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Self-Similar Symmetry Model and Cosmic Microwave Background

Tomohide Sonoda *

Graduate School of Natural Sciences, International Christian University, Tokyo, Japan

In this paper, we present the self-similar symmetry (SSS) model that describes the hierarchical structure of the universe. The model is based on the concept of self-similarity, which explains the symmetry of the cosmic microwave background (CMB). The approximate length and time scales of the six hierarchies of the universe—grand unification, electroweak unification, the atom, the pulsar, the solar system, and the galactic system—are derived from the SSS model. In addition, the model implies that the electron mass and gravitational constant could vary with the CMB radiation temperature.

Keywords: self-similarity, symmetry, cosmic microwave background, large numbers hypothesis, large numbers coincidences

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****Correspondence:***
Tomohide Sonoda
tomo@alm.icu.ac.jp

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1. INTRODUCTION

What determines the values of the physical constants and whether they will remain constant over time are fundamental questions in physics. A long-standing conundrum associated with the physical constants is that large dimensionless numbers that are seemingly unrelated can be linked by a scale factor of 10^{39} [1–5]. The Dirac large numbers hypothesis (LNH) tackles this problem. It claims that the gravitational constant G is inversely proportional to the age of the universe. The LNH is based on the coincidences between three very large dimensionless numbers $N_1 \simeq N_2 \simeq \sqrt{N}$. N_1, the ratio of the radius of the observable universe to the radius of the electron, is approximately 10^{39}; N_2, the ratio of the electromagnetic and gravitational forces between a proton and an electron, is also approximately 10^{39}; and N, the number of protons in the observable universe, is approximately 10^{78}. The Dirac LNH argues that "any two of the very large dimensionless numbers occurring in Nature are connected by a simple mathematical relation, in which the coefficients are of the order of unity"[1].

In this paper, we present the self-similar symmetry (SSS) model in which the relationships among these seemingly unrelated physical quantities are represented using a simple geometric sequence for which the first term and the geometric ratio are given by dimensionless ratios of masses. Based on the LNH, the first term of the geometric sequence corresponds to the cosmic microwave background (CMB) radiation temperature, which points to the possibility that the values of the physical constants are determined by the CMB radiation temperature.

2. THE SELF-SIMILAR SYMMETRY MODEL

In the SSS model, the CMB has a symmetrical self-similar structure and the physical constants are dimensionless, otherwise they would not have universality. Therefore, the fundamental dimensionless mass ratios are defined as follows:

$$A = \log\alpha = \log\left(\frac{m_{\rm pl}}{m_{\rm pr}}\right),\ B = \log\beta = \log\left(\frac{m_{\rm e}}{m_{\rm pr}}\right), \tag{1}$$

where m_{pr} is the proton mass, m_e is the electron mass, and m_{pl} is the Planck mass, and the fundamental dimensionless time and length ratios are defined as

$$T = \log\left(\frac{t}{t_{pl}}\right),\ L = \log\left(\frac{l}{l_{pl}}\right), \tag{2}$$

where t and l are the time and length scales of the hierarchies and t_{pl} and l_{pl} are the Planck time and Planck length, respectively. The similarity dimension D is defined using these dimensionless parameters:

$$D = \left(\frac{T}{L}\right)^3 = \frac{A}{A+B} \approx 1.206. \tag{3}$$

We then assume that hierarchical structures are constructed according to the following sequences:

$$L_0 = 2(A+B) \approx 31.701, \tag{4}$$
$$L_n = D^n L_0 \quad \text{for} \quad L > L_0, \tag{5}$$
$$L_m = (2 - D^m)L_0 \quad \text{for} \quad L < L_0, \tag{6}$$

where n and m are natural numbers that represent the hierarchical level. In addition, the time scales of each hierarchy are calculated using Equation (3).

3. VERIFICATION OF THE SSS MODEL

To verify the SSS model, we compared values obtained with it against reference values. **Tables 1**, **2** summarize the length and time scales, respectively, of the grand unification, electroweak unification, atom, pulsar, solar system, and galactic system hierarchies of the universe. The SSS model values agree well with the reference values. **Figure 1** shows the hierarchy time scale as a function of the length scale. The coincidences in the figure confirm the validity of the SSS model.

TABLE 1 | Length scales of the hierarchies of the universe.

Hierarchy	l (m)	L	SSS model	Error (%)
Planck[a]	1.6×10^{-35}	0	–	–
Grand unification[b]	10^{-27}	7.79	7.81 ($m = 3$)	0.2
Electroweak unification[b]	10^{-17}	17.79	17.30 ($m = 2$)	−2.7
Atom[c]	2.4×10^{-10}	25.17	25.17 ($m = 1$)	0.0
Pulsar[d]	2.4×10^{4}	39.17	38.23 ($n = 1$)	−2.4
Solar system[e]	3.0×10^{11}	46.27	46.10 ($n = 2$)	−0.3
Galaxy[f]	5.3×10^{20}	55.52	55.59 ($n = 3$)	0.1

[a] *The Planck length l_{pl} is defined as $l_{pl} = \sqrt{\hbar G/c^3}$, where $\hbar$ is the Dirac's constant, G is the gravitational constant, and c is the speed of light in a vacuum.*
[b] *Values are taken from a magnetic monopole structure in grand unified theories (GUTs) [2].*
[c] *Assumed to be twice the van der Waals radius of a hydrogen atom [6].*
[d] *Estimated to be 1.5 times the mass of the Sun with a radius of 12 km [7].*
[e] *Based on the average diameter of the Earth's orbit around the Sun; 2 astronomical units (AUs) [6].*
[f] *Taken as twice the distance from the center of the galaxy to the solar system, which is 28,000 light years [6].*

4. DISCUSSION

From Equation (1), $2A = -\log \alpha_G$, where $\alpha_G = Gm_{pr}^2/\hbar c$ is the gravitational coupling constant, the following coincidences occurs:

$$L_{n=2} - L_{m=3} = L_{n=3} - L_{m=2} \approx 2A. \tag{7}$$

Equation (7) shows that α_G plays an important role in forming the hierarchical structure of the universe. In addition,

$$L_{m=1} - L_0 = L_0 - L_{n=1} = 2B. \tag{8}$$

TABLE 2 | Time Scales of the Hierarchies of the Universe[a].

Hierarchy	t (s)	T	SSS model	Error (%)
Planck[b]	5.4×10^{-44}	0	–	–
Grand unification[c]	2.2×10^{-35}	8.61	8.31 ($m = 3$)	−3.5
Electroweak unification[d]	6.6×10^{-27}	17.09	18.42 ($m = 2$)	7.7
Atom[e]	4.8×10^{-17}	26.95	26.79 ($m = 1$)	−0.6
Pulsar[f]	2.9×10^{-2}	41.72	40.69 ($n = 1$)	−2.5
Solar system[g]	3.2×10^{7}	50.77	49.07 ($n = 2$)	−3.3
Galaxy[h]	7.6×10^{15}	59.15	59.17 ($n = 3$)	0.0

[a] *The orbital motion and magnetic fields of the Earth and Sun cause emitted light to have a long period. Therefore, although the physical structures are different in each hierarchy, their time scales can be compared in a unifying manner using the period of light.*
[b] *Planck time $t_{pl} = \sqrt{\hbar G/c^5}$.*
[c] *Grand unification at the intermediate mass scale $M'_{GUT} \approx 3 \times 10^{10}$ GeV proposed by Dienes et al. [8]; $t = \hbar/(3 \times 10^{19})$ s.*
[d] *Electromagnetic force and weak force unify at 10^2 GeV [9]; $t = \hbar/10^{11}$ s.*
[e] *Based on the first ionization energy of hydrogen [6]; $t = \hbar/13.6$ s.*
[f] *Inverse of the average observed frequency of $f_{pulsar} \approx 35$Hz ($N = 2{,}307$ pulsars) determined from [10].*
[g] *Period of the Earth's revolution around the Sun [6].*
[h] *The revolution of the Sun around the center of the Milky Way, i.e., 1 galactic year ≈ 240 million years, based on a galactic rotational speed of approximately 220 km/s [6].*

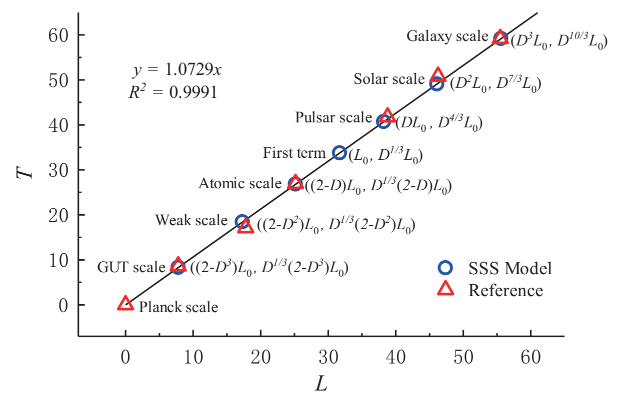

FIGURE 1 | Time scale as a function of the length scale for the SSS model and reference values. The gradient of the fit of the SSS model values to the reference values is 1.0729 ($R^2 = 0.9991$). Note the symmetry about the first term L_0, which corresponds to the CMB radiation temperature. This symmetry leads to speculation about the approximate length and time scale of the universe, which are 4.1×10^{28} m and 5.2×10^{16} years, respectively.

Thus, if L_u is the length of the universe, the following hierarchy holds:

$$L_u = 2L_0. \tag{9}$$

Therefore, the ratios of the coincidences between the length scales of the hierarchies are

$$r_a = \frac{L_{n=2} - L_{m=3}}{L_{n=1}} \approx 1.002, \tag{10}$$

$$r_b = \frac{L_u - L_{n=3}}{L_{n=2} - L_{n=1}} \approx 0.991, \tag{11}$$

from which we get

$$D = \frac{r_a - r_b}{1 - r_b}. \tag{12}$$

From Equation (12), we see that $r_a \neq r_b \neq 1$.

With respect to the first term of the geometric sequence, L_0, we find that

$$(\alpha\beta)^{-2} T_{pl} \approx 2.821 \text{ K}, \tag{13}$$

where T_{pl} is the Planck temperature. The value in Equation (13) is consistent with the CMB radiation temperature T_{CMB}[11]. Assuming the right-hand side of Equation (13) represents the T_{CMB}, if LNH is applied to Equation (13) and we define the dimensionless temperature ratio $\tau_{CMB} = T_{CMB}/T_{pl}$, we get

$$\alpha_G \approx \tau_{CMB}^{D}. \tag{14}$$

Similarly,

$$\beta^2 \approx \tau_{CMB}^{D-1}. \tag{15}$$

Thus, both α_G and β are power functions of T_{CMB}.

Substituting $T_{CMB} = T_{pl}$, an initial condition of the universe, into Equations (14) and (15) yields $\alpha = \beta = 1$, which means that the entire hierarchy was contained in a single point and that the electron, proton, and Planck masses were equivalent. These masses have varied since that initial single point such that $m_e \ll m_{pr} \ll m_{pl}$, in response to the changing T_{CMB}, where $T_{CMB} \ll T_{pl}$. Assuming that $T_{CMB} \rightarrow 0$ is the ultimate fate of the universe, then $\alpha \rightarrow \infty$, $\beta \rightarrow 0$, and $L_0 \rightarrow \infty$, indicating that $m_e \rightarrow 0$ and $G \rightarrow 0$ as the universe expands to infinity.

5. CONCLUSIONS

Our SSS model describes the large-scale structure of the universe and shows that the six hierarchies of the universe are self-similar to the CMB, indicating that the CMB is key to unifying quantum theory with general relativity. In addition, the SSS model leads to the conclusion that m_e and G vary with T_{CMB}. Any errors arising from the SSS model are problems to be tackled in the future.

AUTHOR CONTRIBUTIONS

TS conceived the study and prepared the manuscript.

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