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Wave Effects of the Fractional Shallow Water Equation and the Fractional Optical Fiber Equation

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The non-linear space-time fractional Estevez-Mansfield-Clarkson (EMC) equation and the non-linear space-time fractional Ablowitz-Kaup-Newell-Segur (AKNS) equation showed the motion of waves in the shallow water equation and the optical fiber equation, respectively. The process used to solve these equations is to transform the non-linear fractional partial differential equations (PDEs) into the non-linear ordinary differential equations by using the Jumarie's Riemann-Liouville derivative and setting the solution in the finite series combined with the simple equation (SE) method with the Bernoulli equation. The new traveling wave solutions were the exponential functions resulting in the physical wave effects are produced in the form of kink waves and represented by the two-dimensional graph, three-dimensional graph, and contour graph. In addition, the comparison of the solutions revealed that the new solutions have a more convenient and easier format.

Keywords: simple equation method, fractional partial differential equations, traveling wave solution, Estevez-Mansfield-Clarkson equation, Ablowitz-Kaup-Newell-Segur equation

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1. INTRODUCTION

The partial differential equations (PDEs) are very important in studying and explaining the phenomena of mathematical physics. The most widely used scenarios are plasma physics, optical fibers, fluid mechanics, solid state physics, plasma waves, capillary-gravity waves, and water waves. At present, the researchers are interested in studying the exact solutions or the numerical solutions [1–3] in order to apply the results obtained in the above studies. To study the effects of these matters more closely, it is necessary to study the form of the exact traveling wave solution of non-linear fractional PDEs. There are a variety of methods to solve non-linear fractional PDEs such as generalized Kudryashov method [4, 5], extended Kudryashov method [6], modified Kudryashov method [7, 8], first integral method [9–11], G'/G-expansion method [12–14], fractional sub-equation method [15–17], and Poincaré-Lighthill-Kuo method [18, 19].

In 2006, the Jumarie's Riemann-Liouville derivative [20] was given as follows,

$$D_t^{\alpha} f(t) = \begin{cases} f(t) & ,\alpha = 0 \\ \dfrac{1}{\Gamma(1-\alpha)} \dfrac{d}{dt} \displaystyle\int_0^t (t-\psi)^{-\alpha} [f(\psi) - f(0)] d\psi & ,0 < \alpha < 1 \\ \dfrac{d^n}{dt^n} D_t^{\alpha - n} f(t) & ,n \le \alpha < n+1, n \ge 1, \end{cases} \tag{1}$$

where α is an order of the fractional derivative.

The important properties of fractional Riemann-Liouville derivatives [21] were discovered in 2009 as follows,

$$D_t^{\alpha} t^k = \frac{\Gamma(k+1)}{\Gamma(k-\alpha+1)} t^{(k-\alpha)}, \qquad k \geq 0, \tag{2}$$

$$D_t^{\alpha}[f(t)g(t)] = f(t)D_t^{\alpha}g(t) + g(t)D_t^{\alpha}f(t), \tag{3}$$

$$D_t^{\alpha}f[g(t)] = D_g^{\alpha}f[g(t)][g'(t)]^{\alpha} = f_g'[g(t)]D_t^{\alpha}g(t). \tag{4}$$

Mansfield and Clarkson [22] introduced the Estevez-Mansfield-Clarkson (EMC) equation in 1997. This equation was applied to understand the non-linear dispersion of patterns in liquid drops. Zhen-Ya [23] suggested the two-parameter family of fully EMC equation $E(m,n)$ in 2002 as follows,

$$(u_y^m)_{yyt} + \delta(u_y^n u_t)_y + u_{tt} = 0 \tag{5}$$

where δ is a constant. If $(m,n) = (1,1)$ then Equation (5) reduces to the EMC equation

$$u_{yyyt} + \delta u_y u_{yt} + \delta u_{yy} u_t + u_{tt} = 0. \tag{6}$$

In 1970, Ablowitz, Kaup, Newell, and Segur [24] purposed the Ablowitz-Kaup-Newell-Segur (AKNS) equations which were motivated by the applications to non-linear optics. The AKNS equation can be reduced to some non-linear evolution equations such as the non-linear Schrödinger, the sine-Gordon equations, the kdV equation, and others. The fourth-order non-linear AKNS equation with the parameter β in this form,

$$4u_{xt} + u_{xxxt} + 8u_x u_{xy} + 4u_{xx}u_y - \beta u_{xx} = 0. \tag{7}$$

In this article, we have solved the non-linear space-time fractional EMC equation and the non-linear space-time fractional AKNS equation by applying Jumarie's Riemann-Liouville derivative and the SE method with the Bernoulli equation. We have shown the new exact solutions and the wave effects in a two-dimensional graph, three-dimensional graph, and contour graph. Finally, we have compared the new solutions with some effective articles [25] to show that our solutions had a more convenient and simpler form.

2. ALGORITHM OF SE METHOD

In this section, we discuss the SE method with the Bernoulli Equation [26] for solving fractional PDEs. The general form of fractional PDEs is shown as

$$G(u, D_x^{\alpha}u, D_y^{\alpha}u, D_t^{\alpha}u, D_x^{2\alpha}u, D_y^{\alpha}D_x^{\alpha}u, D_t^{\alpha}D_x^{\alpha}u, \ldots) = 0, t > 0, \\ 0 < \alpha \leq 1. \tag{8}$$

This method is described in the following steps,

Step 1. Wave transformation

The traveling wave solution of fractional PDEs is a solution that satisfies

$$u(x,y,t) = U(\psi), \psi = \frac{kx^{\alpha}}{\Gamma(\alpha+1)} + \frac{ly^{\alpha}}{\Gamma(\alpha+1)} - \frac{ct^{\alpha}}{\Gamma(\alpha+1)}, \tag{9}$$

where ψ is a general term for a traveling wave, c is a wave velocity constant. We called stationary wave when $c = 0$. For $c > 0$, the wave moves in the positive direction, and for $c < 0$, the wave moves in the negative direction [27]. Reducing Equation (8) into an ODE,

$$Q\left(U, \frac{dU}{d\psi}, \frac{d^2U}{d\psi^2}, \frac{d^3U}{d\psi^3}, \ldots\right) = 0, \tag{10}$$

where Q is a polynomial in $U(\psi)$ and its derivatives.

Step 2. Solution supposition

The solution of Equation (10) can be written in the finite series,

$$U(\psi) = \sum_{i=0}^{N} a_i H^i(\psi), \tag{11}$$

where a_i are real constants with $a_N \neq 0$ and $H(\psi)$ depends on the simple equation (SE) method with Bernoulli equation which states as follows.

$$H'(\psi) = \mu H(\psi) + \eta H^2(\psi) \tag{12}$$

where μ and η are the non-zero constant. The solutions of Equation (12) have two cases with ψ_0 as an integration constant,

Case 1 : $\mu > 0, \eta < 0$,

$$H(\psi) = \frac{\mu e^{\mu(\psi+\psi_0)}}{1 - \eta e^{\mu(\psi+\psi_0)}}. \tag{13}$$

Case 2 : $\mu < 0, \eta > 0$,

$$H(\psi) = -\frac{\mu e^{\mu(\psi+\psi_0)}}{1 + \eta e^{\mu(\psi+\psi_0)}}. \tag{14}$$

Step 3. Finding the integer N

Balance the highest order derivative and non-linear terms in Equation (10) to get the integer N in Equation (10).

Step 4. Solution obtaining

Find the parameters $a_i, (i = 1, 2, 3, \ldots, N)$ and c by collecting the coefficients all terms with the same order of $H^i, (i = 1, 2, 3, \ldots, N)$ and setting them to zero [28]. Hence, we constitute the analytical solutions of Equation 10.

3. APPLICATIONS

We present the traveling wave effects of the non-linear space-time fractional EMC equation and the non-linear space-time fractional AKNS equation.

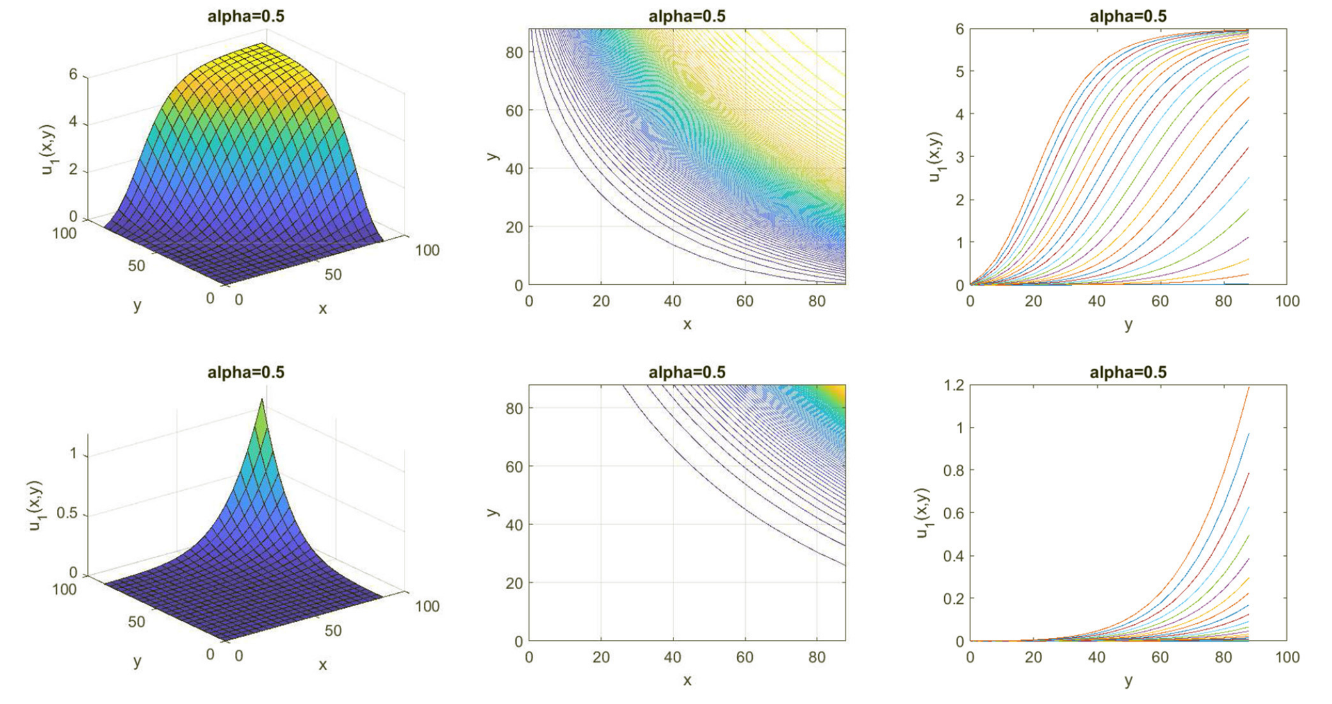

FIGURE 1 | Kink wave solution of Equation (25) in 3D, contour and 2D for Case I.

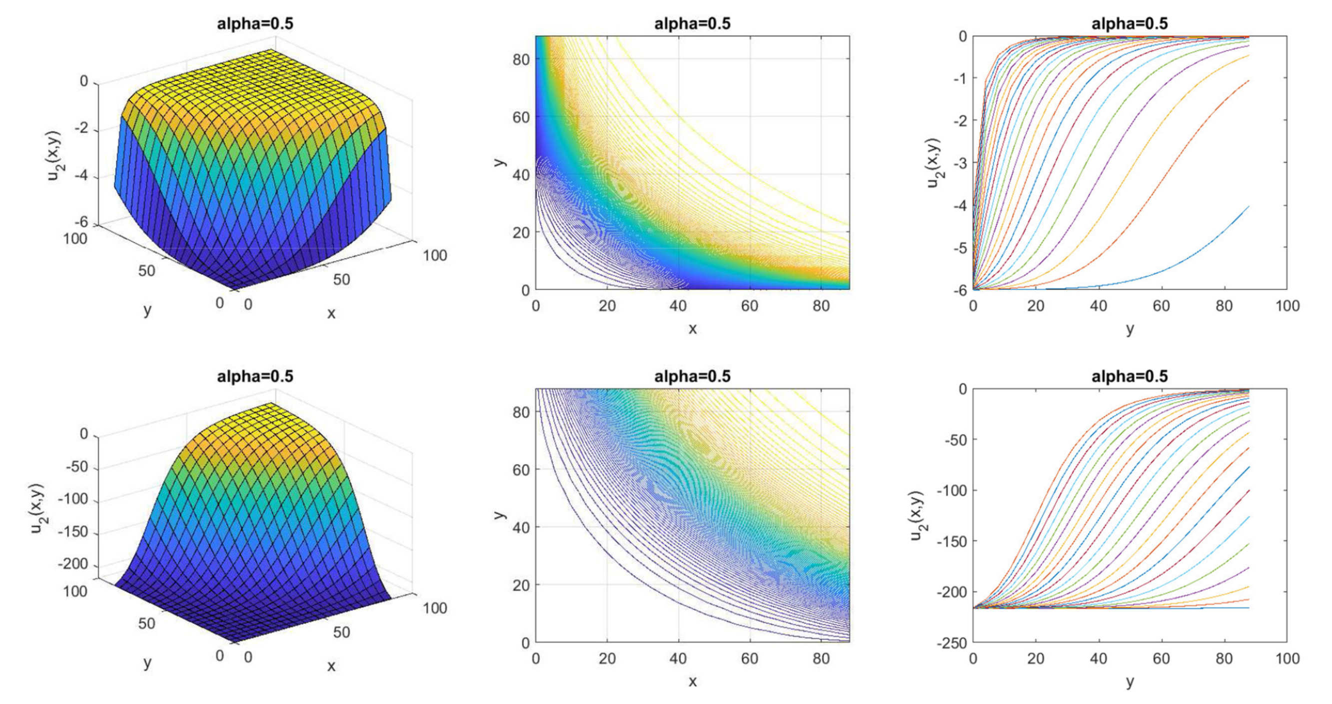

FIGURE 2 | Kink wave solution of Equation (26) in 3D, contour and 2D for Case II.

3.1. The Space-Time Fractional EMC Equation

The fourth-order non-linear space-time fractional EMC Equations [22, 25] is stated as follows,

$$D_y^{3\alpha} D_t^{\alpha} u + \delta D_y^{\alpha} u D_y^{\alpha} D_t^{\alpha} u + \delta D_y^{2\alpha} u D_t^{\alpha} u + D_t^{2\alpha} u = 0, t > 0, 0 < \alpha \leq 1, \quad (15)$$

where $u = u(x, y, t)$ and δ is constant. Supposing the solution $u(x, y, t) = U(\psi)$ and applying the transformation

$$\psi = \frac{kx^{\alpha}}{\Gamma(\alpha+1)} + \frac{ly^{\alpha}}{\Gamma(\alpha+1)} - \frac{ct^{\alpha}}{\Gamma(\alpha+1)}, \quad (16)$$

where k, l, and c are non-zero constants. The Equation (15) changed into an ODE,

$$-l^3 \frac{d^4U}{d\psi^4} - 2l^2\delta \frac{dU}{d\psi} \cdot \frac{d^2U}{d\psi^2} + c\frac{d^2U}{d\psi^2} = 0. \quad (17)$$

Integrating Equation (17) with zero constant, we get

$$-l^3 \frac{d^3U}{d\psi^3} - l^2\delta \left(\frac{dU}{d\psi}\right)^2 + c\frac{dU}{d\psi} = 0. \quad (18)$$

Using the SE method. The solution was set in the form of Equation (11). Next, we balanced the non-linear terms and the highest order derivative of Equation (18). Thus, $N = 1$. The Equation (11) turned into

$$U(\psi) = a_0 + a_1 H(\psi). \quad (19)$$

Replacing Equation (18) with Equation (19). We collected all terms of the same power of $H(\psi)$ and set each coefficient to zero as follows,

$$H^1(\psi): \quad -a_1 l^3\mu^3 + a_1 c\mu = 0, \quad (20)$$
$$H^2(\psi): \quad -7a_1 l^3\mu^2\eta - a_1^2 l^2\delta\mu^2 + a_1 c\eta = 0, \quad (21)$$
$$H^3(\psi): \quad -12a_1 l^3\mu\eta^2 - 2a_1^2 l^3\delta\mu\eta = 0, \quad (22)$$
$$H^4(\psi): \quad -6a_1 l^3\eta^3 - a_1^3 l^2\delta\eta^2 = 0. \quad (23)$$

Solving the system of Equations (20)–(23), we get

$$a_1 = \frac{-6l\eta}{\delta}, c = l^3\mu^2. \quad (24)$$

By Equations (13), (14), (16), and (24), the exact traveling wave solutions of the non-linear space-time fractional EMC equations are represented in two cases with arbitrary constant ψ_0:

Case I: $\mu > 0, \eta < 0$,

$$u(x, y, t) = a_0 - \frac{6l\eta}{\delta}\left(\frac{\mu e^{\mu(\psi+\psi_0)}}{1 - \eta e^{\mu(\psi+\psi_0)}}\right), \quad (25)$$

Case II: $\mu < 0, \eta > 0$,

$$u(x, y, t) = a_0 + \frac{6l\eta}{\delta}\left(\frac{\mu e^{\mu(\psi+\psi_0)}}{1 + \eta e^{\mu(\psi+\psi_0)}}\right), \quad (26)$$

where $\psi = \frac{kx^{\alpha}}{\Gamma(\alpha+1)} + \frac{ly^{\alpha}}{\Gamma(\alpha+1)} - \frac{l^3\mu^2 t^{\alpha}}{\Gamma(\alpha+1)}$.

Next, we present the example graph of wave effects of the non-linear space-time fractional EMC equation by setting some parameter to obtain proper graph. Equation (25) shows the exact exponential solutions which forms a kink wave effect, increasing from one state to another, with $a_0 = 0$, $\delta = 1$, $k = 1$, $l = 1$, $\mu = 1$, $\eta = -1$, $\alpha = 0.5$, $0 \leq x \leq 90$, $0 \leq y \leq 90$, and $t = 200, 400$ shown in **Figure 1**. We set the parameters $a_0 = 0$, $\delta = 1$, $k = 1$, $l = 1$, $\mu = -1$, $\eta = 1$, $\alpha = 0.5$, $0 \leq x \leq 90$, $0 \leq y \leq 90$, and $t = 100, 300$ into an Equation (26), kink wave effect shown in **Figure 2**.

3.2. The Space-Time Fractional AKNS Equation

The fourth-order non-linear space-time fractional AKNS equation is defined as follows [25],

$$4D_x^{\alpha} D_t^{\alpha} u + D_x^{3\alpha} D_t^{\alpha} u + 8D_x^{\alpha} u D_x^{\alpha} D_y^{\alpha} u + 4D_x^{2\alpha} u D_y^{\alpha} u - \beta D_x^{2\alpha} u = 0, t > 0, 0 < \alpha \leq 1, \quad (27)$$

where $u = u(x, y, t)$ and β is constant. Similar to the previous equation, we set the solution and transform Equation (27) by Equation (16), we obtain

$$-4ck\frac{d^2U}{d\psi^2} - ck^3\frac{d^4U}{d\psi^4} + 8k^2l\frac{dU}{d\psi} \cdot \frac{d^2U}{d\psi^2} + 4k^2l\frac{dU}{d\psi} \cdot \frac{d^2U}{d\psi^2} - \beta k^2\frac{d^2U}{d\psi^2} = 0$$
$$-(4c + \beta k)\frac{d^2U}{d\psi^2} - ck^2\frac{d^4U}{d\psi^4} + 12kl\frac{dU}{d\psi} \cdot \frac{d^2U}{d\psi^2} = 0. \quad (28)$$

Integrating Equation (28) and set constant to zero,

$$-(4c + \beta k)\frac{dU}{d\psi} - ck^2\frac{d^3U}{d\psi^3} + 6kl\left(\frac{dU}{d\psi}\right)^2 = 0. \quad (29)$$

Balancing Equation (29), we got $N = 1$. Thus, Equation (11) changed to

$$U(\psi) = a_0 + a_1 H(\psi). \quad (30)$$

Replacing Equation (29) with Equation (30). Collecting all terms which have the same power of $H(\psi)$. Setting each coefficient of them to zero, we have

$$H^1(\psi): \quad -a_1 c\mu - a_1\beta k\mu - a_1 ck^2\mu^3 = 0, \quad (31)$$
$$H^2(\psi): \quad -4a_1 c\eta - a_1\beta k\eta - 7a_1 ck^2\mu^2\eta + 6a_1^2 kl\mu^2 = 0, \quad (32)$$
$$H^3(\psi): \quad -12a_1 ck^2\mu\eta^2 + 12a_1^2 kl\mu\eta = 0, \quad (33)$$
$$H^4(\psi): \quad -6a_1 ck^2\eta^3 + 6a_1^2 kl\eta^2 = 0. \quad (34)$$

Solving Equations (31)–(34), we get

$$a_1 = -\frac{\beta k^2\eta^2}{4l\eta + k^2 l\mu^2}, c = -\frac{\beta k\eta}{4\eta + k^2\mu^2}. \quad (35)$$

Substituting Equation (35) into Equations (16), (25), and (26), the exact traveling wave solutions of the non-linear space-time fractional AKNS equation can be explained as follows with

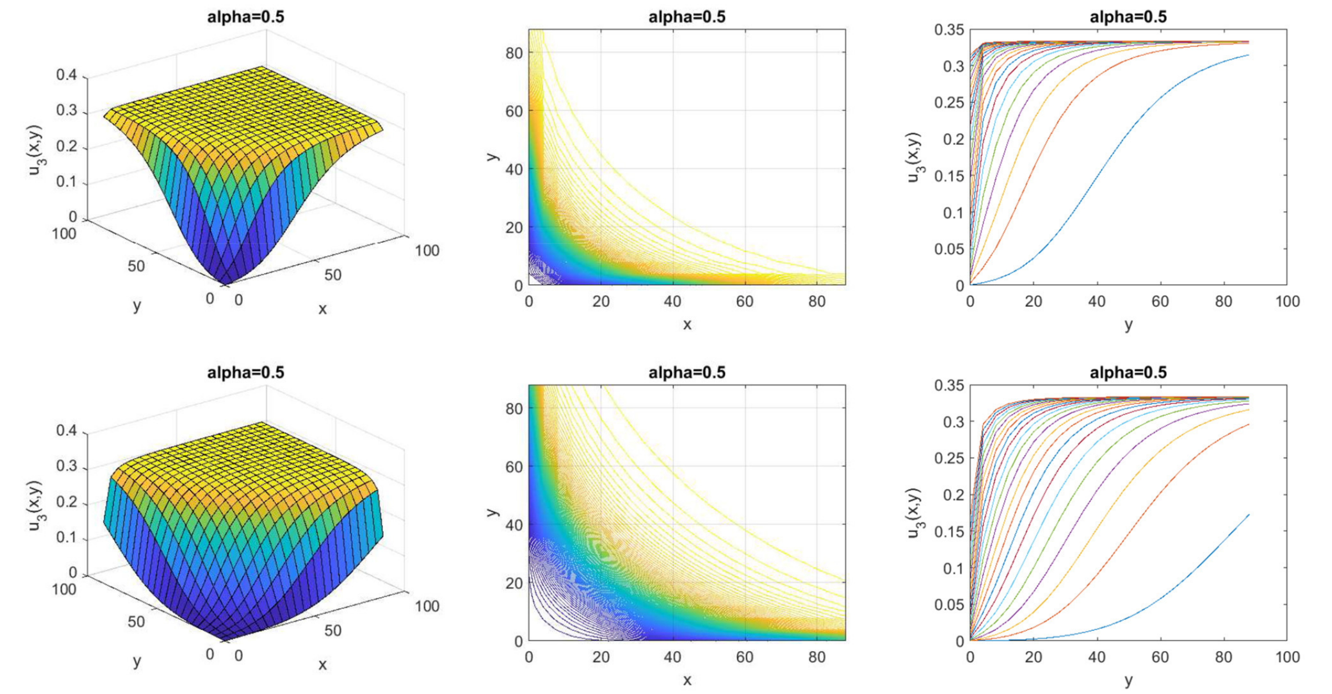

FIGURE 3 | Kink wave solution of Equation (36) in 3D, contour and 2D for Case I.

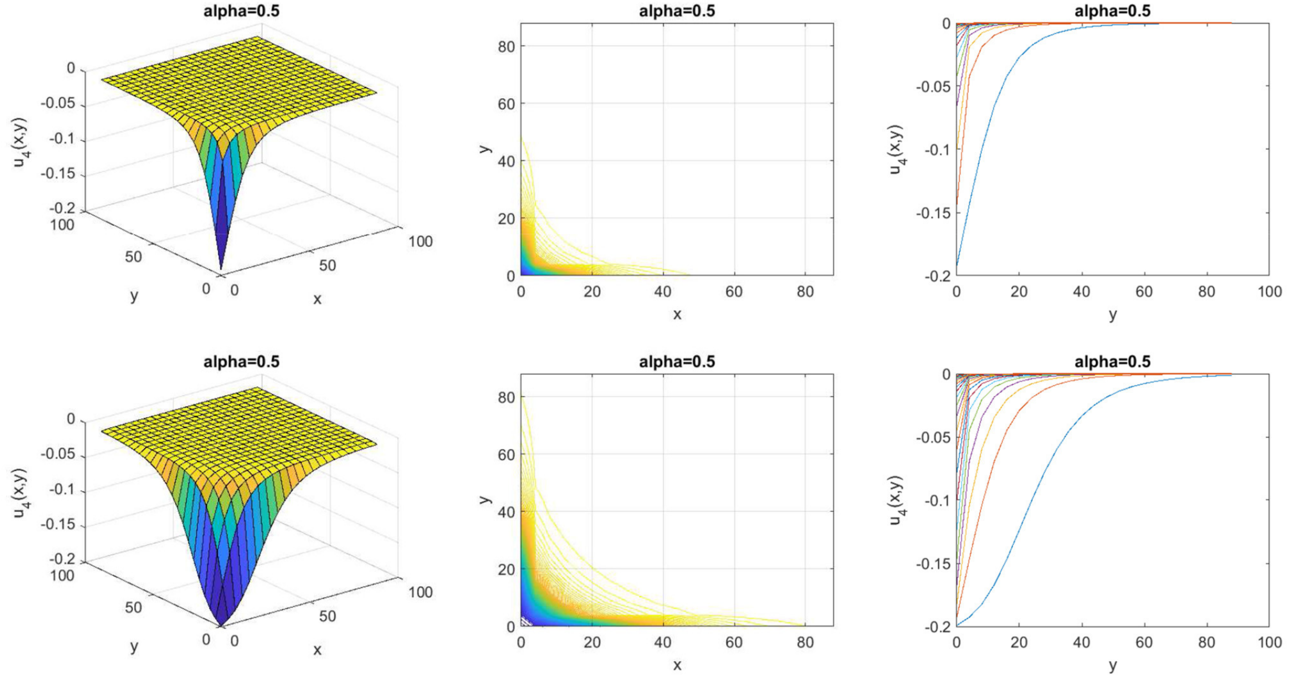

FIGURE 4 | Kink wave solution of Equation (37) in 3D, contour, and 2D for Case II.

TABLE 1 | Solutions comparison of the non-linear space-time frational Estevez-Mansfield-Clarkson (EMC) equation between G'/G-expansion method and simple equation (SE) method.

G'/G-expansion method	Simple equation method
Case I: $\lambda^2 - 4\mu > 0$ $u = a_0 + \frac{6l}{\beta}\left[\frac{-\lambda}{2} + \vartheta_1\left(\frac{c_1\sinh(\vartheta_1\zeta) + c_2\cosh(\vartheta_1\zeta)}{c_1\cosh(\vartheta_1\zeta) + c_2\sinh(\vartheta_1\zeta)}\right)\right]$	**Case I:** $\mu > 0, \eta < 0,$ $u = a_0 - \frac{6l\eta}{\delta}\left(\frac{\mu e^{\mu(\psi+\psi_0)}}{1 - \eta e^{\mu(\psi+\psi_0)}}\right)$
Case II: $\lambda^2 - 4\mu < 0$ $u = a_0 + \frac{6l}{\beta}\left[\frac{-\lambda}{2} + \vartheta_2\left(\frac{-c_1\sin(\vartheta_2\zeta) + c_2\cos(\vartheta_2\zeta)}{c_1\cos(\vartheta_2\zeta) + c_2\sin(\vartheta_2\zeta)}\right)\right]$	**Case II:** $\mu < 0, \eta > 0,$ $u = a_0 + \frac{6l\eta}{\delta}\left(\frac{\mu e^{\mu(\psi+\psi_0)}}{1 + \eta e^{\mu(\psi+\psi_0)}}\right)$
Case III: $\lambda^2 - 4\mu = c = 0$ $u = a_0 + \frac{6l}{\beta}\left(\frac{-\lambda}{2} + \frac{c_2}{c_1 + c_2\zeta}\right)$ where $\zeta = \frac{kx^\alpha}{\Gamma(\alpha+1)} + \frac{ly^\alpha}{\Gamma(\alpha+1)} - \frac{ct^\alpha}{\Gamma(\alpha+1)}$, $\vartheta_1 = \frac{\sqrt{c/l^3}}{2}, \vartheta_2 = \frac{\sqrt{-c/l^3}}{2}$	where $\psi = \frac{kx^\alpha}{\Gamma(\alpha+1)} + \frac{ly^\alpha}{\Gamma(\alpha+1)} - \frac{l^3\mu^2t^\alpha}{\Gamma(\alpha+1)}$

TABLE 2 | Solutions comparison of the non-linear space-time fractional Ablowitz-Kaup-Newell-Segur (AKNS) equation between G'/G-expansion method and simple equation (SE) method.

G'/G-expansion method	Simple equation method
Case I: $\lambda^2 - 4\mu > 0$ $u = a_0 - \frac{ck}{l}\left[\frac{-\lambda}{2} + \varrho_1\left(\frac{c_1\sinh\left(\frac{\zeta\varrho_1}{2}\right) + c_2\cosh\left(\frac{\zeta\varrho_1}{2}\right)}{c_1\cosh\left(\frac{\zeta\varrho_1}{2}\right) + c_2\sinh\left(\frac{\zeta\varrho_1}{2}\right)}\right)\right]$	**Case I:** $\mu > 0, \eta < 0,$ $u = a_0 - \left(\frac{\beta k^2\eta^2}{4l\eta + k^2l\mu^2}\right)\left(\frac{\mu e^{\mu(\psi+\psi_0)}}{1 - \eta e^{\mu(\psi+\psi_0)}}\right)$
Case II: $\lambda^2 - 4\mu < 0$ $u = a_0 - \frac{ck}{l}\left[\frac{-\lambda}{2} + \varrho_2\left(\frac{-c_1\sin\left(\frac{\zeta\varrho_2}{2}\right) + c_2\cos\left(\frac{\zeta\varrho_2}{2}\right)}{c_1\cos\left(\frac{\zeta\varrho_2}{2}\right) + c_2\sin\left(\frac{\zeta\varrho_2}{2}\right)}\right)\right]$	**Case II:** $\mu < 0, \eta > 0,$ $u = a_0 + \left(\frac{\beta k^2\eta^2}{4l\eta + k^2l\mu^2}\right)\left(\frac{\mu e^{\mu(\psi+\psi_0)}}{1 + \eta e^{\mu(\psi+\psi_0)}}\right)$
Case III: $\lambda^2 - 4\mu = \frac{4c + \gamma k}{ck^2} = 0$ $u = a_0 - \frac{ck}{l}\left(\frac{-\lambda}{2} + \frac{c_2}{c_1 + c_2\zeta}\right)$ where $\zeta = \frac{kx^\alpha}{\Gamma(\alpha+1)} + \frac{ly^\alpha}{\Gamma(\alpha+1)} - \frac{ct^\alpha}{\Gamma(\alpha+1)}$, $\varrho_1 = \sqrt{\frac{4c + \gamma k}{4ck^2}}, \varrho_2 = \sqrt{-\frac{4c + \gamma k}{4ck^2}}$	where $\psi = \frac{kx^\alpha}{\Gamma(\alpha+1)} + \frac{ly^\alpha}{\Gamma(\alpha+1)} + \frac{\beta k\eta t^\alpha}{\Gamma(\alpha+1)(4\eta + k^2\mu^2)}$

arbitrary constant ψ_0:
Case I: $\mu > 0, \eta < 0,$

$$u(x, y, t) = a_0 - \left(\frac{\beta k^2\eta^2}{4l\eta + k^2l\mu^2}\right)\left(\frac{\mu e^{\mu(\psi+\psi_0)}}{1 - \eta e^{\mu(\psi+\psi_0)}}\right), \quad (36)$$

Case II: $\mu < 0, \eta > 0,$

$$u(x, y, t) = a_0 + \left(\frac{\beta k^2\eta^2}{4l\eta + k^2l\mu^2}\right)\left(\frac{\mu e^{\mu(\psi+\psi_0)}}{1 + \eta e^{\mu(\psi+\psi_0)}}\right), \quad (37)$$

where $\psi = \frac{kx^\alpha}{\Gamma(\alpha+1)} + \frac{ly^\alpha}{\Gamma(\alpha+1)} + \frac{\beta k\eta t^\alpha}{(4\eta + k^2\mu^2)\Gamma(\alpha+1)}$.

We set some parameter to get the example graph of wave effects of the non-linear space-time fractional AKNS equation. Equation (36) gives a kink wave effect when setting $a_0 = 0$, $\beta = 1, k = 1, l = 1, \mu = 1, \eta = -1, \alpha = 0.5, 0 \le x \le 90$, $0 \le y \le 90$, and $t = 400, 800$ shown in **Figure 3**. Substituting $a_0 = 0, \beta = 1, k = 1, l = 1, \mu = -1, \eta = 1, \alpha = 0.5$, $0 \le x \le 90, 0 \le y \le 90$, and $t = 200, 600$ into an Equation (37), kink wave effect shown in **Figure 4**.

4. SOLUTIONS COMPARISON

In this section, the analytical solutions of the non-linear space-time fractional EMC equation and the non-linear space-time fractional AKNS equation obtained by the SE method can be expressed in a simpler form than the G'/G-expansion method [25] as in **Tables 1**, **2**.

5. CONCLUSION

In this study, we solved some fractional shallow water equations and fractional optical fiber equations which are the non-linear space-time fractional EMC equation and the non-linear space-time fractional AKNS equation, respectively. We converted these equations to nODEs by Jumarie's Riemann-Liouville derivative and defined the solutions by an efficient method, the SE method with the Bernoulli equation. The new analytical solutions of the non-linear space-time fractional EMC equation are presented in 2 cases of the exponential solutions as shown in Equations (25)-(26). After we set some parameters, the wave effects of this equation were kink waves as displayed in **Figures 1**, **2**. The 2 cases of the new analytical solutions of the non-linear space-time fractional AKNS equation were exponential solutions as appeared in Equations (36)-(37). The kink wave effects of this equation are displayed in **Figures 3**, **4**. **Tables 1**, **2** showed our solutions have a simpler form compared to the solution obtained by the G'/G-expansion method [25].

DATA AVAILABILITY STATEMENT

The original contributions presented in the study are included in the article/supplementary material, further inquiries can be directed to the corresponding author.

AUTHOR CONTRIBUTIONS

Both authors listed have made a substantial, direct, and intellectual contribution to the work and approved it for publication.

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