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# Joint modeling of rainfall and temperature in Bahir Dar, Ethiopia: Application of copula

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**Objective:** Climate change has effects on the economy development of any country. This paper aimed to fit the best marginal and joint distribution models of rainfall with minimum and maximum temperatures.

**Methods:** The average values of minimum and maximum monthly temperature, and rainfall were used in this study. For the marginal model, five probability distributions and five families of copula models were employed to show the interdependence between the maximum and minimum average annual temperature with rainfall. The Kendall's tau ( $\tau$ ) correlation coefficient was used to find out the correlations between rainfall with minimum and maximum temperature. Both the Akai Information Criteria (AIC) and Bayesian information criteria (BIC) were used to select the best marginal and copula.

**Results:** The result revealed that there is a significant negative relationship between the maximum temperature and rainfall. The maximum average rainfall was obtained from June to August and the maximum temperature is almost consistent in all months. Based on AIC/BIC, the Weibull distribution for rainfall, the Beta for minimum, and the Gaussian for maximum temperature were identified as the best marginal distributions. The Clayton copula distribution was identified as the best copula for rainfall and minimum temperature (with parameter of  $\theta = 1.21$ , tau correlation =  $-0.41$ ,  $p < 0.001$ ), and Frank copula was identified for rainfall and maximum temperature (with unique Frank parameter of  $\theta = -3.94$ , correlation =  $-0.38$ ,  $p < 0.001$ ).

**Conclusion:** The result showed that there is a significant positive relationship between the average annual minimum temperature and rainfall; whereas a negative relationship occurred between the maximum temperature and rainfall. The Clayton and Frank copula were found to be the most appropriate to the model of a bivariate distribution of mean annual rainfall with minimum/maximum temperature respectively.

## KEYWORDS

copula models, rainfall, dependence, temperature, Kendall tau, Bahir Dar

## Introduction

For the last decades, the risks that are associated with climatic change are the most discussed issues in the scientific world. The temperature variation has many impacts on the economy, ecosystems, and society [1–3]. The measurement of climate includes the estimates of the average value of weather parameters and measures of variability near the average value [4–7]. It also includes patterns of temperature, precipitation, humidity, wind, and seasons. There are a number of literature that have been studying the effects of temperature and rainfall on crop production [4–6, 8].

The effects of climate change and variabilities such as rising temperature and changes in precipitation are undeniably clear its impacts already affecting ecosystems, biodiversity, and people [8]. Associated with global warming, there are strong indications that rainfall and temperature changes are taking place on both the global and regional scales [7]. For instance, the average global surface temperature has warmed by 0.8°C in the past century and 0.6°C in the past three decades [9].

Among the poorest regions, Africa's biophysical and socio-economic environments are highly vulnerable to the impacts of climate change and weather extremes due to high dependency on natural resources and climatic sensitive livelihoods [10]. Like other African countries, Ethiopia has a long history of being affected by extreme weather events.

## Temperature and rainfall

For the last three decades, Ethiopia has been affected by climate-related hazards. Agriculture, the most dominant sector in the national economy, has been mostly at risk because of its dependence on seasonal rainfall. Ethiopia has the most variable rainfall patterns, and a number of professionals and organizations have documented scientifically interesting reports on the country's rainfall variability by classifying the country into various wide temporal and spatial rainfall categories and many others. According to the Dinku et al. [11], the mean minimum temperatures have been increasing throughout the country particularly during the cold months by 0.37°C per decade in the last 60 years.

Even if temperature and rainfall are important determinants of crop yield, it is difficult to simulate rainfall and temperature simultaneously because of the interdependence between them [12–14]. It is assumed that there is a correlation between rainfall and temperature changes over months. According to Rajeevan et al. [15] and Cong and Brady [16], temperature and rainfall were positively correlated during January and May but they negatively correlated in July.

Rainfall and temperature play a vital role in climate change such as droughts and floods [11–14, 17, 18]. Both rainfall and temperature are the main climatic parameters that affect

agricultural productivity and weather-related industries [4, 11–14]. The copula approach for the joint modeling of precipitation and temperature, the structure of the interdependence between these variables in the current and future climate change can be determined. This further assists the risk assessment of extreme hydrological and meteorological events [15, 19]. Moreover, the copula approach is robust in the estimation that it enables the independent modeling of marginal behavior and interdependence between the variables besides revealing information on the degree of dependence. The copula modeling approach provides a flexible method to model the bivariate distributions in which variables might not follow the normality assumptions. Usually, the temperature and rainfall has a skewed distribution. Hence, the copula models have evolved into a popular tool for modeling dependence in a large number of statistical models. This approach is flexible in that it enables independent modeling of marginal behavior and dependence between the variables besides providing information on both the structure and degree of dependence. Besides, the Parametric copula models are more efficient than their corresponding non-parametric or semi-parametric models because their estimation is based on time information [20]. This is because, the parametric models lead to smaller standard errors and more precise estimates [21].

Moreover, little is known about the trends in rainfall and temperature as well as the correlation between these types of three variables (minimum temperature, maximum temperature and rainfall) in Ethiopia, particularly in the study area. Therefore, the present study aimed to see the trends, modeling the marginal distributions of mean annual rainfall, minimum and maximum temperature, and their joint copulas based on 55 years (January 1961–December, 2015) data available in Bahir Dar Meteorological Agency. It provides information that would be helpful for decision-makers in formulating policies to alleviate problems related to temperature and rainfall.

## Data statistical analysis

### Study area and data source

This study was conducted in Bahir Dar city, Ethiopia in order to model the marginal and joint model of rainfall, minimum and maximum temperature. The data used secondary data on climate parameters such as monthly Rainfall in (mm) and Temperature (Tmax and Tmin) in °C from 1961 to 2015 obtained from Bahir Dar Meteorological Agency. There were some missing records for days of the month. But our interest was focused on the monthly datasets so that the missing values were substituted by the mean of near points since the sequential observations are highly dependent on all the variables.

Bahir Dar City is one of the largest and most rapidly expanding cities in Ethiopia. It is the political, economic,

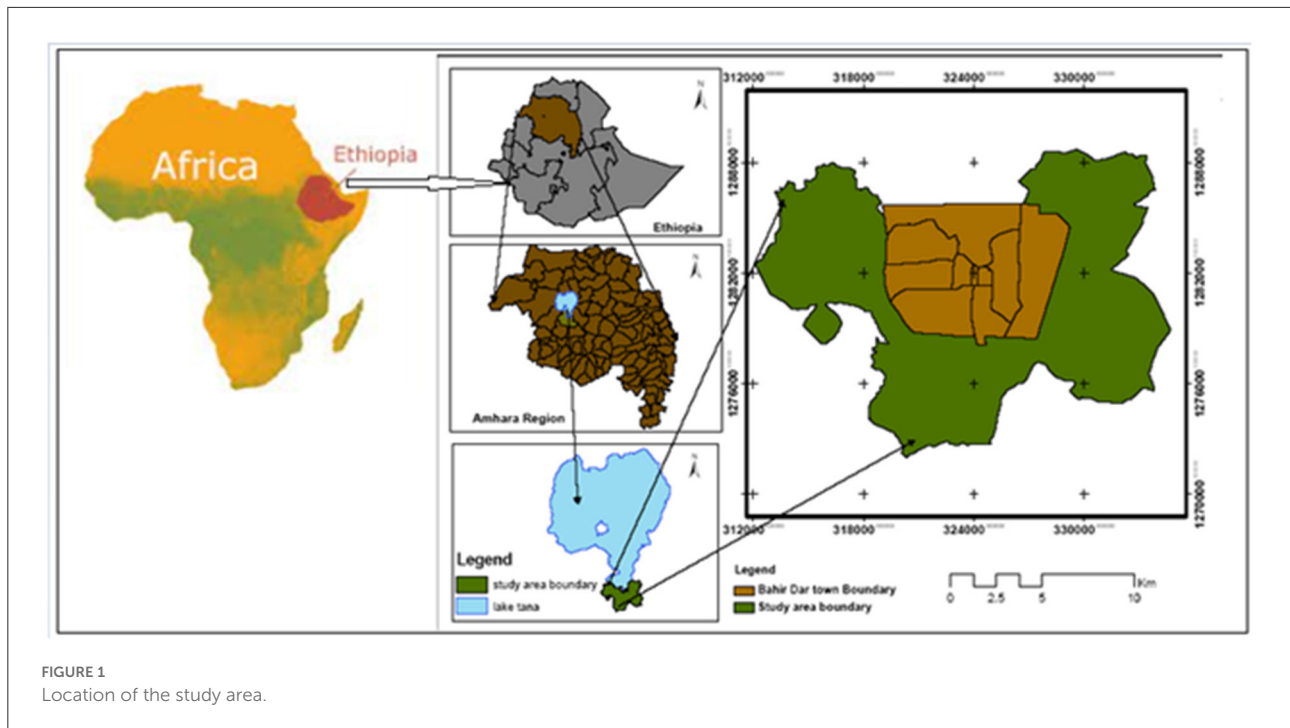


FIGURE 1 Location of the study area.

TABLE 1 The first five univariate continuous probability distributions [23–28].

Distributions	$f(x)$	Range of parameters	Description of the parameters
Normal	$\frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{x-\mu}{2\sigma}\right)^2$	$-\infty < x, \mu < \infty, \sigma > 0$	$\mu$ and $\sigma$ are the location and scale parameters
Exponential	$\frac{1}{\beta} e^{-x/\beta}$	$\alpha, \beta > 0$	$\beta$ ( $\frac{1}{\lambda}$ ) are the scale parameter
Gamma	$\frac{1}{\Gamma(\alpha)} \beta^\alpha x^{\alpha-1} e^{-\beta x}$	$x, \alpha, \beta > 0$	$\alpha$ and $\beta$ are the shape and scale parameters
Weibull	$\alpha\beta^\alpha x^{\alpha-1} e^{-(\beta x)^\alpha}$	$x, \alpha, \beta > 0$	$\alpha$ and $\beta$ are the location and shape parameters
Beta	$\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1} (1-x)^{\beta-1}$	$a \leq x \leq b; \alpha, \beta > 0$	$\alpha, \beta$ are shape parameters, $a$ and $b$ are the lower and upper bounds and $B(\alpha, \beta) = \int_0^1 x^{\alpha-1} (1-x)^{\beta-1} dx$ is the beta function.

and cultural center of Amhara National Regional State (ANRS), the second-most populous region in the country. In addition, the city is one of the major tourist centers in the country because of its cultural heritage (such as the Lake Tana Monasteries and religious festivals) and natural attractions (such as the Blue Nile Falls, birds, and hippos). The area is also characterized by rich biodiversity, and, as such, recognized as a Biosphere Reserve by UNESCO. According to data from the Central Statistics Agency [22], in 2017, about 350,000 people live in the city. The city is located at 11°36'N latitude and 37°23'E longitude, at the southern shores of Lake Tana, the largest lake in Ethiopia (Figure 1).

Both the temperature and rainfall variables followed non-normal probability density functions and non-linear dependence relationships. Practically, temperature belongs to

bounded and skewed distribution, rainfall is characterized by the high zero frequency, and the probability distribution explains its discontinuity between zero and non-zero observations [15, 19]. Therefore, the most commonly used univariate probability distributions such as Normal, Exponential, Gamma, Weibull, and Beta distributions have been used to fit the marginal distributions for the variables, and information criteria methods (AIC, BIC, and logLik) were used to identify the best fit (Table 1).

The copula (joint) distributions were obtained by combining the identified marginal distributions of each variable by using the selected best copula function. A copula is a multivariate distribution function that links joint probability distributions to their one-dimensional marginal distributions [23]. Let X, Z and Y be the continuous random variables representing the mean yearly minimum/maximum temperature and rainfall, with

TABLE 2 The first five families of copulas.

Family	$C(u,v)$	$\theta \in$	Copula families
Normal	$N_\theta(\phi^{-1}(u), \phi^{-1}(v))$	$[-1, 1]$	Elliptical copula
Student	$T_{\theta,\gamma}(T_\gamma^{-1}(u), T_\gamma^{-1}(v))$	$[-1, 1]$	
Clayton	$(u^{-\theta} + v^{-\theta} - 1)^{-\frac{1}{\theta}}$	$(0, \infty)$	Archimedean copula
Frank	$-\theta^{-1} \ln\{1 + (e^{-\theta u} - 1)(e^{-\theta v} - 1)/(e^{-\theta} - 1)\}$	$(-\infty, \infty)$	
Gumbel	$\exp\left\{-\left[(-\ln u)^\theta + (-\ln v)^\theta\right]^{\frac{1}{\theta}}\right\}$	$[1, \infty)$	

TABLE 3 Descriptive statistics for monthly minimum, maximum temperature (unit: °C), and rainfall (mm) from 1961 to 2015.

Variables	Measures	Jan.	Feb.	Mar.	Apr.	May.	Jun.	Jul.	Aug.	Sep.	Oct.	Nov.	Dec.
Minimum temperature	Minimum	3.00	4.80	7.50	7.60	10.80	6.5	8.3	8.4	6.3	7.3	4.5	3.00
	Maximum	28.9	31.2	<b>33.2</b>	<b>32.2</b>	<b>35.2</b>	27.1	26.8	27.5	28.1	28.0	28.0	28.1
	Mean	26.6	28.1	<b>28.9</b>	<b>29.3</b>	<b>28.5</b>	26.9	23.9	24.0	25.2	26.2	25.9	25.7
	SD	0.9	1.3	4.1	4.2	4.1	1.5	1.6	0.95	0.82	0.82	3.65	3.64
	(CV)	3.37	4.71	14.2	14.3	14.3	5.53	6.76	3.95	3.25	3.11	14.1	14.2
Maximum temperature	Minimum	24.80	24.80	25.40	26.60	26.60	<b>14.90</b>	<b>21.60</b>	<b>23.40</b>	24.30	24.30	24.20	24.30
	Maximum	10.3	12.6	<b>15.3</b>	<b>16.7</b>	<b>17.0</b>	17.9	15.2	15.0	14.8	15.0	13.7	12.3
	Mean	<b>7.4</b>	<b>8.9</b>	11.6	12.9	13.5	<b>13.9</b>	<b>13.7</b>	13.5	12.9	12.6	10.5	<b>7.9</b>
	SD	2.22	2.62	2.96	3.41	4.09	1.95	1.26	1.32	1.49	1.49	1.83	2.18
	(CV)	30.1	29.6	25.5	26.4	30.4	13.9	9.2	9.7	11.5	11.8	17.4	27.6
Rainfall	Minimum	0.00	0.00	0.00	0.00	0.00	60.3	208	106	0.00	0.00	0.00	0.00
	Maximum	20.4	26.9	85.6	110	363	<b>437</b>	<b>649</b>	<b>648</b>	378	209	107	34
	Mean	<b>2.46</b>	<b>2.02</b>	9.25	23.9	79.3	<b>182</b>	<b>432</b>	<b>390</b>	199	91.8	18.0	<b>3.69</b>
	SD	4.73	5.17	16.4	28.4	70.2	75.2	109	108	54	56.1	24.9	7.4
	(CV)	<b>193</b>	<b>256</b>	<b>177</b>	<b>118</b>	88.5	41.1	25.3	27.7	27.2	61.1	<b>138.5</b>	<b>202</b>

CV, Coefficients of variation; SD, standard deviation. The bold figures indicate the maximum and minimum values among others.

cumulative distribution functions respectively.

$$F_x(x) = \Pr(X \leq x) \tag{1.1}$$

$$F_z(z) = \Pr(Z \leq z) \tag{1.2}$$

$$F_y(y) = \Pr(Y \leq y) \tag{1.3}$$

Whose margins are Uniform (0.1) [24] and the function is called a copula. Where u and v are equal to CDFs of X and Y, respectively.

$$u = F(x), v = G(y) \tag{1.7}$$

According to [26], there is a unique function C such as,

$$\Pr(X \leq x, Y \leq y) = C(F(x), G(y)) \tag{1.4}$$

Where,

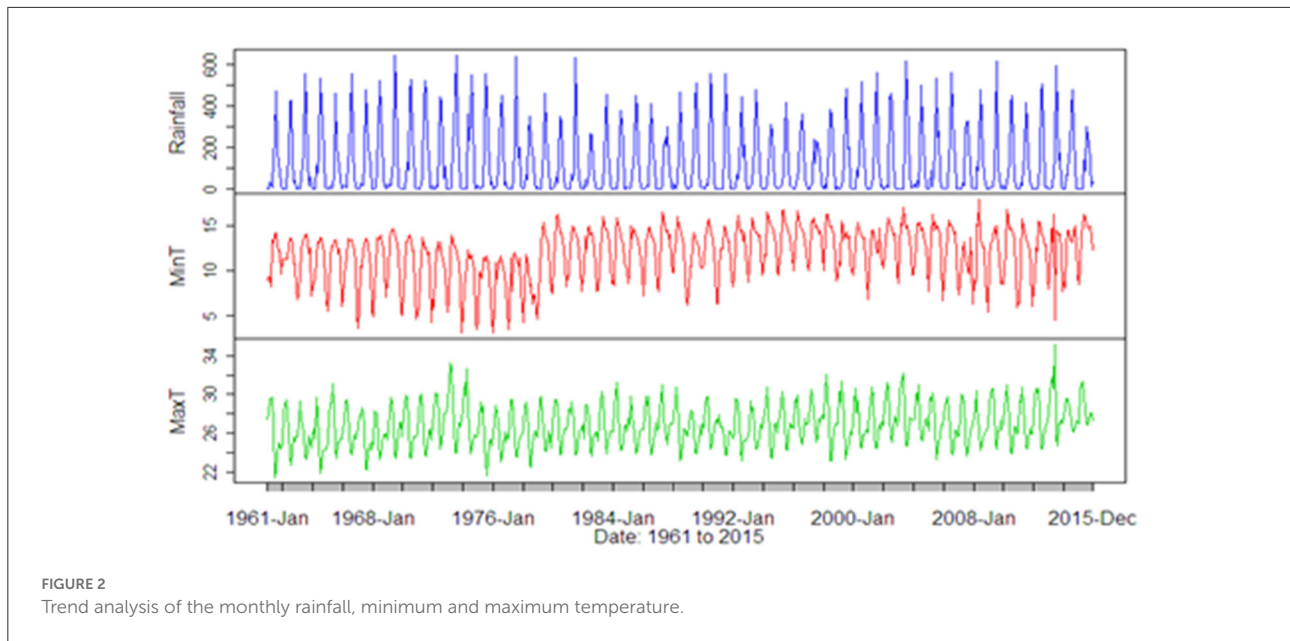
$$C(u, v) = \Pr(U \leq u, V \leq v) \tag{1.5}$$

is the distribution of the pair.

$$(U, V) = (F(X), G(Y)) \tag{1.6}$$

From 0 to 1 (u and v are strictly increasing functions). According to [25], Copulas are advantageous because they can model joint distributions of random variables with greater flexibility both in terms of marginal distributions and the dependence structure.

**Kendall tau ( $\tau$ ):** the measure of association [24, 26], can be expressed in terms of concordance and dis-concordance between random variables. The  $\tau$  is the difference between probability of concordance and dis-concordance of  $(x_1, x_2)$  and



$(y_1, y_2)$ . Hence the Kendall can be described as:

$$\tau_{XY} = [P(x_1 - x_2) - (y_1 - y) > 0] - [P(x_1 - x_2) - (y_1 - y) < 0]$$

The relation of Kendall tau measure with copula can be expressed as:

$$\tau(C_{X,Y}) = \tau_{XY} = 4 \int_0^1 \int_0^1 C(u,v) dC(u,v) - 1$$

Where,  $u \in F_X(x)$  and  $v \in F_Y(y)$ .

Hence the  $\tau$  correlation can be described by copula function and is termed as copula-correlation (Ccorr) in this study.

There are many parametric copula families available, which usually have parameters that control the strength of dependence. For this study two Elliptical [27] and three Archimedean [28], copula families are used (Table 2).

Where,

$\theta$ , measures dependency between  $u$  and  $v$ .

$\phi$ : the cumulative distribution function (CDF) of a  $N(0,1)$ ,  $N_\theta$ : CDF of the standard bivariate normal distribution with Pearson correlation  $\theta$ .  $T_\gamma$ : CDF of a student distribution with  $\gamma$  degrees of freedom and  $T_{\theta,\gamma}$ : [29].

After we have calculated the parameters of each copula, we decided which family is the best representation of the dependence structure between the variables of interest. In order to select the best copula, we used Akaike and Bayesian information criteria [30, 31]. The data was analyzed with the statistical package R 3.3.2.

## Results

Table 3 present the statistical characteristics of rainfall, minimum and maximum temperature in the area between 1961 and 2015.

The average minimum temperature usually reaches its bottom peak from December to February and the maximum peak from June to August. The coefficient of variation reveals that the maximum temperature is more consistent than the minimum temperature recorded in all months. Moreover, more variability of the rainfall was recorded in February (CV = 256) and the least variation was in July (25.3). From June to August, the minimum values of the maximum temperature and the monthly maximum rainfall were recorded (Table 3).

Like the minimum and maximum temperature, monthly average rainfall in the study area shows a clear seasonal cycle from 1961 to 2015. Moreover, both the minimum and maximum temperature increases from time to time (Figure 2).

The mean maximum and minimum temperature have been increasing in the study area by 0.3792 and 0.3714°C per decade in the last six decades, respectively. The rainfall is decreased by 1.48 (mm) per decade from 1961 to 2015 years (Table 4).

The skewness of rainfall, minimum, and maximum temperature respectively are 1.39, -0.73, and 0.06. Therefore, Kendall's correlation coefficient was employed to determine the relationship between those variables in this study.

According to the directives of Benth and Saltyte-Benth [32], the time series of temperature and rainfall are tested for auto-correlation using the Q-statistics (Figure 3). The term "autocorrelation" refers to the relationship between temperature (or rainfall) data at various times in relation to the function of time. The effectiveness of the model is decreased because

TABLE 4 Basic descriptive statistics for rainfall, minimum and maximum temperature based on decades from 1961 to 2015.

Variables	Year category	Mean	Mean difference	SE mean	CV	Minimum	Maximum
Rainfall	1960–1970	124.6	6.2	15.1	132.84	0.0	649.5
	1971–1980	130.8	−22.8	15.4	129.29	0.0	648.3
	1981–1990	108.0	6.8	13.6	137.51	0.0	636.4
	1991–2000	114.8	5.9	12.8	122.52	0.0	557.2
	2001–2010	120.7	−3.5	15.2	138.03	0.0	618.5
	2011–2020	117.2		19.8	130.75	0.0	594.0
Average change			−1.48				
Minimum temperature	1960–1970	10.538	−0.874	0.324	33.70	0.0	14.6
	1971–1980	9.664	2.656	0.304	34.44	0.0	16.3
	1981–1990	12.320	0.480	0.227	20.16	6.1	16.5
	1991–2000	12.800	−0.446	0.278	23.83	0.0	16.7
	2001–2010	12.354	0.041	0.267	23.66	0.0	17.9
	2011–2020	12.395		0.376	23.51	4.5	16.3
Average change			0.371				
Maximum temperature	1960–1970	26.051	0.597	0.194	8.16	21.4	31.1
	1971–1980	26.648	0.100	0.207	8.50	21.6	33.2
	1981–1990	26.748	−0.538	0.171	7.00	23.2	31.2
	1991–2000	26.210	0.519	0.425	17.75	0.0	32.1
	2001–2010	26.729	1.218	0.384	15.74	0.0	32.2
	2011–2020	27.947		0.274	7.74	24.3	35.2
Average change			0.379				

autocorrelation raises the variances of the predicted coefficients and residuals. The Ljung-Box Q test is a type of statistical test of whether autocorrelations of a time series are found to be statistically insignificant for maximum temperature (Q-stat = 1.767,  $P = 0.184$ ), minimum temperature (Q-stat = 1.830,  $P = 0.176$ ) and rainfall (Q-stat = 0.445,  $P = 0.505$ ) as shown in Figure 3.

The result indicated that there is a significant negative relationship between the mean monthly maximum temperature and rainfall (Kendall’s correlation coefficient =  $-0.388$ ,  $p \leq 0.001$ ). Particularly, from April to June (Kendall correlation coefficient is  $-0.283$ ,  $-0.33$ ,  $-0.396$ ,  $P = 0.003$ ,  $\leq 0.001$ ,  $\leq 0.001$  and respectively) and October to November ( $-0.251$ ,  $-0.260$ ,  $p = 0.022$  and  $0.006$ ). However, there is a positive significant relationship between the average monthly minimum temperature and rainfall (Kendall’s correlation coefficient =  $0.425$ ,  $p \leq 0.001$ ), from February to March (Kendall’s correlation coefficient =  $0.353$ ,  $0.250$ ,  $p \leq 0.001$ ,  $\leq 0.010$ ) (Table 5 and Figure 4).

Based on the minimum information criteria AIC and BIC values, the Weibull distribution for rainfall with parameters ( $\alpha = 0.41$ ,  $\beta = 0.004$ ) mm, Beta for average yearly minimum

temperature with parameters ( $\alpha = 2.95$ ,  $\beta = 5.02$  and  $a = 0$  and  $b = 18.3$ ) oC and Gaussian distribution for average yearly maximum temperature with parameters ( $\mu = 26.8$ ,  $\sigma = 2.189$ ) oC were identified as the best marginal distributions, indicated by bold (Table 6).

The marginal distribution function is given below.

$$f(y) = 0.04y^{-0.59} \exp(0.002y), y > 0 \tag{2.1}$$

$$f(x) = \frac{x^{1.95}(18.4 - x)^{4.02}}{6172824}, x > 0 \tag{2.2}$$

$$f(z) = 0.183 \exp \left\{ \frac{(z - 26.8)^2}{4.36} \right\}, z > 0 \tag{2.3}$$

The above Equations (2.1–2.3) refer to the marginal/univariate distribution function of average yearly rainfall (Weibull), minimum (Beta), and maximum temperature (Gaussian), respectively.

According to the AIC, BIC, and logLik values, the best copula for the joint modeling of average annual rainfall with minimum temperature is the Clayton, whose parameter was  $\theta = 1.21$ . However, the Frank copula model fits well for average annual rainfall and maximum temperature, the parameters



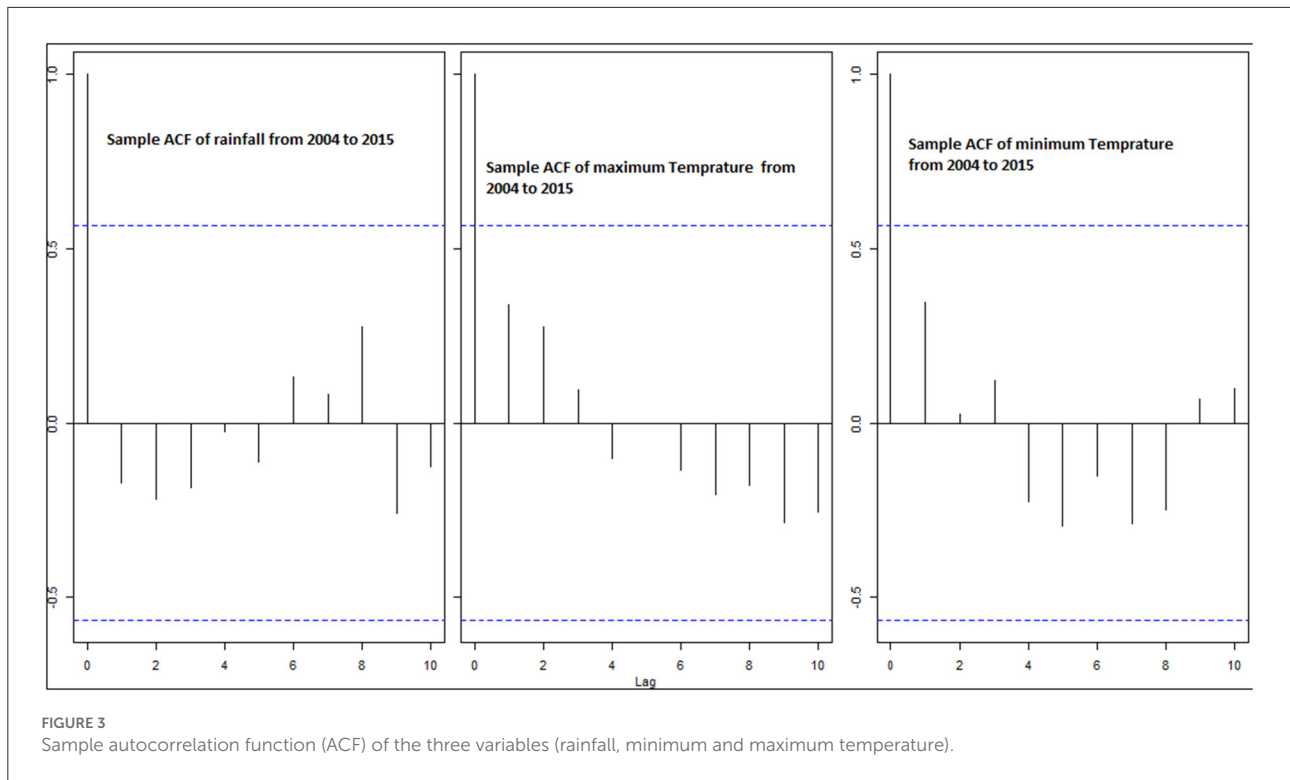


FIGURE 3 Sample autocorrelation function (ACF) of the three variables (rainfall, minimum and maximum temperature).

TABLE 5 Correlation analysis for monthly minimum and maximum temperature with rainfall from 1961 to 2015.

		Jan.	Feb.	Mar.	Apr.	May.	Jun.	Correlation
Minimum temperature	Kendall correlation coefficients	0.023	<b>0.353</b>	<b>0.250</b>	0.161	0.049	-0.138	<b>0.425</b> <b>&lt;0.001</b>
	p-value	0.826	<b>0.001</b>	<b>0.010</b>	0.086	0.601	0.142	
		<b>Jul</b>	<b>Aug.</b>	<b>Sep.</b>	<b>Oct.</b>	<b>Nov.</b>	<b>Dec.</b>	
	Kendall correlation coefficients	-0.025	-0.153	-0.007	0.165	0.035	0.161	
	p-value	0.794	0.103	0.936	0.079	0.711	0.101	
		<b>Jan.</b>	<b>Feb.</b>	<b>Mar.</b>	<b>Apr.</b>	<b>May</b>	<b>Jun</b>	<b>-0.388</b> <b>&lt;0.001</b>
Maximum temperature	Kendall correlation coefficients	-0.063	-0.128	-0.022	<b>-0.283</b>	<b>-0.33</b>	<b>-0.396</b>	
	p-value	0.548	0.220	0.819	<b>0.003</b>	<b>0.000</b>	<b>0.000</b>	
		<b>Jul</b>	<b>Aug.</b>	<b>Sep.</b>	<b>Oct.</b>	<b>Nov.</b>	<b>Dec.</b>	
	Kendall correlation coefficients	-0.094	-0.009	-0.146	<b>-0.251</b>	<b>-0.26</b>	-0.098	
	p-value	0.319	0.925	0.124	<b>0.022</b>	<b>0.006</b>	0.342	

The bold values indicates the significant values.

for the selected copula family is computed and statistically significant (Table 7).

$$H(X, Y) = \left[ (F(x))^{-1.21} + (F(y))^{-1.21} \right]^{\frac{-1}{1.21}}, x, y > 0 \quad (2.4)$$

$$H(Z, Y) = \frac{1}{3.94} \ln \left\{ 1 + \frac{(e^{3.94F(z)-1})(e^{3.94F(y)-1})}{e^{3.94}-1 = 18.92} \right\}, x, y > 0 \quad (2.5)$$

The Equations (4 and 5) given above show the joint/copula model for the average yearly rainfall with minimum temperature and maximum temperature respectively.

## Conclusion

This study has been done in the Bahir Dar city with the aim of investigating the relationship between temperature and rainfall variability. Monthly rainfall and temperature data for the period 1961 to 2015 years were used for this study. Because the sample data is non-Gaussian distributed and skewed, the Kendall correlation coefficient is employed to calculate the correlation between monthly rain-fall and temperature. Our result revealed that there is a significant negative relationship between the mean monthly maximum temperature and rainfall.

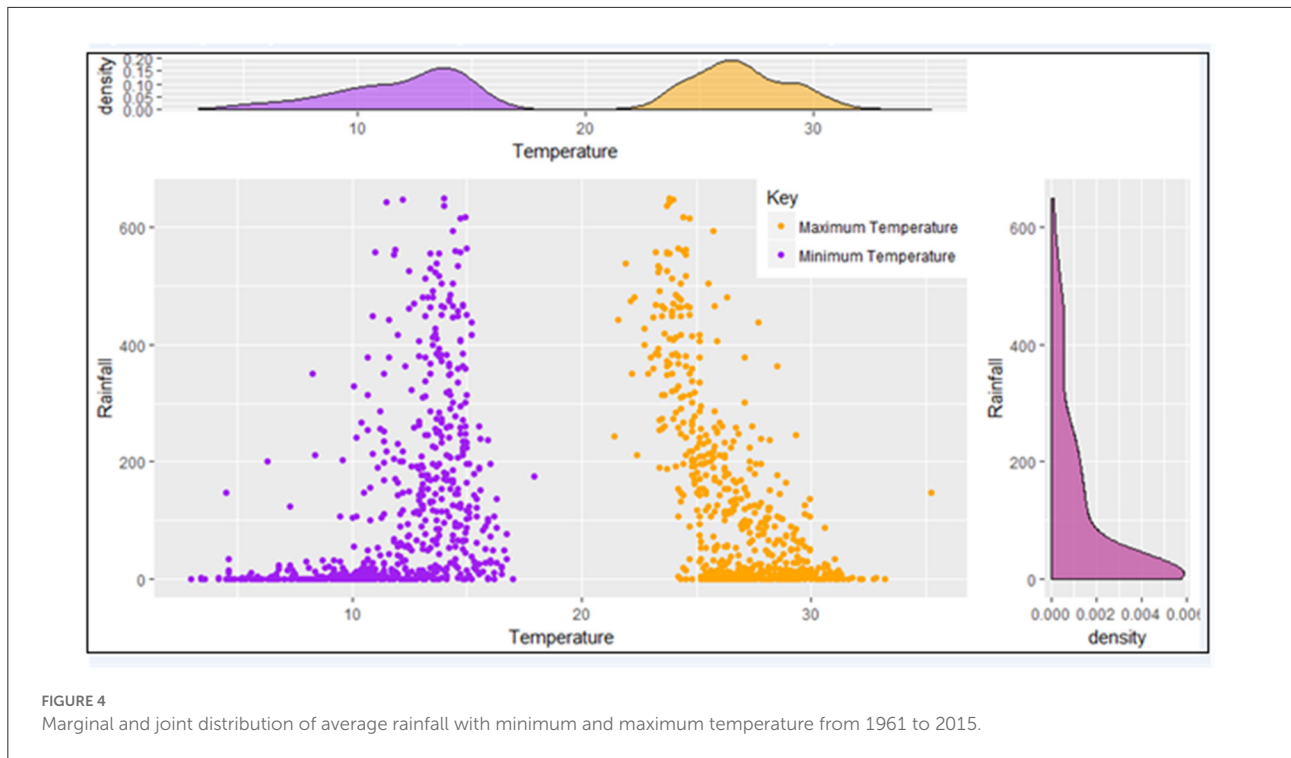


FIGURE 4 Marginal and joint distribution of average rainfall with minimum and maximum temperature from 1961 to 2015.

TABLE 6 The goodness of fit values of AIC, BIC and log likelihood of marginal distributions of the mean rainfall, maximum and minimum temperature.

Probability distribution	Rainfall			Minimum temperature			Maximum temperature		
	AIC	BIC	LogLik	AIC	BIC	logLik	AIC	BIC	logLik
Normal	3,932.9	3,941.9	-1,964.4	<b>3,291.7</b>	<b>3,300.6</b>	<b>-1,643.8</b>	2,873.4	2,882.4	-1,434.7
Exponential	4,808.9	4,808.9	-2,403.5	4,580.9	4,585.3	-2,289.4	5,664.2	5,668.7	-2,831.1
Gamma	3,801.5	3,810.4	-1,898.7	3,428.8	3,437.8	-1,712.4	2,864.9	2,873.9	-1,430.5
Weibull	<b>3,794.8</b>	<b>3,803.7</b>	<b>-1,895.4</b>	3,409.6	3,418.5	-1,702.8	2,864.4	2,873.9	-1,430.4
Beta	4,042.5	4,160.4	-2,345.5	3,198.9	3,216.9	-1,701.8	<b>2,861.9</b>	<b>2,879.9</b>	<b>-1,426.9</b>

TABLE 7 Results of different copula models for mean annual rainfall with minimum and maximum temperature.

Copula families	Between rainfall and minimum temperature				Between rainfall and maximum temperature			
	AIC	BIC	logLik	Parameters	AIC	BIC	logLik	Parameters
Normal	-122.8	-118.9	62.41	for selected Copula	-141.5	-137.5	71.7	For selected Copula
Student	-121.3	-113.5	62.7		-137.8	-130.0	70.9	
<b>Clayton</b>	<b>-182.3</b>	<b>-178.3</b>	<b>92.7</b>	$\theta = 1.21, \tau = 0.41, p < 0.01$	-130.7	-126.8	66.4	
<b>Frank</b>	-160.4	-156.5	81.2		<b>-185.6</b>	<b>-181.7</b>	<b>93.8</b>	$\theta = -3.94, \tau = -0.38, p < 0.01$
Gumbel	-153.4	-149.4	77.7		-167.3	-163.3	84.62	

The bold values indicated to show the selected models.



However, there is a positive significant relationship between the average monthly minimum temperature and rainfall. This is in lined with the climatology correlations that in the study area the correlation between the maximum and minimum temperature, respectively with rainfall is  $-0.58$  and  $0.36$ .

This paper presents modeling the marginal distributions of average annual rainfall, minimum and maximum temperature, and developing copula-based methodologies for rainfall with minimum/maximum temperature, which are very important for climate changes. Moreover, the study presented joint models of rainfall with temperature into the bivariate copula distribution.

The mean maximum and minimum temperature have been increasing in the study area by  $0.3792$  and  $0.3714^{\circ}\text{C}$  per decade in the last six decades, respectively. That is almost consistent with the study conducted throughout the country [11], and less compared with the world in which mean global surface temperature has warmed by  $0.6^{\circ}\text{C}$  in the past three decades [9].

This approach provides a flexible method to model the bivariate distributions in which variables might not follow the normality assumptions. Based on the criteria, the result of this study shows that, for average yearly rainfall Weibull, for maximum temperature Beta, and minimum temperature Gaussian marginal distributions were identified as the best fit. Moreover, in this study, the copula theory was adopted for modeling rainfall and temperature jointly which are mainly the two basic climatic factors affecting agricultural production and meteorological phenomena. Knowing the nature of the relationship of the temporal change of the temperature with other climatic parameters such as rainfall is crucial for climate planning in the future. Using the AIC, BIC, and logLik values, Clayton and Frank Copula were the best fit for the joint (copula) distribution of rainfall with minimum and maximum temperatures respectively.

The result showed that the relationship between the temperature and the rainfall is not linear. However, no studies have been carried out to determine the structure of the interdependency between these two variables and hence we used the copula theory to determine the relationship and their joint probability distributions. To conclude, the simulated results show that the selected copula leads Kendall's correlation to result in close to the real observations. For instance, for the Clayton model of rainfall and minimum temperature ( $\rho = 0.42$  for real observations,  $\rho = 0.44$  for simulated observations). Our result revealed that the joint modeling between the variables are consistent with previous research [16, 33–35].

## Limitations of the study

the study was only based on methodological data for a particular station, the most suitable copula family for both rainfall and temperatures may change from one station to another due to the change in geographical and geographical conditions. The other important limitation is, it impossible to obtain the daily datasets so we focused on the monthly aggregated values over the study period.

## Data availability statement

The original contributions presented in the study are included in the article/supplementary material, further inquiries can be directed to the corresponding author.

## Author contributions

HF was involved in this study from data management, data analysis, drafting, and revising the final manuscript. DW and DZ contributed to the conception, design, and interpretation of data, as well as to manuscript reviews and revisions. All authors contributed to the article and approved the submitted version.

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## Conflict of interest

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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