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Hybrid Fourier series and smoothing spline path non-parametrics estimation model

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Pathway analysis is one way to determine whether there is a causal relationship between extrinsic and intrinsic factors. The linearity assumption is something that can change the model. The shape of the model is subject to linearity assumptions. Path analysis is parametric when the linearity assumption is true, whereas non-parametric path analysis is used when the non-linear shape is unknown and there is no knowledge of the data pattern. Non-linear path analysis is used when the non-linear shape and data pattern are unknown. This work aimed to combine the smoothing spline method and the Fourier series method to compute non-parametric path function and it is believed that they would be able to produce more flexible function estimations for data patterns since both have the benefit of being accurate or close to the real data pattern. As a result, we found that Fourier series and smoothing splines can be used in non-parametric path analysis only if the linearity assumption is violated. Non-parametric regression-based path analysis estimators were then obtained using the ordinary least squares (OLS) approach. It uses a non-parametric approach and therefore gives non-unique estimation results.

KEYWORDS

Fourier series, non-parametric path analysis, regression analysis, ensemble models, smoothing spline

1. Introduction

In 1934, Wright first developed route analysis [1]. Path analysis is used to evaluate models of relationships between variables in the form of cause and effect [2]. Pathway analysis is a method of determining whether there is a causal relationship between extrinsic and intrinsic components [3]. In addition to determining the direct influence of extrinsic factors on intrinsic variables, path analysis is used to determine the indirect influence of extrinsic variables on intrinsic variables by mediating them [4]. A mediation model is a model in which the effect of a leading or independent variable (X) on the dependent variable (Y) is transmitted through a third mediating or mediating variable (M) [5].

Non-parametric path analysis, a method for applying regression, assumes that the shape of the regression function's curve is unknown. Non-parametric route analysis curves only consider a smooth curve [6]. It is assumed that the function curve is present in the function space [7]. The distinction between parametric and non-parametric techniques is that the former attempts to compel the data to follow a specific pattern, whilst the latter gives the data the flexibility to discover its regression curve pattern, making the latter more adaptable and objective [8].

Smoothing parameters are necessary for non-parametric path analysis to establish the size of the curve's smoothness or roughness in characterizing the data [9]. A very rough estimate of the regression curve is provided by the smoothing parameter's relatively modest value. On the other hand, a very smooth non-parametric regression curve estimator will be generated if the smoothing parameter value is quite large. To achieve the best estimate for the data, it is required to select the best smoothing parameter when estimating the non-parametric regression function. The GCV and CV approaches were applied by Dani and Adrianingsih [10] as smoothing parameters in non-parametric regression analysis with Fourier series.

The foundation of non-parametric path analysis is non-parametric regression, which examines the relationships between exogenous, endogenous dependent, and endogenous mediating factors [11]. Non-parametric path analysis employs a variety of methods, including moving averages, Fourier series, splines, kernels, local polynomials, and wavelets [12, 13].

Splines have special characteristics, namely their ability to adjust to changes in data behavior very well. The approaches for estimating non-parametric regression functions include the penalized spline approach and the smoothing spline approach [14]. For smoothing splines, no knot selection is required, because the estimation function is based on the criteria of model accuracy and curve smoothness that has been set by the smoothing parameter. The non-parametric regression model using the Fourier series estimator has been shown in research by Dani and Adrianingsih [10] to tend to mimic the real data pattern. When the two techniques are combined, it is believed that they would be able to produce more flexible function estimations for data patterns since both have the benefit of being accurate or close to the real data pattern. Based on the foregoing context, we will use Fourier series and smoothing spline techniques in this work to estimate a function estimator for a non-parametric route function.

2. Literature review

2.1. Non-parametric regression analysis

Non-parametric regression modeling is incredibly flexible and reduces the researcher's subjectivity [15]. Non-parametric regression analysis was used if the parametric regression analysis's conditions of normality, non-multicollinearity, and homoscedasticity were not fulfilled [16]. This strategy works effectively for concluding when there is little to no prior knowledge about the regression curve or data pattern [7].

There is a chance that using parametric regression on unknown data will result in an unrepresentative regression model, which will cause hypothesis testing to yield incorrect results. When the form of the curve for the response variable (Y), whose pattern of association with the predictor variable (X), is unknown, it can be determined using a non-parametric regression model:

$$Y_i = \hat{f}\,(x_i) + \varepsilon_i \tag{1}$$

If the linearity assumption is correct, parametric route analysis is used to conduct the investigation [17]. However, if the linearity presumption is broken, the analysis employs non-linear and/or non-parametric paths.

Where:

y_i: The response variable's value.

x_i: The predictor variable's value.

$\hat{f}$: Regression curve.

i: 1, 2, . . . , n.

n: The abundance of observations.

ε_i: Errors in the i-th observation.

2.2. Non-parametric path analysis based on Fourier series

One technique for non-parametric path analysis is Fourier series route analysis. The Fourier series may quickly adapt to the local character of the data since it is a versatile trigonometric polynomial [18]. The Fourier series has the benefit of being able to overcome data with a trigonometric distribution (sine and cosine) [12]. According to the Non-parametric Regression model in Equation 1, the Fourier series approximates $f\,(x_i)$ as follows:

$$\begin{gathered}\text{Minimize } {\varepsilon_i}^2\\ Min\left\{\sum_{i=1}^{n}\varepsilon_i^2\right\} = Min\left\{\sum_{i=1}^{n}\left(y_i - f\,(x_i)\right)^2\right\}\end{gathered} \tag{2}$$

Depending on how smooth the function f is, the following penalty is also applied in addition to minimizing Equation 2:

$$\int_0^{\pi} \frac{2}{\pi}\left(f^{(2)}\,(x)\right)^2 dx \tag{3}$$

This means that by using Penalized Least Squares (PLS) to finish the optimization, the estimator for the regression curve f can be obtained.

$$Min\left\{n^{-1}\sum_{i=1}^{n}\left(y_i - f\,(x_i)\right)^2 + \lambda\int_0^{\pi}\frac{2}{\pi}\left(f^{(2)}\,(x)\right)^2 dx\right\} \tag{4}$$

To resolve Equation 4, start by determining the value of P(a):

$$
\begin{aligned}
qualP\left(a\right) &= \int_0^{\pi} \frac{2}{\pi}\left(f^{(2)}\left(x\right)\right)^2 dxP\left(a\right) \\
&= \frac{2}{\pi}\int_0^{\pi}\left(\sum\nolimits_{k=1}^{K}\left(k^2 a_k coskx\right)^2\right. \\
&+ \left. 2\sum\nolimits_{k<j}^{K}\sum\nolimits_{k<j}^{K}\left(k^2 a_k coskx\right)\left(j^2 a_j cosjx\right)\right)dx \\
P\left(a\right) &= \sum\nolimits_{k=1}^{K} k^4 a_k^2
\end{aligned}
\tag{5}
$$

Which is a smoothing parameter that controls between the goodness of fit and smoothness of the function. For a very large, a very smooth solution function will be obtained, while for a very small, a very coarse solution will be obtained. f can be approximated by the function x because it is a continuous function, with:

$$f\left(x\right) = bx + \frac{1}{2}a_0 + \sum\nolimits_{k=1}^{K} a_k coskx \tag{6}$$

Using Equation 6, it is possible to write

$$
\begin{aligned}
&Min\left\{n^{-1}\sum\nolimits_{i=1}^{n}\left(y_i - f\left(x_i\right)\right)^2 + \lambda\int_0^p \frac{2}{p}\left(f^{(2)}\left(x\right)\right)^2 dt\right\} \\
&Min\left\{n^{-1}\sum\nolimits_{i=1}^{n}\left[y_i - bx - \frac{1}{2}a_0 - \sum\nolimits_{k=1}^{K} a_k coskx\right]^2\right. \\
&\quad \left. +\lambda\sum\nolimits_{k=1}^{K} k^4 a_k^2\right\} \\
&Min\left\{n^{-1}\left(y - \boldsymbol{Xa}\right)^{'(y-\boldsymbol{Xa})} + \lambda \boldsymbol{a}'\boldsymbol{Da}\right\} \\
&= Min\left\{n^{-1}y'y - n^{-1}a'X'y - n^{-1}\left(a'X'y\right)'\right. \\
&\quad \left. +a'\left(n^{-1}X'X + \lambda D\right)a\right\}
\end{aligned}
\tag{7}
$$

where:

$$D = diag(0, 0, 1^4, 2^4, \ldots, K^4) \tag{8}$$

If Equation 7 is known as $Q(a)$, then we can obtain it by partially subtracting $Q(a)$ from a and equating it to zero:

$$\frac{\partial Q(a)}{\partial a} = 0 - 2n^{-1}\boldsymbol{X}'y + 2\left(n^{-1}\boldsymbol{X}'\boldsymbol{X} + \lambda\boldsymbol{D}\right)\boldsymbol{as}$$

$$\hat{a}\left(\lambda\right) = \left(n^{-1}X'X + \lambda D\right)^{-1} n^{-1}X'y \tag{9}$$

It can be expressed as a matrix based on the characteristics of the Fourier series estimator in Equation 9.

$$\underset{\sim}{f} = \boldsymbol{X}\underset{\sim}{a} + \underset{\sim}{e} \tag{10}$$

$$\hat{\underset{\sim}{f}} = \boldsymbol{X}\underset{\sim}{a} \tag{11}$$

where:

$$\underset{\sim}{a} = \left(b, \frac{1}{2}a_0, a_1, \ldots, a_K\right) \tag{12}$$

$$\boldsymbol{X} = \begin{pmatrix} x_1 & 1 & \cos x_1 & \cos 2x_1 & \cdots & \cos Kx_1 \\ x_2 & 1 & \cos x_2 & \cos 2x_2 & \cdots & \cos Kx_2 \\ x_3 & 1 & \cos x_3 & \cos 2x_3 & \cdots & \cos Kx_3 \\ \vdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ x_n & 1 & \cos x_n & \cos 2x_n & \cdots & \cos Kx_n \end{pmatrix} \tag{13}$$

If Equation 11 is translated it will look as follows

$$\begin{pmatrix} y_1 \\ y_2 \\ y_1 \\ \vdots \\ y_M \end{pmatrix} = \begin{pmatrix} x_1 & 1 & \cos x_1 & \cos 2x_1 & \cdots & \cos Kx_1 \\ x_2 & 1 & \cos x_2 & \cos 2x_2 & \cdots & \cos Kx_2 \\ x_3 & 1 & \cos x_3 & \cos 2x_3 & \cdots & \cos Kx_3 \\ \vdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ x_n & 1 & \cos x_n & \cos 2x_n & \cdots & \cos Kx_n \end{pmatrix}\begin{pmatrix} b \\ \frac{1}{2}a_0 \\ a_1 \\ \vdots \\ a_K \end{pmatrix} + \begin{pmatrix} e_1 \\ e_2 \\ e_3 \\ \vdots \\ e_K \end{pmatrix}$$

Consequently, the following is how the estimator for the Fourier series' non-parametric path function is obtained:

$$\hat{f}_\lambda(x_i) = \hat{b}(\lambda)x_i + \frac{1}{2}\hat{a}_0(\lambda) + \sum_{k=1}^{K}\hat{a}_k(\lambda)\cos kx_i \tag{14}$$

2.3. Smoothing spline non-parametric path analysis

Regression analysis includes spline, more precisely non-parametric and semiparametric regression [19]. Spline research, which is independent and has personality, necessitates a thorough procedure with many steps that takes a very long time [20]. Non-parametric spline regression offers the following benefits for modeling data patterns: (a) Splines have very unique and excellent statistical interpretations. The Penalized Least Square (PLS) approach was optimized to produce the spline model. (b) Smooth data or functions can be handled by the spline. (c) The spline handles data whose behavior varies at particular sub-intervals quite well. (d) Spline excels at generalizing large and detailed statistical modeling. One of the spline models that can estimate the non-parametric regression curve is the Smoothing Spline [21].

The form of the relationship pattern between the response variable and the variable predictors can be estimated using the

Smoothing Spline function. The Estimator of Smoothing Spline can be obtained in the following way to minimize the Penalized Least Squares (PLS) in the regression function. A roughness penalty multiplied by the positive smoothing parameter added to the sum of the squares of the residuals in the Smoothing Spline regression. Therefore, the estimation of the function depends on the smoothing parameter.

The non-parametric path function based on the Smoothing Spline according to Budiantara [22] can be presented like the matrix in Equation 15.

$$.\hat{\underset{\sim}{f}} = \mathbf{T}\hat{\underset{\sim}{d}} + \mathbf{V}\hat{\underset{\sim}{c}} \tag{15}$$

Matrix $\mathbf{T}$ in Equation 15 is $n \times m$ in size with the following description.

$$\mathbf{T} = \begin{bmatrix} \langle \eta_{k1}, \phi_{k1} \rangle & \langle \eta_{k1}, \phi_{k2} \rangle & \cdots & \langle \eta_{k1}, \phi_{km} \rangle \\ \langle \eta_{k2}, \phi_{k1} \rangle & \langle \eta_{k2}, \phi_{k2} \rangle & \cdots & \langle \eta_{k2}, \phi_{km} \rangle \\ \vdots & \vdots & \ddots & \vdots \\ \langle \eta_{kn}, \phi_{k1} \rangle & \langle \eta_{kn}, \phi_{k2} \rangle & \cdots & \langle \eta_{kn}, \phi_{km} \rangle \end{bmatrix}_{n \times m}$$

Then, the element $\left\langle \eta_{ki}, \phi_{kj} \right\rangle$ is obtained from the Equation 16.

$$\left\langle \eta_{ki}, \phi_{kj} \right\rangle = \frac{x_i^{j-1}}{(j-1)!};\ i = 1,\ 2,\ \ldots,\ n;\ j = 1,\ 2,\ \ldots,\ m \tag{16}$$

Meanwhile, the $n \times n$ matrix $\boldsymbol{B}$ is described as follows.

$$\mathbf{V} = \begin{bmatrix} \langle \xi_{k1}, \xi_{k1} \rangle & \langle \xi_{k1}, \xi_{k2} \rangle & \cdots & \langle \xi_{k1}, \xi_{kn} \rangle \\ \langle \xi_{k2}, \xi_{k1} \rangle & \langle \xi_{k2}, \xi_{k2} \rangle & \cdots & \langle \xi_{k2}, \xi_{kn} \rangle \\ \vdots & \vdots & \ddots & \vdots \\ \langle \xi_{kn}, \xi_{k1} \rangle & \langle \xi_{kn}, \xi_{k2} \rangle & \cdots & \langle \xi_{kn}, \xi_{kn} \rangle \end{bmatrix}_{n \times n}$$

With element $\langle \xi_{ki}, \xi_{ks} \rangle$ obtained from Equation 17.

$$\langle \xi_{ki}, \xi_{ks} \rangle = \int_a^b \frac{(x_i - u)_+^{m-1} (x_s - u)_+^{m-1}}{((m-1)!)^2} du \tag{17}$$

With:

$$(x_i - u)_+^{m-1} = \begin{cases} (x_i - u)_+^{m-1}, & x_i \geq u \\ 0, & x_i < u \end{cases}$$

$$a = \min(x_i)\ \ dan\ b = \max(x_i)$$

For $x \in [0, 1]$ the results of calculations in Equation 18 are obtained as follows.

$$\langle \xi_{ki}, \xi_{ks} \rangle = x_i x_s - \frac{1}{2}(x_i + x_s) + \frac{1}{3} \tag{18}$$

then $d_\sim$ and $c_\sim$ vectors are obtained from Equations 19, 20.

$$\hat{\underset{\sim}{d}} = \mathbf{T}(\mathbf{T'M}^{-1}\mathbf{T})^{-1}\mathbf{T'M}^{-1}\underset{\sim}{y} \tag{19}$$

$$\hat{\underset{\sim}{c}} = \mathbf{M}^{-1}[\mathbf{I} - \mathbf{T}\left(\mathbf{T'M}^{-1}\mathbf{T}\right)^{-1}\mathbf{T'M}^{-1}]\underset{\sim}{y} \tag{20}$$

3. Method

Regression modeling in particular was the focus of this project, which aimed to develop statistical modeling theory. The lemma theorem from non-parametric route analysis was derived in this paper by combining the Smoothing Spline and Fourier series techniques. The Fourier series function's non-parametric path analysis was calculated using the Ordinary Least Square (OLS) approach.

4. Result and discussion

The Fourier series is a polynomial with flexibility that may effectively adapt to the local nature of the data. It is based on a cosine function. Periodic curves like sine and cosine waves may be accurately described by the Fourier series. The polynomial functions with a shortened function are added to create the smoothing spline. The function or data when there is a change in the behavior pattern of the curve that varies at various intervals is represented by the smoothing spline.

Lemma 4.1. Forms of a simple Fourier series and smoothing spline non-parametric path analysis model.

If given paired data $(X_{1i}, X_{2i}, Y_{1i}, Y_{2i})$, the relationship between $(X_{1i}, X_{2i}, Y_{1i}, Y_{2i})$ them is modeled by additive non-parametric path analysis. Equation 21 shows a combined non-parametric path analysis model of the Fourier series and smoothing spline.

$$\begin{aligned} y_{1i} &= f(x_{1i}) + \hat{g}(x_{2i}) + \varepsilon_i \\ y_{2i} &= f(x_{1i}, y_{1i}) + \hat{g}(x_{2i}, y_{1i}) + \varepsilon_i \end{aligned} \tag{21}$$

Where y_{1i} and y_{2i} are response variables, $f(x_{1i})$, $\hat{g}(x_{2i})$, $f(x_{1i}, y_{1i})$, and $\hat{g}(x_{2i}, y_{1i})$ are path curves of unknown shape and ε_i is a random error, with a mean of zero and a variance σ^2 of assumed to have an independent normal distribution. Let $f(x_{1i})$ be approximated by a Fourier series function and $\hat{g}(x_{2i})$ be modeled by a short spline function. The model of Fourier series can be seen in Equation 22.

$$\begin{aligned} \hat{f}_{1i} &= \frac{1}{2}a_{01} + \sum\nolimits_{j=1}^{p} b_{1j}X_{1i} + \sum\nolimits_{j=1}^{p} b_{1j}X_{2i} \\ &\quad + \sum\nolimits_{k=1}^{K} \gamma_{1(2\times k)} cosKY_{1i} \\ \hat{f}_{2i} &= \frac{1}{2}a_{02} + \sum\nolimits_{j=1}^{p} b_{2j}X_{1i} + \sum\nolimits_{j=1}^{p} b_{2j}X_{2i} \\ &\quad + \sum\nolimits_{k=1}^{K} \gamma_{1(2\times k)} cosKY_{1i} + \sum\nolimits_{k=1}^{K} \gamma_{1(k)} cosKY_{1i} \end{aligned} \tag{22}$$

When translated, equation Fourier series and smoothing spline is shown in the equation below:

If $K = 2$ and $m = 2$

$$\hat{f}_{1i} = \frac{1}{2}a_{01} + b_{11j}X_{1i} + \gamma_{11}\cos X_{1i} + \gamma_{21}\cos 2X_{1i} + b_{21} + b_{31}(X_{2i}) + \sum_{s=1}^{n} c_{1s}\left(X_{2i}X_{2s} - \frac{1}{2}(X_{2i} + X_{2s}) + \frac{1}{3}\right)$$

$$\hat{f}_{2i} = \frac{1}{2}a_{02} + b_{12j}X_{1i} + \gamma_{12}\cos X_{1i} + \gamma_{22}\cos 2X_{1i} + b_{22} + b_{32}(X_{2i} + Y_{1i}) + \sum_{s=1}^{n} c_{2s}\left(X_{2i}X_{2s} - \frac{1}{2}(X_{2i} + X_{2s}) + \frac{1}{3}\right)\left(Y_{1i}Y_{1s} - \frac{1}{2}(Y_{1i} + Y_{1s}) + \frac{1}{3}\right) \quad (23)$$

If $K = 3$ and $m = 2$

$$\hat{f}_{1i} = \frac{1}{2}a_{01} + b_{11j}X_{1i} + \gamma_{11}\cos X_{1i} + \gamma_{21}\cos 2X_{1i} + \gamma_{31}\cos 3X_{1i} + b_{21} + b_{31}(X_{2i}) + \sum_{s=1}^{n} c_{1s}\left(X_{2i}X_{2s} - \frac{1}{2}(X_{2i} + X_{2s}) + \frac{1}{3}\right)$$

$$\hat{f}_{2i} = \frac{1}{2}a_{02} + b_{12j}X_{1i} + \gamma_{12}\cos X_{1i} + \gamma_{22}\cos 2X_{1i} + \gamma_{32}\cos 3X_{1i} + b_{22} + b_{32}(X_{2i} + Y_{1i}) + \sum_{s=1}^{n} c_{2s}\left(X_{2i}X_{2s} - \frac{1}{2}(X_{2i} + X_{2s}) + \frac{1}{3}\right)(Y_{1i}Y_{1s} - \frac{1}{2}(Y_{1i} + Y_{1s}) + \frac{1}{3}\Big) \quad (24)$$

Jika $K = 4$ and $m = 2$

$$\hat{f}_{1i} = \frac{1}{2}a_{01} + b_{11j}X_{1i} + \gamma_{11}\cos X_{1i} + \gamma_{21}\cos 2X_{1i} + \gamma_{31}\cos 3X_{1i} + \gamma_{41}\cos 4X_{1i} + b_{21} + b_{31}(X_{2i}) + \sum_{s=1}^{n} c_{1s}\left(X_{2i}X_{2s} - \frac{1}{2}(X_{2i} + X_{2s}) + \frac{1}{3}\right)$$

$$\hat{f}_{2i} = \frac{1}{2}a_{02} + b_{12j}X_{1i} + \gamma_{12}\cos X_{1i} + \gamma_{22}\cos 2X_{1i} + \gamma_{32}\cos 3X_{1i} + \gamma_{42}\cos 4X_{1i} + b_{22} + b_{32}(X_{2i} + Y_{1i}) + \sum_{s=1}^{n} c_{2s}\left(X_{2i}X_{2s} - \frac{1}{2}(X_{2i} + X_{2s}) + \frac{1}{3}\right)(Y_{1i}Y_{1s} - \frac{1}{2}(Y_{1i} + Y_{1s}) + \frac{1}{3}\Big) \quad (25)$$

Jika $K = 5$ and $m = 2$

$$\hat{f}_{1i} = \frac{1}{2}a_{01} + b_{11j}X_{1i} + \gamma_{11}\cos X_{1i} + \gamma_{21}\cos 2X_{1i} + \gamma_{31}\cos 3X_{1i} + \gamma_{41}\cos 4X_{1i} + \gamma_{51}\cos 5X_{1i} + +b_{21} + b_{31}(X_{2i}) + \sum_{s=1}^{n} c_{1s}\left(X_{2i}X_{2s} - \frac{1}{2}(X_{2i} + X_{2s}) + \frac{1}{3}\right)$$

$$\hat{f}_{2i} = \frac{1}{2}a_{02} + b_{12j}X_{1i} + \gamma_{12}\cos X_{1i} + \gamma_{22}\cos 2X_{1i} + \gamma_{32}\cos 3X_{1i} + \gamma_{42}\cos 4X_{1i} + \gamma_{52}\cos 5X_{1i} + b_{22} + b_{32}(X_{2i} + Y_{1i}) + \sum_{s=1}^{n} c_{2s}\left(X_{2i}X_{2s} - \frac{1}{2}(X_{2i} + X_{2s}) + \frac{1}{3}\right)\left(Y_{1i}Y_{1s} - \frac{1}{2}(Y_{1i} + Y_{1s}) + \frac{1}{3}\right) \quad (26)$$

Proof

To create the model for non-parametric path analysis of the Fourier series and smoothing spline, multiple linear regression analysis, simple linear path analysis, and non-parametric regression analysis were used.

First part:

The following is how the simple linear regression model can be expressed:

$$Y_i = \beta_0 + \beta_i X_i + \varepsilon_i \quad (27)$$

where:

Y_i: The response variable's value for observation i.

β_0: Intercept parameters.

β_1: Parameter slope.

X_i: The predictor variable's value for observation i.

ε_i: Remainder in i-th observation.

Multiple linear regression analysis is used when there are multiple predictor variables. The equation can be used to express the multiple linear regression model (Equation 28).

$$Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \varepsilon_i \quad (28)$$

The equation can be used to represent the general linear regression model (29).

$$Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \ldots + \beta_p X_{i(p-1)} + \varepsilon_i \quad (29)$$

In multiple linear regression analysis with more than two predictor variables, the matrix technique can be utilized to address the parameter estimation problem. Equation 24 is a generic equation for a population multiple linear regression model with p-1 predictor variables. If there are n observations and p predictor variables, the regression equation is as follows:

$$\begin{gathered} Y_1 = \beta_0 + \beta_1 X_{11} + \beta_2 X_{12} + \cdots + \beta_p X_{1p-1} + \varepsilon_1 \\ Y_2 = \beta_0 + \beta_1 X_{21} + \beta_2 X_{22} + \cdots + \beta_p X_{2p-1} + \varepsilon_2 \\ Y_3 = \beta_0 + \beta_1 X_{31} + \beta_2 X_{32} + \cdots + \beta_p X_{3p-1} + \varepsilon_3 \\ \vdots \\ Y_n = \beta_0 + \beta_1 X_{n1} + \beta_2 X_{n2} + \cdots + \beta_p X_{np-1} + \varepsilon_n \end{gathered} \quad (30)$$

From the equation above, the following can be written in matrix form:

$$\begin{pmatrix} Y_1 \\ Y_2 \\ Y_3 \\ \vdots \\ Y_n \end{pmatrix} = \begin{pmatrix} 1 & X_{11} & X_{12} & \cdots & X_{1p-1} \\ 1 & X_{21} & X_{22} & \cdots & X_{2p-1} \\ 1 & X_{31} & X_{32} & \cdots & X_{3p-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & X_{n1} & X_{n2} & \ldots & X_{np-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \vdots \\ \varepsilon_n \end{pmatrix} \tag{31}$$

Second part:

It is well-known that the model in Equation 32 and the simple path analysis model are (Equation 33),

$$\begin{aligned} Y_{1i} &= f_1(X_{1i}, X_{2i}) + \varepsilon_{1i} \\ Y_{2i} &= f_2(X_{1i}, X_{2i}, Y_{1i}) + \varepsilon_{2i} \end{aligned} \tag{32}$$

$$\begin{aligned} Y_{1i} &= \beta_{10} + \beta_{11}X_1 + \beta_{12}X_2 + \varepsilon_{1i} \\ Y_{2i} &= \beta_{20} + \beta_{21}X_1 + \beta_{22}X_2 + \beta_{23}Y_1 + \varepsilon_{2i} \end{aligned} \tag{33}$$

With matrix form:

$$\underset{\sim}{Y}_{2nx1} = X_{2nx7}\underset{\sim}{\beta}_{7x1} + \underset{\sim}{\varepsilon}_{2nx1} \tag{34}$$

$$\begin{bmatrix} Y_{11} \\ Y_{12} \\ \vdots \\ Y_{1n} \\ Y_{21} \\ Y_{22} \\ \vdots \\ Y_{2n} \end{bmatrix} = \begin{bmatrix} X_X & \underset{\sim}{0}_{nx4} \\ \underset{\sim}{0}_{nx3} & X_{XY} \end{bmatrix} \begin{bmatrix} \beta_{10} \\ \beta_{11} \\ \beta_{12} \\ \beta_{20} \\ \beta_{21} \\ \beta_{22} \\ \beta_{23} \end{bmatrix} + \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{12} \\ \vdots \\ \varepsilon_{1n} \\ \varepsilon_{21} \\ \varepsilon_{22} \\ \vdots \\ \varepsilon_{2n} \end{bmatrix}$$

where

$$X_X = \begin{bmatrix} 1 & X_{11} & X_{21} \\ 1 & X_{12} & X_{22} \\ & \vdots & \\ 1 & X_{1n} & X_{2n} \end{bmatrix}; \; X_{XY} = \begin{bmatrix} 1 & X_{11} & X_{21} & Y_{11} \\ 1 & X_{12} & X_{22} & Y_{12} \\ & & \vdots & \\ 1 & X_{1n} & X_{2n} & Y_{1n} \end{bmatrix}$$

With:

Y_{hi}: Endogenous variable h-th, observation i-th.

X_i: Exogenous variable observation i-th.

β : Parameters for predictor variables.

ε_{hi}: Random error endogenous variable h-th, observation i-th.

Third Part:

Equations 35, 36 present non-parametric regression models that can be created after understanding the equations and multiple linear regression models (Equation 36).

$$Y_{1i} = f_1\left(X_{1i}, X_{2i}\right) + \varepsilon_{1i} \tag{35}$$

$$\hat{f}_\lambda(x_i) = \hat{b}(\lambda)x_i + \frac{1}{2}\hat{a}_0(\lambda) + \sum_{k=1}^{2}\hat{a}_k(\lambda)\cos kx_i \tag{36}$$

With the equation and matrix form like the following equation:

$$\underset{\sim}{f}_{2nx1} = X_{2nx7}\alpha_{7x1} \tag{37}$$

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ \vdots \\ y_n \end{pmatrix} = \begin{pmatrix} x_{11} & 1 & cosx_{11} & \cos 2x_{11} & x_{21} & cosx_{21} & \cos 2x_{21} \\ x_{12} & 1 & cosx_{12} & \cos 2x_{12} & x_{22} & cosx_{22} & \cos 2x_{22} \\ x_{13} & 1 & cosx_{13} & \cos 2x_{13} & x_{23} & cosx_{23} & \cos 2x_{23} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x_{1n} & 1 & cosx_{1n} & \cos 2x_{1n} & x_{2n} & cosx_{2n} & \cos 2x_{2n} \end{pmatrix} \begin{pmatrix} b \\ \frac{1}{2}a_0 \\ a_1 \\ \vdots \\ a_7 \end{pmatrix} \tag{38}$$

From the equations in the basic linear regression analysis model, simple path analysis, and non-parametric regression analysis that have been explained, a function constructed as in Equation 21 may be obtained, resulting in the matrix shown below:

$$\underset{\sim}{f}_{2nx1} = X_{2nx17}\alpha_{17x1} \tag{39}$$

$$\mathbf{X} = \begin{pmatrix} \frac{1}{2} & x_{11} & \cos x_{11} & \cdots & \cos 2x_{21} & 0 & 0 & 0 & \cdots & 0 \\ \frac{1}{2} & x_{12} & \cos x_{12} & \cdots & \cos 2x_{22} & 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{1}{2} & x_{1n} & \cos x_{1n} & \cdots & \cos 2x_{2n} & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 & \frac{1}{2} & x_{11} & \cos x_{11} & \cdots & \cos 2y_{11} \\ 0 & 0 & 0 & \cdots & 0 & \frac{1}{2} & x_{12} & \cos x_{12} & \cdots & \cos 2y_{12} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & \frac{1}{2} & x_{1n} & \cos x_{1n} & \cdots & \cos 2y_{1n} \end{pmatrix} \tag{40}$$

Where:

$\underset{\sim}{f}\left(X_{ij}\right)$: The i-th observation's non-parametric j-th exogenous variable's vector non-parametric regression function.

X_{ij}: The i-th observation's j-th exogenous variable matrix.

$\underset{\sim}{\alpha}_{ij}$: The j-th observation's parameter vector for the i-th exogenous variable.

Theorem 4.1. Ordinary least square.

Given the data from the non-parametric path analysis model on cross-section data stated in Lemma 4.1, the least square approach is the way of estimating parameters that may reduce the number of squares of errors (ordinary least square). As a result, the minimizing estimator of the Fourier and smoothing spline series is:

$$\begin{aligned}
Min_{\beta\epsilon R}\left(\underset{\sim}{\varepsilon}'\underset{\sim}{\varepsilon}\right) &= \left\{n^{-1}\sum_{i=1}^{n}\left(y_i - f_1\left(x_{1i}\right)\right)^2 \right.\\
&\quad \left. + \lambda\int_0^{\pi}\frac{2}{\pi}\left(f_1^{(2)}\left(x_{1i}\right)\right)^2 \partial x_{1i}\right\}\\
Min_{\beta\epsilon R}\left(\underset{\sim}{\varepsilon}'\underset{\sim}{\varepsilon}\right) &= \left\{n^{-1}\left(y - Wa - X\beta\right)'\left(y - Wa - X\beta\right)\right.\\
&\quad \left. + \lambda a' Da\right\}\\
Min_{\beta\epsilon R}\left(\underset{\sim}{\varepsilon}'\underset{\sim}{\varepsilon}\right) &= \left\{Q\left(\mathbf{a},\ \beta\right)\right\}
\end{aligned} \tag{41}$$

Equation 42 needs to be described to obtain an estimate of its function. The decomposition of Equation 42 is as follows:

$$\begin{aligned}
Q\left(a,\ \beta\right) &= n^{-1}\left(y - Wa - X\beta\right)'\left(y - Wa - X\beta\right) + \lambda a' Da\\
&= n^{-1}\left(y' - a'W' - \beta'X'\right)\left(y - Wa - X\beta\right) + \lambda a' Da\\
&= n^{-1}\begin{pmatrix} y'y - a'W'y - \beta'X'y - y'Wa + a'W'Wa + \\ \beta'X'Wa - y'X\beta + a'W'X\beta + \beta'X'X\beta \end{pmatrix}\\
&\quad + \lambda a' Da\\
&= n^{-1}y'y - 2n^{-1}a'W'y - n^{-1}\beta'X'y + 2n^{-1}a'W'X\beta\\
&\quad - n^{-1}y'X\beta + n^{-1}\beta'X'X\beta + a'\left(n^{-1}W'W + \lambda D\right)a
\end{aligned} \tag{42}$$

Estimates of **a** and β can be obtained using the ordinary least squares method, by reducing the total squares of error as follows:

$$\begin{aligned}
&\frac{\partial Q\left(\boldsymbol{a},\ \beta\right)}{\partial \boldsymbol{a}} =\\
&\left(\frac{\boldsymbol{1}}{\partial \boldsymbol{a}}\right)\partial\begin{pmatrix} n^{-1}y'y - 2n^{-1}\boldsymbol{a}'W'y - n^{-1}\beta'X'y + 2n^{-1}\boldsymbol{a}'W'X\beta \\ -n^{-1}y'X\beta + n^{-1}\beta'X'X\beta + \boldsymbol{a}'^{\left(n^{-1}W'W+\lambda D\right)}\boldsymbol{a} \end{pmatrix}\\
&= -2n^{-1}\boldsymbol{W}'y + 2n^{-1}\boldsymbol{W}'\boldsymbol{X}\beta + 2\left(n^{-1}\boldsymbol{W}'\boldsymbol{W} + \lambda\boldsymbol{D}\right)\boldsymbol{a}\\
&= 2(-n^{-1}\boldsymbol{W}'y + n^{-1}\boldsymbol{W}'\boldsymbol{X}\beta + n^{-1}\boldsymbol{W}'\boldsymbol{W} + \lambda\boldsymbol{D})\boldsymbol{a}\\
&0 = \left(-n^{-1}\boldsymbol{W}'y + n^{-1}\boldsymbol{W}'\boldsymbol{X}\hat{\beta}\right)\left(n^{-1}\boldsymbol{W}'\boldsymbol{W} + \lambda\boldsymbol{D}\right)\hat{\boldsymbol{a}}\\
&\left(n^{-1}\boldsymbol{W}'y - n^{-1}\boldsymbol{W}'\boldsymbol{X}\hat{\beta}\right) = \left(n^{-1}\boldsymbol{W}'\boldsymbol{W} + \lambda\boldsymbol{D}\right)\hat{\boldsymbol{a}}\\
&\hat{\boldsymbol{a}} = \left(n^{-1}\boldsymbol{W}'\boldsymbol{W} + \lambda\boldsymbol{D}\right)^{-1}\left(n^{-1}\boldsymbol{W}'y - n^{-1}\boldsymbol{W}'\boldsymbol{X}\hat{\beta}\right)\\
&\hat{\boldsymbol{a}} = \left(\boldsymbol{W}'\boldsymbol{W} + n\lambda\boldsymbol{D}\right)^{-1}\boldsymbol{W}'\left(y - \boldsymbol{X}\hat{\beta}\right)\\
&\hat{\boldsymbol{a}} = S\left(K,\lambda\right)\boldsymbol{W}'\left(y - \boldsymbol{X}\hat{\beta}\right)
\end{aligned} \tag{43}$$

Equation 44 can be minimized by partially deriving $Q\left(\mathbf{a},\beta\right)$ relating to and equaling zero.

$$\begin{aligned}
\frac{\partial Q\left(\boldsymbol{a},\ \beta\right)}{\partial \beta} &= -2n^{-1}\boldsymbol{X}'\boldsymbol{y} + 2n^{-1}\boldsymbol{X}'\boldsymbol{Wa} + 2n^{-1}\boldsymbol{X}'\boldsymbol{X}\beta\\
&= 2n^{-1}\left(-\boldsymbol{X}'\boldsymbol{y} + \boldsymbol{X}'\boldsymbol{Wa} + \boldsymbol{X}'\boldsymbol{X}\beta\right)\\
0 &= 2n^{-1}\left(-\boldsymbol{X}'\boldsymbol{y} + \boldsymbol{X}'\boldsymbol{W}\hat{\boldsymbol{a}} + \boldsymbol{X}'\boldsymbol{X}\hat{\beta}\right)\\
\boldsymbol{X}'\boldsymbol{X}\hat{\beta} &= \boldsymbol{X}'\boldsymbol{y} - \boldsymbol{X}'\boldsymbol{W}\hat{\boldsymbol{a}}\\
\hat{\beta} &= \left(\boldsymbol{X}'\boldsymbol{X}\right)^{-1}\left(\boldsymbol{X}'\boldsymbol{y} - \boldsymbol{X}'\boldsymbol{W}\hat{\boldsymbol{a}}\right)
\end{aligned}$$

$$\hat{\beta} = \left(\boldsymbol{X}'\boldsymbol{X}\right)^{-1}\boldsymbol{X}'\left(\boldsymbol{y} - \boldsymbol{W}\hat{\boldsymbol{a}}\right) \tag{44}$$

By substituting Equation 43 into Equation 44, the estimator of the function is obtained as follows:

$$\begin{aligned}
\hat{\beta} &= \left(\boldsymbol{X}'\boldsymbol{X}\right)^{-1}\boldsymbol{X}'\left(\boldsymbol{y} - \boldsymbol{W}\left(S\left(K,\lambda\right)\boldsymbol{W}'\left(\boldsymbol{y} - \boldsymbol{X}\hat{\beta}\right)\right)\right)\\
&= \left(\boldsymbol{X}'\boldsymbol{X}\right)^{-1}\boldsymbol{X}'\left(\boldsymbol{y} - \boldsymbol{W}S\left(K,\lambda\right)\boldsymbol{W}'\left(\boldsymbol{y} - \boldsymbol{X}\hat{\beta}\right)\right)\\
&= \left(\boldsymbol{X}'\boldsymbol{X}\right)^{-1}\boldsymbol{X}'\boldsymbol{y} - \left(\boldsymbol{X}'\boldsymbol{X}\right)^{-1}\boldsymbol{X}'\boldsymbol{W}S\left(K,\lambda\right)\boldsymbol{W}'\boldsymbol{y}\\
&+ \left(\boldsymbol{X}'\boldsymbol{X}\right)^{-1}\boldsymbol{X}'\boldsymbol{W}S\left(K,\lambda\right)\boldsymbol{W}'\boldsymbol{X}\hat{\beta}
\end{aligned} \tag{45}$$

To obtain the result of the function estimator, it is necessary to subtract both sides $\left(\mathbf{X'X}\right)^{-1}\mathbf{X'W}S\left(K,\lambda\right)\mathbf{W}'\mathbf{X}\hat{\beta}$ in Equation 46 as follows:

$$\begin{aligned}
&\hat{\beta} - \left(\boldsymbol{X}'\boldsymbol{X}\right)^{-1}\boldsymbol{X}'\boldsymbol{W}S\left(K,\lambda\right)\boldsymbol{W}'\boldsymbol{X}\hat{\beta} = \left(\boldsymbol{X}'\boldsymbol{X}\right)^{-1}\boldsymbol{X}'\boldsymbol{y}\\
&\quad - \left(\boldsymbol{X}'\boldsymbol{X}\right)^{-1}\boldsymbol{X}'\boldsymbol{W}S\left(K,\lambda\right)\boldsymbol{W}'\boldsymbol{y}\\
&\left(\boldsymbol{I} - \left(\boldsymbol{X}'\boldsymbol{X}\right)^{-1}\boldsymbol{X}'\boldsymbol{W}S\left(K,\lambda\right)\boldsymbol{W}'\boldsymbol{X}\right)\hat{\beta} = \left(\boldsymbol{X}'\boldsymbol{X}\right)^{-1}\\
&\quad \boldsymbol{X}'\left(\boldsymbol{y} - \boldsymbol{W}S\left(K,\lambda\right)\boldsymbol{W}'\boldsymbol{y}\right)\\
&\left(\boldsymbol{I} - \left(\boldsymbol{X}'\boldsymbol{X}\right)^{-1}\boldsymbol{X}'\boldsymbol{W}S\left(K,\lambda\right)\boldsymbol{W}'\boldsymbol{X}\right)\hat{\beta} = \left(\boldsymbol{X}'\boldsymbol{X}\right)^{-1}\\
&\quad \boldsymbol{X}'\left(\boldsymbol{I} - \boldsymbol{W}S\left(K,\lambda\right)\boldsymbol{W}'\right)\boldsymbol{y}\\
&\left(\boldsymbol{I} - \left(\boldsymbol{X}'\boldsymbol{X}\right)^{-1}\boldsymbol{X}'\boldsymbol{W}S\left(K,\lambda\right)\boldsymbol{W}'\boldsymbol{X}\right)\hat{\beta} = \left(\boldsymbol{X}'\boldsymbol{X}\right)^{-1}\\
&\quad \boldsymbol{X}'\left(\boldsymbol{I} - \boldsymbol{W}S\left(K,\lambda\right)\boldsymbol{W}'\right)\boldsymbol{y}
\end{aligned} \tag{46}$$

5. Conclusion

The use of non-parametric route analysis leads to the following that the Fourier series and smoothing spline can only be used when the linearity presumption is broken. The estimator of the Non-parametric Regression-Based Path Analysis, which coupled Fourier series and smoothing spline using the Ordinary Least Square (OLS) technique, then delivers a nonsingular estimate result since it applies a non-parametric methodology. Different combinations of lambdas, oscillations, orders, and knots will provide different results. This non-unique conclusion, on the other hand, will provide a graphical depiction that is more similar to the initial data distribution.

Data availability statement

The raw data supporting the conclusions of this article will be made available by the authors, without undue reservation.

Author contributions

All authors listed have made a substantial, direct, and intellectual contribution to the work and approved it for publication.

Conflict of interest

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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