



Fast Rotating Neutron Stars: Oscillations and Instabilities

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In this review article, we present the main results from our most recent research concerning the oscillations of fast rotating neutron stars. We derive a set of time evolution equations for the investigation of non-axisymmetric oscillations of rapidly rotating compact objects in full general relativity, taking into account the contribution of a dynamic spacetime. Using our code, which features high accuracy at comparably low computational expense, we are able to extract the frequencies of non-axisymmetric modes of compact objects with rotation rates up to the Kepler limit. We propose various universal relations combining bulk properties of isolated neutron stars as well as of binary systems before and after merger; these relations are independent of the true equation of state and may serve as a valuable tool for gravitational wave asteroseismology. We also present an introductory example using a Bayesian analysis.

Keywords: neutron star, equation of state, stars, rotation, stars, Bayesian analysis, binary merger

1 INTRODUCTION

Oscillations and instabilities of neutron stars were always considered among the promising sources for gravitational waves. The systematic study of non-axisymmetric neutron star oscillations began in the 1960s with the pioneering works of Thorne and collaborators (cf. Thorne and Campolattaro (1967); Thorne (1969) (and subsequent papers)), in which they laid out the equations governing the perturbations of compact stars in general relativity. The numerical solution of these equations has proven highly challenging and it has taken nearly two decades before Lindblom and Detweiler found an advantageous formulation of the eigenvalue problem that allowed them to determine the complex frequencies of the acoustic modes of sufficiently realistic stellar models (Lindblom and Detweiler (1983); Detweiler and Lindblom (1985)). These results did not conclude the investigation of these modes; in particular Chandrasekhar and Ferrari (1991a); Chandrasekhar and Ferrari (1991b) turned to the perturbations of relativistic stars and studied their oscillations as a scattering problem.

In the mid-1980s, inspired by a toy model Kokkotas and Schutz (1986) suggested that the dynamic spacetime of a neutron star exhibits its very own class of modes and christened them w -modes (Kokkotas and Schutz (1992)). The short damping times of these modes (comparable to the ones of black-holes) pose numerical challenges, but the application of the continued fraction method (Leaver (1985)) and numerical integration along anti-Stokes lines (Andersson *et al.* (1995)) improved considerably the accuracy in the calculation of both *fluid* and *spacetime* modes and led to the discovery of the new class of w -modes, the so-called w_{II} -modes (Leins *et al.* (1993)). This advancement in computational accuracy allowed the study of many neutron star equations of state leading to the discovery of universal relations involving frequencies and damping times of oscillation modes. These universal relations provided ways for solving the inverse problem within the so-called *gravitational wave asteroseismology*, allowing to set constraints on the mass and/or radius of

the neutron star, and eventually the nuclear equation of state, via the observation of oscillations (see, e.g., Andersson and Kokkotas (1996, 1998); Kokkotas *et al.* (2001); Benhar *et al.* (2004)).

With increasing computational power during the 1990s, the first attempts were made to increase the dimensionality of the hitherto (due to the restriction to spherical symmetry) purely radial problem and include time as a second dimension. The first successful time evolutions of perturbations of relativistic neutron stars were reported by Allen *et al.* (1998). Ruoff (2000, 2001) reformulated those evolution equations by means of the ADM-formalism (Arnowitt *et al.* (1962)) and introduced a non-uniform radial grid to ensure stable numerical evolution when using realistic equations of state.

1.1 Asteroseismology

The different oscillation patterns of a neutron star are characterized by their restoring force, e.g., p (pressure)-modes, g (gravity)-modes, i (Coriolis)-modes, s (shear)-modes or w (wave)-modes. The f -mode is the fundamental mode of the p -mode sequence and it is the oscillation mode most likely to be excited in violent processes such as neutron star formation by supernova core collapse (Torres-Forné *et al.* (2018; 2019)), the pre-merger interaction of neutron stars (see, e.g., Lai (1994); Kokkotas and Schafer (1995); Fuller and Lai (2011); Gold *et al.* (2012); Chirenti *et al.* (2017); Chaurasia *et al.* (2018); Suvorov and Kokkotas (2020); Kuan *et al.* (2021a,b)), the early-post-merger oscillations of the final object (Shibata and Taniguchi (2006); Bauswein and Janka (2012); Hotokezaka *et al.* (2013); Bernuzzi *et al.* (2014); Bauswein *et al.* (2014); Lehner *et al.* (2016); Bauswein *et al.* (2017); Rezzolla and Takami (2016); Takami *et al.* (2015); De Pietri *et al.* (2018); Breschi *et al.* (2019)). In the case that the merging neutron stars are of relatively small mass and the post-merger object is a fast spinning neutron star, the unstable f -mode oscillations can lead to its spin-down (Doneva *et al.* (2015)). The f -mode is associated with major density variations and thus can potentially be an emitter of copious amounts of gravitational radiation. The emission of gravitational waves is the primary reason for the mode's rapid damping, at least for newly born neutron stars.

The efforts to associate the patterns of oscillations with the bulk parameters of the stars, e.g., their mass, radius or equation of state (henceforth EoS) was initiated in the mid-1990s and continued for almost two decades, advancing the field of gravitational wave asteroseismology (Andersson and Kokkotas (1996, 1998); Kokkotas *et al.* (2001); Benhar *et al.* (2004); Tsui and Leung (2005); Lau *et al.* (2010)). To date, very robust empirical relations have been derived for non-rotating neutron stars, connecting observables such as frequency, damping time, or moment of inertia I to the bulk properties; for example, relations of the form $\sigma_0 = \alpha + \beta \sqrt{M_0/R_0^3}$ or $M\sigma_0 = F(M_0^3/I)$ (cf. Andersson and Kokkotas (1996); Lau *et al.* (2010)) could provide the average density or the moment of inertia of the star if the f -mode frequency σ_0 is known.

In the era of gravitational wave astronomy, the various oscillation patterns (traced already in non-linear numerical simulations, e.g., Stergioulas *et al.* (2004); Kastaun (2006);

Baiotti *et al.* (2008, 2009); Zink *et al.* (2010); Stergioulas *et al.* (2011); Bernuzzi *et al.* (2015)), if observed, can provide a wealth of information about the emitting sources, and their effects can leave their imprints both in the gravitational but also in the electromagnetic spectrum. Moreover, recent studies relate the f -mode frequencies to the Love numbers (Wen *et al.* (2019); Pratten *et al.* (2020); Andersson and Pnigouras (2021); Manoharan *et al.* (2021)) and even to the postmerger short gamma-ray bursts (Chirenti *et al.* (2019)).

1.2 Rotation

Nearly all the previously mentioned studies were concerned with non-rotating stars. In nature, neutron stars will always rotate and their rotation rate may reach extreme values. The inclusion of rotation proves difficult since the extreme rotation rates that neutron stars may (and do) reach do not allow to neglect the star's oblateness which removes the spherical symmetry from the system; this in turn makes the mathematical formulation much more involved. As a first approximation, rotation was treated perturbatively, too, which allowed considering even rotating stars as spherically symmetric (Hartle (1967)). In this so-called *slow-rotation approximation*, the perturbation equations gain considerably in complexity and have been written down in a gauge introduced by Regge and Wheeler (1957) first by Kojima (1992). Even though the problem remains one-dimensional, its solution is not straightforward as (among other technicalities) the outgoing-wave boundary condition at infinity is elusive. Notwithstanding, Andersson (1998) successfully applied this formalism and discovered that r -modes are prone to the so-called CFS-instability, named after their discoverers Chandrasekhar (1970) and Friedman and Schutz (1975, 1978), at any rotation rate. There has been continuing effort using the slow-rotation approximation concerning rotational modes (Lockitch *et al.* (2000; 2003)), and also employing different gauges (Ruoff *et al.* (2002); Vavoulidis *et al.* (2008); Passamonti *et al.* (2008)), but with the pressing need for frequencies of *rapidly* rotating neutron stars, the interest slowly faded.

Even though the slow-rotation approximation has proven fruitful in the understanding of neutron star physics, it is no longer applicable when considering neutron stars at arbitrary rotation rates, which is essential for nascent neutron stars or post-merger configurations in the immediate aftermath of a binary merger. Without the spherical symmetry of the problem, one has to account for at least two spatial dimensions which complicates the equations further and amplifies the computational expense; furthermore, it remains elusive how to formulate the outgoing-wave boundary condition at infinity for the spacetime perturbations in two spatial dimensions which essentially removes the possibility to formulate a corresponding eigenvalue problem. This issue can be circumvented by adopting the *Cowling approximation* (Cowling (1941)), in which the spacetime is considered static, also leading to a considerable simplification of the perturbation equations. Ignoring the impact of a dynamic spacetime (which is most severe for the quadrupolar f -mode), Yoshida and Eriguchi (1997, 1999) computed quadrupolar f -mode frequencies of rapidly

rotating neutron stars and studied the associated CFS-instability in the late 1990s. Boutloukos and Nollert (2007) revived this approach and investigated the general properties of the spectrum of neutron stars regarding the acoustic and Coriolis-driven modes. As a further step toward a more general relativistic treatment, Yoshida (2012) revisited the problem in the *conformal flatness approximation*. However, with the mathematical difficulties of extending the eigenvalue formulation to include a dynamic spacetime, the focus shifted to study the oscillation spectra by evolving the perturbation equations in time.

Despite their complexity, the perturbation equations for rapidly rotating relativistic stars have been written down by Priou (1992), even though they were not approached numerically at that time. Gaertig and Kokkotas (2008, 2009) worked in the Cowling approximation and successfully extracted f - and g -mode frequencies of arbitrarily uniformly and fast rotating neutron stars and even of differentially rotating ones Krüger et al. (2010) by adding *artificial viscosity* (also known as Kreiss and Olinger (1973) dissipation) to their evolution equations in order to stabilise their time evolutions.

During the first decade of the new millennium, substantial advances were made in the time evolution of the unperturbed, non-linear Einstein equations, mostly driven by the aim to simulate compact binary mergers but also applicable to isolated neutron stars. These systems have hardly any symmetries that can be exploited to reduce the complexity of the problem, requiring to carry out the time evolutions on a three-dimensional grid. The upside of which is that essentially no constraints have to be placed on the rotational profile when simulating the dynamics of a neutron star. Such codes have been seen as a promising new approach to the calculation of mode frequencies of rapidly (and differentially) rotating neutron stars and already at the beginning of the decade, the frequencies of axisymmetric modes in the Cowling approximation (Font et al. (2000; 2001)) and those of (quasi-)radial modes in full general relativity (Font et al. (2002)) had been reported. The non-linear codes kept evolving and were used to generate mode frequencies of f -modes in the conformal flatness approximation (Dimmelmeier et al. (2006)) or those of inertial modes in the Cowling approximation (Kastaun (2008)). Baiotti et al. (2009). Not much later, the frequencies of non-axisymmetric modes in full general relativity of non-rotating polytropic neutron stars (Baiotti et al. (2009)) and soon those of rapidly rotating polytropic neutron stars (Zink et al. (2010)) were obtained from fully non-linear simulations. Even though successful, this approach to computing the frequencies of non-axisymmetric modes, however, has not been followed closely, which is also due to the computational expense associated with such numerical simulations and the accompanying limited accuracy.

In fact, from the point of view of gravitational wave detectability of oscillation modes, the most relevant scenarios are likely to involve rapidly rotating stars. Unfortunately, the aforementioned empirical relations cannot be trivially extended to rotating stars. Rotation splits the oscillation spectra in a similar fashion as the Zeeman splitting of the spectral lines due to the presence of magnetic fields. In rotating stars, the splitting

leads to perturbations propagating in the direction of rotation (so-called *co-rotating modes*) and perturbations traveling in the opposite direction (*counter-rotating modes*). The oscillation frequency as observed by an observer at infinity will either increase or decrease depending on the propagation direction of the waves; for slow rotation there will be a shift of the form $\sigma = \sigma_0 \pm \kappa m \Omega + \mathcal{O}(\Omega^2)$ where m is the angular harmonic index, κ a mode and stellar model-dependent constant, and Ω the angular rotation rate of the star. If the spin of the star exceeds a critical value, which depends on, e.g., the EoS and its mass—i.e., when the pattern velocity σ/m of the backward moving mode becomes smaller than the star's rotation rate $\Omega/2\pi$ —then the star becomes unstable to the emission of gravitational radiation; this is the aforementioned CFS instability. This instability is generic (independent of the degree of rotation) for the r -modes (Andersson (1998); Friedman and Morsink (1998)) while it can be excited only for relatively high spin values ($\Omega \gtrsim 0.8\Omega_K$, with Ω_K the Kepler velocity) for the quadrupolar f -modes. An extensive discussion can be found in Paschalidis and Stergioulas (2017) and Glampedakis and Gualtieri (2018).

This review is based on the highlights of four recent articles published by the authors, which are Krüger and Kokkotas (2020a,b); Völkel et al. (2021); Manoharan et al. (2021).

Throughout this article, we employ units in which $c = G = M_\odot = 1$.

2 PERTURBATION EQUATIONS

2.1 Background Configuration

We are going to work with the Einstein equations along with the law for the conservation of energy-momentum,

$$G_{\mu\nu} = 8\pi T_{\mu\nu} \quad \text{and} \quad \nabla_\mu T^{\mu\nu} = 0, \quad (1)$$

where $G_{\mu\nu}$ is the Einstein tensor and $T_{\mu\nu}$ is the energy-momentum tensor.

We restrict ourselves to the study of the dynamics of small perturbations around an equilibrium configuration which allows us to linearise **Eq. 1**. We assume an axisymmetric, stationary background configuration for which the metric written in quasi-isotropic coordinates takes the form

$$\begin{aligned} ds^2 &= g_{\mu\nu}^{(0)} dx^\mu dx^\nu \\ &= -e^{2\nu} dt^2 + e^{2\psi} r^2 \sin^2 \theta (d\varphi - \omega dt)^2 + e^{2\mu} (dr^2 + r^2 d\theta^2). \end{aligned} \quad (2)$$

Here, ν , ψ , μ , and ω are the four unknown metric potentials, depending only on r and θ .

We model the neutron star to be a perfect fluid without viscosity for which the corresponding energy-momentum tensor takes the form

$$T^{\mu\nu} = (\epsilon + p)u^\mu u^\nu + pg^{\mu\nu}, \quad (3)$$

where ϵ is the energy density, p is the pressure, and u^μ the 4-velocity of the fluid. The only two non-vanishing components of the 4-velocity are linked via the star's angular rotation rate, $u^\varphi =$

Ωu^t , and by means of the normalisation of the 4-velocity they are given by

$$u^t = \frac{1}{\sqrt{e^{2\nu} - e^{2\psi} r^2 \sin^2 \theta (\Omega - \omega)^2}} \quad (4)$$

After specifying an EoS, which may be a polytropic or a tabulated one, linking energy density and pressure to each other, we generate uniformly rotating equilibrium configurations using the `rns-code` (Stergioulas and Friedman (1995); Nozawa et al. (1998); Stergioulas (1995)).

We considered sequences of neutron stars along which we keep either the central energy density constant or the baryon mass fixed (the latter are also known as *evolutionary sequences*) with rotation rates up to their respective mass-shedding limit. Our neutron star models were based on polytropic and realistic EoSs. We considered polytropic models with three different polytropic indices $N = 0.6849, 0.7463, \text{ and } 1.0$. Furthermore, we employed piecewise-polytropic approximations, introduced by Read et al. (2009), for the four tabulated EoSs (APR4, H4, SLy, and WFF1) that we used. Our nonrotating configurations have gravitational masses $M \in [1.17, 2.19]M_\odot$. Even though current astrophysical constraints play a role in our particular choice of EoSs, it is largely motivated by our desire to provide robust universal relations by covering a wide part of the parameter space.

2.2 Perturbation Equations

As usual in perturbative studies, we decompose the metric as

$$g_{\mu\nu} = g_{\mu\nu}^{(0)} + h_{\mu\nu}, \quad (5)$$

where $g_{\mu\nu}^{(0)}$ is the background metric and $h_{\mu\nu}$ its perturbation. As we will work in the Hilbert gauge, it will be advantageous to work instead with the trace-reversed metric perturbation, defined by

$$\phi_{\mu\nu} := h_{\mu\nu} - \frac{1}{2}g_{\mu\nu}^{(0)}h, \quad (6)$$

where $h := h^t{}_\mu{}^\mu$ is the trace of the metric perturbations.

The metric perturbations are not unique but possess gauge freedom which can be utilised in different ways. Often, the gauge freedom is used to eliminate some of the spacetime perturbations, e.g., by using the well-known gauge by Regge and Wheeler (1957), and hence to reduce the number of perturbation equations. In our studies, however, we followed a different approach (which we will reason below) and opt for the Hilbert gauge, which is the gravitational equivalent to the well-known Lorenz gauge in electromagnetism, specified by

$$f_\mu := \nabla^\nu \phi_{\mu\nu} = 0. \quad (7)$$

In the Hilbert gauge, the perturbed Einstein tensor takes the form.

$$\begin{aligned} -2\delta G_{\mu\nu} = & \square \phi_{\mu\nu} + 2R^\alpha{}_\beta{}^\nu{}_\mu \phi_{\alpha\beta} + R\phi_{\mu\nu} - (R^\alpha{}_\mu \phi_{\nu\alpha} + R^\alpha{}_\nu \phi_{\mu\alpha}) \\ & - g_{\mu\nu} R^{\alpha\beta} \phi_{\alpha\beta}, \end{aligned}$$

where $R^\alpha{}_\beta{}^\nu{}_\mu$, $R^{\alpha\beta}$, and R are the background Riemann tensor, Ricci tensor and scalar curvature, respectively. The advantage of the Hilbert gauge is that the evolution equations for the metric

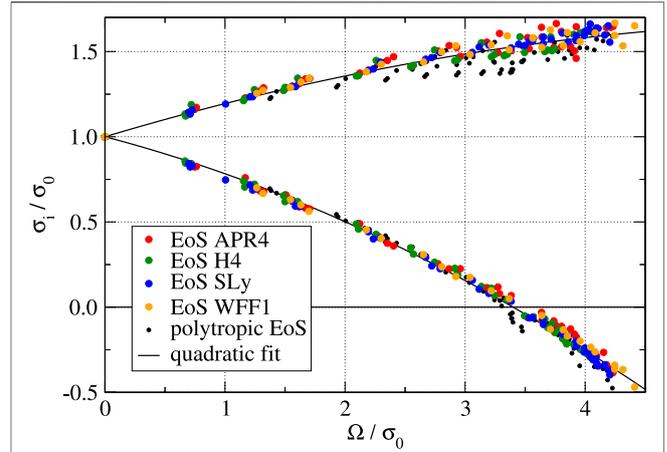


FIGURE 1 | Universal relations for the $l=|m|=2$ f -mode frequencies for sequences of constant central energy density as observed in the inertial frame. The graph shows the results from 21 such sequences (three sequences per EoS; three polytropic and four realistic EoSs). The potentially unstable f -mode branch displays a strikingly universal behavior; the largest deviations from a quadratic fit occur close to the mass-shedding limit of the sequences, mainly for the polytropic EoSs. Figure taken from Krüger and Kokkotas (2020a) with permission by APS.

perturbations will take the form of ten coupled wavelike equations (note that in the above expression, the d'Alembert operator, defined with respect to the background metric, is the only differential operator acting on the metric perturbations) while the mixing of temporal and spatial derivatives is avoided. This is in contrast to other common gauge choices (Allen et al. (1998); Battiston et al. (1971); Ruoff (2001); Ruoff et al. (2002)) or the ones without any gauge choice (Priou (1992)) where the field equations split into subsets of hyperbolic and elliptic equations which have to be either solved simultaneously or by quite cumbersome manipulations to bring them in a hyperbolic form, something that is technically very difficult for the perturbations of rotating stars if not impossible. The fully hyperbolic character of the perturbation equations in the Hilbert gauge makes this gauge particularly convenient for the numerical implementation in a time evolution.

Our choice of gauge, namely the Hilbert gauge, does not eliminate any of the metric perturbations; hence, we need 10 variables to describe the spacetime perturbations. As in previous studies of rotating neutron stars (Vavoulidis (2007); Krüger et al. (2010)), we use four variables for the fluid. In total, we need to evolve 14 variables in time. The evolution equations follow in a very straightforward manner from the perturbed Einstein equations

$$\delta G_{\mu\nu} = 8\pi\delta T_{\mu\nu}, \quad (8)$$

and the perturbed law for the conservation of energy-momentum

$$\delta(\nabla_\mu T^{\mu\nu}) = 0. \quad (9)$$

The evolution equations themselves are quite lengthy and not very enlightening and we refer the reader to Krüger and Kokkotas (2020b) for details in the derivation and their implementation.

3 RESULTS

3.1 Universal Relations for Single Neutron Stars

As shown in Krüger and Kokkotas (2020b), our code produces results in excellent agreement with previously published values (Zink et al. (2010); Chaurasia et al. (2018)) and our convergence tests demonstrate an accuracy of the obtained frequencies of 1–2%. In this article, we will provide some highlighted results in order to demonstrate the existence of asteroseismological relations of various types and we lay out the way that one can make use of these relations in analyzing gravitational wave signals.

More specifically, we will show different universal relations providing accurate estimates for the f -mode frequency given some bulk parameters of the star and vice versa. First, we observe a universal behavior of the f -mode frequency σ_i as observed in the inertial frame as a function of the star's angular spinning frequency Ω along sequences of *fixed central energy density* models when we normalize both frequencies with the f -mode frequency σ_0 of the corresponding non-rotating star. **Figure 1** displays this behavior for more than 230 different neutron star models of each the co-rotating (i.e., stable) and counter-rotating (i.e., potentially unstable) branches of the f -mode for seven EoSs and various central energy densities (with corresponding central rest mass densities $\rho_c \in [2.2, 7.3]\rho_0$, where $\rho_0 = 2.7 \times 10^{14} \text{ g/cm}^3$ is the nuclear saturation density); we model the universal behavior using the quadratic function

$$\frac{\sigma_i}{\sigma_0} = 1 + a_1 \left(\frac{\Omega}{\sigma_0} \right) + a_2 \left(\frac{\Omega}{\sigma_0} \right)^2. \quad (10)$$

The results of a least squares fit are $a_1^u = -0.193$ and $a_2^u = -0.0294$ for the potentially unstable branch and $a_1^s = 0.220$ and $a_2^s = -0.0170$ for the stable branch of the f -mode. The quadratic fit accounts well for the increasing oblateness of the star with its rotation; however, close to the Kepler limit, deviations from this simple model become visible. As this deviation is most pronounced for the less realistic polytropic EoSs, we do not take them into account for the quadratic fits. The root mean square of the residuals is 0.024 for the counter-rotating branch and 0.048 for the co-rotating branch.

We point out that our model predicts that the unstable branch of the quadrupolar f -mode becomes susceptible to the CFS instability once the angular rotation rate of the star exceeds $\Omega \approx (3.4 \pm 0.1)\sigma_0$ (when considering sequences of constant central energy density); note that the given uncertainty is a bound, not a confidence interval. This finding regarding the critical value complements the well-known threshold of $T/|W| \approx 0.08 \pm 0.01$ in terms of the ratio of rotational to gravitational potential energy (Stergioulas and Friedman (1998); Morsink et al. (1999)), which is confirmed in our simulations and is in contrast to the widely used Newtonian result of $T/|W| \approx 0.14$.

The stable branch of the f -mode can be fitted more accurately when switching to the comoving frame and considering sequences of constant baryon mass. The frequency σ_c observed

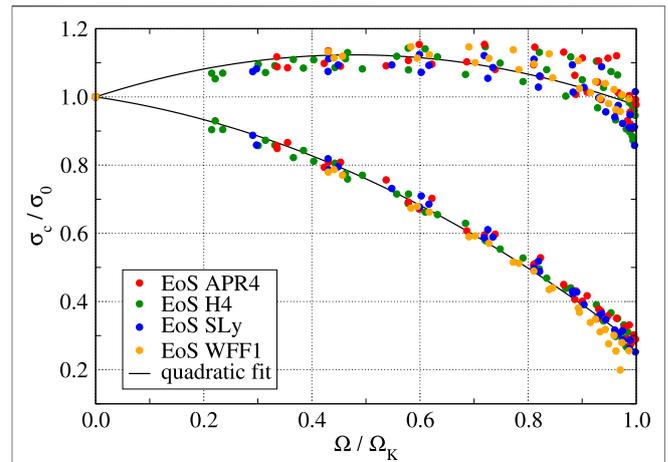


FIGURE 2 | Universal relations for the $l=|m|=2$ f -mode frequencies for sequences of constant baryon mass as observed in the comoving frame. The graph shows the results from 12 such sequences (three sequences per EoS). The stable f -mode branch displays universal behavior. Figure taken from Krüger and Kokkotas (2020a) with permission by APS.

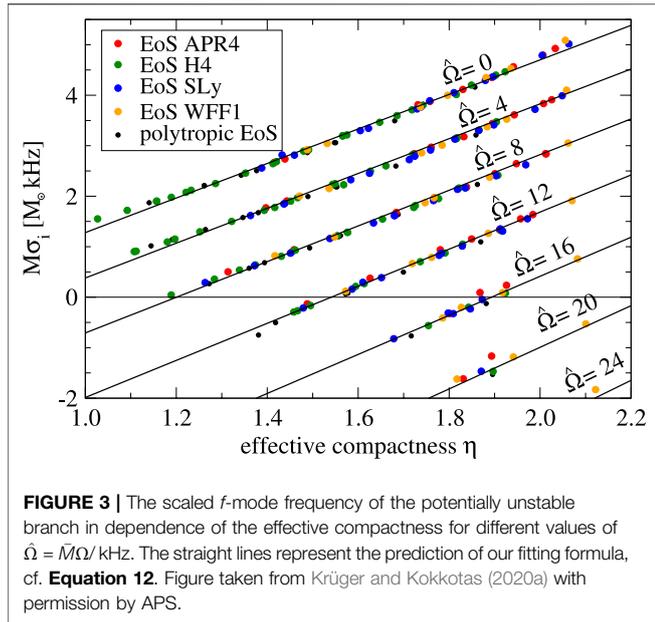
in the comoving frame is related to the frequency observed in the inertial frame via $\sigma_c = \sigma_i + m\Omega/2\pi$. We show our results for more than 120 different neutron star models using four realistic EoSs in **Figure 2**. We fit our results to the quadratic function

$$\frac{\sigma_c}{\sigma_0} = 1 + b_1 \left(\frac{\Omega}{\Omega_K} \right) + b_2 \left(\frac{\Omega}{\Omega_K} \right)^2; \quad (11)$$

note that we use the Kepler velocity Ω_K to normalize the star's rotation rate in this formula. The results of a least squares fit are $b_1^u = 0.517$ and $b_2^u = -0.542$ for the potentially unstable branch (which in the comoving frame exhibits the higher frequencies) and $b_1^s = -0.235$ and $b_2^s = -0.491$ for the stable branch of the f -mode. The root mean square of the residuals is 0.024 for the co-rotating branch and 0.051 for the counter-rotating branch.¹

In earlier studies for *non-rotating* models, fitting relations of the form $\sigma_0 = \alpha + \beta\sqrt{M_0/R_0^3}$ were derived (Andersson and Kokkotas (1998); Gaertig and Kokkotas (2011); Doneva et al. (2013)). Here, α and β can be estimated for the EoSs that fulfill the constraints at the time of observation while M_0 and R_0 correspond to the mass and radius of the non-rotating model. Thus, this relation in combination with **Eqs 10, 11** connects three fundamental parameters of the sequence, i.e., mass and radius of the non-rotating member with the spin of the observed model. Obviously, from a single observation of the f -mode frequency, one cannot extract these values but can put constraints among the three of them. Any extra observed oscillation frequency, e.g., both co- and counter-rotating frequencies or knowledge of some parameters of the star, such as its mass, will place more stringent constraints.

¹Similar relations were presented in Gaertig and Kokkotas (2011); Doneva et al. (2013) but in the Cowling approximation.



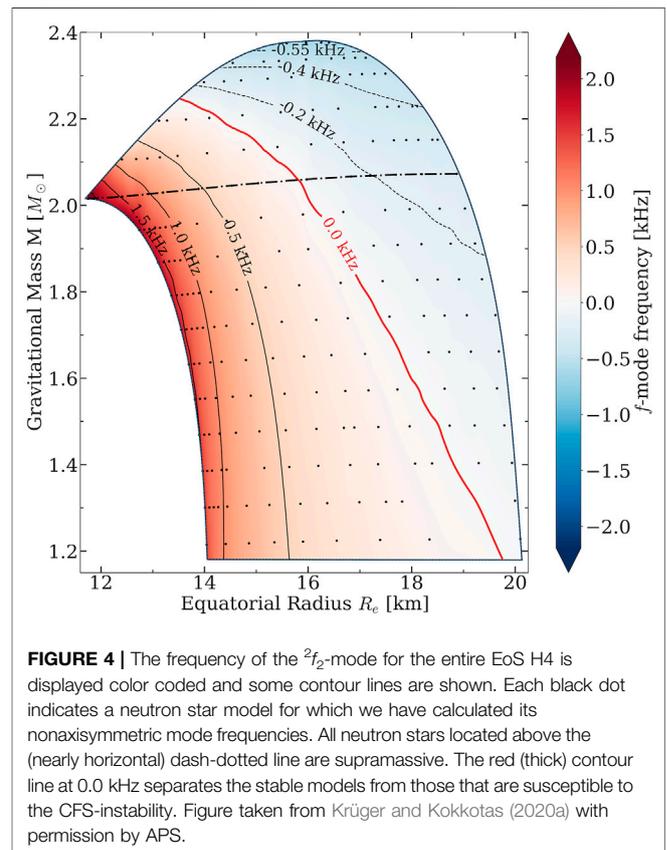
Another fitting relation which can easily be implemented in solving the inverse problem is incorporating the *effective compactness* $\eta := \sqrt{\bar{M}^3/I_{45}}$ (which is closely related to the compactness M/R), where $\bar{M} := M/M_\odot$ and $I_{45} := I/10^{45} \text{ g cm}^2$ are the star's scaled gravitational mass and moment of inertia, inspired by Lau et al. (2010). We will be guided by the model employed in the Cowling approximation by Doneva and Kokkotas (2015) which reproduces the f -mode frequency of a particular neutron star from its rotation rate, gravitational mass, and effective compactness. We propose the fitting formula

$$\hat{\sigma}_i = \left(c_1 + c_2 \hat{\Omega} + c_3 \hat{\Omega}^2 \right) + \left(d_1 + d_3 \hat{\Omega}^2 \right) \eta, \quad (12)$$

where $\hat{\sigma}_i := \bar{M}\sigma_i/\text{kHz}$ and $\hat{\Omega} := \bar{M}\Omega/\text{kHz}$; note that we set $d_2 = 0$ as it turns out that this coefficient would be afflicted with a large uncertainty. Using around 100 models based on polytropic as well as around 400 models based on realistic EoSs, the resulting coefficients from a least-squares fit for the counter-rotating branch of the f -mode are $(c_1, c_2, c_3)^u = (-2.14, -0.201, -7.68 \times 10^{-3})$ and $(d_1, d_2, d_3)^u = (3.42, 0, 1.75 \times 10^{-3})$; for the co-rotating branch, we find the coefficients $(c_1, c_2, c_3)^s = (-2.14, 0.220, -14.6 \times 10^{-3})$ and $(d_1, d_2, d_3)^s = (3.42, 0, 6.86 \times 10^{-3})$. The error in the above reported coefficients is less than 10% and the fitting formula recovers the frequencies with a deviation of less than 20%, with considerably higher accuracy (below 5%) where the f -mode frequency is larger than $\approx 500 \text{ Hz}$. We show the obtained frequencies along with the predictions from our proposed fitting formula for a few select values of $\hat{\Omega}$, spanning the parameter space up to the Kepler limit, in **Figure 3**. Qualitatively, our coefficients for the counter-rotating branch agree in order of magnitude with those published by Doneva and Kokkotas (2015) in the Cowling approximation; comparing the special case of no rotation, $\hat{\Omega} = 0$, our fitting formula yields roughly 20% lower frequencies in our fully general relativistic setup, which is in accordance with expectations.

The lines of constant $\hat{\Omega}$ in **Figure 3** may give the impression that the CFS instability operates more easily in stars with low (effective) compactness, seemingly in contrast to the finding that post-Newtonian effects tend to enhance this instability Cutler and Lindblom (1992). This paradox can be resolved by noting that relativistic effects mainly shift the f -mode frequency to lower values while the inclination of the lines of constant $\hat{\Omega}$ is largely unaltered (cf. **Figure 3** in Doneva and Kokkotas (2015)). Furthermore, while stars of lower (effective) compactness may reach the neutral point of the f -mode indeed at a lower rotation rate, this happens considerably closer to the Kepler limit (if at all) than it would do in more compact stars.

The fitting **formula (12)** has the advantage that it does not rely on specifically defined sequences of neutron stars, along which a particular property is held constant. For example, **Eq. 11** depends on the f -mode frequency σ_0 of the (in a very particular fashion) corresponding non-rotating configuration, which may not even exist in some cases (e.g., for supramassive neutron stars supported by rotation); the latter model, cf. **Eq. 12**, is satisfied with bulk properties of the star of which we want to know the oscillation frequency and vice versa. Another benefit of this formulation is that (as demonstrated by Doneva and Kokkotas (2015)) a similar formula can be derived for higher multipoles, i.e., $l \geq 3$. Fitting **formula (12)** can be useful in imposing further constraints on the parameters of the postmerger objects since it combines the mass and spin of the resulting object with the f -mode frequency and, *via* η , the



moment of inertia I or the compactness M/R . Thus, the latter two can be further constrained by an observation of an f -mode signal, as mass and potentially spin can be extracted from the premerger and early postmerger analysis of the signal. The situation becomes more attractive if both co- and counter-rotating modes or other combination of modes are observed since only the mass of the postmerger object will be needed to constrain its parameters by using only the asteroseismological relations (see Gaertig and Kokkotas (2011); Doneva and Kokkotas (2015); Vretinaris et al. (2020)). This will be an independent yet complementary constraint in the estimation of the radius in addition to those based on the Love numbers (see Dudi et al. (2018); Abbott et al. (2018); De et al. (2018); Radice et al. (2018)). As a graphical illustration of the behavior of the f -mode frequency across the entire parameter space of stable equilibrium models, we present in **Figure 4** the frequency of the counter-rotating branch as obtained from the time evolutions exemplary for EoS H4; the graph will be qualitatively similar for other EoSs. We constructed several hundred neutron star models across the entire M - R_e plane along so-called *evolutionary sequences*; those are sequences of differently fast rotating neutron stars that share the same baryon mass. The two-dimensional plane of equilibrium models has four distinct boundaries: the static limit at the lower left along which the non-rotating models are located; second, the mass-shedding limit bounds the equatorial radius to the right; next, in the top left of the graph, the limit of stability with respect to quasi-radial perturbations connects the (non-rotating) maximum mass TOV model to (approximately) the heaviest model²; equilibrium configurations that are located to the left of this line will collapse to a black hole upon quasi-radial perturbation. Last, in line with current theory and observations (Suwa et al. (2018); Martinez et al. (2015)), we limit ourselves to neutron stars with masses $M \geq 1.17M_\odot$. The dots in **Figure 4** depict the considered equilibrium models. The dash-dotted line slightly above $M = 2.0M_\odot$ is the evolutionary sequence with the baryon mass $M_0 = 2.3M_\odot$ and it separates the supramassive neutron stars (above that line) which are merely supported by centrifugal force from those (below that line) which may spin down ending up at a stable non-rotating configuration. Other evolutionary sequences can be imagined by combining the dots with lines parallel to the dash-dotted one.

The graph shows that for this particular EoS each neutron star model may become CFS-unstable if it is sufficiently spun up. For heavier neutron stars, this happens considerably below the mass-shedding limit and sufficiently heavy supramassive models (approximately $M \geq 2.25M_\odot$) will inevitably be CFS-unstable (with respect to the quadrupolar f -mode); i.e. those stars will be stabilized merely by viscous mechanisms counteracting the CFS-instability.

²It is well-known that the uniformly rotating equilibrium models with the fastest rotation rate, the highest angular momentum, or the largest gravitational mass may be distinct and it depends on the EoS whether or not those are stable with respect to quasi-radial perturbations (see Cook et al. (1994)).

3.2 Universal Relations Involving Long-Lived Remnants From BNS Mergers

Inspired by the works on universal relations for single neutron stars (Andersson and Kokkotas (1998); Benhar et al. (2004); Yagi and Yunes (2017)), the last 5 years have given rise to universal relations for BNS: they relate the pre-merger neutron stars to the early post-merger remnant, and have been developed using numerical relativity simulations (Bernuzzi et al. (2015); Rezzolla and Takami (2016); Kiuchi et al. (2020)).

These works have primarily focused on relating the tidal deformability of the pre-merger stars, which impact the dynamics of the pre-merger gravitational waves at leading order through the *reduced tidal deformability* or *binary tidal deformability* $\tilde{\Lambda}$ (Flanagan and Hinderer (2008); Favata (2014)), to various stellar parameters of the early remnant.

Recently, Vretinaris et al. (2020) investigated empirical relations for BNS mergers based on the extensive CoRe data set of numerical relativity gravitational wave simulations (Dietrich et al. (2018)). Covering a wide range of mass ratios, they find an extensive set of universal relations involving the various peak frequencies of the post-merger gravitational wave signal, involving, e.g., the chirp mass and characteristic radius of a $1.6M_\odot$ neutron star. In particular, they also find universal relations between the binary tidal deformability and the primary f -mode frequency of the post-merger signal (as in Kiuchi et al. (2020)), however, this time involving the chirp mass of the BNS. Other established universal relations connect the post-merger peak frequency to the tidal coupling constant (Bernuzzi et al. (2015); Breschi et al. (2019)).

The universal relation between the binary tidal deformability of the BNS and the stable, co-rotating f -mode frequency σ^s of the early, differentially rotating remnant proposed in Kiuchi et al. (2020), led us in Manoharan et al. (2021) to derive a similar relation for a potentially long-lived, uniformly rotating remnant: the relation takes the form

$$\log_{10} \hat{\sigma}^s = a(q) \cdot \tilde{\Lambda}^{-\frac{1}{3}} + b(q), \quad (13)$$

where $\hat{\sigma}^s = \frac{M}{M_\odot} \frac{\sigma^s}{\text{kHz}}$ is the normalized co-rotating f -mode frequency, and $q = \frac{M_1}{M_2} \leq 1$ the gravitational mass ratio of the pre-merger stars. For rapidly rotating, long-lived remnants (with rotation frequency $\tilde{\Omega} \geq 800$ Hz), this relation achieves an average relative error of 1.3%.

We also derive a relation for the potentially unstable, counter-rotating f -mode frequency of the long-lived remnant, presenting the possibility of predicting the onset of the earlier mentioned CFS-instability.

Combining these results with the universal relation, **Eq. 11**, for fast rotating neutron stars between the stable, co-rotating f -mode frequency and the effective compactness $\eta = \sqrt{\tilde{M}^3/I_{45}}$ (as defined in the previous section), we also derived a combined relation of the form.

$$\eta = \frac{10^{a(q) \cdot \tilde{\Lambda}^{-\frac{1}{3}} + b(q)} - (c_1 + c_2 \hat{\Omega} + c_3 \hat{\Omega}^2)}{d_1 + d_3 \hat{\Omega}} \quad (14)$$

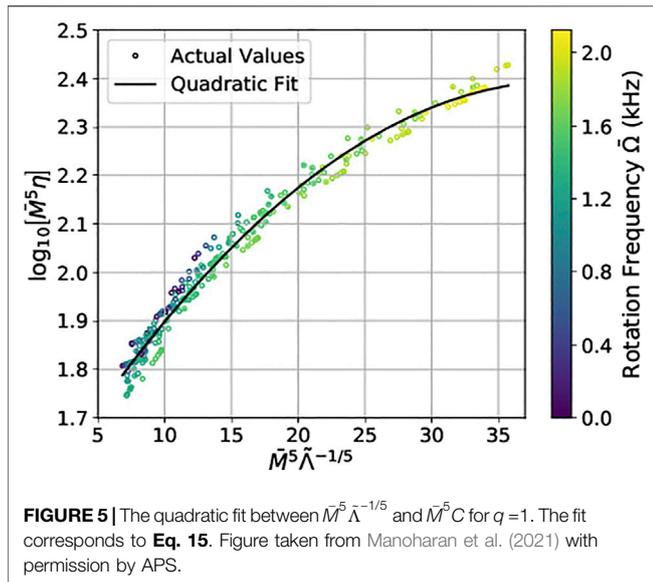


FIGURE 5 | The quadratic fit between $\bar{M}^5 \bar{\Lambda}^{-1/5}$ and $\bar{M}^5 C$ for $q=1$. The fit corresponds to Eq. 15. Figure taken from Manoharan et al. (2021) with permission by APS.

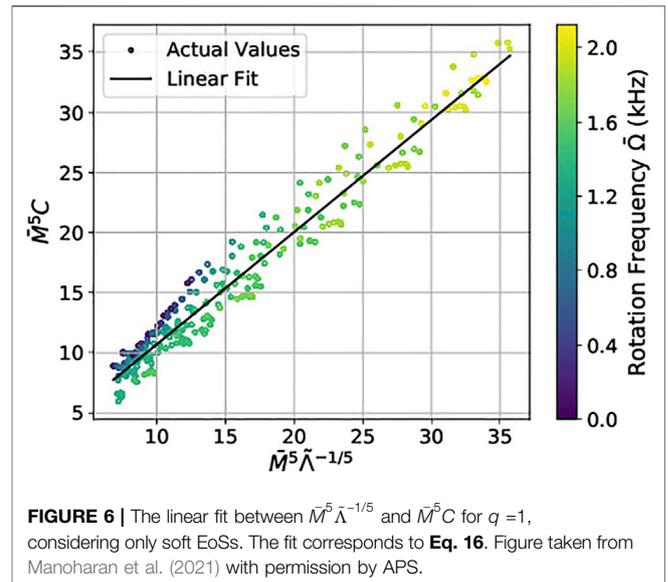


FIGURE 6 | The linear fit between $\bar{M}^5 \bar{\Lambda}^{-1/5}$ and $\bar{M}^5 C$ for $q=1$, considering only soft EoSs. The fit corresponds to Eq. 16. Figure taken from Manoharan et al. (2021) with permission by APS.

that relates the pre-merger binary tidal deformability of the BNS with the effective compactness of the long-lived remnant. For rapidly rotating remnants, this relation achieves an average relative error of 2.4%.

Finally, by directly relating these quantities without going via the f -mode, we obtain a universal relation of the form

$$\log[\bar{M}^5 \eta] = a(q) \left(\bar{M}^5 \bar{\Lambda}^{-1/5} \right)^2 + b(q) \bar{M}^5 \bar{\Lambda}^{-1/5} + c(q). \quad (15)$$

This relation achieves improved accuracy, reaching an average relative error of $\sim 1.5\%$ for remnants with any rotation frequency. We show the quadratic fit in Figure 5 for the symmetric case of $q = 1$.

We finally also consider a direct relation between the binary tidal deformability and the compactness $C = M/R$ of the long-lived remnant. Such a relation would allow the direct estimation of the remnant’s radius R using independent estimates of its gravitational mass. We propose a relation of the form

$$\bar{M}^5 C = a(q) \bar{M}^5 \bar{\Lambda}^{-1/5} + b(q) \quad (16)$$

which, however, only achieves an accuracy an order of magnitude worse than for the effective compactness relation, reaching an average relative error of $\sim 8.8\%$. A graphic representation of this fit along with the data is shown in Figure 6.

The results presented in Manoharan et al. (2021) represent a first step towards finding universal relations between the pre-merger neutron stars and the potential long-lived remnant of a BNS merger using perturbative calculations. Our approach can be freely extended to, e.g., hot EoSs, phase transitions, as well as differential rotation for the remnant to, to cover, e.g., earlier parts of the post-merger phase.

The various functional expressions for $a(q)$, $b(q)$ and $c(q)$ used in Equations 13 and 14, 15, 16 as well as the coefficients c_1 , c_2 , c_3 , d_1 and d_2 are given in Manoharan et al. (2021).

4 APPROACHING THE INVERSE PROBLEM

Having laid out the procedure to extract fluid oscillation frequencies from time evolution in a general relativistic framework and having developed various universal relations between bulk properties of neutron stars and their f -mode frequencies, we now turn to one of the possible applications. The field of gravitational wave asteroseismology attempts to invert the above outlined procedure and aims to constrain bulk properties of neutron stars such as mass and radius given one or more oscillations frequencies. The above derived universal relations are a valuable tool for this as they do not depend on the underlying and hitherto poorly understood nuclear equation of state. However, even without universal relations at hand, the inverse problem can be approached.

We will summarise an approach of the inverse problem using Bayesian methods as laid out by Völkel et al. (2021). Here, the fundamental idea is tested on the f -mode frequency, but it may easily be extended to other fluid modes or combined with their damping times. The use of the Bayesian framework is advantageous (compared to, e.g., an analytical inversion of the non-linear universal relation) as it directly allows to incorporate error bars of the measurement of the f -mode into the calculation. Besides the Bayesian approach and universal relations, there are in principle also semi-classical techniques, e.g., WKB theory, which can be used to address the inverse spectrum problem in simplified cases; see Völkel and Kokkotas (2019), for using axial w -modes of spherically symmetric neutron stars and Völkel and Kokkotas (2017) using a similar approach for ultra compact stars.

In a first step, we pick an EoS and a particular (rotating) neutron star model for which we then calculate two f -mode frequencies. We assume those to have a relative error of 3% (Gaussian), while our prior knowledge on M and R is uninformative. Two data points should in principle suffice to uniquely pinpoint one neutron star model (assuming a cold EoS). We show the result of the Bayesian analysis in Figure 7. The

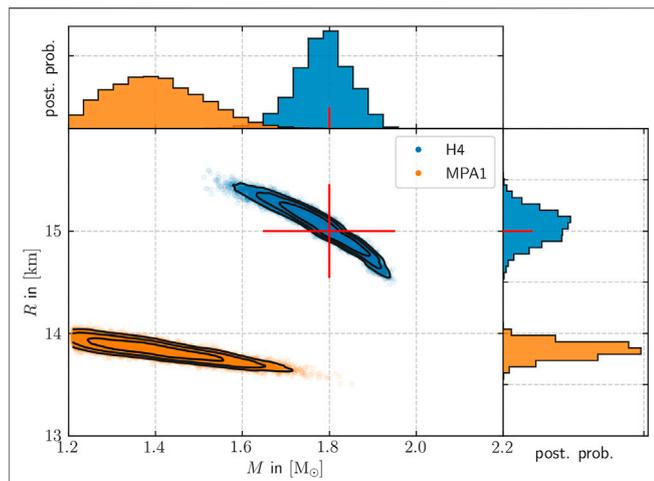


FIGURE 7 | Here we compare the EoS method assuming the H4 EoS (blue) and the MPA1 EoS (orange). The diagonal panels show the sampled posterior distribution of M and R , while the off diagonal panel combines a scatter plot with logarithmic contour lines. The red cross and red line indicate the true H4 parameters that belong to the assumed 3% f -mode data. For both cases we assume that the mass is known by 10%. Figure taken from Völkel et al. (2021) with permission by APS.

initially chosen model is depicted by the red cross at $M = 1.8 M_{\odot}$ and $R = 15$ km. The blue shaded area in the big panel shows the correlations of mass and radius during sampling, whose posterior distributions are shown in the side panels. While the radius of the star can be reconstructed with fairly small error bars, the reconstructed mass is considerably less constrained. Nonetheless, the peak of the probability distribution nicely resembles the initially chosen model. If the f -mode frequencies were to be known more accurately, the corresponding error bars of reconstructed mass and radius would be smaller, too.

This analysis required the assumption of a particular EoS and so far we used the same EoS to reconstruct the model that we also used to generate the underlying neutron star model. If we use a different EoS to perform the Bayesian analysis, we will obviously reconstruct a different model; this is shown in orange in **Figure 7**. The reconstructed star in this case is considerably smaller and lighter. Note that the apparent cut of the MPA1 EoS posteriors (in particular for the mass) is not an indication that this EoS has not been used for the injection, since a similar behavior can also be found for the H4 EoS, if the injected parameters are closer to the edge of the H4 EoS neutron star parameter space. One needs further information, such as the precise mass, the tidal deformability, or perhaps a damping time, in order to rule out this EoS.

Instead of making an educated guess for the “correct” EoS and reconstructing mass and radius as just described, we may also use the universal relation proposed in **Eq. 12**. We will not need to assume any EoS but the universal relation will provide us—given two f -mode frequencies and a good estimate of the star’s mass—with an estimate for the stars’s rotation rate Ω and its effective compactness η .

We again use the Bayesian method to invert the universal relation and we apply it to the same model as in the previous analysis; the H4 model with $M = 1.8 M_{\odot}$ and $R = 15$ km. In **Figure 8** we show the results of the analysis. On the x -axis, we show the star’s effective compactness and rotation rate, both normalised to the actual value corresponding to the equilibrium configuration. The two graphs then show the posterior distributions. We ran the same analysis twice, once assuming a 30% relative error on the prior mass (solid lines) and once assuming a 10% relative error (dashed lines). As expected, with a less informed prior on the mass, the posterior distribution for the desired quantities have a considerably larger variance.

The blue and orange curves correspond to the previously describe method where we assume a particular EoS. We have described this above and here we show the posteriors for η and $1/\Omega$. The green curves show the posterior distribution employing the universal relation.

It is evident that the H4 EoS method (blue lines) and universal relation (UR) method (green lines) yield very similar results for the rotation rate Ω , while assuming the MPA1 EoS (orange lines) indicates a value that is larger than the correct one. Note that both observations hold independent of the specific prior knowledge of M assumed here (30 and 10%).

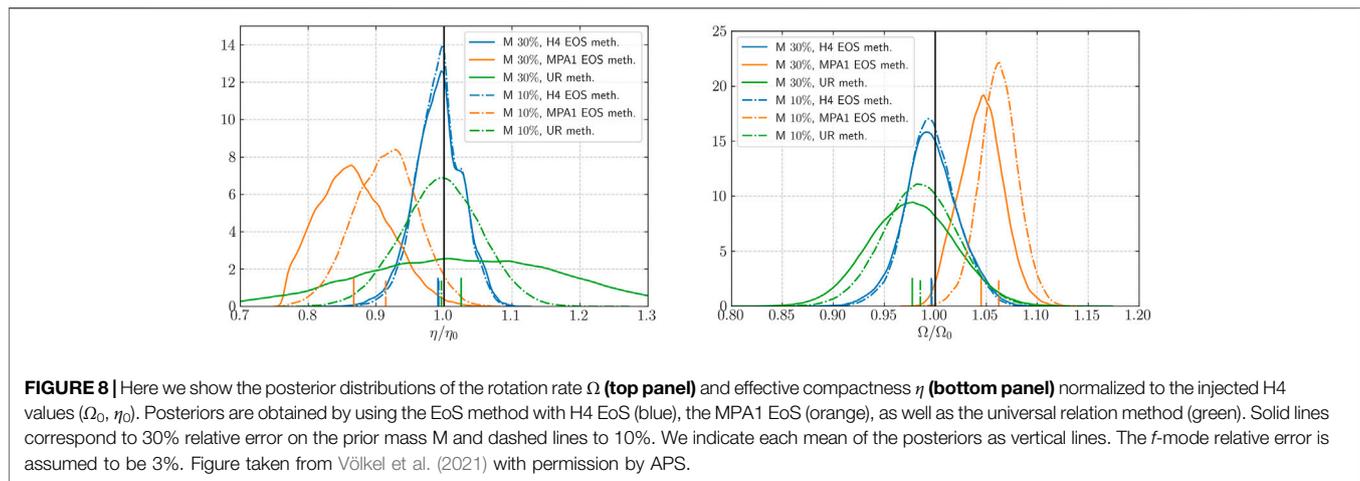
The situation for the effective compactness η is qualitatively different. First, the prior knowledge of M plays a big role for the UR method, but is less important for the EoS methods. For those we find that the correctly assumed H4 EoS is almost independent of uncertainties in M , while the posterior distribution obtained by the MPA1 EoS is shifted. Note that the rather different scaling behavior of the UR method is in agreement with the findings described above.

Finally, while the posteriors of Ω are very smooth, one observes small “bumps” for the H4 EoS, e.g., at $\eta/\eta_0 \approx 1.02$. We have verified that this likely is an artifact from the finite resolution and particular range of the used H4 f -mode data that is available to us. This directly sets the scale of how precise our currently implemented EoS data can be used to resolve the underlying parameters, which is of order percent level.

5 SUMMARY AND OUTLOOK

We report the first extraction of frequencies of the $l = |m| = 2$ f -mode of general relativistic, rapidly rotating neutron stars *without* the commonly used *slow-rotation* or *Cowling approximation* to an extent that allows us to generalize the findings into universal relations. This concludes a long-standing open problem, building upon the effort from numerous studies throughout the past 5 decades.

For this, we have derived a set of time evolution equations governing the perturbations of the fluid of a rapidly rotating neutron star as well as its surrounding spacetime, derived in a perturbative framework in full general relativity. We have opted for the Hilbert gauge in order to arrive at a set of fully hyperbolic equations for the spacetime perturbations whose implementation does not pose major obstacles.



Convergence tests to study the accuracy of our code reveal that in a typical simulation, the obtained frequency usually deviates less than 1–2% from the limiting value or has an accuracy of about 10 Hz. As we model axisymmetric configurations only, we can reduce the problem to two spatial dimensions which drastically lowers the computational expense of our numerical time evolutions in comparison to non-linear codes that perform three-dimensional simulations. The evolution of the perturbations of a neutron star for 15 ms on a grid with $3,120 \times 50$ points, which facilitates a decent frequency resolution, requires only a few dozen of CPU hours; this enables us to study broad ranges of parameters and various EoSs. We expect to further reduce the computational cost even further by deriving a simplified set of perturbation equations.

As a result, we provide different universal relations for the frequencies of the $l = |m| = 2$ f -modes of uniformly rotating neutron stars at zero temperature which are independent of the EoS; the proposed formulae are calibrated to several hundred neutron star models that are constructed using both polytropic and realistic EoSs and are scattered across the entire parameter space of equilibrium solutions. Furthermore, it is possible to link the pre-merger binary tidal deformability to the effective compactness of the late post-merger remnant in a universal manner, too. Such universal relations will be an essential piece in the asteroseismological toolkit once the third-generation GW observatories will be able to pick up the ring-down and fluid ringing signal following the merger of a binary neutron star system; they allow to solve the inverse problem, leading to significantly tighter constraints for mass and radius of the postmerger object. For this task, it is elementary to have a smorgasbord of universal relations at hand, which allows to make a practical choice, depending on which observables are available or in which of the star's properties one is interested. We will extend the present list of such universal relations in future articles, utilizing different combinations of bulk properties of the star; while we may obviously be (and already have been) inspired by previously published fitting formulae that were derived using different approximative frameworks, we need to be open-minded about models involving novel combinations of observables.

We also report the discovery of an accurate estimate for the onset of the CFS-instability when the f -mode frequency of the

non-rotating member of the family is known and verified the corresponding critical value of $T/|W|$.

A natural extension of the present study will be a more comprehensive investigation of the spectrum of neutron stars (i.e. higher multipole f -modes, low p -modes and g -modes as well as w -modes) which may be excited in different astrophysical processes. Furthermore, we are going to extend our code to account for differentially rotating neutron stars and hot EoSs that are particularly relevant for nascent neutron stars or postmerger configurations in the immediate aftermath of a binary merger, both of which will have a considerable impact on the vibration frequencies or the onset of the CFS-instability (and *via* two further scaling parameters also on the universal relations) during a very short but dynamic interval of their lives.

Furthermore, a precise knowledge of the spectrum of compact objects is invaluable as also isolated neutron stars as well as those in inspiraling binary systems may possess high spin rates and various oscillation modes, which may be excited, e.g., via glitches or tidal coupling, may impact their electromagnetic emission or the dynamic of the whole system.

AUTHOR CONTRIBUTIONS

CK and KK contributed the majority of the manuscript. PM contributes the section on “Universal relations involving long-lived remnants from BNS mergers”. SV contributes the section on “Approaching the Inverse Problem”.

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REFERENCES

- Abbott, B. P., Abbott, R., Abbott, T. D., Acernese, F., Ackley, K., Adams, C., et al. (2018). Erratum: Photothermal Engineering of Graphene Plasmons. *Phys. Rev. Lett.* 121, 161101. doi:10.1103/PhysRevLett.121.161101
- Allen, G., Andersson, N., Kokkotas, K. D., and Schutz, B. F. (1998). Gravitational Waves from Pulsating Stars: Evolving the Perturbation Equations for a Relativistic star. *Phys. Rev. D* 58, 124012. doi:10.1103/physrevd.58.124012
- Andersson, N. (1998). A New Class of Unstable Modes of Rotating Relativistic Stars. *ApJ* 502, 708–713. doi:10.1086/305919
- Andersson, N., and Kokkotas, K. D. (1996). Gravitational Waves and Pulsating Stars: What Can We Learn from Future Observations? *Phys. Rev. Lett.* 77, 4134–4137. doi:10.1103/physrevlett.77.4134
- Andersson, N., Kokkotas, K. D., and Schutz, B. F. (1995). Monthly Notices of the Royal Astronomical Society. *MNRAS* 274, 1039. doi:10.1093/mnras/274.4.1039
- Andersson, N., and Kokkotas, K. D. (1998). Towards Gravitational Wave Asteroseismology. *Monthly Notices R. Astronomical Soc.* 299, 1059–1068. doi:10.1046/j.1365-8711.1998.01840.x
- Andersson, N., and Pnigouras, P. (2021). The Phenomenology of Dynamical Neutron star Tides. *MNRAS* 503, 533–539. doi:10.1093/mnras/stab371
- Arnowitz, R., Deser, S., and Misner, C. W. (1962). “The Dynamics of General Relativity,” in *Gravitation: An Introduction to Current Research*. Editor L. Witten (New York: Wiley), 227–265.
- Baiotti, L., Bernuzzi, S., Corvino, G., de Pietri, R., and Nagar, A. (2009). Gravitational-wave Extraction from Neutron-star Oscillations: Comparing Linear and Nonlinear Techniques. *Phys. Rev. D* 79, 024002, 2009. arXiv: 0808.4002 [gr-qc]. doi:10.1103/physrevd.79.024002
- Baiotti, L., Giacomazzo, B., and Rezzolla, L. (2008). Accurate Evolutions of Inspiral Neutron-star Binaries: Assessment of the Truncation Error. *Phys. Rev. D* 78, 084033. doi:10.1103/physrevd.78.084033
- Battiston, L., Cazzola, P., and Lucaroni, L. (1971). Stability of Nonradial Oscillations of Cold Nonrotating Neutron Stars. *Nuov Cim B* 3, 295–318. doi:10.1007/bf02815343
- Bauswein, A., and Janka, H.-T. (2012). Measuring Neutron-star Properties via Gravitational Waves from Neutron-star Mergers. *Phys. Rev. Lett.* 108, 011101. doi:10.1103/physrevlett.108.011101
- Bauswein, A., Just, O., Janka, H.-T., and Stergioulas, N. (2017). Neutron-star Radius Constraints from GW170817 and Future Detections. *ApJ* 850, L34. doi:10.3847/2041-8213/aa9994
- Bauswein, A., Stergioulas, N., and Janka, H.-T. (2014). Revealing the High-Density Equation of State through Binary Neutron star Mergers. *Phys. Rev. D* 90, 023002. doi:10.1103/physrevd.90.023002
- Benhar, O., Ferrari, V., and Gualtieri, L. (2004). Gravitational Wave Asteroseismology Reexamined. *Phys. Rev. D* 70, 124015. doi:10.1103/physrevd.70.124015
- Bernuzzi, S., Dietrich, T., and Nagar, A. (2015). Modeling the Complete Gravitational Wave Spectrum of Neutron Star Mergers. *Phys. Rev. Lett.* 115, 091101. doi:10.1103/physrevlett.115.091101
- Bernuzzi, S., Dietrich, T., Tichy, W., and Brügmann, B. (2014). Mergers of Binary Neutron Stars with Realistic Spin. *Phys. Rev. D* 89, 104021, 2014. arXiv: 1311.4443 [gr-qc]. doi:10.1103/physrevd.89.104021
- Boutloukos, S., and Nollert, H.-P. (2007). Eigenmode Frequency Distribution of Rapidly Rotating Neutron Stars. *Phys. Rev. D* 75, 043007. doi:10.1103/physrevd.75.043007
- Breschi, M., Bernuzzi, S., Zappa, F., Agathos, M., Perego, A., Radice, D., et al. (2019). KiloHertz Gravitational Waves from Binary Neutron star Remnants: Time-Domain Model and Constraints on Extreme Matter. *Phys. Rev. D* 100, 104029, 2019. arXiv:1908.11418 [gr-qc]. doi:10.1103/physrevd.100.104029
- Chandrasekhar, S., and Ferrari, V. (1991a). On the Non-radial Oscillations of a star. *Proc. R. Soc. Lond. A* 432, 247–279. doi:10.1098/rspa.1991.0016
- Chandrasekhar, S., and Ferrari, V. (1991b). On the Non-radial Oscillations of a star. III. A Reconsideration of the Axial Modes. *Proc. R. Soc. Lond. A* 434, 449–457. doi:10.1098/rspa.1991.0104
- Chandrasekhar, S. (1970). Solutions of Two Problems in the Theory of Gravitational Radiation. *Phys. Rev. Lett.* 24, 611–615. doi:10.1103/physrevlett.24.611
- Chaurasia, S. V., Dietrich, T., Johnson-McDaniel, N. K., Ujevic, M., Tichy, W., and Brügmann, B. (2018). Gravitational Waves and Mass Ejecta from Binary Neutron star Mergers: Effect of Large Eccentricities. *Phys. Rev. D* 98, 104005. doi:10.1103/physrevd.98.104005
- Chirenti, C., Gold, R., and Miller, M. C. (2017). Gravitational Waves from F-Modes Excited by the Inspiral of Highly Eccentric Neutron Star Binaries. *ApJ* 837, 67. doi:10.3847/1538-4357/aa5ebb
- Chirenti, C., Miller, M. C., Strohmayer, T., and Camp, J. (2019). Searching for Hypermassive Neutron Stars with Short Gamma-Ray Bursts. *ApJ* 884, L16. doi:10.3847/2041-8213/ab43e0
- Cook, G. B., Shapiro, S. L., and Teukolsky, S. A. (1994). Rapidly Rotating Neutron Stars in General Relativity: Realistic Equations of State. *ApJ* 424, 823. doi:10.1086/173934
- Cowling, T. G. (1941). The Non-radial Oscillations of Polytopic Stars. *Monthly Notices R. Astronomical Soc.* 101, 367–375. doi:10.1093/mnras/101.8.367
- Cutler, C., and Lindblom, L. (1992). Post-newtonian Frequencies for the Pulsations of Rapidly Rotating Neutron Stars. *ApJ* 385, 630. doi:10.1086/170968
- De Pietri, R., Feo, A., Font, J. A., Löffler, F., Maione, F., Pasquali, M., et al. (2018). Convective Excitation of Inertial Modes in Binary Neutron Star Mergers. *Phys. Rev. Lett.* 120, 221101. doi:10.1103/physrevlett.120.221101
- De, S., Finstad, D., Lattimer, J. M., Brown, D. A., Berger, E., and Biwer, C. M. (2018). *Phys. Rev. Lett.* 121, 091102. doi:10.1103/physrevlett.121.259902
- Detweiler, S., and Lindblom, L. (1985). On the Nonradial Pulsations of General Relativistic Stellar Models. *ApJ* 292, 12. doi:10.1086/163127
- Dietrich, T., Radice, D., Bernuzzi, S., Zappa, F., Perego, A., Brügmann, B., et al. (2018). CoRe Database of Binary Neutron star Merger Waveforms. *Class. Quan. Grav.* 35, 24LT01. doi:10.1088/1361-6382/aaebc0
- Dimmelmeier, H., Stergioulas, N., and Font, J. A. (2006). Non-linear Axisymmetric Pulsations of Rotating Relativistic Stars in the Conformal Flatness Approximation. *Monthly Notices R. Astronomical Soc.* 368, 1609–1630. doi:10.1111/j.1365-2966.2006.10274.x
- Doneva, D. D., Gaertig, E., Kokkotas, K. D., and Krüger, C. (2013). Gravitational Wave Asteroseismology of Fast Rotating Neutron Stars with Realistic Equations of State. *Phys. Rev. D* 88, 044052. doi:10.1103/physrevd.88.044052
- Doneva, D. D., and Kokkotas, K. D. (2015). Asteroseismology of Rapidly Rotating Neutron Stars: An Alternative Approach. *Phys. Rev. D* 92, 124004. doi:10.1103/physrevd.92.124004
- Doneva, D. D., Kokkotas, K. D., and Pnigouras, P. (2015). Gravitational Wave Afterglow in Binary Neutron star Mergers. *Phys. Rev. D* 92, 104040. doi:10.1103/physrevd.92.104040
- Dudi, R., Pannarale, F., Dietrich, T., Hannam, M., Bernuzzi, S., Ohme, F., et al. (2018). Relevance of Tidal Effects and post-merger Dynamics for Binary Neutron star Parameter Estimation. *Phys. Rev. D* 98, 084061. doi:10.1103/physrevd.98.084061
- Favata, M. (2014). Systematic Parameter Errors in Inspiral Neutron Star Binaries. *Phys. Rev. Lett.* 112, 101101. doi:10.1103/physrevlett.112.101101
- Flanagan, É. É., and Hinderer, T. (2008). Constraining Neutron star Tidal Love Numbers with Gravitational Wave Detectors. *Phys. Rev. D* 77, 021502. doi:10.1103/physrevd.77.021502
- Font, J. A., Dimmelmeier, H., Gupta, A., and Stergioulas, N. (2001). Axisymmetric Modes of Rotating Relativistic Stars in the Cowling Approximation. *Monthly Notices R. Astronomical Soc.* 325, 1463–1470. doi:10.1046/j.1365-8711.2001.04555.x
- Font, J. A., Goodale, T., Iyer, S., Miller, M., Rezzolla, L., Seidel, E., et al. (2002). Three-dimensional General Relativistic Hydrodynamics. 2. Long Term Dynamics of Single Relativistic Stars. *Phys. Rev. D* 65, 084024. doi:10.1103/physrevd.65.084024
- Font, J. A., Stergioulas, N., and Kokkotas, K. D. (2000). Non-linear Hydrodynamical Evolution of Rotating Relativistic Stars: Numerical Methods and Code Tests. *MNRAS* 313, 678–688. doi:10.1046/j.1365-8711.2000.03254.x
- Friedman, J. L., and Morsink, S. M. (1998). Axial Instability of Rotating Relativistic Stars. *ApJ* 502, 714–720. doi:10.1086/305920
- Friedman, J. L., and Schutz, B. F. (1975). Gravitational Radiation Instability in Rotating Stars. *ApJ* 199, L157. doi:10.1086/181872
- Friedman, J. L., and Schutz, B. F. (1978). Secular Instability of Rotating Newtonian Stars. *ApJ* 222, 281. doi:10.1086/156143
- Fuller, J., and Lai, D. (2011). Tidal Excitations of Oscillation Modes in Compact white dwarf Binaries. *MNRAS* 41b2, 1331. doi:10.1111/j.1365-2966.2010.18017.x

- Gaertig, E., and Kokkotas, K. D. (2011). Gravitational Wave Asteroseismology with Fast Rotating Neutron Stars. *Phys. Rev. D* 83, 064031. doi:10.1103/physrevd.83.064031
- Gaertig, E., and Kokkotas, K. D. (2008). Oscillations of Rapidly Rotating Relativistic Stars. *Phys. Rev. D* 78, 064063. doi:10.1103/physrevd.78.064063
- Gaertig, E., and Kokkotas, K. D. (2009). Relativistic G-Modes in Rapidly Rotating Neutron Stars. *Phys. Rev. D* 80, 064026. doi:10.1103/physrevd.80.064026
- Glampedakis, K., and Gualtieri, L. (2018). "Gravitational Waves from Single Neutron Stars: An Advanced Detector Era Survey," in *Astrophysics and Space Science Library, Astrophysics and Space Science Library*. Editors L. Rezzolla, P. Pizzochero, D. I. Jones, N. Rea, and I. Vidaña, 673–736. doi:10.1007/978-3-319-97616-7_12
- Gold, R., Bernuzzi, S., Thierfelder, M., Brüggmann, B., and Pretorius, F. (2012). Eccentric Binary Neutron star Mergers. *Phys. Rev. D* 86, 121501, 2012. arXiv:1109.5128 [gr-qc]. doi:10.1103/physrevd.86.121501
- Hartle, J. B. (1967). Slowly Rotating Relativistic Stars. I. Equations of Structure. *ApJ* 150, 1005. doi:10.1086/149400
- Hotokezaka, K., Kiuchi, K., Kyutoku, K., Muranushi, T., Sekiguchi, Y.-i., Shibata, M., et al. (2013). Remnant Massive Neutron Stars of Binary Neutron star Mergers: Evolution Process and Gravitational Waveform. *Phys. Rev. D* 88, 044026, 2013. arXiv:1307.5888 [astro-ph.HE]. doi:10.1103/physrevd.88.044026
- Kastaun, W. (2006). High-resolution Shock Capturing Scheme for Ideal Hydrodynamics in General Relativity Optimized for Quasistationary Solutions. *Phys. Rev. D* 74, 124024. doi:10.1103/physrevd.74.124024
- Kastaun, W. (2008). Inertial Modes of Rigidly Rotating Neutron Stars in Cowling Approximation. *Phys. Rev. D* 77, 124019. doi:10.1103/physrevd.77.124019
- Kiuchi, K., Kawaguchi, K., Kyutoku, K., Sekiguchi, Y., and Shibata, M. (2020). Sub-radian-accuracy Gravitational Waves from Coalescing Binary Neutron Stars in Numerical Relativity. II. Systematic Study on the Equation of State, Binary Mass, and Mass Ratio. *Phys. Rev. D* 101, 084006. doi:10.1103/physrevd.101.084006
- Kojima, Y. (1992). Equations Governing the Nonradial Oscillations of a Slowly Rotating Relativistic star. *Phys. Rev. D* 46, 4289–4303. doi:10.1103/physrevd.46.4289
- Kokkotas, K. D., Apostolatos, T. A., and Andersson, N. (2001). The Inverse Problem for Pulsating Neutron Stars: a 'fingerprint Analysis' for the Supranuclear Equation of State. *Monthly Notices R. Astronomical Soc.* 320, 307–315. doi:10.1046/j.1365-8711.2001.03945.x
- Kokkotas, K. D., and Schäfer, G. (1995). Tidal and Tidal-Resonant Effects in Coalescing Binaries. *MNRAS* 275, 301–308. doi:10.1093/mnras/275.2.301
- Kokkotas, K. D., and Schutz, B. F. (1986). Normal Modes of a Model Radiating System. *Gen. Relat. Gravit.* 18, 913–921. doi:10.1007/bf00773556
- Kokkotas, K. D., and Schutz, B. F. (1992). W-modes: a New Family of normal Modes of Pulsating Relativistic Stars. *MNRAS* 255, 119–128. doi:10.1093/mnras/255.1.119
- Kreiss, H.-O., and Olinger, J. (1973). *Methods for the Approximate Solution of Time Dependent Problems, GARP Publications Series*. Geneva: World Meteorological Organization.
- Krüger, C., Gaertig, E., and Kokkotas, K. D. (2010). Oscillations and Instabilities of Fast and Differentially Rotating Relativistic Stars. *Phys. Rev. D* 81, 084019. doi:10.1103/physrevd.81.084019
- Krüger, C. J., and Kokkotas, K. D. (2020a). Fast Rotating Relativistic Stars: Spectra and Stability without Approximation. *Phys. Rev. Lett.* 125, 111106. doi:10.1103/physrevlett.125.111106
- Krüger, C. J., and Kokkotas, K. D. (2020b). High Energy Physics - Phenomenology. *Phys. Rev. D* 102, 064026. doi:10.1103/PhysRevD.102.064026
- Kuan, H.-J., Suvorov, A. G., and Kokkotas, K. D. (2021b). *General-relativistic Treatment of Tidal G-Mode Resonances in Coalescing Binaries of Neutron Stars. II. As Triggers for Precursor Flares of Short Gamma-ray Bursts*. arXiv e-prints, arXiv:2107.00533. arXiv:2107.00533 [astro-ph.HE].
- Kuan, H.-J., Suvorov, A. G., and Kokkotas, K. D. (2021a). *General-relativistic Treatment of Tidal G-Mode Resonances in Coalescing Binaries of Neutron Stars. I. Theoretical Framework and Crust Breaking*. arXiv e-prints, arXiv:2106.16123. arXiv:2106.16123 [gr-qc].
- Lai, D. (1994). Resonant Oscillations and Tidal Heating in Coalescing Binary Neutron Stars. *MNRAS* 270, 611–629. doi:10.1093/mnras/270.3.611
- Lau, H. K., Leung, P. T., and Lin, L. M. (2010). Inferring Physical Parameters of Compact Stars from Their-Mode Gravitational Wave Signals. *ApJ* 714, 1234–1238. doi:10.1088/0004-637x/714/2/1234
- Leaver, E. W. (1985). An Analytic Representation for the Quasi-normal Modes of Kerr Black Holes. *Proc. R. Soc. Lond. A* 402, 285–298. doi:10.1098/rspa.1985.0119
- Lehner, L., Liebling, S. L., Palenzuela, C., Caballero, O. L., O'Connor, E., Anderson, M., et al. (2016). Unequal Mass Binary Neutron star Mergers and Multimessenger Signals. *Class. Quant. Grav.* 33, 184002, 2016. arXiv:1603.00501 [gr-qc]. doi:10.1088/0264-9381/33/18/184002
- Leins, M., Nollert, H.-P., and Soffel, M. H. (1993). Nonradial Oscillations of Neutron Stars: A New branch of Strongly Damped normal Modes. *Phys. Rev. D* 48, 3467–3472. doi:10.1103/physrevd.48.3467
- Lindblom, L., and Detweiler, S. L. (1983). The Quadrupole Oscillations of Neutron Stars. *ApJS* 53, 73. doi:10.1086/190884
- Lockitch, K. H., Andersson, N., and Friedman, J. L. (2000). The Rotational Modes of Relativistic Stars: Analytic Results. *Phys. Rev. D* 63, 024019. doi:10.1103/physrevd.63.024019
- Lockitch, K. H., Friedman, J. L., and Andersson, N. (2003). Rotational Modes of Relativistic Stars: Numerical Results. *Phys. Rev. D* 68, 124010. doi:10.1103/physrevd.68.124010
- Manoharan, P., Krüger, C. J., and Kokkotas, K. D. (2021). Universal Relations for Binary Neutron Star Mergers with Long-Lived Remnants. *Phys. Rev. D* 103, 11. doi:10.1103/PhysRevD.103.023005
- Martinez, J. G., Stovall, K., Freire, P. C. C., Deneva, J. S., Jenet, F. A., McLaughlin, M. A., et al. (2015). Pulsar J0453+1559: a Double Neutron Star System with a Large Mass Asymmetry. *ApJ* 812, 143. doi:10.1088/0004-637x/812/2/143
- Morsink, S. M., Stergioulas, N., and Blattnig, S. R. (1999). Quasi-normal Modes of Rotating Relativistic Stars: Neutral Modes for Realistic Equations of State. *ApJ* 510, 854–861. doi:10.1086/306630
- Nozawa, T., Stergioulas, N., Gourgoulhon, E., and Eriguchi, Y. (1998). Construction of Highly Accurate Models of Rotating Neutron Stars - Comparison of Three Different Numerical Schemes. *Astron. Astrophys. Suppl. Ser.* 132, 431–454. doi:10.1051/aas:1998304
- Paschalidis, V., and Stergioulas, N. (2017). Rotating Stars in Relativity. *Living Rev. Relativ* 20, 7. doi:10.1007/s41114-017-0008-x
- Passamonti, A., Stavridis, A., and Kokkotas, K. D. (2008). Nonaxisymmetric Oscillations of Differentially Rotating Relativistic Stars. *Phys. Rev. D* 77, 024029. doi:10.1103/physrevd.77.024029
- Pratten, G., Schmidt, P., and Hinderer, T. (2020). Gravitational-wave Asteroseismology with Fundamental Modes from Compact Binary Inspirals. *Nat. Commun.* 11, 2553. doi:10.1038/s41467-020-15984-5
- Priou, D. (1992). The Perturbations of a Fully General Relativistic and Rapidly Rotating Neutron star - I. Equations of Motion for the Solid Crust. *Monthly Notices R. Astronomical Soc.* 254, 435–452. doi:10.1093/mnras/254.3.435
- Radice, D., Perego, A., Zappa, F., and Bernuzzi, S. (2018). GW170817: Joint Constraint on the Neutron Star Equation of State from Multimessenger Observations. *ApJ* 852, L29. doi:10.3847/2041-8213/aaa402
- Read, J. S., Lackey, B. D., Owen, B. J., and Friedman, J. L. (2009). Constraints on a Phenomenologically Parametrized Neutron-star Equation of State. *Phys. Rev. D* 79, 124032. doi:10.1103/physrevd.79.124032
- Regge, T., and Wheeler, J. A. (1957). Stability of a Schwarzschild Singularity. *Phys. Rev.* 108, 1063–1069. doi:10.1103/physrev.108.1063
- Rezzolla, L., and Takami, K. (2016). Gravitational-wave Signal from Binary Neutron Stars: A Systematic Analysis of the Spectral Properties. *Phys. Rev. D* 93, 124051. doi:10.1103/physrevd.93.124051
- Ruoff, J. (2001). New Approach to the Evolution of Neutron star Oscillations. *Phys. Rev. D* 63, 064018. doi:10.1103/physrevd.63.064018
- Ruoff, J., Stavridis, A., and Kokkotas, K. D. (2002). Evolution Equations for the Perturbations of Slowly Rotating Relativistic Stars. *Monthly Notices R. Astronomical Soc.* 332, 676–688. doi:10.1046/j.1365-8711.2002.05329.x
- Ruoff, J. (2000). *The Numerical Evolution Of Neutron Star Oscillations*. Tübingen: Eberhard Karls Universität Tübingen. Ph.D. thesis.
- Shibata, M., and Taniguchi, K. (2006). Merger of Binary Neutron Stars to a Black Hole: Disk Mass, Short Gamma-ray Bursts, and Quasinormal Mode Ringing. *Phys. Rev. D* 73, 064027. doi:10.1103/physrevd.73.064027
- Stergioulas, N., Apostolatos, T. A., and Font, J. A. (2004). Non-linear Pulsations in Differentially Rotating Neutron Stars: Mass-Shedding-Induced Damping and

- Splitting of the Fundamental Mode. *MNRAS* 352, 1089–1101. doi:10.1111/j.1365-2966.2004.07973.x
- Stergioulas, N., Bauswein, A., Zagkouris, K., and Janka, H.-T. (2011). Gravitational Waves and Non-axisymmetric Oscillation Modes in Mergers of Compact Object Binaries. *MNRAS* 418, 427–436. doi:10.1111/j.1365-2966.2011.19493.x
- Stergioulas, N. (1995). Center for Gravitation, Cosmology & Astrophysics. Available at: <http://www.gravity.phys.uwm.edu/rns/>.
- Stergioulas, N., and Friedman, J. L. (1995). Comparing Models of Rapidly Rotating Relativistic Stars Constructed by Two Numerical Methods. *ApJ* 444, 306. doi:10.1086/175605
- Stergioulas, N., and Friedman, J. L. (1998). Nonaxisymmetric Neutral Modes in Rotating Relativistic Stars. *ApJ* 492, 301–322. doi:10.1086/305030
- Suvorov, A. G., and Kokkotas, K. D. (2020). Precursor Flares of Short Gamma-ray Bursts from Crust Yielding Due to Tidal Resonances in Coalescing Binaries of Rotating, Magnetized Neutron Stars. *Phys. Rev. D* 101, 083002. doi:10.1103/physrevd.101.083002
- Suwa, Y., Yoshida, T., Shibata, M., Umeda, H., and Takahashi, K. (2018). On the Minimum Mass of Neutron Stars. *MNRAS* 481, 3305–3312. doi:10.1093/mnras/sty2460
- Takami, K., Rezzolla, L., and Baiotti, L. (2015). Inferring Neutron-star Properties from Gravitational-Wave Signals of Binary Mergers. *Phys. Rev. D* 91, 064001. doi:10.1103/physrevd.91.064001
- Thorne, K. S., and Campolattaro, A. (1967). Non-Radial Pulsation of General-Relativistic Stellar Models. I. Analytic Analysis for $L \geq 2$. *ApJ* 149, 591. doi:10.1086/149288
- Thorne, K. S. (1969). Nonradial Pulsation of General-Relativistic Stellar Models. III. Analytic and Numerical Results for Neutron Stars. *ApJ* 158, 1. doi:10.1086/150168
- Torres-Forné, A., Cerdá-Durán, P., Passamonti, A., and Font, J. A. (2018). Towards Asteroseismology of Core-Collapse Supernovae with Gravitational-Wave Observations - I. Cowling Approximation. *MNRAS* 474, 5272–5286. doi:10.1093/mnras/stx3067
- Torres-Forné, A., Cerdá-Durán, P., Passamonti, A., Obergaulinger, M., and Font, J. A. (2019). Towards Asteroseismology of Core-Collapse Supernovae with Gravitational Wave Observations - II. Inclusion of Space-Time Perturbations. *MNRAS* 482, 3967–3988. doi:10.1093/mnras/sty2854
- Tsui, L. K., and Leung, P. T. (2005). Probing the Interior of Neutron Stars with Gravitational Waves. *Phys. Rev. Lett.* 95, 151101. doi:10.1103/physrevlett.95.151101
- Vavoulidis, M. (2007). *Aristotle University of Thessaloniki*. Ph.D. Thesis.
- Vavoulidis, M., Kokkotas, K. D., and Stavridis, A. (2008). Crustal Oscillations of Slowly Rotating Relativistic Stars. *Monthly Notices RAS* 384, 1711–1724. doi:10.1111/j.1365-2966.2007.12835.x
- Völkel, S. H., and Kokkotas, K. D. (2019). On the Inverse Spectrum Problem of Neutron Stars. *Class. Quan. Grav.* 36, 115002. doi:10.1088/1361-6382/ab186e
- Völkel, S. H., and Kokkotas, K. D. (2017). Ultra Compact Stars: Reconstructing the Perturbation Potential. *Class. Quan. Grav.* 34, 175015. doi:10.1088/1361-6382/aa82de
- Völkel, S. H., Krüger, C. J., and Kokkotas, K. D. (2021). Bayesian Inverse Problem of Rotating Neutron Stars. *Phys. Rev. D* 103, 083008. doi:10.1103/physrevd.103.083008
- Vretinaris, S., Stergioulas, N., and Bauswein, A. (2020). Empirical Relations for Gravitational-Wave Asteroseismology of Binary Neutron star Mergers. *Phys. Rev. D* 101, 084039. doi:10.1103/physrevd.101.084039
- Wen, D.-H., Li, B.-A., Chen, H.-Y., and Zhang, N.-B. (2019). GW170817 Implications on the Frequency and Damping Time of f-mode Oscillations of Neutron Stars. *prc* 99, 045806. doi:10.1103/PhysRevC.99.045806
- Yagi, K., and Yunes, N. (2017). Approximate Universal Relations for Neutron Stars and Quark Stars. *Phys. Rep.* 681, 1–72. doi:10.1016/j.physrep.2017.03.002
- Yoshida, S. i., and Eriguchi, Y. (1999). A Numerical Study of Normal Modes of Rotating Neutron Star Models by the Cowling Approximation. *ApJ* 515, 414–422. doi:10.1086/307012
- Yoshida, S. i., and Eriguchi, Y. (1997). Neutral Points of Oscillation Modes along Equilibrium Sequences of Rapidly Rotating Polytropes in General Relativity: Application of the Cowling Approximation. *ApJ* 490, 779–784. doi:10.1086/304918
- Yoshida, S. i. (2012). Nonaxisymmetric Oscillations of Rapidly Rotating Relativistic Stars by Conformal Flatness Approximation. *Phys. Rev. D* 86, 104055. doi:10.1103/physrevd.86.104055
- Zink, B., Korobkin, O., Schnetter, E., and Stergioulas, N. (2010). Frequency Band of the F-Mode Chandrasekhar-Friedman-Schutz Instability. *Phys. Rev. D* 81, 084055. doi:10.1103/physrevd.81.084055

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