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EDITORIAL
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Editorial: Recent Advances in Bifurcation Analysis: Theory, Methods, Applications and Beyond…

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Editorial on the Research Topic

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The description of the phase space of a dynamical system has attracted the attention of the scientific community for decades. With many examples coming from the real world, the motivation to understand the underlying nature of a given dynamical system has led to a rich cooperation between the theoretical and the applied aspects of the subject. Such dynamical models may come in various sizes and shapes: they can be finite dimensional (given by the flow of a vector field or iterations of a map) or infinite dimensional (defined by the evolution operator of a PDE). Of special interest are systems whose solutions undergo topological changes upon variations on their parameters. These events are known as bifurcations and are ubiquitous in every nonlinear system that depends on parameters. These phenomena are characterized by the re-arrangement of invariant objects—such as equilibria, periodic solutions and invariant manifolds—when one or more control parameters are perturbed beyond a critical threshold. Typically, a bifurcation triggers crucial transitions from one kind of qualitative dynamics to completely new different behaviors. This may result in dramatic changes of the dynamics, including passages to chaotic regimes, transforming or creating basins of attraction, shaping and giving rise to families of multiple solution types with a particular set of spatiotemporal features and, ultimately, reorganizing the overall structure of the phase space.

Today, the scope of bifurcation theory has broadened to make an impact on rapidly growing branches of dynamics such as slow-fast systems, piece-wise models, delay differential equations, Hamiltonian systems, stochastic systems, as well as across the pattern formation theory. Recent discoveries in these areas have seen the emergence of new exciting types of bifurcations, some of which have yet to be addressed in all their complexity. This is of special interest to understand the nature of systems near bifurcations in many applications, such as in laser dynamics, nerve impulses in neurons, electrochemical reactions, extinction/survival/synchronizing thresholds in population models in ecology and developmental biology, fluid mechanics and celestial mechanics, to name a few examples.

This special issue has gathered a series of avantgarde works touching on both fundamental and applied aspects of bifurcation theory, while successfully giving deeper insight into old and new challenges arising from these phenomena.

Some of the topics addressed in this collection include: New fundamental results on bifurcation phenomena in differential-algebraic systems, application of advanced methods for numerical bifurcation analysis, the role of bifurcation in chaotic dynamics, bifurcations of slow-fast systems and delay differential equations, global bifurcations and invariant manifolds (including homoclinic and heteroclinic phenomena), among others. Special mention is due to several contributions that

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delve into applications in neuroscience, ecology and population dynamics, biomedical systems, and engineering.

For instance, Creaser et al. investigate the loss of entrainment of our intrinsic body clock and the dynamics of re-entrainment in a model of the human circadian pacemaker forced by a 24 h light to dark cycle. By means of a bifurcation analysis of periodic solutions they identify the boundaries of the parameter regimes in which the circadian oscillator can entrain to external forcing. Also, computation of global invariant manifolds allows the authors to find and characterize a separatrix between phase advancing and phase delaying re-entrainment trajectories.

Ferrario et al. analyse periodic solutions in a system of four delayed differential equations forced by periodic inputs representing two competing neural populations. They follow a slow-fast modeling approach in considering the populations connected with fast mutual excitation and slow delayed inhibition. The authors focus on cycle skipping states and find that they accumulate in an infinite cascade of period-incrementing bifurcations with increasing periods for decreasing values of the local input strength. Moreover, authors examine periodic solutions that alternate between 1:1 locked states that repeat after an integer multiple of the input period and show that such states accumulate in a similar bifurcation cascades fashion with decreasing values of the lateral input strength.

In addition within the wide area of mathematical biology, Arancibia-Ibarra et al. study a Leslie–Gower predator-prey model with a hyperbolic functional response and weak Allee effect. Among the results, conditions that ensure long term coexistence and oscillation of both predator and prey populations can be found. Such a phenomena turn to be sorted out by saddle-node and Hopf bifurcations around a Bogdanov-Takens point as an organizing center of the dynamics.

On the more abstract side, Ovsyannikov and Ruan provide a full list of all codimension-one singular bifurcations of differential-algebraic equations in low-dimensional systems. The authors have pushed the boundaries of bifurcation theory with their fundamental results by constructing the relevant normal forms, finding the associated non-degeneracy conditions and giving a qualitative description of the emerging dynamics.

Finally, Nobukawa et al. offer an enlightening survey paper presenting recent findings concerning control of chaotic resonance. Unlike stochastic resonance, this phenomenon is caused by deterministic chaos while producing a similar effect. The authors examine attractor-merging bifurcations as sources of chaotic resonances and summarize the recent findings and conceivable approaches to address applications in engineering and biomedical systems.

We hope the exciting, let us say, dynamical findings gathered in this Research Topic will help the community to learn from such a sample of recent advances in the field, as well as to help out and shed light on new bifurcation archetypes.

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