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# Composite fuzzy learning finite-time prescribed performance control of uncertain nonlinear systems with dead-zone inputs

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This paper presents a composite fuzzy learning finite-time prescribed performance control (PPC) approach for uncertain nonlinear systems with dead-zone inputs. First, a finite-time performance function is constructed by a quartic polynomial. Subsequently, with the help of an error transformation function, the restriction problem of the tracking error performance is transformed into a stability problem of an equivalent transformation system. In order to ensure that all signals of the closed-loop system are bounded, a finite-time PPC method combined with fuzzy logic systems (FLSs) is proposed. Although the tracking error can be guaranteed to be limited within a predefined range, the proposed finite-time PPC method only uses instantaneous data, which cannot guarantee the accurate estimation of unknown functions under the influence of dead-zone inputs. Therefore, based on the persistent excitation (PE) condition, a predictive error is defined by using online recorded data and instantaneous data, and a corresponding composite learning finite-time PPC method with parameter updating the law, which can not only achieve the control aim of the former method but also improve the control effect, is designed. The simulation results verified that the composite learning finite-time PPC method is more effective than the finite-time PPC method without learning.

## KEYWORDS

nonlinear system, performance function, partial persistent excitation, dead-zone input, finite-time

## 1. Introduction

The traditional control methods, such as adaptive control [1–3], feedback control [4–6], active control [7–9], impulsive control [10, 11], can ensure that the tracking error converges to a residual set whose size is usually unknown. Although these controllers can achieve satisfactory steady-state error performance, the transient error performance (including settling time and maximum overshoot) cannot be guaranteed. Thus, many researchers focused on the transient performance of control systems, and a lot of methods were proposed, for example, in Bechlioulis and Rovithakis [12], Niu and Zhao [13], Li and Tong [14], Yao and Tomizuka [15], and Zhi and Xu [16]. In order to ensure that the

tracking error satisfies certain transient performance, a prescribed performance control (PPC) method was proposed by Bechlioulis and Rovithakis [12]. It is concluded that the characteristic of the PPC method is that the tracking error is limited within a small pre-defined range, and its convergence rate is not less than a predefined constant. Thus, the PPC method can ensure that the tracking error satisfies both steady-state performance and transient performance. Some recent research works on PPC can be referred to Kostarigka et al. [17], Zhang et al. [18], Zhang and Cheng [19], Bu et al. [20], Xiang and Liu [21], and Liu et al. [22]. Kostarigka et al. [17] designed a PPC method for the flexible joint robot with unknown or possibly variable elasticity in order to make the link position error meet the pre-set performance index. In Zhang et al. [18] proposed a PPC control strategy for a generic flexible air-breathing hypersonic vehicle, which ensures that the velocity and altitude tracking errors of the vehicle have ideal transient performance. For non-strict feedback systems with unmeasurable states, an observer-based neural adaptive prescribed performance control approach [19] was proposed to achieve expected output tracking performance. The PPC methods in Kostarigka et al. [17], Zhang et al. [18], Xiang et al. [23], and Zhang and Cheng [19] can not achieve the convergence of tracking error with sufficiently small overshoot. To solve this problem, an improved PPC control strategy combined with backstepping technology is proposed by Bu et al. [20] to guarantee the convergence of tracking error with small overshoot for uncertain nonlinear dynamic systems. Using a similar PPC strategy, the transient performance of tracking errors for uncertain systems with unknown control gain signs was discussed by Xiang and Liu [21]. In order to realize the finite-time control of tracking error, a finite-time performance function and a corresponding finite-time PPC method were proposed for strict-feedback nonlinear systems by Liu et al. [22]. However, it should be pointed out that the above PPC methods mainly focus on driving the tracking error to meet certain transient performances, in addition, although fuzzy logic systems (FLSs) or neural networks (NNs) are used to approximate the nonlinear functions of the controlled system, the approximation errors are not further discussed.

In practical control systems, using FLS or NN to deal with system uncertainty has brought us great help. However, if the parameter updating law of FLS or NN is designed based on instantaneous data, it may not guarantee the accurate estimation of the unknown function. To obtain an accurate estimation of system uncertainty, a valid strategy is to define a prediction error by using online recorded data together with instantaneous data, and then design a composite learning parameter updating law [24–32]. Based on a partial persistent excitation (PE) condition, an NN composite learning control scheme was proposed by Wang and Hill [24], which can accurately approximate the unknown function. In Pan and Yu [32], a composite learning robot control approach was developed to achieve accurate parameter estimation under an

interval excitation (IE) condition. To the best of our knowledge, most of the composite learning control methods only consider linear input, and the existing results are not valid for nonlinear input, such as dead-zone and saturation. In addition, the composite learning control method can achieve the steady-state performance of the tracking error; however, whether it can be combined with PPC technology to achieve transient performance will become an interesting topic.

Inspired by the above discussion, a composite learning finite-time PPC was proposed for an uncertain multi-input multi-output (MIMO) nonlinear second-order system. Compared with Xiang et al. [23], the highlights of this paper are presented as follows: 1) The proposed method in this paper ensures that the tracking error is limited within the predefined region at the settling time. 2) By using online recorded data and instantaneous data, composite learning parameter updating laws are designed. 3) The proposed control method overcomes the influence of dead-zone inputs and realizes accurate estimation of unknown functions.

The paper is arranged as follows. The control problem and some preliminaries are presented in Section 2. The composite learning finite-time PPC method is designed in Section 3. Some simulation results are shown in Section 4. The conclusion is provided in Section 5.

## 2. Preliminaries

### 2.1. System description

Consider a MIMO nonlinear second-order system whose dynamics can be described as

$$\ddot{\xi} = F(\xi, \dot{\xi}, t) + \Gamma(u(t)), \tag{1}$$

where  $\xi = [\xi_1, \xi_2, \dots, \xi_n]^T \in \mathbb{R}^n$  is a state vector,  $\dot{\xi} = [\dot{\xi}_1, \dot{\xi}_2, \dots, \dot{\xi}_n]^T$  and  $\ddot{\xi} = [\ddot{\xi}_1, \ddot{\xi}_2, \dots, \ddot{\xi}_n]^T$  are the first derivative and the second derivative of  $\xi$ , respectively.  $F(\xi, \dot{\xi}, t) = [f_1(\xi, \dot{\xi}, t), f_2(\xi, \dot{\xi}, t), \dots, f_n(\xi, \dot{\xi}, t)]^T \in \mathbb{R}^n$  is the nonlinear function vector;  $u(t) = [u_1(t), u_2(t), \dots, u_n(t)]^T \in \mathbb{R}^n$  is the control input vector and  $\Gamma(u(t)) = [\Gamma_1(u_1(t)), \Gamma_2(u_2(t)), \dots, \Gamma_n(u_n(t))]^T \in \mathbb{R}^n$  is the output of dead-zone function vector in the actuator, which is described as

$$\Gamma_i(u_i(t)) = \begin{cases} m_{i1}(u_i(t) - \bar{b}_i), & \text{if } u_i(t) \geq \bar{b}_i, \\ 0, & \text{if } -\underline{b}_i < u_i(t) < \bar{b}_i, \\ m_{i2}(u_i(t) + \underline{b}_i), & \text{if } u_i(t) \leq -\underline{b}_i, \end{cases} \tag{2}$$

where  $m_{i1}$  and  $m_{i2}$  represent the right and the left slopes of the dead-zone and are assumed to satisfy that  $m_{i1} = m_{i2} = m_i$ .  $\bar{b}_i$  and  $\underline{b}_i$  stand for the breakpoints of the dead-zone.

From Equation (2), the equivalent form of  $\Gamma(u(t))$  can be expressed as

$$\Gamma(u(t)) = Mu(t) + \Delta u(t), \tag{3}$$

where  $\mathbf{M} = \text{diag}(m_1, m_2, \dots, m_n)$ ,  $\Delta \mathbf{u}(t) = [\Delta u_1(t), \Delta u_2(t), \dots, \Delta u_n(t)]^T$  and  $\Delta u_i(t)$  is described as

$$\Delta u_i(t) = \begin{cases} -m_i \bar{b}_i, & \text{if } u_i(t) \geq \bar{b}_i, \\ -m_i u_i(t), & \text{if } -\underline{b}_i < u_i(t) < \bar{b}_i, \\ m_i \underline{b}_i, & \text{if } u_i(t) \leq -\underline{b}_i. \end{cases} \quad (4)$$

where  $\bar{\xi}$  is defined as  $\bar{\xi} = [\xi_1^T, \xi_2^T]^T = [\xi^T, \dot{\xi}^T]^T$ . Substituting (Equations 3–1), the system (Equation 1) can be rewritten as

$$\begin{cases} \dot{\xi}_1 = \xi_2, \\ \dot{\xi}_2 = \bar{F}(\bar{\xi}, t) + \mathbf{M}u(t), \end{cases} \quad (5)$$

where  $\bar{F}(\bar{\xi}, t) = [\bar{f}_1(\bar{\xi}, t), \bar{f}_2(\bar{\xi}, t), \dots, \bar{f}_n(\bar{\xi}, t)]^T = [f_1(\xi, \dot{\xi}, t) + \Delta u_1(t), f_2(\xi, \dot{\xi}, t) + \Delta u_2(t), \dots, f_n(\xi, \dot{\xi}, t) + \Delta u_n(t)]^T$ .

The following assumptions and lemmas are provided to facilitate control analysis.

**Assumption 1.** State vectors  $\xi$  and  $\dot{\xi}$  are measurable.

**Assumption 2.**  $\bar{f}_i(\bar{\xi}, t), i = 1, 2, \dots, n$  is an unknown continuous function.

**Remark 1.** Assumptions 1 and 2 are common assumptions for the second-order nonlinear system [8, 23].

**Lemma 1.** Assume that  $f(\mathbf{x})$  is a continuous function, which is defined on a compact set  $\Omega_{\mathbf{x}}$ , then there exists an optimal FLS  $\varphi^T(\mathbf{x})\theta^*$  satisfies

$$\sup_{\mathbf{x} \in \Omega_{\mathbf{x}}} |f(\mathbf{x}) - \varphi^T(\mathbf{x})\theta^*| \leq \varepsilon_f^*, \quad (6)$$

where  $\theta^* = [\theta_1^*, \theta_2^*, \dots, \theta_m^*]^T \in \mathbb{R}^m$  is the ideal weight vector,  $m \in \mathbb{N}$  is the number of the fuzzy rules,  $\varepsilon_f^*$  is the desired level, and  $\varphi(\mathbf{x})$  is a basis function vector, which can be expressed as

$$\varphi(\mathbf{x}) = \frac{[\varphi_1(\mathbf{x}), \varphi_2(\mathbf{x}), \dots, \varphi_m(\mathbf{x})]^T}{\sum_{j=1}^m \varphi_j(\mathbf{x})} \quad (7)$$

where  $\varphi_j(\mathbf{x}) = \exp\left(-\frac{\|\mathbf{x} - \mathbf{c}_j\|^2}{\mathbf{b}_j^T \mathbf{b}_j}\right)$  is a Gaussian function,  $\mathbf{c}_j = [c_{j1}, c_{j2}, \dots, c_{jm}]^T$ , and  $\mathbf{b}_j = [b_{j1}, b_{j2}, \dots, b_{jm}]^T$  are the center vector and the width of  $\varphi_j(\mathbf{x})$ , respectively.

According to Lemma 1, the unknown function  $\bar{f}_i(\bar{\xi}, t)$  can be written as

$$\bar{f}_i(\bar{\xi}, t) = \varphi_{\bar{f}_i}^T(\bar{\xi})\theta_{\bar{f}_i}^* + \varepsilon_{\bar{f}_i}(\bar{\xi}), \quad (8)$$

where  $\varphi_{\bar{f}_i}(\bar{\xi})$  is the basis function vector and  $\varepsilon_{\bar{f}_i}(\bar{\xi})$  is the estimation error satisfying  $|\varepsilon_{\bar{f}_i}(\bar{\xi})| \leq \varepsilon_{\bar{f}_i}^*$ ,  $\varepsilon_{\bar{f}_i}^*$  is a positive constant.  $\theta_{\bar{f}_i}^*$  can be defined as

$$\theta_{\bar{f}_i}^* = \arg \min_{\hat{\theta}_{\bar{f}_i} \in \mathbb{R}^m} \left\{ \sup_{\bar{\xi} \in D_{\bar{\xi}}} |\bar{f}_i(\bar{\xi}, t) - \varphi_{\bar{f}_i}^T(\bar{\xi})\hat{\theta}_{\bar{f}_i}| \right\}, \quad (9)$$

where  $\hat{\theta}_{\bar{f}_i}$  is an adjustable parameter vector. In this paper,  $\varphi_{\bar{f}_i}^T(\bar{\xi})\hat{\theta}_{\bar{f}_i}$  is applied to approximate  $\bar{f}_i(\bar{\xi}, t)$ .

Let  $\varphi_{\bar{F}}^T(\bar{\xi})\theta_{\bar{F}}^* = [\varphi_{\bar{f}_1}^T(\bar{\xi})\theta_{\bar{f}_1}^*, \varphi_{\bar{f}_2}^T(\bar{\xi})\theta_{\bar{f}_2}^*, \dots, \varphi_{\bar{f}_n}^T(\bar{\xi})\theta_{\bar{f}_n}^*]^T$ ,  $\varphi_{\bar{F}}^T(\bar{\xi})\hat{\theta}_{\bar{F}} = [\varphi_{\bar{f}_1}^T(\bar{\xi})\hat{\theta}_{\bar{f}_1}, \varphi_{\bar{f}_2}^T(\bar{\xi})\hat{\theta}_{\bar{f}_2}, \dots, \varphi_{\bar{f}_n}^T(\bar{\xi})\hat{\theta}_{\bar{f}_n}]^T$ ,  $\varphi_{\bar{F}}^T(\bar{\xi})\tilde{\theta}_{\bar{F}} = [\varphi_{\bar{f}_1}^T(\bar{\xi})\tilde{\theta}_{\bar{f}_1}, \varphi_{\bar{f}_2}^T(\bar{\xi})\tilde{\theta}_{\bar{f}_2}, \dots, \varphi_{\bar{f}_n}^T(\bar{\xi})\tilde{\theta}_{\bar{f}_n}]^T$ ,  $\varepsilon_{\bar{F}}(\bar{\xi}) = [\varepsilon_{\bar{f}_1}(\bar{\xi}), \varepsilon_{\bar{f}_2}(\bar{\xi}), \dots, \varepsilon_{\bar{f}_n}(\bar{\xi})]^T$  and  $\tilde{\theta}_{\bar{F}} = \theta_{\bar{F}}^* - \hat{\theta}_{\bar{F}}$  is the estimated parameter error, where  $\theta_{\bar{F}}^* = [\theta_{\bar{f}_1}^{*T}, \theta_{\bar{f}_2}^{*T}, \dots, \theta_{\bar{f}_n}^{*T}]^T$  and  $\hat{\theta}_{\bar{F}} = [\hat{\theta}_{\bar{f}_1}^T, \hat{\theta}_{\bar{f}_2}^T, \dots, \hat{\theta}_{\bar{f}_n}^T]^T$ . Thus, the system (Equation 5) can be modified as

$$\begin{cases} \dot{\xi}_1 = \xi_2, \\ \dot{\xi}_2 = \varphi_{\bar{F}}^T(\bar{\xi})\tilde{\theta}_{\bar{F}} + \varphi_{\bar{F}}^T(\bar{\xi})\theta_{\bar{F}}^* + \varepsilon_{\bar{F}}(\bar{\xi}) + \mathbf{M}u(t). \end{cases} \quad (10)$$

The tracking error  $\mathbf{e}$  is defined as  $\mathbf{e} = [e_1, e_2, \dots, e_n]^T = \xi_1 - \xi_d$  and  $\xi_d = [\xi_{d1}, \xi_{d2}, \dots, \xi_{dn}]^T$  is a reference signal vector, where  $\xi_{d1}, \xi_{d2}$ , and  $\xi_{dn}$  are continuous, recurrent, and available.

**Definition 1.** Pan and Yu [27] proposed that if the inequality  $\int_{t-\tau_d}^t \varphi(\tau)\varphi^T(\tau)d\tau \geq \mu \mathbf{I}$  holds for a bounded signal  $\varphi(t)$ , then it is said that  $\varphi(t)$  satisfies the PE condition, where  $\mu, \tau_d$  are positive constants and  $\mathbf{I}$  is an identity matrix.

**Lemma 2.** Pan and Yu [27] proposed that if  $\mathbf{x}$  is recurrent, then there exists a regression vector  $\varphi(\mathbf{x})$  in Equation (7) such that the PE condition is satisfied.

**Remark 2.** In Pan et al. [26, 31], Liu et al. [29], and Pan and Yu [32], a PE condition criterion for  $\varphi_{\bar{f}_i}(\bar{\xi})$  in Equation (8) is given, that is, by calculating the minimum singular value of  $\int_{t-\tau_d}^t \varphi_{\bar{f}_i}(\bar{\xi})\varphi_{\bar{f}_i}^T(\bar{\xi})d\tau$ . If the minimum singular value of  $\int_{t-\tau_d}^t \varphi_{\bar{f}_i}(\bar{\xi})\varphi_{\bar{f}_i}^T(\bar{\xi})d\tau$  is greater than zero,  $\varphi_{\bar{f}_i}(\bar{\xi})$  can be regarded as satisfying the PE condition, i.e., there exists a positive constant  $\mu_i$  such that  $\int_{t-\tau_d}^t \varphi_{\bar{f}_i}(\bar{\xi})\varphi_{\bar{f}_i}^T(\bar{\xi})d\tau > \mu_i \mathbf{I}$ , and  $\mu_i$  is only used for the theoretical proof of the proposed control method in this paper.

**Remark 3.** In Wang and Yang [33], a recurrent trajectory is described as periodic or period-like trajectory, that is, given a  $\varnothing$ -neighborhood and a constant time  $T(\varnothing)$ , the return time of the recurrent trajectory to any point in the  $\varnothing$ -neighborhood will not exceed  $T(\varnothing)$ . If a recurrent trajectory  $\xi$  is given in advance and  $|\zeta - \xi| < \varnothing$  holds after a certain time,  $\zeta$  can also be regarded as a recurrent trajectory.

**Lemma 3.** (Boundedness of basis function). There exists a positive constant  $\psi$ , which is independent of  $\mathbf{x}$ , such that  $\varphi(\mathbf{x})$  in Equation (7) satisfies  $\max\{\|\varphi(\mathbf{x})\|, \|\dot{\varphi}(\mathbf{x})\|\} \leq \psi$ .

## 2.2. Prescribed performance

First, a finite-time performance function  $p(t, \alpha_0, \alpha_\infty, T_e)$  is introduced as

$$\Pi_i = \frac{p_i(t, \underline{\alpha}_{i0}, \underline{\alpha}_{i\infty}, T_e) \dot{p}_i(t, \bar{\alpha}_{i0}, \bar{\alpha}_{i\infty}, T_e) - \dot{p}_i(t, \underline{\alpha}_{i0}, \underline{\alpha}_{i\infty}, T_e) p_i(t, \bar{\alpha}_{i0}, \bar{\alpha}_{i\infty}, T_e) - e_i(t) (\dot{p}_i(t, \bar{\alpha}_{i0}, \bar{\alpha}_{i\infty}, T_e) - \dot{p}_i(t, \underline{\alpha}_{i0}, \underline{\alpha}_{i\infty}, T_e))}{p_i(t, \bar{\alpha}_{i0}, \bar{\alpha}_{i\infty}, T_e) - p_i(t, \underline{\alpha}_{i0}, \underline{\alpha}_{i\infty}, T_e)}. \quad (16)$$

$$p(t, \alpha_0, \alpha_\infty, T_e) = \begin{cases} \alpha_4 t^4 + \alpha_3 t^3 + \alpha_2 t^2 + \alpha_1 t + \alpha_0, & 0 \leq t < T_e, \\ \alpha_\infty, & t \geq T_e, \end{cases} \quad (11)$$

where  $\alpha_0$  and  $\alpha_\infty$  are the initial value and boundary value of the performance function  $p(t, \alpha_0, \alpha_\infty, T_e)$ , respectively.  $T_e$  is the predefined experienced time of  $p(t, \alpha_0, \alpha_\infty, T_e)$  than that from  $\alpha_0$  to  $\alpha_\infty$ . To guarantee that  $p(t, \alpha_0, \alpha_\infty, T_e)$ ,  $\dot{p}(t, \alpha_0, \alpha_\infty, T_e)$ , and  $\ddot{p}(t, \alpha_0, \alpha_\infty, T_e)$  are continuous at  $t = T_e$ , parameters  $\alpha_1, \alpha_2, \alpha_3$ , and  $\alpha_4$  are designed as

$$\begin{cases} \alpha_1 = \frac{4(\alpha_\infty - \alpha_0)}{T_e}, \\ \alpha_2 = \frac{6(\alpha_0 - \alpha_\infty)}{T_e^2}, \\ \alpha_3 = \frac{4(\alpha_\infty - \alpha_0)}{T_e^3}, \\ \alpha_4 = \frac{\alpha_0 - \alpha_\infty}{T_e^4}. \end{cases} \quad (12)$$

For the tracking error  $\mathbf{e} = [e_1, e_2, \dots, e_n]^T$ , assume that each  $e_i$  satisfies the following prescribed performance boundary (PPB):

$$p_i(t, \underline{\alpha}_{i0}, \underline{\alpha}_{i\infty}, T_e) < e_i < p_i(t, \bar{\alpha}_{i0}, \bar{\alpha}_{i\infty}, T_e), \quad (13)$$

where  $p_i(t, \underline{\alpha}_{i0}, \underline{\alpha}_{i\infty}, T_e)$  and  $p_i(t, \bar{\alpha}_{i0}, \bar{\alpha}_{i\infty}, T_e)$  are the lower boundary and the upper boundary of  $e_i$ , respectively. The designed parameters  $\underline{\alpha}_{i0}, \underline{\alpha}_{i\infty}, \bar{\alpha}_{i0}, \bar{\alpha}_{i\infty}$  satisfy  $\underline{\alpha}_{i\infty} < \underline{\alpha}_{i0} < \bar{\alpha}_{i0}$  and  $\underline{\alpha}_\infty < \bar{\alpha}_\infty < \bar{\alpha}_0$ .

In this paper, a transformation variable  $z_i$  is defined as

$$z_i = \ln \left( \frac{\varrho_i(t)}{1 - \varrho_i(t)} \right), \quad (14)$$

where  $\varrho_i(t) = \frac{e_i - p_i(t, \underline{\alpha}_{i0}, \underline{\alpha}_{i\infty}, T_e)}{p_i(t, \bar{\alpha}_{i0}, \bar{\alpha}_{i\infty}, T_e) - p_i(t, \underline{\alpha}_{i0}, \underline{\alpha}_{i\infty}, T_e)}$ .

Obviously, the following result holds:

**Lemma 4.** *The boundedness of  $z_i$  can guarantee that  $e_i$  satisfies the PPB (13).*

*Proof.* From Equation (14), one obtains that  $\varrho_i(t) = \frac{e^{\tilde{z}_i}}{1 + e^{\tilde{z}_i}}$ . Since  $z_i$  is bounded, which means  $0 < \varrho_i(t) = \frac{e_i - p_i(t, \underline{\alpha}_{i0}, \underline{\alpha}_{i\infty}, T_e)}{p_i(t, \bar{\alpha}_{i0}, \bar{\alpha}_{i\infty}, T_e) - p_i(t, \underline{\alpha}_{i0}, \underline{\alpha}_{i\infty}, T_e)} < 1$ . Obviously,  $p_i(t, \underline{\alpha}_{i0}, \underline{\alpha}_{i\infty}, T_e) < e_i < p_i(t, \bar{\alpha}_{i0}, \bar{\alpha}_{i\infty}, T_e)$ . The proof is completed.

$\mathbf{z}$  is defined as  $\mathbf{z} = [z_1, z_2, \dots, z_n]^T$ , the derivative of  $\mathbf{z}$  becomes

$$\dot{\mathbf{z}} = \mathbf{r}(\boldsymbol{\xi}_2 - \dot{\boldsymbol{\xi}}_d + \boldsymbol{\Pi}), \quad (15)$$

where  $\mathbf{r} = \text{diag}(r_1, r_2, \dots, r_n)$ ,  $\boldsymbol{\Pi} = [\Pi_1, \Pi_2, \dots, \Pi_n]^T$ , and

$$r_i = \frac{1}{\varrho_i(t)(1 - \varrho_i(t))(p_i(t, \bar{\alpha}_{i0}, \bar{\alpha}_{i\infty}, T_e) - p_i(t, \underline{\alpha}_{i0}, \underline{\alpha}_{i\infty}, T_e))},$$

**Remark 4.** *If  $e_i$  is limited within PPB (Equation 13), then  $0 < \varrho_i(t) < 1$  and  $0 < 1 - \varrho_i(t) < 1$ , which implies that  $r_i$  is negative and bounded. For  $t \geq T_e$ , one has  $p_i(t, \underline{\alpha}_{i0}, \underline{\alpha}_{i\infty}, T_e) = \underline{\alpha}_{i\infty}$ ,  $\dot{p}_i(t, \underline{\alpha}_{i0}, \underline{\alpha}_{i\infty}, T_e) = 0$ ,  $p_i(t, \bar{\alpha}_{i0}, \bar{\alpha}_{i\infty}, T_e) = \bar{\alpha}_{i\infty}$ , and  $\dot{p}_i(t, \bar{\alpha}_{i0}, \bar{\alpha}_{i\infty}, T_e) = 0$ , then  $r_i = \frac{\bar{\alpha}_{i\infty} - \underline{\alpha}_{i\infty}}{(\bar{\alpha}_{i\infty} - e_i)(e_i - \underline{\alpha}_{i\infty})}$  and  $\Pi_i = 0$ . Obviously, there exists a minimum value  $r^* = \frac{4}{\bar{\alpha}_{i\infty} - \underline{\alpha}_{i\infty}}$  such that  $r_i \geq r^*$  during  $t \in [T_e, \infty)$ .*

$\mathbf{z}_1$  and  $\mathbf{z}_2$  are defined as  $\mathbf{z}_1 = \mathbf{z}$ ,  $\mathbf{z}_2 = \dot{\mathbf{z}}$ , the transformation system is given as follows:

$$\begin{cases} \dot{\mathbf{z}}_1 = \mathbf{z}_2, \\ \dot{\mathbf{z}}_2 = \mathbf{r}\boldsymbol{\varphi}_F^T(\bar{\boldsymbol{\xi}})\tilde{\boldsymbol{\theta}}_F + \mathbf{r}\boldsymbol{\varphi}_F^T(\bar{\boldsymbol{\xi}})\hat{\boldsymbol{\theta}}_F + \mathbf{r}\boldsymbol{\varepsilon}_F(\bar{\boldsymbol{\xi}}) + \mathbf{r}\mathbf{M}\mathbf{u}(t) - \mathbf{r}\dot{\boldsymbol{\xi}}_d + \mathbf{r}\dot{\boldsymbol{\Pi}} + \boldsymbol{\Lambda}, \end{cases} \quad (17)$$

where  $\dot{\boldsymbol{\Pi}} = [\dot{\Pi}_1, \dot{\Pi}_2, \dots, \dot{\Pi}_n]^T$ ,  $\boldsymbol{\Lambda} = \dot{\mathbf{r}}(\boldsymbol{\xi}_2 - \dot{\boldsymbol{\xi}}_d + \boldsymbol{\Pi})$ , and  $\dot{\mathbf{r}} = \text{diag}(\dot{r}_1, \dot{r}_2, \dots, \dot{r}_n)$ .

## 3. Composite learning control design

First, introduce a variable  $\boldsymbol{\sigma} = [\sigma_1, \sigma_2, \dots, \sigma_n]^T$  as

$$\boldsymbol{\sigma} = \mathbf{z}_2 + \mathbf{c}\mathbf{z}_1, \quad (18)$$

where  $\mathbf{c} = \text{diag}(c_1, c_2, \dots, c_n)$ ,  $c_i$  is designed as positive constant. From Equations (15) and (18), the time derivative of  $\boldsymbol{\sigma}$  becomes

$$\begin{aligned} \dot{\boldsymbol{\sigma}} &= \mathbf{r}\boldsymbol{\varphi}_F^T(\bar{\boldsymbol{\xi}})\tilde{\boldsymbol{\theta}}_F + \mathbf{r}\boldsymbol{\varphi}_F^T(\bar{\boldsymbol{\xi}})\hat{\boldsymbol{\theta}}_F + \mathbf{r}\boldsymbol{\varepsilon}_F(\bar{\boldsymbol{\xi}}) + \mathbf{r}\mathbf{M}\mathbf{u}(t) \\ &\quad - \mathbf{r}\dot{\boldsymbol{\xi}}_d + \mathbf{r}\dot{\boldsymbol{\Pi}} + \boldsymbol{\Lambda} + \mathbf{c}\mathbf{z}_2, \end{aligned} \quad (19)$$

In order to make variable  $\boldsymbol{\sigma}$  and the estimated error  $\tilde{\boldsymbol{\theta}}_F$  bounded, the following theorem shows the first main result of this paper.

**Theorem 1.** *For the MIMO nonlinear second-order system (Equation 1) with Assumptions 1 and 2, if the controller  $\mathbf{u}$  is designed as*

$$\mathbf{u} = (\mathbf{r}\mathbf{M})^{-1}[-\mathbf{r}\boldsymbol{\varphi}_F^T(\bar{\boldsymbol{\xi}})\tilde{\boldsymbol{\theta}}_F + \mathbf{r}\dot{\boldsymbol{\xi}}_d - \mathbf{r}\dot{\boldsymbol{\Pi}} - \boldsymbol{\Lambda} - \mathbf{c}\mathbf{z}_2 - \mathbf{K}\boldsymbol{\sigma}], \quad (20)$$

where  $\mathbf{K} = \text{diag}(k_1, k_2, \dots, k_n)$ ,  $k_i$  is the positive parameter. And the updating law  $\hat{\boldsymbol{\theta}}_{f_i}$  is chosen as

$$\dot{\hat{\boldsymbol{\theta}}}_{f_i} = \eta_{f_i}(r_i\sigma_i\boldsymbol{\varphi}_{f_i}(\bar{\boldsymbol{\xi}}) - \gamma_{f_i}\hat{\boldsymbol{\theta}}_{f_i}), \quad (21)$$

where  $\eta_{\bar{F}}$  and  $\gamma_{\bar{F}}$  are designed positive constants. All signals in Equation (22) are bounded, and then  $e_i$  satisfies PPB (Equation 13). Meanwhile,  $\xi_1$  and  $\xi_2$  are recurrent after  $T_e$ .

**Proof.** Consider the following Lyapunov function

$$V_1 = \frac{1}{2} \left( \sigma^T \sigma + \frac{1}{\eta_{\bar{F}}} \tilde{\theta}_{\bar{F}}^T \tilde{\theta}_{\bar{F}} \right), \tag{22}$$

Substituting Equations (19)–(21) into  $\dot{V}_1$ , one gets

$$\begin{aligned} \dot{V}_1 &= -\mathbf{K}\sigma^T \sigma + \sigma^T \mathbf{r} \boldsymbol{\varepsilon}_{\bar{F}}(\bar{\xi}) + \sigma^T \mathbf{r} \boldsymbol{\varphi}_{\bar{F}}^T(\bar{\xi}) \tilde{\theta}_{\bar{F}} \\ &\quad - \sum_{i=1}^n \frac{1}{\eta_{\bar{F}}} \tilde{\theta}_{\bar{F}_i}^T(\bar{\xi}) \dot{\hat{\theta}}_{\bar{F}_i} \\ &= -\mathbf{K}\sigma^T \sigma + \sigma^T \mathbf{r} \boldsymbol{\varepsilon}_{\bar{F}}(\bar{\xi}) + \sum_{i=1}^n \sigma_i r_i \boldsymbol{\varphi}_{\bar{F}_i}^T(\bar{\xi}) \tilde{\theta}_{\bar{F}_i} \\ &\quad - \sum_{i=1}^n \frac{1}{\eta_{\bar{F}}} \tilde{\theta}_{\bar{F}_i}^T \left[ \eta_{\bar{F}} (r_i \sigma_i \boldsymbol{\varphi}_{\bar{F}_i}(\bar{\xi}) - \gamma_{\bar{F}} \hat{\theta}_{\bar{F}_i}) \right] \\ &= -\mathbf{K}\sigma^T \sigma + \sigma^T \mathbf{r} \boldsymbol{\varepsilon}_{\bar{F}}(\bar{\xi}) + \sum_{i=1}^n \gamma_{\bar{F}} \tilde{\theta}_{\bar{F}_i}^T \hat{\theta}_{\bar{F}_i} \\ &= -\mathbf{K}\sigma^T \sigma + \sigma^T \mathbf{r} \boldsymbol{\varepsilon}_{\bar{F}}(\bar{\xi}) + \gamma_{\bar{F}} \tilde{\theta}_{\bar{F}}^T \hat{\theta}_{\bar{F}}. \end{aligned} \tag{23}$$

By using Young's inequality, one obtains

$$\begin{aligned} \sigma^T \mathbf{r} \boldsymbol{\varepsilon}_{\bar{F}}(\bar{\xi}) &\leq \frac{\sigma^T \sigma}{4} + \|\mathbf{r} \boldsymbol{\varepsilon}_{\bar{F}}(\bar{\xi})\|^2, \\ \gamma_{\bar{F}} \tilde{\theta}_{\bar{F}}^T \hat{\theta}_{\bar{F}} &\leq -\frac{\gamma_{\bar{F}}}{2} \tilde{\theta}_{\bar{F}}^T \tilde{\theta}_{\bar{F}} + \frac{\gamma_{\bar{F}}}{2} \|\boldsymbol{\theta}_{\bar{F}}^*\|^2, \end{aligned} \tag{24}$$

Because  $\mathbf{r}$  and  $\boldsymbol{\varepsilon}_{\bar{F}}(\bar{\xi})$  are bounded, there exists an unknown upper bound  $b_{\varepsilon_r}^*$  such that  $\|\mathbf{r} \boldsymbol{\varepsilon}_{\bar{F}}(\bar{\xi})\| \leq b_{\varepsilon_r}^*$ . Therefore,

$$\dot{V}_1 \leq -\bar{k}\sigma^T \sigma - \frac{\gamma_{\bar{F}}}{2} \tilde{\theta}_{\bar{F}}^T \tilde{\theta}_{\bar{F}} + R_1^* \tag{25}$$

where  $\bar{k} = \min\{k_i - \frac{1}{4}\} > 0$ ,  $R_1^* = b_{\varepsilon_r}^{*2} + \frac{\gamma_{\bar{F}}}{2} \|\boldsymbol{\theta}_{\bar{F}}^*\|^2$ . The following compact sets are defined as follows:

$$\begin{aligned} \Omega_{\sigma} &= \left\{ \sigma \mid \|\sigma\| \leq \sqrt{\frac{R_1^*}{\bar{k}}} \right\}, \\ \Omega_{\theta} &= \left\{ \|\tilde{\theta}_{\bar{F}}\| \mid \|\tilde{\theta}_{\bar{F}}\| \leq \sqrt{\frac{2R_1^*}{\gamma_{\bar{F}}}} \right\}. \end{aligned} \tag{26}$$

Obviously, if  $\sigma \notin \Omega_{\sigma}$  or  $\tilde{\theta}_{\bar{F}} \notin \Omega_{\theta}$ , one has  $\dot{V}_1 < 0$ . Thus,  $\sigma$  and  $\tilde{\theta}_{\bar{F}}$  are semiglobally uniformly bounded. Notice that  $\mathbf{z}_2 = \dot{\mathbf{z}}_1 = \sigma - \mathbf{c}\mathbf{z}_1$ , selecting Lyapunov function. Let  $V_2 = \frac{1}{2} \mathbf{z}_1^T \mathbf{z}_1$ , one has

$$\begin{aligned} \dot{V}_2 &= \mathbf{z}_1^T \dot{\mathbf{z}}_1 = \mathbf{z}_1^T \sigma - \mathbf{z}_1^T \mathbf{c}\mathbf{z}_1 \\ &\leq -c_{\min} \mathbf{z}_1^T \mathbf{z}_1 + \frac{\mathbf{z}_1^T \mathbf{z}_1}{4} + \|\sigma\|^2 \\ &\leq -\bar{c} \mathbf{z}_1^T \mathbf{z}_1 + \frac{R_1^*}{k}, \end{aligned} \tag{27}$$

where  $\bar{c} = c_{\min} - \frac{1}{4} > 0$ ,  $c_{\min} = \min_i \{c_i\}$ . The compact set is defined as  $\Omega_{\mathbf{z}_1} = \left\{ \mathbf{z}_1 \mid \|\mathbf{z}_1\| \leq \sqrt{\frac{R_1^*}{\bar{c}k}} \right\}$ , if  $\mathbf{z}_1 \notin \Omega_{\mathbf{z}_1}$ ,  $\dot{V}_2 < 0$ . So,  $\mathbf{z}_1$  is bounded, and then

$$\begin{aligned} \|\mathbf{z}_2\| &\leq \|\sigma\| + \|\mathbf{c}\mathbf{z}_1\| \\ &\leq \sqrt{\frac{R_1^*}{k}} + c_{\max} \|\mathbf{z}_1\| \\ &= \sqrt{\frac{R_1^*}{k}} \left( 1 + \frac{c_{\max}}{\sqrt{\bar{c}}} \right), \end{aligned} \tag{28}$$

where  $c_{\max} = \max_i \{c_i\}$ . According to Lemma 4, the boundedness of  $\mathbf{z}_1$  ensures that all tracking errors  $e_i$  meet PPB (Equation 13), which means that  $\alpha_{\infty} < \xi_i - \xi_{di} < \bar{\alpha}_{\infty}$  after  $T_e$ . Because  $\xi_{di}$  is recurrent, according to the description of recurrent trajectory in Remark 3, we know that  $\xi_i$  is also recurrent.

Form Equation (15), one gets  $\dot{\xi}_i - \dot{\xi}_{di} = \frac{\dot{z}_i}{r_i} - \Pi_i$ , and according to Remark 4, one has  $r_i \geq r^*$ ,  $\Pi_i = 0$  after  $T_e$ . Thus

$$|\dot{\xi}_i - \dot{\xi}_{di}| = \frac{|\dot{z}_i|}{r_i} \leq \frac{\|\mathbf{z}_2\|}{r^*} = \sqrt{\frac{R_1^*}{k}} \left( 1 + \frac{c_{\max}}{\sqrt{\bar{c}}} \right) \frac{1}{r^*}, t \geq T_e. \tag{29}$$

It can be found that  $\dot{\xi}_i$  is limited within the  $\sqrt{\frac{R_1^*}{k}} \left( 1 + \frac{c_{\max}}{\sqrt{\bar{c}}} \right) \frac{1}{r^*}$ -neighborhood of  $\dot{\xi}_{di}$  after  $T_e$ . Since  $\dot{\xi}_{di}$  is recurrent, therefore,  $\dot{\xi}_i$  is also recurrent. Therefore, we can conclude that  $\xi_1$  and  $\xi_2$  are recurrent vector after time  $T_e$ . □

**Remark 5.** Notice that the parameter updating law (Equation 21) in Theorem 1 only uses instantaneous data, which may not estimate the unknown function  $\bar{\mathbf{F}}(\bar{\xi}, t)$  accurately. Therefore, we need to use online recorded data and instantaneous data to design composite learning parameter updating law.

Define  $G_i(t) = \int_{t-\tau_d}^t \boldsymbol{\varphi}_{\bar{F}_i}(\bar{\xi}) \boldsymbol{\varphi}_{\bar{F}_i}^T(\bar{\xi}) d\tau$ ,  $t \geq \tau_d + T_e$ . Notice that  $\bar{\xi} = [\xi_1^T, \xi_2^T]^T$  is recurrent vector after  $T_e$ , according to Lemma 2 and Remark 2, if the selected fuzzy basis function  $\boldsymbol{\varphi}_{\bar{F}_i}(\bar{\xi})$  in Equation (8) satisfies that the minimum singular value of  $G_i(t)$  is greater than zero, it means that  $G_i(t)$  satisfies the PE condition, i.e. there exists a positive constant  $\mu_i$  such that  $G_i(t) \geq \mu_i \mathbf{I}$ .

From Equation (5), one gets

$$\ddot{f}_i(\bar{\xi}, t) = \ddot{\xi}_i - m_i u_i(t). \tag{30}$$

Multiplying  $\boldsymbol{\varphi}_{\bar{F}_i}(\bar{\xi})$  to both sides of Equation (30) and integrating over  $[t - \tau_d, t]$ , one has

$$\begin{aligned} \int_{t-\tau_d}^t \boldsymbol{\varphi}_{\bar{F}_i}(\bar{\xi}) \ddot{f}_i(\bar{\xi}, \tau) d\tau &= \int_{t-\tau_d}^t \boldsymbol{\varphi}_{\bar{F}_i}(\bar{\xi}) (\boldsymbol{\varphi}_{\bar{F}_i}^T(\bar{\xi}) \boldsymbol{\theta}_{\bar{F}_i}^* + \varepsilon_{\bar{F}_i}(\bar{\xi})) d\tau \\ &= G_i(t) \boldsymbol{\theta}_{\bar{F}_i}^* + \int_{t-\tau_d}^t \boldsymbol{\varphi}_{\bar{F}_i}(\bar{\xi}) \varepsilon_{\bar{F}_i}(\bar{\xi}) d\tau \\ &= \int_{t-\tau_d}^t \boldsymbol{\varphi}_{\bar{F}_i}(\bar{\xi}) (\ddot{\xi}_i - m_i u_i(\tau)) d\tau. \end{aligned} \tag{31}$$



Therefore, one obtains

$$G_i(t)\tilde{\theta}_{f_i} = \int_{t-\tau_d}^t \varphi_{f_i}(\bar{\xi})(\ddot{\xi}_i - m_i u_i(\tau) - \varepsilon_{f_i}(\bar{\xi}))d\tau. \quad (32)$$

Because  $\ddot{\xi}_i$  is not measurable, a second-order filter is used to estimate  $\ddot{\xi}_i$ :

$$\begin{cases} \dot{\omega}_{i1} = \omega_{i2}, \\ \dot{\omega}_{i2} = -\alpha(2\beta\omega_{i2} + \alpha(\omega_{i1} - \dot{\xi}_i)) \end{cases} \quad (33)$$

with  $\omega_{i1}(0) = \dot{\xi}_i(0)$  and  $\omega_{i2}(0) = 0$ , where  $\alpha$  denotes the natural frequency and  $\beta$  stands for the damping factor. According to Lemma 2 in Hu and Zhang [34],  $\dot{\xi}_i$  and  $\ddot{\xi}_i$  can be estimated by  $\omega_{i1}$  and  $\omega_{i2}$ , respectively. Moreover, for any given positive constant  $\pi_i$ , there exist corresponding  $\alpha$  and  $\beta$  so that  $|\omega_{i2} - \ddot{\xi}_i| < \pi_i$  holds.

Define a prediction error  $\epsilon_i(t)$  as

$$\epsilon_i(t) = G_i(t)\tilde{\theta}_{f_i} + \varepsilon_{e_i}(t), \quad (34)$$

where  $\varepsilon_{e_i}(t)$  is a lumped approximation error given by

$$\varepsilon_{e_i}(t) = \int_{t-\tau_d}^t \varphi_{f_i}(\bar{\xi})(\omega_{i2} - \ddot{\xi}_i + \varepsilon_{f_i}(\bar{\xi}))d\tau. \quad (35)$$

From Equations (32) and (35),  $\epsilon_i(t)$  can be computed by

$$\epsilon_i(t) = \int_{t-\tau_d}^t \varphi_{f_i}(\bar{\xi})(\omega_{i2} - m_i u_i(\tau))d\tau. \quad (36)$$

Obviously,  $\|\varepsilon_{e_i}(t)\| \leq \int_{t-\tau_d}^t \|\varphi_{f_i}(\bar{\xi})\| \cdot (|\omega_{i2} - \ddot{\xi}_i| + |\varepsilon_{f_i}(\bar{\xi})|)d\tau \leq \psi(\pi_i + \varepsilon_{f_i}^*)\tau_d$  by using Lemma 3 and Lemma 2 in Hu and Zhang [34]. Therefore, we modify the updating law (Equation 21) as follows

$$\dot{\hat{\theta}}_{f_i} = \begin{cases} \eta_{f_i}(r_i\sigma_i\varphi_{f_i}(\bar{\xi}) - \gamma_{f_i}\hat{\theta}_{f_i}), & t < \tau_d + T_e, \\ \eta_{f_i}(r_i\sigma_i\varphi_{f_i}(\bar{\xi}) + \gamma_{f_i}'\epsilon_i(t)), & t \geq \tau_d + T_e, \end{cases} \quad (37)$$

where  $\gamma_{f_i}'$  is a designed positive constant.

The following theorem shows the second main result of this paper.

**Theorem 2.** For the MIMO nonlinear second-order system (Equation 1) with assumptions 1 and 2, if the selected based function  $\varphi_{f_i}(\bar{\xi})$  in Equation (8) satisfies  $G_i(t) \geq \mu_i \mathbf{I}$  after  $T_e + \tau_d$ , then, the controller (Equation 20) and the composite learning law (Equation 37) guarantee that each tracking error  $e_i$  satisfies PPB (Equation 13) at  $t \in [0, \infty)$  and  $\tilde{\theta}_{f_i}$  converges to a small neighborhood of zero during  $t \in [T_e + \tau_d, \infty)$ .

**Proof.** Theorem 1 has proved that  $e_i$  satisfies PPB (Equation 13) at  $t \in [0, T_e + \tau_d]$  and  $\bar{\xi}_i = [\xi_i^T, \dot{\xi}_i^T]^T$  is recurrent after  $T_e$ . If the selected based function  $\varphi_{f_i}(\bar{\xi})$  in Equation (8) satisfies

$G_i(t) \geq \mu_i \mathbf{I}$  after  $T_e + \tau_d$ , then we just need to prove the stability of  $\sigma$  and  $\tilde{\theta}_{f_i}$  at  $t \in [T_e + \tau_d, \infty)$ .

Consider the following Lyapunov function

$$V_3 = \frac{1}{2} \left( \sigma^T \sigma + \frac{1}{\eta_{f_i}} \tilde{\theta}_{f_i}^T \tilde{\theta}_{f_i} \right), \quad t \geq T_e + \tau_d. \quad (38)$$

Substituting Equations (19), (20), and (37) into  $\dot{V}_3$ , one gets

$$\begin{aligned} \dot{V}_3 &= -\mathbf{K}\sigma^T \sigma + \sigma^T r \varepsilon_{f_i}(\bar{\xi}) - \sum_{i=1}^n \gamma_{f_i}' \tilde{\theta}_{f_i}^T \epsilon_i(t) \\ &= -\mathbf{K}\sigma^T \sigma + \sigma^T r \varepsilon_{f_i}(\bar{\xi}) \\ &\quad - \sum_{i=1}^n \gamma_{f_i}' \tilde{\theta}_{f_i}^T G_i(t) \tilde{\theta}_{f_i} - \sum_{i=1}^n \gamma_{f_i}' \tilde{\theta}_{f_i}^T \varepsilon_{e_i}(t). \end{aligned} \quad (39)$$

Substituting  $G_i(t) \geq \mu_i \mathbf{I}$ ,  $t \in [T_e + \tau_d, \infty)$  to  $\dot{V}_3$ , one has

$$\begin{aligned} \dot{V}_3 &\leq -\mathbf{K}\sigma^T \sigma + \sigma^T r \varepsilon_{f_i}(\bar{\xi}) \\ &\quad - \sum_{i=1}^n \mu_i \gamma_{f_i}' \tilde{\theta}_{f_i}^T \tilde{\theta}_{f_i} - \sum_{i=1}^n \gamma_{f_i}' \tilde{\theta}_{f_i}^T \varepsilon_{e_i}(t) \\ &= -\mathbf{K}\sigma^T \sigma + \sigma^T r \varepsilon_{f_i}(\bar{\xi}) - \frac{\bar{\mu} \gamma_{f_i}'}{2} \tilde{\theta}_{f_i}^T \tilde{\theta}_{f_i} \\ &\quad - \sum_{i=1}^n \frac{\mu_i \gamma_{f_i}'}{2} [\tilde{\theta}_{f_i}^T \tilde{\theta}_{f_i} + \frac{2}{\mu_i} \tilde{\theta}_{f_i}^T \varepsilon_{e_i}(t)], \end{aligned} \quad (40)$$

where  $\bar{\mu} = \min_i \{\mu_i\}$ . Applying Young's inequality, one gets

$$\begin{aligned} & - \sum_{i=1}^n \frac{\mu_i \gamma_{f_i}'}{2} [\tilde{\theta}_{f_i}^T \tilde{\theta}_{f_i} + \frac{2}{\mu_i} \tilde{\theta}_{f_i}^T \varepsilon_{e_i}(t)] \leq \sum_{i=1}^n \frac{\mu_i \gamma_{f_i}'}{2} \cdot \frac{\|\varepsilon_{e_i}\|^2}{\mu_i^2} \\ & \leq \sum_{i=1}^n \frac{\gamma_{f_i}' [\psi(\pi_i + \varepsilon_{f_i}^*)\tau_d]^2}{2\mu_i}. \end{aligned} \quad (41)$$

Substituting the first inequality of Equations (24) and (41) into Equation (40) shows that

$$\dot{V}_3 \leq -\bar{k}\sigma^T \sigma - \frac{\bar{\mu} \gamma_{f_i}'}{2} \tilde{\theta}_{f_i}^T \tilde{\theta}_{f_i} + R_2^*, \quad (42)$$

where  $\bar{k} = \min_i \{k_i - \frac{1}{4}\} > 0$ ,  $R_2^* = b_{\varepsilon_r}^{*2} + \sum_{i=1}^n \frac{\gamma_{f_i}' [\psi_i(\pi_i + \varepsilon_{f_i}^*)\tau_d]^2}{2\mu_i}$ .

Let  $\kappa = \min\{2\bar{k}, \frac{\bar{\mu} \gamma_{f_i}'}{\eta_{f_i}}\}$ . From Equation (42), it yields

$$\dot{V}_3 \leq -\kappa V_3 + R_2^*. \quad (43)$$

Solving the inequality (Equation 43) leads to

$$V_3(t) \leq V_3(T_e + \tau_d)e^{-\kappa t} + \frac{R_2^*}{\kappa}, \quad \forall t \in [T_e + \tau_d, \infty), \quad (44)$$

which means  $\sigma$  and  $\tilde{\theta}_{f_i}$  tend to  $\sqrt{\frac{R_2^*}{\kappa}}$ —neighborhood of zero that can be arbitrarily diminished by properly designed parameters  $k_i$ ,  $\eta_{f_i}$ , and  $\gamma_{f_i}'$  are chosen. Since  $\sigma$  is bounded, similar to the derivation of Theorem 1, we can conclude that  $e_i$  is limited within  $[\underline{\alpha}_\infty, \bar{\alpha}_\infty]$ .

**Remark 6.** Theorem 1 shows a finite-time PPC method without learning. The parameter adaptive law uses instantaneous data, which does not guarantee the accurate estimation of the unknown function (see Figures 2A–D in Section 4). Theorem 2 improves the parameter adaptive law by using online recorded data and instantaneous data after the preset time  $T_e$ . Figures 4A–C in Section 4 show the control effect. Obviously, the unknown functions are not affected by the dead-zone inputs and are accurately estimated.

### 4. Example

In this section, an inverted double pendulum system [35] (shown in Figure 1) was studied to illustrate the different control effectiveness of the proposed two methods. The dynamic model of the inverted double pendulum system (IBPS) was expressed as

$$\begin{cases} \ddot{\theta}_1 = f_1(\theta_1, \dot{\theta}_1, \theta_2, \dot{\theta}_2, t) + \Gamma_1(\zeta u_1(t)), \\ \ddot{\theta}_2 = f_2(\theta_1, \dot{\theta}_1, \theta_2, \dot{\theta}_2, t) + \Gamma_2(\zeta u_2(t)), \end{cases} \quad (45)$$

where  $\theta_i$  and  $\dot{\theta}_i$  were the angles and angular velocities of the inverted double pendulum.  $f_1(\theta_1, \dot{\theta}_1, \theta_2, \dot{\theta}_2, t) = \chi\theta_1 + h(t)(-\alpha(t)\theta_1 + \alpha(t)\theta_2 - \delta_1 + \delta_2) + d_1(\theta_1, \dot{\theta}_1)$ ,  $f_2(\theta_1, \dot{\theta}_1, \theta_2, \dot{\theta}_2, t) = \chi\theta_2 + h(t)(-\alpha(t)\theta_2 + \alpha(t)\theta_1 - \delta_1 + \delta_2) + d_2(\theta_2, \dot{\theta}_2)$ ,  $\chi = \frac{g_r}{c_m l}$ ,  $c_m = \frac{m_b}{m_a + m_b}$ ,  $h(t) = \frac{k(\alpha(t) - c_m l)}{m_b c_m l^2}$ ,  $d_1(\theta_1, \dot{\theta}_1) = -\frac{m_b}{m_a} \dot{\theta}_1^2 \sin(\theta_1)$ , and  $d_2(\theta_2, \dot{\theta}_2) = -\frac{m_b}{m_a} \dot{\theta}_2^2 \sin(\theta_2)$  were external disturbances for the IBPS (Equation 45), and control coefficient  $\zeta = \frac{1}{m_b c_m l^2}$ . The parameter values of the IBPS were selected as  $m_a = m_b = 50$ ,  $\underline{L} = 2g_r = 2k = 2l = 2$ ,  $\delta_1 = \sin(2t)$ ,  $\delta_2 = \sin(3t) + \underline{L}$ . Let  $\xi_1 = [\theta_1, \theta_2]^T$ ,  $\xi_2 = [\dot{\theta}_1, \dot{\theta}_2]^T$ , the reference signal  $\xi_d =$

$[\xi_{d1}, \xi_{d2}]^T$ ,  $\xi_{d1} = -\xi_{d2} = \frac{\pi}{6.28} [\sin(t) + 0.3 \sin(3t)]$ . The initial angles and angular velocities were set as  $\xi_1(0) = [0.8, -0.8]^T$  and  $\xi_2(0) = [0, 0]^T$ . The control parameters were chosen as  $m_1 = m_2 = 5$ ,  $\bar{b}_1 = \bar{b}_2 = 3$ ,  $k_1 = k_2 = 10$ , and  $c_1 = c_2 = 5$ . Let  $f_1(\xi, t) = f_1(\theta_1, \dot{\theta}_1, \theta_2, \dot{\theta}_2, t) + \Delta_1(\zeta u_1(t))$  and  $f_2(\xi, t) = f_2(\theta_1, \dot{\theta}_1, \theta_2, \dot{\theta}_2, t) + \Delta_2(\zeta u_2(t))$  and  $\xi = [\theta_1, \dot{\theta}_1, \theta_2, \dot{\theta}_2]^T$ . The fuzzy membership functions were defined as

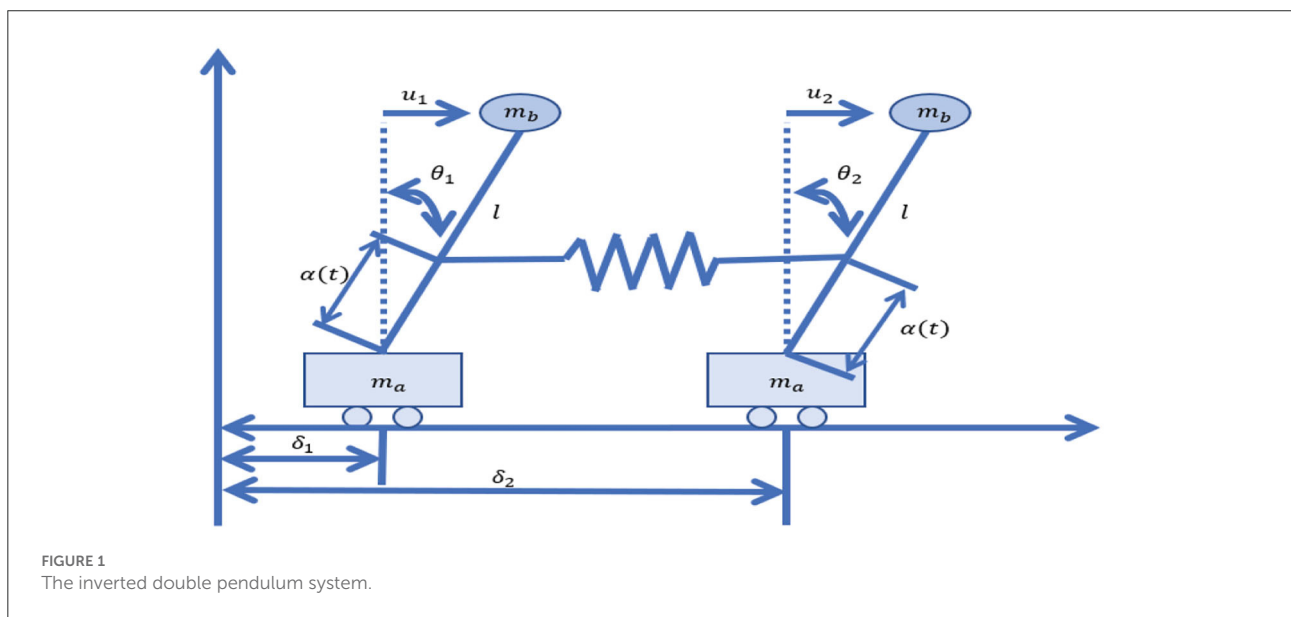
$$\varphi_{f_1^j} = \varphi_{f_2^j} = \exp\left(-\frac{(\bar{\xi} - \xi_j)^T (\bar{\xi} - \xi_j)}{0.5}\right), \quad (46)$$

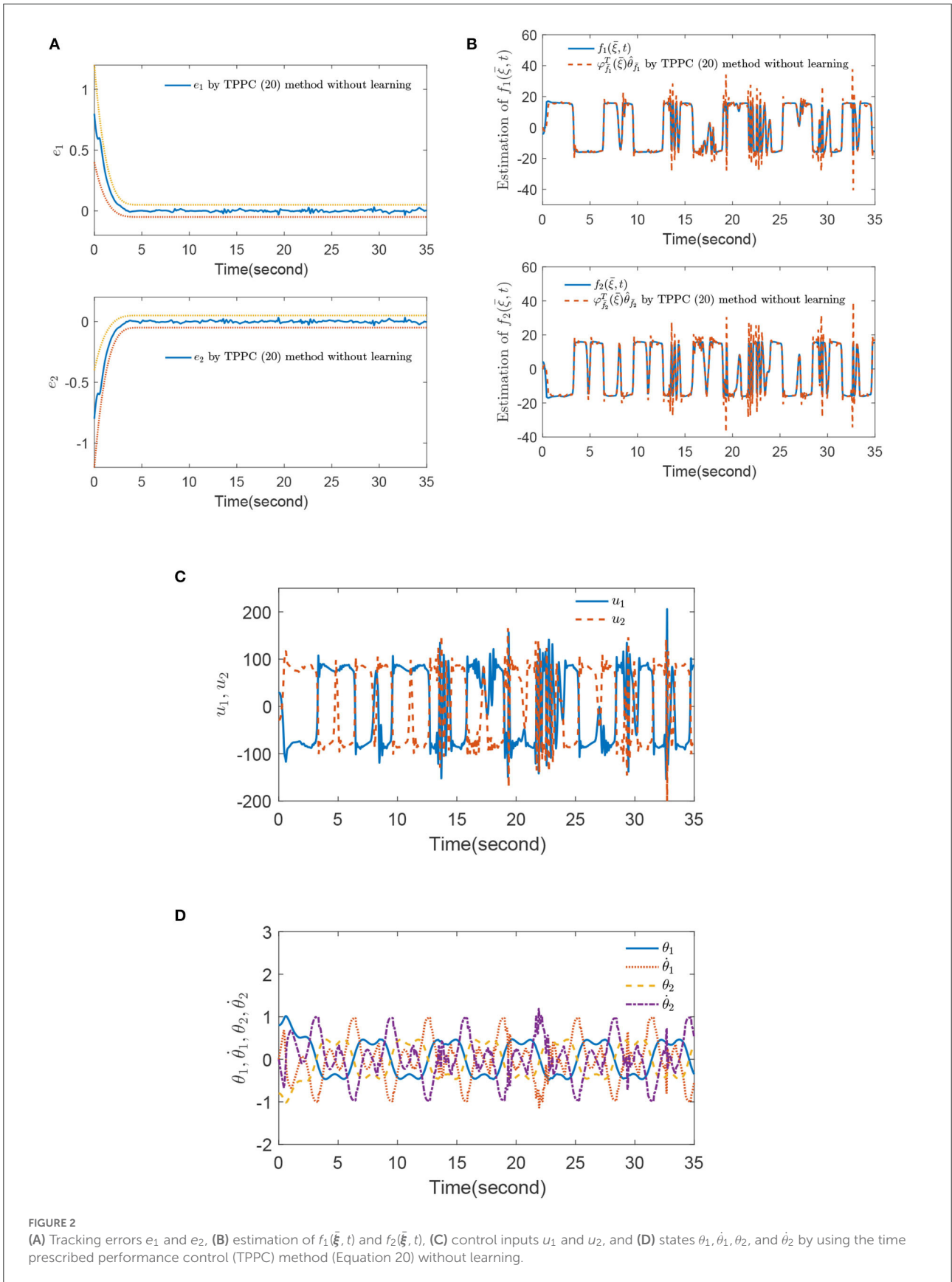
where  $\xi_j = [-2 + j, -2 + j, -2 + j, -2 + j]^T$ ,  $j = 1, 2$ , and 3. The initial values of  $\hat{\theta}_{f_1}(t)$  and  $\hat{\theta}_{f_2}(t)$  were designed as zero, parameters  $\eta_{\bar{F}}$ ,  $\gamma_{\bar{F}}$ , and  $\gamma'_{\bar{F}}$  in Equations (21) and (37) were set as  $\eta_{\bar{F}} = 10$ ,  $\gamma_{\bar{F}} = 0.01$ , and  $\gamma'_{\bar{F}} = 6$ . The parameters  $\alpha$  and  $\beta$  in Equation (33) were designed as  $\alpha = 100$  and  $\beta = 0.7$ . The tracking errors were defined as  $e_1 = \theta_1 - \xi_{d1}$  and  $e_2 = \theta_2 - \xi_{d2}$ , and the following prescribed performance boundary conditions were chosen:

$$\begin{cases} p_1(t, 0.4, -0.05, T_e) < e_1 < p_1(t, 1.2, 0.05, T_e), \\ p_2(t, -1.2, -0.05, T_e) < e_2 < p_2(t, -0.4, 0.05, T_e), \end{cases} \quad (47)$$

where  $T_e = 5$ . The method in Theorem 1 was denoted as the TPPC method (Equation 20) without learning. Based on the above designed parameters and initial values, the simulation results of the IBPS by using the TPPC method (Equation 20) without learning are shown in Figures 2A–D.

Figure 2A shows that tracking errors  $e_1$  and  $e_2$  are limited within PPBs (Equation 47). However, it was found that  $\varphi_{f_1}^T(\xi) \hat{\theta}_{f_1}$  and  $\varphi_{f_2}^T(\xi) \hat{\theta}_{f_2}$  appeared as some large jumps after 10 s in Figure 2B, indicating that the parameter updating law (Equation







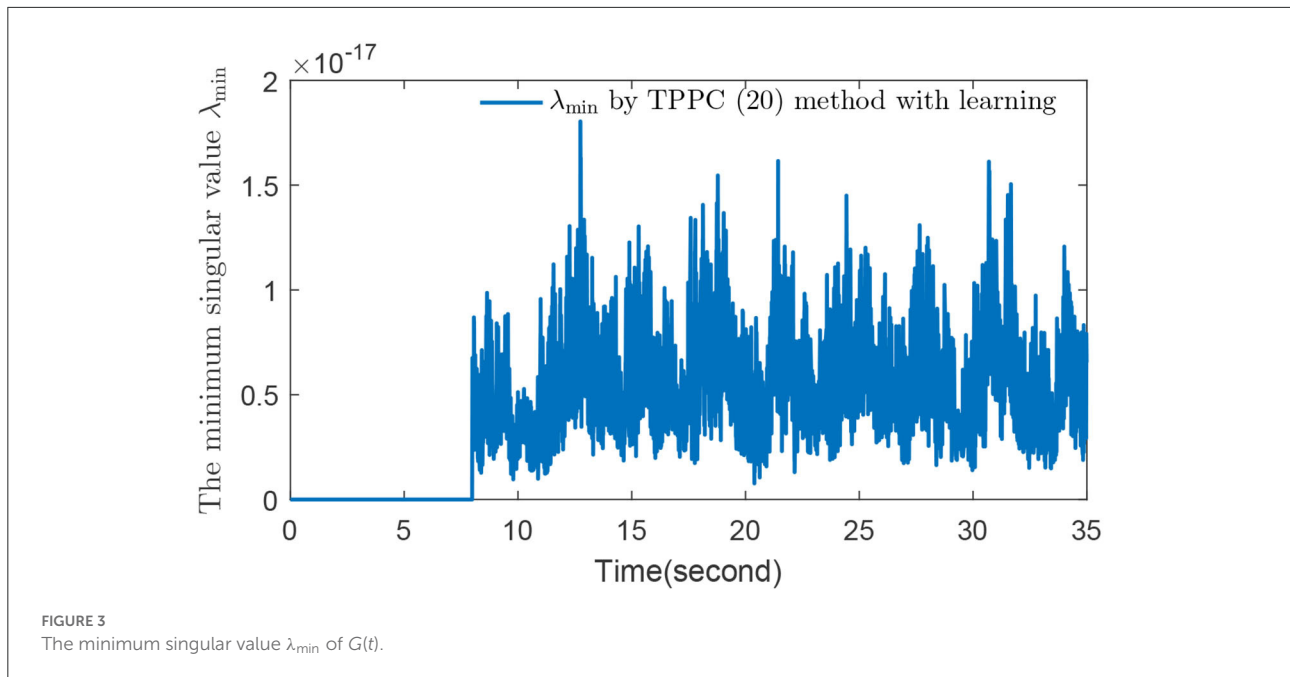


FIGURE 3  
The minimum singular value  $\lambda_{\min}$  of  $G(t)$ .

21) cannot ensure that the unknown functions  $f_1(\bar{\xi}, t)$  and  $f_2(\bar{\xi}, t)$  are accurately estimated. From Figure 2C, one finds that each jump of controllers  $u_1$  and  $u_2$  is consistent with each fluctuation of  $\varphi_{f_1}^T(\xi)\hat{\theta}_{f_1}$  and  $\varphi_{f_2}^T(\xi)\hat{\theta}_{f_2}$ , which indicates that the TPPC method (Equation 20) without learning cannot overcome the influence of dead-zone inputs  $\Gamma_1(\varsigma u_1)$  and  $\Gamma_2(\varsigma u_2)$ . Meanwhile, Figure 2D shows that states  $\theta_1, \hat{\theta}_1, \theta_2$ , and  $\hat{\theta}_2$  are recurrent, which also confirms the conclusion of Theorem 1.

Before using the method in Theorem 2, we need to verify whether  $G(t) = \int_{t-\tau_d}^t \varphi_{\bar{F}}^T(\bar{\xi})\varphi_{\bar{F}}(\bar{\xi})d\tau \geq \mu I$  is always true after time  $T_e + \tau_d$ . According to Remark 2, we only need to verify that the minimum singular value of  $G(t)$  is greater than zero. The method in Theorem 2 was denoted as the TPPC (Equation 20) method with learning and let  $\tau_d = 3$  and

$$\lambda_{\min} = \begin{cases} 0, & t < T_d + \tau_d, \\ \lambda_{\min}\{G(t)\}, & t \geq T_d + \tau_d. \end{cases} \quad (48)$$

Figure 3 shows that the selected fuzzy basic vector  $\varphi_{\bar{F}}(\bar{\xi})$  satisfies  $\lambda_{\min}\{G(t)\} > 0$  for  $t \geq T_d + \tau_d$ , which means that there is a constant  $\mu$  such that  $G(t) \geq \mu I$  holds. The simulation results of the IBPS by using the TPPC method (Equation 20) with learning are displayed in Figures 4A–C. By comparing Figure 2A with Figure 4A, the tracking error control effect of the TPPC method (Equation 20) with learning is obviously better than that of the TPPC method (Equation 20) without learning after  $T_e + \tau_d$ . By comparing the estimation effect of unknown functions in Figures 2B, 4B,  $\varphi_{f_1}^T(\xi)\hat{\theta}_{f_1}$  and  $\varphi_{f_2}^T(\xi)\hat{\theta}_{f_2}$

do not show big fluctuation in Figure 4 by using the TPPC method (Equation 20) with learning and unknown functions  $f_1(\bar{\xi}, t)$  and  $f_2(\bar{\xi}, t)$  are estimated accurately. Figure 4C shows that controllers  $u_1$  and  $u_2$  were stable without large fluctuation by the TPPC method (Equation 20) with learning. Through the comparison of simulation results, the control effect of the TPPC method (Equation 20) with learning was obviously better than that of the TPPC method (Equation 20) without learning, which also confirmed the theoretical analysis of this paper.

## 5. Conclusion

In this paper, a composite learning finite-time PPC method was proposed for uncertain nonlinear systems with dead-zone inputs. A finite-time performance function and a transformation function were introduced, which can ensure that the tracking error can be limited within a predefined region at a settling time. Then, in order to improve the accurate estimation effect of the unknown function, a prediction error was defined and a corresponding composite learning parameter law was designed. The simulation comparison of IBPS showed the superiority of the control effect of the proposed composite learning finite-time PPC method. Meanwhile, one can notice the limitation of the proposed method in this paper is that all states must be measurable. If some states are unmeasurable, the results of this paper cannot be obtained. Therefore, it is necessary to further study the problem of effective estimation of uncertainties based on partially measurable states.

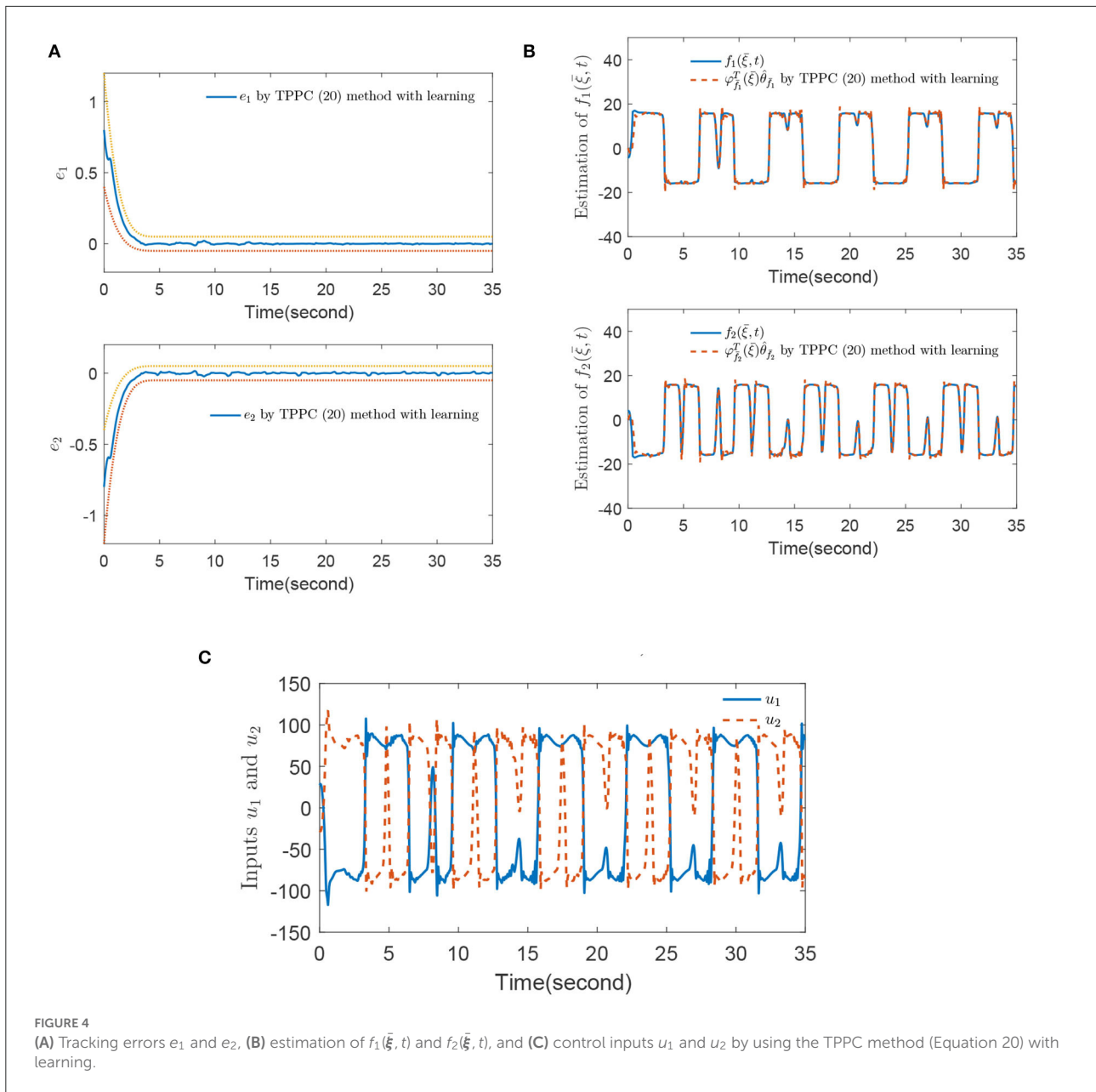


FIGURE 4 (A) Tracking errors  $e_1$  and  $e_2$ , (B) estimation of  $f_1(\bar{\xi}, t)$  and  $f_2(\bar{\xi}, t)$ , and (C) control inputs  $u_1$  and  $u_2$  by using the TPPC method (Equation 20) with learning.

### Data availability statement

The original contributions presented in the study are included in the article/supplementary material, further inquiries can be directed to the corresponding author.

### Author contributions

All authors listed have made a substantial, direct, and intellectual contribution to the work and approved it for publication.

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### Conflict of interest

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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