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Control of Chimera States in Multilayer Networks

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Chimera states are intriguing complex spatio-temporal patterns of coexisting coherent and incoherent domains. They can often be observed in networks with non-local coupling topology, where each element interacts with its neighbors within a fixed range. In small-size non-locally coupled networks, chimera states usually exhibit short lifetimes and erratic drifting of the spatial position of the incoherent domain. This problem can be solved with a tweezer feedback control which can stabilize and fix the position of chimera states. We analyse the action of the tweezer control in two-layer networks, where each layer is a small non-locally coupled ring of Van der Pol oscillators. We demonstrate that tweezer control, applied to only one layer, successfully stabilizes chimera patterns in the other, uncontrolled layer, even in the case of non-identical layers. These results might be useful for applications in multilayer networks, where one of the layers cannot be directly accessed, thus it can be effectively controlled via a neighboring layer.

Keywords: dynamical systems, synchronization, chimera states, multilayer networks, feedback control, Van der Pol oscillators

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1. INTRODUCTION

Networks of coupled oscillators are an intensively studied topic in non-linear science, they have a wide range of applications in physics, biology, chemistry, technology, and social sciences. Special interest has been paid to synchronization and partial synchronization of oscillators, including *chimera states* which are characterized by a hybrid nature of coexisting spatially coherent and incoherent domains [1–7]. Theoretical studies of chimera states have considered a wide range of networks with different local dynamics and a variety of regular and irregular coupling topologies: rings of phase oscillators with non-local coupling [8–12], interacting globally coupled populations of phase oscillators [13, 14], non-locally coupled maps [15, 16], oscillators with phase-amplitude dynamics [17–21], neural oscillators [22–26], two- and three-dimensional lattices of oscillators [27–31], networks with adaptive topologies [32, 33], fractal complex topologies [34–38], oscillators with local or global interaction [39–42], and networks with multiple layers[43–47]. Experimentally, chimera states were demonstrated in optical [48] and chemical [49, 50] systems, as well as in mechanical [51], electronic [52, 53], optoelectronic [54, 55], electrochemical [56, 57] oscillator systems, and Boolean networks [58]. Possible analytical insights and bifurcation analysis of chimera states have been obtained in the continuum limit, which explains the behavior of very large ensembles of coupled oscillators [59–63]. In contrast, lab experiments are commonly performed with small-size networks, where chimera states are more difficult to observe [64–67].

Ring networks with non-local coupling, where each element interacts with its neighbors within a certain range, are a prominent example of a topology allowing for the observation of chimera states. However, the size of the network is essential. In small-size rings of non-locally coupled oscillators, chimera states are often short-living chaotic transients, which eventually collapse to the synchronized state. Their mean lifetime decreases rapidly with decreasing system size [10]. In addition to this, chimera states exhibit a chaotic spatial motion of the position of the coherent and incoherent domains, which is more pronounced with decreasing of the system size [68]. These two effects are weakly noticeable in large networks, but they strongly impede the observation of chimera states in small systems. Only in some special cases beyond simple non-local topologies, chimera states can be observed. For instance, when phase interaction involves higher order harmonics [69, 70], or oscillators are organized in globally coupled interacting subpopulations, the observation of stable chimeras that are not transients is possible in small phase oscillator networks [64, 70].

Control of non-linear systems is an important topic in applied complex systems science [71]. Some control techniques, which allow to stabilize chimera patterns in non-locally coupled oscillator networks, have been proposed recently. The lifetime of amplitude chimeras can be greatly enhanced by time-delayed coupling [72]. For Kuramoto phase oscillators the lifetime of chimera states can be extended by proportional feedback control based on the measurement of the global order parameter [73]. The spatial position of the coherent and incoherent domains of the chimera states can be fixed by a feedback loop inducing a state-dependent asymmetry of the coupling topology [74], defined by a finite difference derivative for a local mean field. Moreover, in one-dimensional arrays of identical oscillators, a self-feedback control applied to a subpopulation of the array can be used for the stabilization of the spatial positions of the coherent and incoherent domains of the chimeras [75]. Recently, we introduced a *tweezer control* scheme for stabilization of chimera states [76] in small-size non-locally coupled networks. This control scheme consists of two parts, symmetric and asymmetric, and effectively stabilizes chimera states in small networks of oscillators exhibiting both phase and amplitude dynamics. Note, that in contrast to pure phase oscillators, a simple analytical study for the continuum limit ($N \to \infty$) is not possible for non-linear phase-amplitude oscillators, therefore we concentrated mainly on the numerical stability analysis. In small networks of Van der Pol and FitzHugh-Nagumo oscillators, we demonstrated that tweezer control allows for stabilization of variable chimera patterns with different sizes of coherent domains [77].

Current research in the field of complex systems is moving beyond simple network structures to more complicated, realistic topologies. One of them are multilayer networks, which find a wide range of applications in nature and technology, such as neuronal and genetic networks, social networks, power grids, transportation networks [78–91]. Recent studies have been focused on various synchronization scenarios in multilayer structures, including remote and relay synchronization [92–94]. Moreover, it has been reported that multiplexing can be used to control spatio-temporal patterns in networks [86, 88, 95]. The advantage of control schemes based on multiplexing is that they allow to achieve the desired state in a certain layer without manipulating its parameters, and they can work for weak inter-layer coupling. For example, it has been shown that weak multiplexing can induce coherence resonance [96] as well as chimera states and solitary states [95] in neural networks. However, multiplexing has not been previously combined with tweezer control.

In many real multi-layer networks some of the layers cannot be easily accessed. An urgent issue, therefore, is the question whether it is possible to control or stabilize spatio-temporal patterns in one layer of the network by applying control to the other layer. We aim to answer this question by an analysis of a simple two-layer network of Van der Pol oscillators. We demonstrate that chimera states which are not observable in small isolated networks, can be efficiently stabilized by the combined action of multiplexing and tweezer control.

2. TWEEZER CONTROL IN TWO-LAYER NETWORK OF VAN DER POL OSCILLATORS

We consider a network of $2N$ coupled Van der Pol oscillators, organized in two layers, each of which contains N oscillators, with non-local ring topology within each layer:

$$
\begin{aligned}
\ddot{x}_k^{(i)} &= (\varepsilon - (x_k^{(i)})^2)\dot{x}_k^{(i)} - x_k^{(i)} \\
&+ \frac{1}{R_i}\sum_{j=1}^{R_i}\left[a_-^{(i)}(x_{k-j}^{(i)} - x_k^{(i)}) + b_-^{(i)}(\dot{x}_{k-j}^{(i)} - \dot{x}_k^{(i)})\right] \\
&+ \frac{1}{R_i}\sum_{j=1}^{R_i}\left[a_+^{(i)}(x_{k+j}^{(i)} - x_k^{(i)}) + b_+^{(i)}(\dot{x}_{k+j}^{(i)} - \dot{x}_k^{(i)})\right] \\
&+ \left[a_{inter}(x_k^{(3-i)} - x_k^{(i)}) + b_{inter}(\dot{x}_k^{(3-i)} - \dot{x}_k^{(i)})\right], \qquad (1)
\end{aligned}
$$

where $x_k^{(i)} \in \mathbb{R}$, $i = 1, 2$ denotes the layer number, $k = 1, ..., N$ is the oscillator index within each layer. The scalar parameter $\varepsilon > 0$ determines the internal dynamics of the individual elements. For small ε the oscillation of a single element is sinusoidal, while for large ε it is a strongly non-linear relaxation oscillation. Each element is coupled with R_i nearest neighbors to the left and to the right, we assume that the oscillators within the layers are arranged on a ring (i.e., periodic boundary conditions). The coupling term inside each layer consists of two parts: the coupling constants with respect to position and velocity to the left and to the right are denoted as $a_-^{(i)}$, $a_+^{(i)}$ and $b_-^{(i)}$, $b_+^{(i)}$, respectively. Such a coupling scheme can be associated with biological [97, 98] and technological applications [99]. Interaction between the layers consists of one-to-one bidirectional connections between the corresponding pairs of oscillators $x_k^{(1)}$ and $x_k^{(2)}$, with inter-layer coupling strength a_{inter} and b_{inter}. **Figure 1** shows schematically the topology of the considered network: both layers consist of non-locally coupled rings of oscillators, corresponding pairs of oscillators in each layer are connected by inter-layer links shown

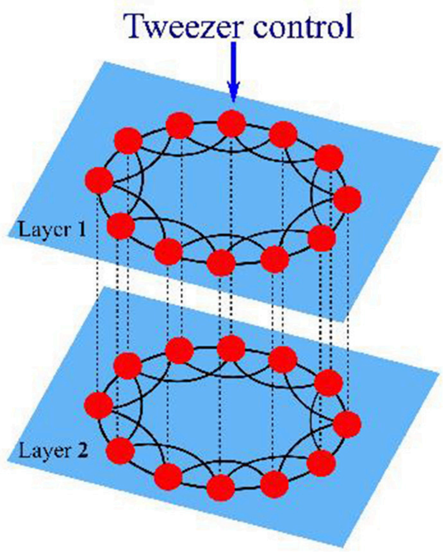

FIGURE 1 | Schematic structure of the two layer networks: red dots denote the oscillators, blue planes correspond to the two layers. Solid lines show intra-layer couplings, dashed lines represent pairwise inter-layer couplings between the oscillators. Tweezer control is applied to layer 1.

by dashed lines. This network structure can also be referred to as a *multiplex*, since only one-to-one inter-layer connections between the layers exist.

For the sake of simplicity we assume

$$a_-^{(i)} = a_+^{(i)} = a^{(i)}, \qquad b_-^{(i)} = a^{(i)}\sigma_-^{(i)}, \qquad b_+^{(i)} = a^{(i)}\sigma_+^{(i)}, \quad (2)$$

with rescaled coupling parameters $a^{(i)}$, $\sigma_-^{(i)}$, and $\sigma_+^{(i)}$.

In order to introduce the tweezer control [76], we define two complex order parameters within each network layer i

$$Z_1^{(i)}(t) = \frac{1}{[N/2]} \sum_{k=1}^{[N/2]} e^{i\phi_k^{(i)}(t)} \quad (3)$$

$$Z_2^{(i)}(t) = \frac{1}{[N/2]} \sum_{k=1}^{[N/2]} e^{i\phi_{N-k+1}^{(i)}(t)}, \quad (4)$$

where $\phi_k^{(i)}(t)$ is the geometric phase of the k-th oscillator computed from

$$e^{i\phi_k^{(i)}(t)} = \left((x_k^{(i)})^2(t) + (\dot{x}_k^{(i)})^2(t)\right)^{-1/2} \left(x_k^{(i)}(t) + \mathrm{i}\dot{x}_k^{(i)}(t)\right). \quad (5)$$

The tweezer feedback control [76] for the non-locally coupled ring of oscillators is defined as

$$\sigma_\pm^{(i)} = K_\mathrm{s}\left(1 - \frac{1}{2}|Z_1^{(i)} + Z_2^{(i)}|\right) \pm K_\mathrm{a}(|Z_1^{(i)}| - |Z_2^{(i)}|). \quad (6)$$

The control term has two parts referred to as *symmetric* and *asymmetric* controls, with corresponding control gains K_s and K_a.

The idea of the symmetric proportional control was suggested for phase oscillators in Sieber et al. [73]. It is defined as a feedback loop between coupling parameters $\sigma_\pm^{(i)}$ and the global Kuramoto order parameter of the oscillators within one layer $|Z_\mathrm{s}^{(i)}| = \frac{|Z_1^{(i)}+Z_2^{(i)}|}{2}$. This feedback loop aims to suppress the collapse of small-size chimera states and extend their lifetime.

The asymmetric control part is realized as a second feedback loop between coupling parameters $\sigma_\pm^{(i)}$ and the difference $Z_\mathrm{a}^{(i)} = |Z_1^{(i)}| - |Z_2^{(i)}|$. It indicates a relative spatial shift of the chimera's incoherent domain with respect to the center of the oscillator array $1, ..., N$. If the incoherent domain of the chimera state is shifted toward larger indices ($|Z_1^{(i)}| > |Z_2^{(i)}|$), then the difference is positive, and as a result $\sigma_+^{(i)} > \sigma_-^{(i)}$. In the opposite case, when the incoherent domain of the chimera state is shifted toward smaller indices ($|Z_1^{(i)}| < |Z_2^{(i)}|$), we will obtain $\sigma_+^{(i)} < \sigma_-^{(i)}$. A discrepancy between $\sigma_+^{(i)}$ and $\sigma_-^{(i)}$ introduces asymmetry in the coupling term, and induces the counterbalancing lateral motion of a chimera state toward dynamically preferable centered position.

In Omelchenko et al. [76, 77] we have demonstrated the effective action of the tweezer control in small rings of non-locally coupled Van der Pol and FitzHugh-Nagumo oscillators. When both the symmetric and asymmetric parts of the control are acting (the control gains K_s and the K_a are positive), a stable chimera state can be observed in the system. When we switch off the asymmetric part of the control, $K_\mathrm{a} = 0$, and keep a positive symmetric gain $K_\mathrm{s} > 0$, the chimera state starts to drift on the ring. Its motion becomes stronger for decreasing system size. To switch off both parts of the control, we keep $\sigma_+^{(i)}$ and $\sigma_-^{(i)}$ constant, and after a short transient time the chimera state collapses to the completely synchronized state.

In the present work, the tweezer control acts in the first layer of our network (1) only, while in the second layer the coupling strength is constant. We will compare patterns obtained in both layers in a network of relatively small size. The characteristic signature of a chimera state is a pronounced difference of the average frequencies for oscillators belonging to the coherent and incoherent domains, respectively. The oscillators from the coherent domain are phase-locked having equal frequencies, while the oscillators from the incoherent domain have different average frequencies which typically form an arc-like profile. The mean phase velocities are obtained as

$$\omega_k^{(i)}(t) = \frac{1}{T_0}\int_0^{T_0} \dot{\phi}_k^{(i)}(t - t')dt', \quad k = 1, \ldots, N, i = 1, 2, \quad (7)$$

averaged over the time window T_0. To visualize the temporal dynamics of the oscillators we plot their mean phase velocities defined by Equation (7) with $T_0 = 50$ for each layer. Throughout this work in our numerical simulations we use random initial conditions.

Figure 2 shows the mean phase velocities for a two-layer network of Van der Pol oscillators with $N = 48$ oscillators within each layer, coupled to their $R_1 = R_2 = 16$ nearest neighbors. Such an intermediate coupling range is the prerequisite of the

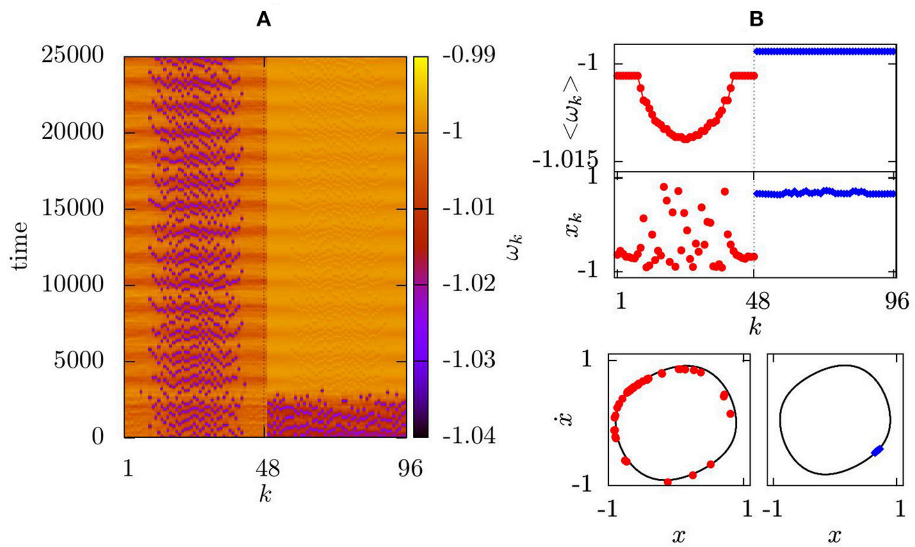

FIGURE 2 | Mean phase velocities for a two-layer system of $N = 48$ oscillators in each layer with $R_1 = R_2 = 16$, $\varepsilon = 0.2$, $a^{(1)} = a^{(2)} = 0.02$, $a_{inter} = 0.002$, $b_{inter} = 0.001$. Layer 1 is controlled with $K_a = 2$, $K_s = 0.5$, layer 2 is uncontrolled with $b_{-}^{(2)} = b_{+}^{(2)} = 0.0027$. Oscillators $k = 1...48$ represent the first layer, $k = 49...96$ the second layer; **(A)** space-time plot; **(B)** mean phase velocity profile averaged over 25,000 time units for the first layer (red dots) and the second layer (blue diamonds) (top panel); snapshot of variables x_k (middle panel) and snapshot in the $(x_k; \dot{x}_k)$ phase space at t =25,000 (bottom panel).

existence of chimera patterns in non-locally coupled rings. In our earlier work [21] we have demonstrated analytically that in the ring of non-locally coupled Van der Pol oscillators the ratio of the coupling constants of position and velocity (in our case $a_{\pm}^{(i)}$ and $b_{\pm}^{(i)}$) can be associated with the phase lag parameter for a reduced phase oscillator network. In order to observe chimera states, the coupling constant of position should be chosen larger than the coupling constant of velocity. In the following, we will use this property for both intra- and inter-layer couplings. As a first step, we consider very weak inter-layer coupling with constants $a_{inter} = 0.002$, $b_{inter} = 0.001$ approximately ten times smaller than intra-layer coupling strengths. **Figure 2A** shows, that in this case the intra-layer coupling dominates, and the two layers perform different dynamics: in the first layer we observe a stable chimera state due to the action of the tweezer control, while in the second, uncontrolled layer after short transient time all oscillators synchronize. **Figure 2B** shows the mean phase velocities of the oscillators averaged over large time interval T_0 =25,000 (upper panel). In the first layer the typical arc-shape profile is formed (shown red), which is one of the prominent features of chimera states. In the second layer, all oscillators are frequency-locked (shown blue). The middle panel demonstrates snapshots for both layers at fixed time, and the bottom panel depicts the same snapshots in the phase space, where the limit cycle of one uncoupled Van der Pol unit is shown in black. The oscillators from the incoherent domain of the chimera state are scattered around this limit cycle.

With increasing inter-layer coupling strength, we observe successful stabilization of the chimera state in the second layer shown in **Figure 3** for $a_{inter} = 0.011$, $b_{inter} = 0.0025$. Due to the fact that our layers are identical, the mean phase velocities profiles have the same shape, and coherent/incoherent domains of chimera states are synchronized spatially in both layers. **Figure 4** presents a diagram in the parameter plane of inter- and intra-layer coupling constants. When the interlayer coupling is too weak, chimera states in the second layer can not be stabilized (blue region). In the red region synchronization of both layers, and thus successful control of the chimera state in the second layer via multiplexing is observed. In the thin hatched region our numerical evidence shows a sensitive dependence on the initial conditions. However, system (1) has numerous parameters which should also be fixed appropriately. For instance, the control gains K_s and K_a can influence the shape of the controlled chimera pattern and the size of its coherent domain [77]. Moreover, in the examples demonstrated in the present manuscript, the nonlinearity parameter ε of the individual Van der Pol oscillator is chosen to be small, corresponding to sinusoidal oscillations. The tweezer control acts successfully also in the case of relaxation oscillations when ε is large [76].

Figure 5 demonstrates the behavior of chimera patterns within two layers under the action of each part of the control (6) separately. When the asymmetric part is deactivated, $K_a = 0$, the lifetime of the chimera state is still extended, but it starts to drift, as shown in **Figure 5A**. The coherent and incoherent domains of the chimera state in the second layer drift along with the chimera in the controlled layer. Thus, extending of lifetime without position control in both layers is possible as well. When we stop to control the lifetime of chimeras, by keeping the coupling coefficients constant in the first layer, after some transient time we observe simultaneous chimera collapse in both layers as depicted in **Figure 5B**.

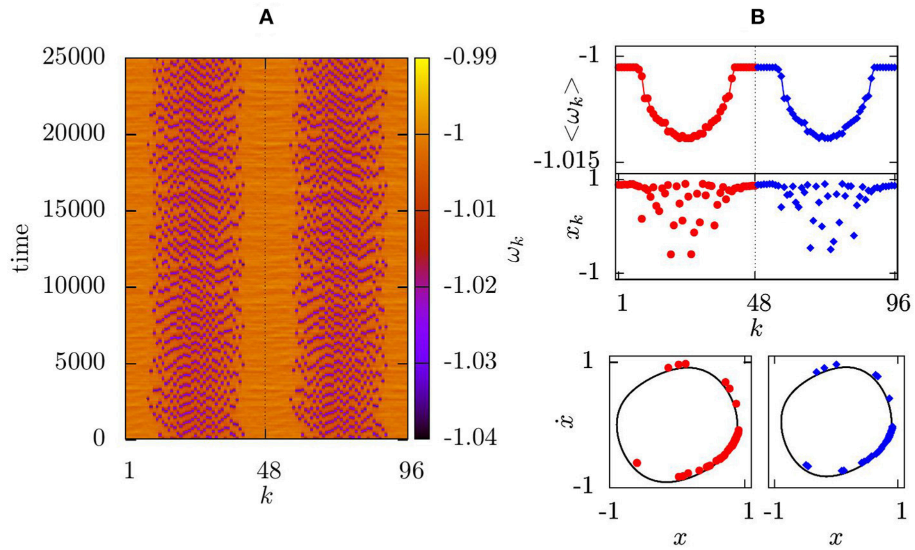

FIGURE 3 | Same as **Figure 2** for a_{inter} =0.011, b_{inter} = 0.0025.

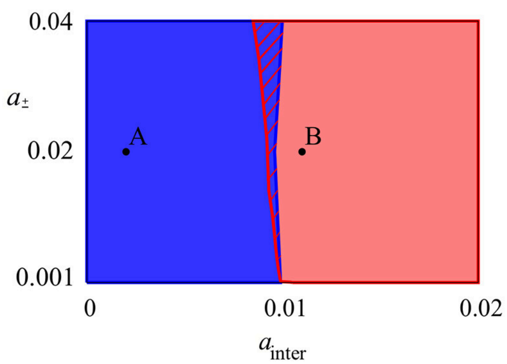

FIGURE 4 | Diagram in the plane of inter-layer a_{inter} and intra-layer a coupling constants for a two-layer system of $N = 48$ oscillators in each layer with $R_1 = R_2 = 16$. The red region corresponds to the successful stabilization of chimera states in both layers, the blue region depicts chimera collapse in the uncontrolled layer. In the hatched region only part of the numerical realizations indicated successful control. Points A and B denote parameter values corresponding to **Figures 2** and **3**, respectively. Other parameters as in **Figure 2**.

Hence, the combination of tweezer control and multiplexing allows for efficient control of chimera states in multiple layers, with the control applied directly to only one layer.

3. ROBUSTNESS OF THE TWEEZER CONTROL IN TWO-LAYER NETWORKS

In real-world networks non-identical layers are more common. Therefore, the analysis of the robustness of the tweezer control in two-layer networks is an important issue. In the following we will consider the topological inhomogeneity of the layers in the network by introducing a coupling range mismatch $R_1 \neq R_2$. In non-locally coupled rings the coupling range is one of the essential parameters for the observation of chimera states. An intermediate coupling range is usually favorable, while too small or too large numbers of coupled neighbors prevent the formation of chimera states. Furthermore, within intermediate values, smaller coupling ranges can cause multiple coherent and incoherent domains of the chimera state. Thus, considering different coupling ranges in two layers will result in competitive patterns formed in each layer. As before, the tweezer control acts only in the first layer of our network.

Figure 6 depicts the stabilization of chimera states in system (1) with $N = 48$ oscillators in each layer, and slightly inhomogeneous topologies with coupling ranges $R_1 = 16$ and $R_2 = 18$. After a short transient time, the interplay of the tweezer control and inter-layer coupling results in the successful spatial alignment of the coherent and incoherent domains, and their mean phase velocity profiles have similar shapes as well, as shown in **Figure 6B**.

As a next step, we increase the layer mismatch and choose $R_1 = 16$, $R_2 = 12$. In the isolated case, the second layer would exhibit a chimera state with multiple incoherent domains [21], which collapses to the completely synchronized state. **Figure 7** demonstrates that by controlling the first layer, we successfully suppress the collapse, and synchronize the chimera states in both layers. However, due to the bidirectional inter-layer interaction, the dynamics of the second layer has indeed an influence on the first one. The chimera states shown in **Figure 7** have larger incoherent domains induced by the smaller coupling range in the second layer. It is worth to note that to stabilize their position, the asymmetric control gain had to be increased ($K_a = 6$). We have shown numerically that we can stabilize the chimera states

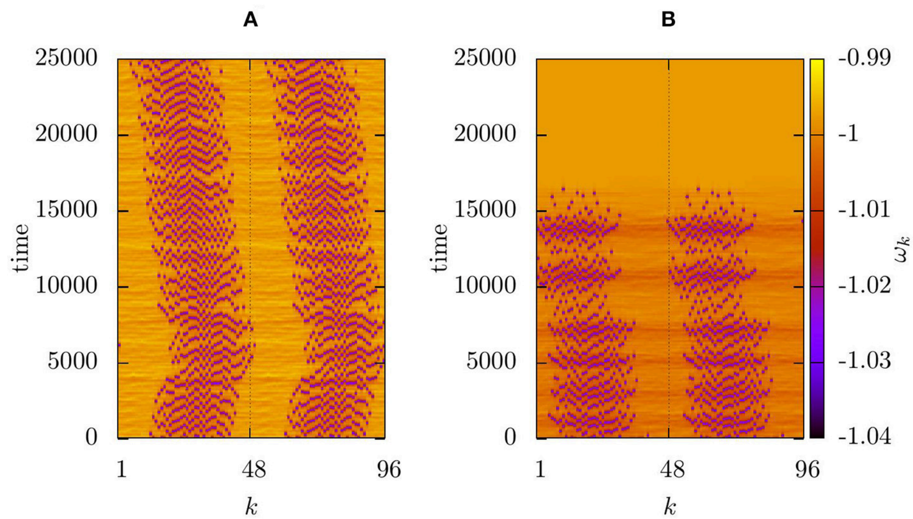

FIGURE 5 | Space-time plot of the mean phase velocities for a two-layer network with one part of the control switched off; **(A)** drifting chimera state in both layers with $K_a = 0$; **(B)** collapse of the uncontrolled chimera state in both layers, with constant coupling coefficients $\sigma_{\pm}^{(1)} = 0.14$. Other parameters as in **Figure 3**.

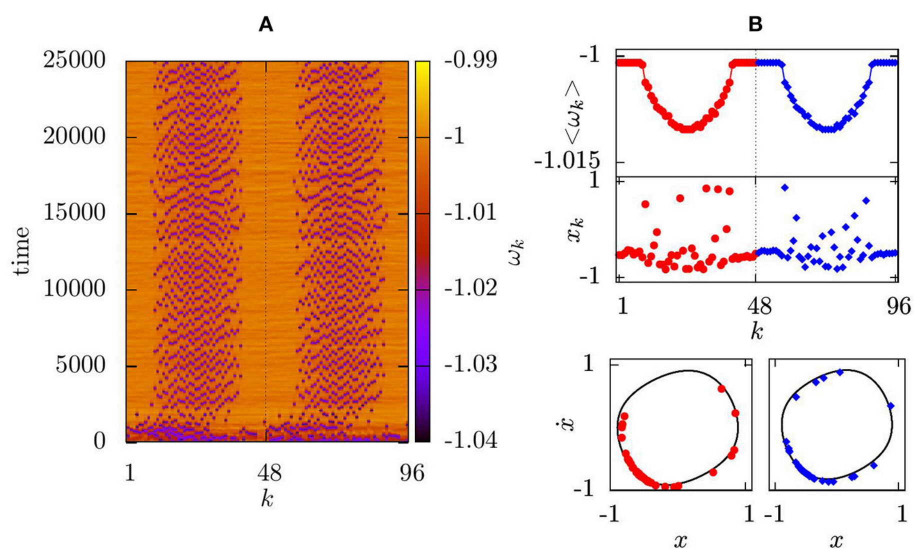

FIGURE 6 | Mean phase velocities for a two-layer system of $N = 48$ oscillators in the layers and non-identical coupling ranges: $R_1 = 16$, $R_2 = 18$, other parameters:$\varepsilon = 0.2$, $a^{(1)} = a^{(2)} = 0.02$, $a_{inter} = 0.009$, $b_{inter} = 0.0023$, $K_a = 2$, $K_s = 0.5$, $b_{-}^{(2)} = b_{+}^{(2)} = 0.0027$. Oscillators $k = 1...48$ represent the first layer, $k = 49...96$ the second layer; **(A)** space-time plot; **(B)** mean phase velocity profile averaged over 25,000 time units for the first layer (red dots) and the second layer (blue diamonds) (top panel); snapshot of variables x_k (middle panel) and snapshot in the $(x_k; \dot{x}_k)$ phase space at t =25,000 (bottom panel).

in two layers for an even larger topology mismatch, however, the coupling parameters and control gains had to be tuned.

4. CONCLUSION

In the present manuscript, we have demonstrated that the combination of the tweezer control for chimera states and multiplexing allows for successful stabilization of chimera states in both layers of two-layer networks of Van der Pol oscillators. Considering a ring topology with non-local interaction between the oscillators within each layer, and one-to-one connections between the corresponding oscillators from the two layers, we have focused on networks of relatively small size, where chimera states are usually hard to observe. Tweezer control, consisting of two parts, extends the lifetime of chimera states, and fixes their spatial position on the ring.

In two-layer networks we have applied the tweezer control to one layer only, and have shown that for sufficiently strong inter-layer coupling the action of the control is transferred to

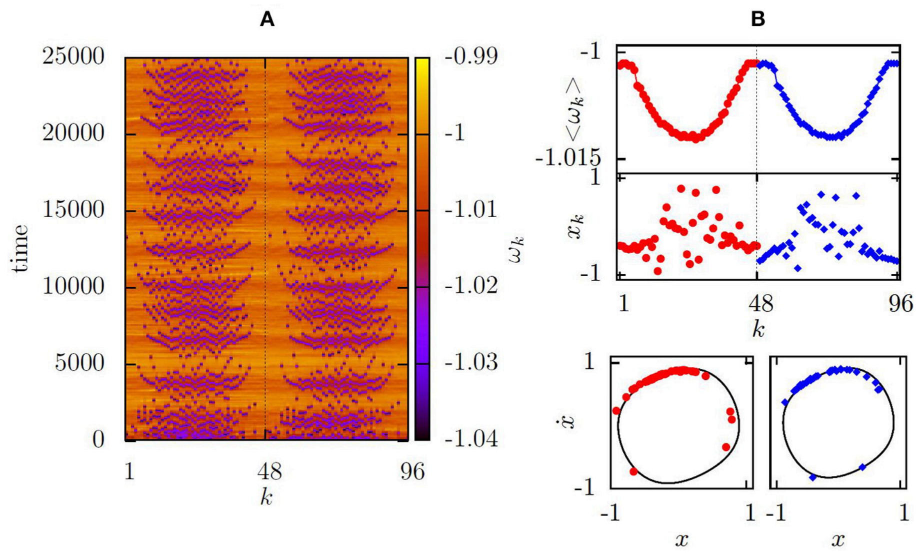

FIGURE 7 | Same as **Figure 6** with $R_1 = 16$, $R_2 = 12$, and $K_a = 6$.

the second layer, where the lifetime of the chimera state is increased and its spatial position if fixed. Without the inter-layer connections, or if their strength is too weak, after a short transient time the chimera state collapses and all oscillators in the second layer synchronize.

We have demonstrated that the combination of tweezer control and multiplexing is robust with respect to the topological inhomogeneity of the layers, and chimera states can be successfully stabilized even in the case of large coupling range mismatch between the layers. Previously, we have demonstrated that tweezer control acts efficiently in non-locally coupled rings consisting of inhomogeneous oscillators [76], therefore the stabilization of chimera states in the two-layer networks with inhomogeneous nodes is plausible as well.

Our results can be useful in real multilayer networks, where the access to some layers is not possible, but there is need to control spatio-temporal patterns. Combination of tweezer control and multiplexing appears to be a powerful and robust tool to solve this problem even for small networks with inhomogeneous layers.

AUTHOR CONTRIBUTIONS

IO, AZ, and ES designed and supervised the study and carried out the analytical work. TH provided numerical simulations and constructed the figures. All authors discussed the results and contributed to writing the manuscript.

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