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# Editorial: Interfaces and mixing – non-equilibrium dynamics and conservation laws at continuous and kinetic scales

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## Editorial on the Research Topic

[Interfaces and mixing – non-equilibrium dynamics and conservation laws at continuous and kinetic scales](#)

Interfacial transport and mixing are non-equilibrium processes coupling kinetic to macroscopic scales. They occur in fluids, plasmas and materials over celestial events to molecules. Examples include supernovae and fusion, planetary convection and reactive fluids, wetting and adhesion, turbulence and turbulent mixing, nano-fabrication and bio-technology. Addressing the societal challenges posed by alternative energy sources, efficient use of non-renewable resources and climate change requires a better understanding of non-equilibrium transport of interfaces and mixing [1–4].

Non-equilibrium dynamics of interfaces and interfacial mixing are exceedingly challenging to study. These processes often involve sharp changes of vector and scalar fields, and may also include strong accelerations and shocks, radiation transport and chemical reactions, diffusion of species and electric charges, among other effects. They are inhomogeneous, anisotropic, non-local, and statistically unsteady. At macroscopic scales, their spectral and invariant properties differ substantially from those of canonical turbulence. At atomistic and meso-scales, the non-equilibrium dynamics depart dramatically from the standard scenario given by Gibbs ensemble averages and the quasi-static Boltzmann equation. At the same time, non-equilibrium dynamics of interfaces and interfacial mixing may lead to self-organization and order, thus providing with new opportunities for diagnostics and control. Capturing properties of interfaces and mixing, enabling their accurate description and conservative properties, solving the boundary value problems within Eulerian and Lagrangian frameworks—can aid better understanding of non-equilibrium dynamics in nature and technology [1–4].

Significant success was recently achieved in understanding of interfaces and interfacial mixing on the sides of theoretical analysis, large-scale numerical simulations, and data analysis. This success opens new opportunities for studies of fundamentals of non-equilibrium dynamics across the scales, for developing a unified description of

particles and fields on the basis of synergy of theory, numeric and data analysis, and for applying these fundamentals to address the contemporary challenges of modern science, technology and society [1–4].

This Research Topic builds upon recent achievements in the understanding of interfacial transport and mixing using theoretical analysis, large-scale numerical simulations, and data analysis, and is focused on conservation laws and boundary value problems, from continuous to kinetic scales. It brings together mathematicians and scientists from applied mathematics and applied analysis, dynamical systems and data analysis, fluid dynamics and industrial mathematics [1–4]. Some of the works in this Research Topic were presented at the 2019 Workshop on “Conservation laws and boundary value problems” at the Matrix Institute for Mathematical Sciences [1]. The Research Topic consists of the following five accepted papers (Samoilova and Nepomnyashchy; Hsu et al.; Chen; Durazo et al.; Hill and Abarzhi).

Samoilova and Nepomnyashchy investigate the Marangoni wave patterns in thin films and focus on instabilities and control associated with longitudinal modulations. The authors consider non-linear Marangoni waves which are generated by the long-wave oscillatory instability of the conductive state in a thin liquid film heated from below. The authors derive the amplitude equation for a modulated traveling wave in case of a substrate of very low conductivity and for a deformable free surface. The equation describes non-linear interaction of the main convective pattern with the perturbations having slightly different wavenumbers. When compared to the conventional complex Ginzburg–Landau equation, the amplitude equation contains an additional term associated with the local rise of the liquid level. The stability analysis reveals two instability modes, which are caused by the amplitude modulation and the phase modulation (Benjamin–Feir), respectively. The influence of the non-linear feedback control is thoroughly investigated, the computations are conducted and the parameter regime is identified where the waves can be stabilized.

Hsu et al. study scaling laws of partially developed turbulence. Their work presents the multi-fractal models developed to capture the complete range of length scales in turbulent flow, including laminar, dissipation, inertial, and stirring ranges. The models are consistent with existing turbulence models in the inertial range. The authors elaborate the multi-fractal scaling laws in the localized range of length scales where the turbulence is either developed only partially, or whether the fully developed turbulence is anisotropic due to, e.g., effect of influence of large-scale turbulent structures. The authors state that in the ranges of partially developed turbulence the flow can be far from universal. The theoretical reasoning is supported by comparison with high resolution simulation data.

Chen presents theoretical foundations of lattice Boltzmann numerical models for a general curvilinear coordinate system. The numerical approach is critically important for modeling a

broad range of natural and technological processes, including realistic environments in automotive, aerodynamic and aerospace industries. The foundations are built on the concept that a particle is moving along a curved path and derive the discrete space-time inertial force ensuring the momentum conservation in the underlying Euclidean space. They elaborate a volumetric representation in which mass and momentum are precisely conserved as in the conventional lattice Boltzmann on a Cartesian lattice. The state-of-the-art numerical approach is demonstrated to fully recover in the hydrodynamic limit and in general curvilinear coordinates the Navier-Stokes equation alone with the mass continuity equation. The approach has a number of important advantages when compared to other conventional numerical methods. Particularly, it preserves the fundamental one-to-one advection feature of a standard lattice Boltzmann method on a uniform Cartesian lattice.

Durazo et al. study the data assimilation for ionospheric space-weather forecasting in the presence of model bias. The authors are particularly interested in the response of the Earth ionosphere during geomagnetic storms and in the predictions of space weather. The work presents the results of observing system simulation experiments that employ the first-principles model of the Earth’s ionosphere and the Thermosphere Ionosphere Electrodynamics Global Circulation Model. These models depend on a number of parameters describing solar activity, geomagnetic conditions, and the state of the thermosphere. The driving parameters can potentially have large errors which can be caused by geomagnetic storms and which, in turn, can lead to systematic biases in the model predictions and challenge data assimilation methods. The authors elaborate the physically accurate and computationally efficient methodology for handling the systematic bias in the model outputs and report the encouraging results. The authors further suggest the methodology refining in order to improve the short-term predictions of ionospheric dynamics during geomagnetic storms.

Hill and Abarzhi investigate the dynamics of Rayleigh-Taylor and Richtmyer-Meshkov instabilities driven by variable accelerations. Rayleigh-Taylor (RT) and Richtmyer-Meshkov (RM) instabilities play an important role in a broad range of processes in Nature and technology from astrophysical to atomic scales, including stellar evolution, oceanic flows, and plasma fusion. While these instabilities are sister phenomena, a link of RT-to-RM dynamics requires better understanding. The authors focus on the long-standing problem of RT/RM instabilities induced by accelerations varying as inverse-quadratic power-laws in time. They apply group theory to obtain theoretical solutions for the early-time linear and late-time non-linear dynamics of RT/RM coherent structure of bubbles and spikes, and investigate the dependence of the solutions on the acceleration’s parameters and initial conditions. The authors find that the dynamics is of RT type for strong accelerations and is of RM type for weak accelerations, and identify the

effects of the acceleration's strength and the fluid density ratio on RT-to-RM transition. The theory achieves good agreement with available observations and elaborate benchmarks for future research.

We expect this Research Topic of papers to explore and to assess the state-of-the-art in the fundamentals of interfaces and mixing and their non-equilibrium dynamics; to elaborate the novel methods of studies of boundary value problems at kinetic and at continuous scales; and to chart new research directions in this fundamental and actively developing research area of mathematical physics.

## Author contributions

The author confirms being the sole contributor of this work and has approved it for publication.

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