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# Editorial: Modelling and numerical simulations with differential equations in mathematical biology, medicine and the environment, volume II

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## Editorial on the Research Topic

[Modelling and numerical simulations with differential equations in mathematical biology, medicine and the environment, volume II](#)

The main objective of this Research Topic has been to bring academics, engineers, researchers and scientists to share recent ideas, methods, trends, problems and solutions in mathematical biology, medicine and the environment. This Research Topic (volume 2) is an extension of the Research Topic; *Modelling and numerical simulations with differential equations in mathematical biology, medicine and the environment*.

In this Research Topic, there were four topic editors and the Research Topic was open from February 2023 to November 2023 with further extension until February 2024. Twelve papers were submitted, out of which five were accepted and published.

In the following paragraphs, we give a short summary of the 5 published papers in that Research Topic.

[Wangari et al.](#) develop a mathematical model for the transmission dynamics of COVID-19 using a non-linear deterministic system that accounts for the unwillingness of both susceptible and partially vaccinated individuals to receive either single-dose or double-dose vaccines. They then apply the Pontryagin's Maximum Principle to establish the optimality conditions associated with the optimal controls.

The approach to solving the ODE is commonly used for optimal control theory. The novelty is the positive results obtained to prove that administering single dose vaccines leads to a significant reduction of COVID-19 prevalence than when double dose vaccines are administered. The impact of vaccine hesitancy comparison against either single dose or double dose on COVID-19 prevalence revealed that vaccine hesitancy against single dose is more detrimental than vaccine hesitancy against a double dose vaccine. This is what the study attained and the community of researchers in this field will benefit from this article.

Appadu and Kelil construct two standard finite difference schemes to solve a non-homogeneous fractional dispersive Korteweg-de Vries equation with specified initial and boundary conditions. The two methods are constructed by approximating the fractional derivative with conformable and Caputo operators and referred to as FDMCO and FDMCA. The first novelty is that an explicit finite difference method with Caputo's operator is constructed as compared to Murio [1] who derived an implicit finite difference scheme for a time-fractional diffusion equation. The second novelty is that we constructed two finite difference methods to solve this problem whereas in published literature, a semi analytic method was used [2]. It is well known that semi analytic methods can be quite slow to converge and their accuracy are problem dependent. Thirdly, it is possible to gauge the performance of the two schemes we constructed over short and longer time propagation and for some fractional parameter values such as 0.40, 0.75, 0.90, and 0.95 by computing L1, L-infinity errors and relative errors hence highlighting the strengths and any weaknesses of the two methods.

FDMCO is satisfactory scheme when:

- (i)  $\alpha = 0.40$  at time 0.01.
- (ii)  $\alpha = 0.90$  at time 0.1.
- (iii)  $\alpha = 0.95$  at time 0.1.

FDMCA is very effective at  $\alpha = 0.95$  at times 0.1, 1.0.

FDMCA is satisfactory when:

- (i)  $\alpha = 0.75$  at times 0.1, 1.0
- (ii)  $\alpha = 0.90$  at times 0.1, 1.0

In general, FDMCA is more efficient than FDMCO for the problem considered.

Ahmed et al. formulate and analyze a cell-level mathematical model of malaria parasites with antimalarial drug treatments. They investigate the within-host dynamics of malaria infection with primary liver-stage antimalarial drug treatment, blood-stage antimalarial drug treatment, and gametocytocidal drug treatment. They also study the sensitivity analysis of the in-host basic reproduction number to investigate most influential parameters using the normalized forward sensitivity index. The novelty here is that the authors consider this study within-host dynamics of malaria parasites with stage-specific antimalarial drug treatment. This study's findings offer guidance for antimalarial medication therapy and malaria control. By reducing the average number of merozoites released from ruptured infected erythrocytes and hepatocytes, the production of malaria parasites within the host can be limited, helping to prevent further infection.

Kitaro et al. develop a mathematical model to investigate the effect of chemoprophylaxis treatment on the transmission of tuberculosis with the drug-resistant compartment. They analyze the stability of the endemic and disease-free equilibriums. Stability analysis is important to see the effectiveness of the model formulated. If the model is stable, slight deviation (error) in the system will have a slight effect in the solution of the model. Otherwise, the model will not accurately predict the future outcome. They proved the existence and positivity of solutions. The sensitivity analysis and numerical simulations of the model show that treatment of latently infected individuals reduces the progression to the infected group.

The novelty of this article is that the authors formulated a mathematical model by incorporating chemoprophylaxis treatment for latent individuals and studying its effect on the transmission of tuberculosis

Appadu et al. have constructed four methods, namely, FTCSCO, FTCSCA, NSFDCO, and NSFDCA, to solve a time fractional diffusion partial differential equation with specified initial and boundary conditions [3], and considered some different values for the fractional parameter. There is no known exact solution for the problem considered. To study the stability of the proposed numerical schemes, they employ von Neumann stability analysis or establish conditions under which the schemes replicate/preserve the positivity of the continuous model.

The first novelty is that such methods were not used before to solve that problem. Second, we compared standard and non-standard finite difference methods using conformable and Caputo operators with regard to the study of stability, CPU time, numerical rate of convergence.

Numerical profiles vs.  $x$  vs.  $t$  were displayed. The numerical results obtained from these different schemes provide valuable insights into their performance and convergence behavior. From the analysis of NSFDCO, we conclude that it offers an easily obtainable condition for positivity with significantly lower CPU time compared to FTCSCO. While both schemes exhibit similar profiles, for smaller coefficients of  $U_{xx}$ , FTCSCO may introduce dispersive oscillations and potential blow-up. FTCSCA, utilizing previous time levels Caputo operator, provides a more accurate representation of non-local and history dependent diffusion processes. It is anticipated that NSFDCA exhibits less susceptibility to non-physical oscillations than FTCSCA, particularly for small coefficients of dissipation and stiff problems.

We find that FTCSCO and NSFDCO give issues with the numerical rate of convergence. FTCSCA and NSFDCA are quite reliable methods to solve the problem we considered.

## Author contributions

AA: Writing – original draft, Writing – review & editing. RL: Writing – original draft, Writing – review & editing. HG: Writing – original draft, Writing – review & editing. SD: Writing – original draft, Writing – review & editing.

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## Conflict of interest

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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