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# Editorial: Advances in computational relativity

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general relativity, software, numerical relativistic hydrodynamics, black holes, gravitational waves

## Editorial on the Research Topic [Advances in computational relativity](#)

The Institute for Computational and Experimental Research in Mathematics (ICERM) at Brown University hosted a semester-long program on [Advances in computational relativity](#) in 2020. The program covered Research Topics in mathematical and computational approaches for solving the Einstein field equation, relativistic hydrodynamics, and methods for the detection, classification, and Bayesian inference of gravitational waves from compact objects such as colliding neutron stars. This program was run 4 years after the historic 2016 announcement on the first-ever detection of gravitational waves from a binary black hole system. Gravitational waves encode detailed information about the astrophysical systems they emerge from and complement what can be learned through traditional light-based observation. Advanced numerical methods are crucial for simulations, models, and data analysis efforts.

This Research Topic *Advances in computational relativity* includes contributions from some of the program participants and work that was discussed and presented at the ICERM program. These contributions cover various Research Topics in computational relativity, focusing on methodology, simulation, and models of gravitational wave signals. Below, we briefly review the scientific contributions of the articles published in this Research Topic.

To understand gravitational waves, scientists study how the fabric of spacetime behaves, especially during events like the collision of compact objects such as black holes. The best place to measure the ripples in spacetime caused by these events is far from where they occur, at a boundary called future null infinity. Hyperboloidal slices are a promising approach to including this boundary within a computational setup. These are smooth, space-like cross-sections that extend all the way to future null infinity. However, incorporating this boundary into simulations is a non-trivial, long-term effort. [Vañó-Viñuales](#) proposes hyperboloidal-adapted gauge conditions and provides stationary black hole solutions for the gauge conditions for two specific sets of parameters. The effectiveness and robustness of these methods are tested by running simulations and observing long-time convergence behavior against the exact solution. As gauge conditions play a crucial role in numerical evolutions, this work makes important progress toward understanding robust gauge choices within the hyperboloidal approach, as well as proposing concrete code tests associated with new initial data that can be used to verify a numerical scheme.

Numerical computations of physical observables (e.g., gravitational waves and optical spectra) are essential in interpreting multi-messenger observations such as neutron star and black hole mergers. [Rosswog et al.](#) introduce the first version of SPHINCS\_BSSN, a new computational tool for numerical simulations of the Einstein equations (written in the BSSN formulation) coupled to matter fields. Unlike traditional codes that use Eulerian relativistic hydrodynamics, SPHINCS\_BSSN adopts a unique approach by treating the matter as individual particles using a technique known as smooth particle hydrodynamics (SPH). For certain kinds of simulations, this method may allow for a more accurate modeling of matter, such as long-term evolution of merger ejecta. The paper documents three key methodological improvements over an earlier version of the code. First, a new regression technique is used to represent the stress-energy tensor on the spacetime grid using its values defined at particle locations. Second, the code now supports additional heuristics for grid refinement in the hydrodynamic sector of the system by tracking when values of the lapse become very small (which typically happens inside an apparent horizon). This leads to higher resolution when simulating the extreme process of a neutron star collapsing into a black hole. Finally, improvements in how the initial positions of the SPH particles are determined and interfacing with a greater variety of initial data solvers are implemented. Using SPHINCS\_BSSN, the paper presents results from simulations of neutron star mergers, a key event for gravitational wave astronomy. These simulations cover different scenarios, including mergers that lead to immediate collapse into a black hole and those involving neutron stars with non-zero spin. This work represents a significant step forward in simulating relativistic matter fields with Lagrangian hydrodynamics.

Gravitational wave signals from extreme mass ratio inspirals are a key scientific target for space-based gravitational wave detectors such as Laser Interferometer Space Antenna (LISA). These systems involve a smaller object orbiting a supermassive black hole, and detectors such as LISA will observe 100's of 1,000's of gravitational wave cycles emitted from such systems. Fast and accurate extreme mass ratio inspiral (EMRI) waveform models are crucial for LISA science goals to be achieved. In a contribution by [Speri et al.](#) researchers developed the first ready-to-use model for EMRI gravitational waves that focuses on orbits around non-spinning (Schwarzschild) black holes with eccentric orbits. This model can efficiently run on both graphics processing units (GPUs) and central processing units (CPUs). This new frequency domain waveform model offers several advantages, including generating a waveform in about 0.04 s, making it sufficiently fast to use in realistic data analysis efforts. This improvement in speed and efficiency will enable researchers to conduct parameter inference studies using fully relativistic EMRI waveforms. Looking ahead, as models evolve to include more complex scenarios like spinning black holes and various orbital geometries, the development of next-generation frequency domain models will greatly benefit from the methods proposed in this paper.

Advanced numerical schemes for solving wave-like differential equations enable accurate, efficient, and robust solvers. Over the years, computational relativity codes have adopted such schemes for their advantageous properties. [Tinoco-Guerrero et al.](#) focuses on stability in numerical simulations when solving the wave equation in complicated and irregularly shaped domains. Long-time stability is paramount in numerical schemes because it ensures that the solutions to equations do not grow uncontrollably or become distorted while propagating, which can happen if the mathematical methods used are not well-suited to the problem. This paper examines a generalized method that uses finite differences for explicit and implicit formulations and derives stability conditions for both settings. The authors point out that their findings on stability are not limited to the specific methods they propose but apply broadly to other numerical schemes for solving the wave equation that is at least second-order accurate in space. This work is particularly relevant for simulations in domains with complex geometries.

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## Conflict of interest

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