominant at the interface and a strong

field-aligned Poynting flux appears. Therefore, the energy transfer from the boundary layer to the Earth via mode-converted shear Alfvén waves occurs, which is similar to observations (Chaston et al., 2007). The wave simulations predict that a stronger mode conversion occurs from the SKHWs than from the PKHWs. However, the wave growth rate should be considered as well. Even though the mode conversion efficiency from the PKHWs is weaker than the SKHWs, the PKHWs amplitude can be strong enough due to the higher growth rate. Thus, a strong transverse component also can be detected from the PKHWs, but the compressional components are still comparable to the transverse components.

Although we clearly show the characteristics of PKHWs and SKHWs, this article only considers that the magnetic field is perpendicular to the flow velocity, and the magnetic field is assumed to be a constant. Indeed, the magnetic field in the magnetosheath and magnetosphere can be perpendicular in the magnetopause, and also the magnetic field and the flow velocity can be parallel in space, such as the solar corona and magnetotail. The secondary KH instability or resonant flow instability can occur under such conditions (Taroyan and Erdélyi, 2002, 2003; Turkakin et al., 2013). Furthermore, compressional waves bounded in the inner magnetosphere can contribute to the generation of the secondary KH instability (Turkakin et al., 2013) and also mode conversion to the shear Alfvén wave (Taroyan and Erdélyi, 2002). The total length of our numerical simulation model, including the collisional layer in Section 4, is somewhat comparable to  $\sim 10R_E$ ; thus, the *bounded* plasma effect should be considered in the future.

We also used a cold plasma approximation in the magnetosheath. The inclusion of thermal effects leads to an additional KH wave branch (Taroyan and Erdélyi, 2003). In warm plasmas, the Alfvén waves propagate as kinetic Alfvén waves (KAW). The KAW can have a larger wavenumber across the magnetic field line and field-aligned electric field and velocity components (Lin et al., 2010, 2012). Similar to Alfvén waves, KAW also transfers the energy away from the mode conversion location along the magnetic field line; thus, it is expected that a strong transverse component at each interface would also be detected with thermal effect.

In addition, a high level of turbulent fluctuations in the magnetosheath is observed in multiple satellites (e.g., Rakhmanova et al., 2021); however, nonlinear effects are not included in our analysis. It is possible that if these modes grow to sufficient amplitude, vortices will form and nonlinear interactions may become important, leading to plasma heating and transport. These nonlinear effects are left for future studies.

## DATA AVAILABILITY STATEMENT

The raw data supporting the conclusions of this article will be made available by the authors, without undue reservation. Digital data can be found in the DataSpace of Princeton University <http://arks.princeton.edu/ark:/88435/dsp013r074z09k>

## AUTHOR CONTRIBUTIONS

E-HK developed the real-time simulation code and ran both eigenfrequency calculation and simulation code, JRJ solved the dispersion relation of the KH waves and built the eigenfrequency calculation code, and KN discussed the observational background.

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