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EDITED BY

Janice Swanson,
Michigan State University, United States

REVIEWED BY

Kenneth Anderson,
North Carolina State University,
United States
Danielle Ufer,
United States Department of Agriculture
(USDA), United States

*CORRESPONDENCE

Maurice Doyon
✉ Maurice.Doyon@eac.ulaval.ca

RECEIVED 07 April 2023

ACCEPTED 30 May 2023

PUBLISHED 03 July 2023

CITATION

Traore OZ and Doyon M (2023) Economic sustainability of extending lay cycle in the supply-managed Canadian egg industry. *Front. Anim. Sci.* 4:1201771. doi: 10.3389/fanim.2023.1201771

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Economic sustainability of extending lay cycle in the supply-managed Canadian egg industry

Ousmane Z. Traore and Maurice Doyon*

Department of Agricultural Economics and Consumer Science, Laval University, Quebec, QC, Canada

The productivity levels of the Canadian egg industry have increased over the years, including hen productivity and feed conversion efficiency. Moreover, genetic improvements combined with improved feed and light management have recently resulted in hens potentially being able to produce 500 eggs in an 80-week laying cycle. Nevertheless, most egg farms in Canada are still on a 51-week production cycle despite high hen productivity levels at culling. Lack of economic impact information, combined with the fact that egg production is under supply management in Canada and that farmers are paid their cost of production reduces the incentive to extend laying cycles despite the savings associated with lower rates of flock replacement. On the other hand, a greater percentage of large eggs is beneficial to the value chain, and the use of fewer resources per egg associated with longer laying cycles generates environmental benefits. This article analyzes the economic sustainability of extending laying cycles in Canada by combining partial budgeting analysis based on farm-level data with a non-linear mathematical programming model to assess the economic costs and benefits of extending laying cycles, while taking into consideration the policy context of supply management in Canada. The results suggest that, for hens housed in an aviary, extending the laying cycles from 51 to 64 weeks would increase profits by approximately 6% per year over a 5-year period. Our optimization model forecast that a laying cycle of 71 weeks would be economically optimal, with an average productivity of 6.7 eggs per hen per week and a cumulative mortality rate of 5.53%. This article, through an innovative methodological approach that combines partial budgeting and non-linear mathematical programming models, generates information to help the egg industry stakeholders to make informed decisions on extending laying cycles while considering the policy context of supply management in Canada.

KEYWORDS

Canada, economic sustainability, egg production, longer lay cycle, cost–benefit

1 Introduction

The Canadian egg industry has evolved considerably in its movement toward specialization which has been taking place since the early 20th century (Pelletier et al., 2018). Farm-level efficiency, including the productivity efficiency of laying hens and feed conversion efficiency, along with bird health and hen losses due to mortality, have also been continuously improving. For example, the annual rate of lay among Canadian laying hens has increased from less than 100 eggs per year in the early 20th century to over 300 at present (Statistics Canada, 2022). Pelletier (2018) reports that the rate of lay increased by more than 50% between 1962 and 2012 and that the combined mortality rates for pullets and laying hens decreased from approximately 13% in the early 1960s to 3.2% in 2012. Over the same period, feed conversion went from an average of 3 kg of feed for 1 kg of eggs (a 3:1 ratio) to an average ratio of 2:1 on contemporary egg farms. Moreover, due to these gains in productivity and feed conversion efficiency, Pelletier (2018) shows that since the early 1960s the Canadian egg industry has seen reductions of 41%, 51%, and 57% in acidification, eutrophication, and greenhouse gas emissions, respectively.

These continued improvements in egg production have been facilitated by several factors, such as technological changes, veterinary progress, genetics, and improved management practices (Siegel et al., 2006; Sharma, 2010; Pelletier et al., 2014). These include the development and adoption of cage systems for housing laying hens, the genetic selection of more productive birds, new treatments for disease, and new barn environment technologies (e.g., artificial ventilation systems and climate control, automated feeding, egg collection, and manure removal). The creation of cage-based systems for housing laying hens in the early 1920s would eventually contribute to the continuous year-round production of eggs (Freidberg, 2008). Its wider adoption after WWII provided egg farmers with cost advantages by reducing issues with cannibalism and allowing for the practice of “positive culling,” or removing birds from the flock that are “laying at a slow unprofitable rate or [have] quit laying altogether” (Hartman, 1958, p. 11). In Canada, the movement from a free-run to a cage-based production system largely took place in the early 1960s (Pelletier et al., 2018) and led to an increase in egg production and improved resource use efficiency.

As productivity increased at a faster rate than consumption, Canadian egg producers were faced with the challenges of overproduction and price volatility. Other factors, such as the cyclical nature of egg production, and both domestic competition (exports of cheap eggs from surplus provinces to deficit provinces in Canada) and international competition (cheap surplus eggs imported from the United States), also played important roles. The persistent instability of egg prices at the farm led egg farmers to create marketing boards in each province; this was followed by the creation of the Canadian Egg Marketing Agency (CEMA) in 1972. In 1976, CEMA successfully implemented a national supply management system with farm price mechanisms, production quotas, and overproduction penalties. This led to egg supply being more closely aligned with domestic demand, and the stabilization of egg prices at farms. In 2008, CEMA was renamed Egg Farmers of Canada (EFC).

Today, the Canadian egg industry is facing two important challenges: (1) increasing consumer concerns regarding animal welfare and the environmental footprint of egg production (Doyon et al., 2023) and (2) the fact that laying hens can now produce 500 eggs in an 80-week laying cycle following significant genetic and management improvements (Gwendolyn, 2018; Dekalb, 2023), which poses a challenge to the status quo, that is, a laying cycle of 52 weeks in Canada (including 1 week of cleaning).

In response to the challenge of consumer concerns with animal welfare, Canadian egg producers are shifting away from battery cages, although battery cages have contributed to the achievement of significant production efficiency. The transition to alternative housing systems such as enriched, aviary/free-run, and free-range systems, should be complete by 2031 (Egg farmers, 2021, p.15)¹. According to EFC data, the proportion of hens housed in enriched housing represents 27% of hens in Canada, while hens in aviary/free-run and free-range production systems constitute approximately another 17% of hens (Egg farmers, 2021, p.15). Canadian egg producers are yet to respond to the second challenge. Despite the fact that hens culled at 51 weeks of lay have a laying rate of over 92% and produce large, higher-value eggs, there are numerous reasons for Canadian farmers’ hesitance to extend laying cycles. The first is the absence of economic impact studies, followed by potential logistical difficulties with other partners of the value chain such as hatcheries, concerns regarding the quality of eggshells, status quo bias, and the particularity of supply management.

This article attempts to address the lack of economic information that Canadian egg farmers have regarding the economic impact of longer laying cycles by developing an economic model that can be interpreted within the context of supply management. More specifically, the objective of this article is to develop a model to assess the economic impacts of longer laying cycles in Canada. Our global model is composed of a partial budgeting model and a mathematical programming model and can be used to determine the optimal number of weeks for a laying cycle, given price parameters such as input price and the farm price of eggs. Although our model structure could be used with different housing systems, our model was developed using real farm data from aviary/free-run farms for the purpose of this article.

This article innovates methodologically by developing a partial budgeting model, based on farm data, which results are used to feed a non-linear mathematical programming model to create optimization scenarios in a policy context, that is, in the context of supply management in Canada. It also generates information

¹ In Canada, we can loosely define four types of housing system for hens: the battery cage system, which restricts birds to a small space; the enriched housing system, which houses hens in a colony of up to 200 birds, with more space per bird compared with the battery cage, allowing more movement, and which provides furnishings such as perches, nesting boxes, and scratching pads; the free-run production system, which houses birds in large common areas (floor or aviary), with access to nesting boxes and perches; and the free-range production system, which is similar to the free-run system, but which has the added benefit of providing hens with outdoor access (weather permitting).

regarding the economic feasibility of extending hens' laying cycle beyond the usual 52 weeks in Canada. Our results, which are for the commonly found Lohmann LSL-LITE white hens in aviary/free-run housing, suggest that extending laying cycles from 51 to 64 weeks would increase profits by approximately 6% per year over a 5-year period. Our optimization model forecast that a laying cycle of 71 weeks would be economically optimal with the input and output prices that were prevalent in Eastern Canada in the fall of 2022. An average productivity of 6.7 eggs per hen per week and a cumulative mortality rate of 5.53% is predicted for this optimal laying cycle. The positive difference in profit comes from a reduction of approximately 20% in the use of some resources, not taking into account the associated environmental benefits. Indeed, we find that a longer laying cycle increases the number of eggs that can be graded as jumbo, extra large, or large, and decreases the cost of pullets, feed, capture, and cleaning, which in turn offsets the increased number of cracked and dirty eggs and additional costs of electricity, maintenance, and repairs.

The rest of the article is organized as follows. Section 2 contains the literature review. Section 3 briefly describes supply management and how it relates to longer laying cycles. The methodological approach is presented in section 4 and the empirical results for aviaries/free-run housing are presented and discussed in section 5, which is the last section. We also conclude the article in section 6.

2 Literature review

Most of the literature on extending laying cycles focuses on its biological feasibility and its impact on the environment and hen welfare. To the best of our knowledge, there is no study in the literature that specifically assesses the economic impacts of extending laying cycles. For instance, Gwendolyn (2018) and Dekalb (2023) report that laying hens can produce 500 eggs in 80 weeks due to genetic selection and appropriate management (Sözcü et al., 2021). In 2016, Bain et al. indicated that a hen capable of producing 500 eggs in a laying cycle would generate environmental benefits in terms of more efficient utilization of diminishing resources, including land, water, and raw materials for feed, a reduction in waste, and an overall reduced carbon footprint. Therefore, a longer laying cycle should, over time, reduce the use of resources such as pullets and pullet-rearing resources.

Although it appears that extending laying cycles is biologically feasible and environmentally beneficial, it comes with certain challenges, including a lack of clarity on what the benefits for egg farms will be. For instance, Bain et al. (2016) found that maintaining egg quality, lay persistency, and hen health is challenging in longer laying cycles, as it requires thorough knowledge and consideration of hens' physiology and their nutritional requirements. On the health front, Aerni et al. (2005) found that older hens have a higher mortality rate (cost increase) than younger hens, but also a lower cannibalism rate (which is positive for productivity). Thus, the nature and magnitude of the economic impact of longer laying cycles will depend on the effects on costs and revenues of variables such as the quality and size of the eggs laid, the laying rate, and the hen mortality rate.

However, as hens progress through their laying cycle, several studies (National Research Council, 1994; Van Den Brand et al., 2004; Silversides et al., 2006; Leenstra et al., 2012; Weeks et al., 2016; TierZucht, 2023) show that these factors are highly variable, depending on management practices and hens' environment. For instance, Van Den Brand et al. (2004) found that there are interaction effects between hens' age and housing systems that impact variables such as egg weight, eggshell quality, and internal egg quality. In particular, they found that egg weight increases with the laying hens' age regardless of housing systems, but that free-range laying hens produce eggs of lower weights than battery cage laying hens at the beginning of laying cycles. On the other hand, eggshell quality seems to be better in free-range systems than in battery cages, although cracked and dirty eggs are more common. Thus, although the age of hens impacts economic variables such as the size and the quality of the eggshell, these effects may differ according to the type of housing (Mench et al., 1986; National Research Council, 1994; Fraser, 1994; Vits et al., 2005). As an illustration, Samiullah et al. (2017) compared the performance of flocks of different ages where pullets were reared together and then placed into three commercial production systems (cage, free run, and free range). They found that shell reflectivity and egg weight significantly increase as hen age increases, whereas the Haugh unit, which measures egg protein quality, significantly decreases with the increasing age of laying hens. The authors showed that the 44-week-old flock has significantly higher values for albumen height and that the intensity of the yolk color was lowest in the 73-week-old hens. Travel et al. (2011) demonstrated that hen age is the single most important factor affecting the egg weight and albumen quality of freshly laid eggs. Another factor that affects the parameters of egg quality is genetics, which varies between strains of hens (Silversides et al., 2006). Thus, while older hens might increase revenues through larger eggs, they might also reduce them through lower egg quality.

From the available literature, it is difficult to get a sense of the economic impact of a longer laying cycle on egg farms since some impacts related to the age of the flock are moving in the opposite direction (size of eggs vs. eggshell quality). In addition, numerous factors that can affect costs or revenues are greatly impacted by management practices and housing systems. For example, as hens progress through their laying cycle, the nutrient content of their feed and feed consumption rates change (National Research Council, 1994; Vits et al., 2005; TierZucht, 2023), impacting feed costs, which account for approximately 70% of egg operational production costs. In particular, Matthews and Sumner (2015) show that feed costs per dozen eggs decrease rapidly between weeks 24 and 28 of the egg-laying cycle, stabilize between weeks 29 and 47, and gradually increase between weeks 48 and 76 of the cycle.

However, because of lower flock replacement rates, longer laying cycles are expected to result in more efficient use of resources in the long run, including land, water, and feed materials (Bain et al., 2016), reducing the cost of production. For example, the flock replacement rate associated with longer laying cycles reduces the number of pullets used over a period of time, similarly reducing cleaning costs.

3 Supply management and extending laying cycle

Supply management in Canada is an agricultural policy that is based on the following three elements:

- a producer price determined by the national cost of production;
- production quotas (right to produce a given quantity) that match Canadian demand;
- tight control (high tariffs) of supply-managed products.

The production of eggs, chicken, turkey, and dairy is under supply management in Canada. The policy applies nationwide and covers all farms involved in a supply-managed sector. Two elements of supply management in the egg sector interact with longer laying cycles. A producer price determined by a national average cost of production is the first. To be more specific, representative national surveys of egg farms are conducted at regular intervals and a percentage of the least efficient farms are removed from the sample used to compute the average targeted egg price. Between surveys, costs of production are updated with indicators, and some regional adjustment between Canadian regions is made. This implies that when the cost of production increases (or decreases), egg prices at the farm also increase (or decrease). Given that the value chain for table eggs is short, changes in farm prices tend to be quickly transferred to buyers (graders). Moreover, in the absence of oversupply or undersupply (i.e., when supply is matched with demand), the price of eggs in Canada is very stable relative, for example, to that in the USA, as illustrated by [Figure 1](#). It is also important to note that, under supply management, provincial boards are in charge of coordination between egg producers, buyers, and to some extent hatcheries. Thus, extending the laying cycle would not be a fully individual decision.

Therefore, under the hypothesis that longer laying cycles would reduce the cost of production, a collective movement implies that

producer gain will be essentially transferred to egg buyers since the average cost of production would be reduced. This, combined with the greater and tighter production management it requires, reduces egg farmers' incentive to extend the laying cycle. In addition, concerns have been expressed regarding the quality of eggshells and yolks from older hens and how it might affect the marketing of eggs in Canada (personal communications with Canadian graders). From behavioral economics, one can also suspect the existence of status quo bias, which is a non-negligible obstacle ([Bergeron et al., 2019](#)). For instance, egg farmers are on a 52-week schedule (51 weeks of production and 1 week of cleaning) and therefore start a new production cycle on approximately the same date every year. Extending laying cycles implies that new production cycles would start at a different date every year, which could generate resistance. On the other hand, lowering the cost of production and thus the retail cost of eggs is beneficial for the value chain; it increases the competitiveness of eggs as a source of protein and is certainly not detrimental to the maintaining of supply management in eggs.

The second element is related to the import of eggs beyond trade agreement requirements due to temporary imbalances between the demand for and production of large eggs. This effect is especially important when additional production quotas are made available to farmers since the added new hens initially produce smaller eggs. Extending the laying cycle implies that there will be an increase in the average size of eggs available ([Van Den Brand et al., 2004](#); [Travel et al., 2011](#); [Samiullah et al., 2017](#)) such that a higher number of large eggs will be available, which could potentially reduce imports of shell eggs. Between 2011 and 2021, approximately 9 million packs of a dozen table eggs over the trade obligations of Canada, on average, were imported every year. [Figure 2](#) indicates an import peak of more than 15 million packs of a dozen table eggs in 2014. Lower imports in 2020 and 2021 are likely reflective of temporary market distortion associated with the COVID-19 pandemic. Thus, extending the laying cycle has the potential to increase production quotas for Canadian egg farmers by reducing the need for imports of table eggs.

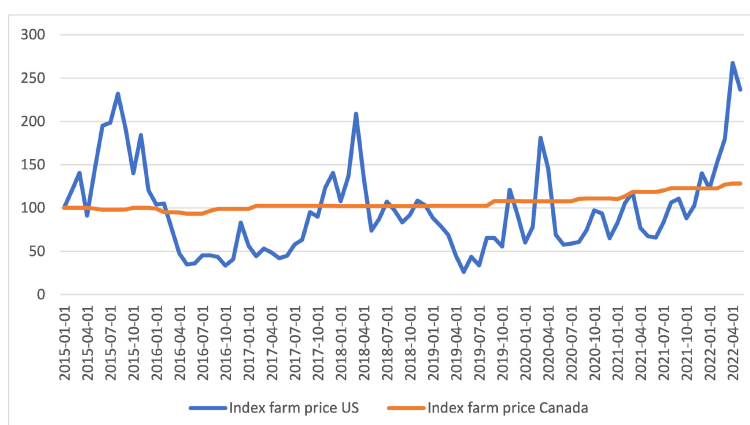


FIGURE 1
Index of egg price at the farm (2015-01-01 = 100) in the USA and Canada, 2015-2022. Source: Egg Industry Center (USA), Egg Farmers of Quebec, and our computation.

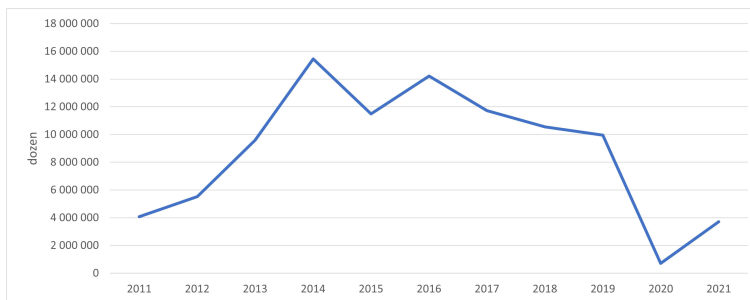


FIGURE 2 Evolution of table egg imports to Canada, 2011–2021. Statistics Canada, Egg Farmers of Canada, and our compilation.

In the Canadian context of supply management in egg production, these factors have to be considered. However, the necessary condition in the collective and individual decision process regarding whether or not hen laying cycles should be extended remains its economic viability, which is not yet determined. The objective of the following section is to present a methodological approach for assessing the economic costs and benefits of extending the laying cycle.

4 Methodology

To analyze the economic impacts of implementing a longer laying cycle in Canada, we first developed a partial budgeting model, describe two analytical production scenarios, and present a normalization procedure to compare the two different production scenarios adequately on a common basis. We then present a non-linear mathematical programming model to determine the economically optimal duration of an egg production cycle for laying hens in aviaries.

4.1 Partial budgeting

4.1.1 Model

This section presents a partial budgeting model for analyzing the economic costs and benefits of an extended laying cycle. A partial budgeting model is a tool that is used to assess how a decision will impact the profitability of a firm. Typically, this can be in the form of a spreadsheet program that compares the cost and benefits of an alternative option vs. the *status quo*. In this study, we compare the costs and benefits of extending the laying cycle in Canada from 52 weeks (including 1 week for cleaning) to $T > 52$ weeks. As in any partial budgeting analysis, we assume that extending the laying cycle will affect the producer’s profit through its impact on cost and income variables in one or more of the following directions: increase in income; reduction or elimination of costs; increase in costs; and reduction or elimination of income.

Within the production cycle, we considered two types of production cost: time-varying cost and time-invariant cost. The main time-varying production costs that we initially considered included the costs of feed, supplements and vaccination, flock capture, labor, electricity, and water, and the time-invariant costs included the costs of pullets, cleaning, maintenance, and repair. Variations in costs and

incomes are due to changes in some performance variables such as feed nutrient content, feed consumption rate, proportions of egg quality grades, hen productivity, and mortality. For instance, nutrient content and feed consumption rate change over time for hens (National Research Council, 1994; ISA, 2000; TierZucht, 2023), and such variations affect feed cost, which accounts for approximately 70% of egg production costs (Matthews and Sumner, 2015). In addition, hen productivity (i.e., rate of lay), hen mortality rate, and egg quality parameters (i.e., egg size and shell thickness) change with the age of hens. The modification of feed cost, hen productivity, hen mortality rate, and proportions of egg quality grades will in turn automatically affect farm profit.

Thus, considering a given laying cycle length (T) of production, one can define an “egg producer” profit² as follows:

$$\pi_T = (\sum_{g=1}^G p_g \bar{e}_{gT} - \bar{c}_T)(\bar{\phi}_T(1 - \bar{\tau}_T) - cov(\phi_T, \tau_T))NT - C_0 \quad (1)$$

where

- π_T : producer total profit during period T
- g : indexes egg grades : jumbo/extra/large; medium; small, peewee, cracked, dirty, rejected
- p_g : producer price per dozen eggs with grade g
- \bar{e}_{gT} : average proportion of eggs with grade(g)produced during period T
- \bar{c}_T : average time – varying production cost per dozen eggs
- $\bar{\phi}_T$: weekly average number of eggs(in dozens) produced per hen during period(T)
- $\bar{\tau}_T$: weekly average cumulative mortality rate(in %) of hens during period(T)
- $cov(\phi_T, \tau_T)$: covariance between productivity and mortality during period(T)
- N : total number of hens housed
- T : total laying cycle length(number of weeks)
- C_0 : total time – invariant production cost

2 Note that we define profit before interest, taxes, depreciation and amortization (EBITDA). The development of the functional form is presented in appendix.

The time-varying production cost per dozen eggs (\bar{c}_T) includes the costs of feed, supplements and vaccination, flock capture, labor, electricity, and water, whereas the total time-invariant costs, represented by (\bar{c}_0) encompass the costs of pullets, cleaning, maintenance, and repair.

Given the producer total profit for a production period (T) in equation (1), one can define the net change in producer unit profit, i.e., the amount per dozen eggs, ($U_{\pi_{T_2}}$) associated with a longer flock cycle (T_2) compared with a shorter production period (T_1) by:

$$\begin{aligned} \underbrace{U_{\pi_{T_2}} - U_{\pi_{T_1}}}_{\Delta U_{\pi_T}} &= \underbrace{\left(\sum_{g=1}^G P_g \bar{e}_{gT_2} - \sum_{g=1}^G P_g \bar{e}_{gT_1} \right)}_{\Delta \bar{p}_g} \\ &+ \underbrace{\left(\bar{c}_{T_1} - \bar{c}_{T_2} \right)}_{\Delta \bar{c}_T} \text{igg} \\ &+ \underbrace{\left(\frac{C_0}{Q_{T_1}} - \frac{C_0}{Q_{T_2}} \right)}_{\Delta \bar{c}_0} \end{aligned} \tag{2}$$

where

$$\begin{aligned} Q_{T_1} &= (\bar{\phi}_{T_1}(1 - \bar{\tau}_{T_1}) - cov(\phi_{T_1}, \tau_{T_1}))NT_1 \\ \text{and} \\ Q_{T_2} &= (\bar{\phi}_{T_2}(1 - \bar{\tau}_{T_2}) - cov(\phi_{T_2}, \tau_{T_2}))NT_2 \end{aligned}$$

indicate the total number of dozen eggs produced during period (T_1) and (T_2), respectively. From equation (2), it follows that the sign of the net change in producer unit profit (i.e., amount per dozen eggs) associated with a longer flock cycle (T_2) will depend on both the positive (increase in revenue and reduction of costs), and negative (decrease in revenue and/or increase in costs) financial changes and on the variation in average fixed cost.

Table 1 illustrates the four types of financial change made possible by longer laying cycles. Note that time-invariant costs, as defined, are always reduced with longer laying cycles in Table 1. In the case of positive financial change only (i.e., increase in revenue and reduction of the two types of costs) as illustrated in the first line of Table 1, the sign of the net change in producer unit profit will be positive. The three other cases of financial change (the last three rows of Table 1) imply that there is an opposing relationship between costs and revenue on producer unit profit. Therefore, changes in producer profits are dependent on the amplitude of positive movements (increase in revenue or decrease in costs) in relation to the negative movements (decrease in revenue or increase in costs) in financial changes when extending laying cycles.

4.1.2 Analytical scenarios

Let us consider two alternative scenarios characterized by the length of the laying cycle and the values taken by different performance variables. Let us define the benchmark scenario as a production cycle of 52 weeks (including 1 week for cleaning), which is currently the norm in Canada. The second scenario implies that in this specific case, the same strain of hens and the same housing system are being investigated. However, these hypotheses could easily be relaxed. Two analytical scenarios for different lengths of laying cycles are illustrated in Table 2. The variation in production cycle length impacts other variables such as nutrient requirements, time-varying costs (vaccination, flock capture, labor, electricity, and water) and time-invariant costs (pullet, cleaning, and maintenance and repair), hen productivity, hen mortality, and proportion of different egg quality grades.

4.1.3 Scenarios normalization for comparison

To compare the costs and benefits of two different production periods, one needs to make comparisons on a common basis. As an illustration, an egg operation with a lay cycle of 65 weeks compared with 52 weeks will remove the need for one flock of pullets and its associated cost every 5 years or 260 weeks. Thus, a longer period of comparison is needed to fully capture the various impacts of longer laying cycles. Therefore, once the performance parameters have been established for both production periods individually, one must normalize the two production periods to a common period of (M) weeks. To continue with our example, one can normalize to 260 weeks or 5 years of production by considering production cycles of 52 and 65 weeks, as 260 weeks is a multiple of 52 ($\times 5$) and 65 ($\times 4$). On the other hand, one could consider comparing the results obtained over a 5-year production period on an annual basis. To do this, one would need to simply divide by 5 the total results of four cycles of 65 weeks, which could then be compared with the 52-week cycle.

4.2 Mathematical programming model

This section presents a mathematical programming model to determine the optimal economic duration of a laying cycle. We define the optimal economic duration in weeks of a single egg production cycle by any argument (T) that equalizes to zero the marginal profit or that maximizes the producer profits. Note that in our model, profit corresponds to earnings before interest, taxes,

TABLE 1 Net change in producer unit profit (amount per dozen eggs) associated with a longer lay cycle (T2-T1).

Revenues $\Delta \bar{p}_g$	Time-varying cost $\Delta \bar{c}_T$	Time-invariant cost $\Delta \bar{c}_0$	Producer profit ΔU_{π_T}
+	-	-	+
+	+	-	±
-	-	-	±
-	+	-	±

- : decrease in revenues or cost.
 + : increase in revenues or in cost.
 +- : either an increase or a decrease in producer profit.

TABLE 2 Analytical scenarios characterization.

Characteristics	Benchmark	Alternative scenario
Length of the production cycle	52 weeks	T >52 weeks
Nutrient requirement (NR)	NR_T	NR_{T_2}
Observed or estimated optimal nutrient level (ONL)	ONL_{T_1}	ONL_{T_2}
Observed or estimated time-varying cost (\bar{c}_T)	\bar{c}_{T_1}	\bar{c}_{T_2}
Observed or estimated time-invariant in varying costs (\bar{c}_0)	\bar{c}_0	\bar{c}_0
Observed or estimated optimal hen productivity ($\bar{\phi}_T$)	$\bar{\phi}_{T_1}$	$\bar{\phi}_{T_2}$
Observed or estimated hen mortality rate ($\bar{\tau}_T$)	$\bar{\tau}_{T_1}$	$\bar{\tau}_{T_2}$
Observed or estimated proportions of egg quality grades (\bar{e}_{gT})	\bar{e}_{gT_1}	\bar{e}_{gT_2}
Estimated producer profit (1)	$\bar{\pi}_{T_1}$	$\bar{\pi}_{T_2}$
The net change in producer unit profit (2)	$\bar{U}_{\pi_{T_2}} - \bar{U}_{\pi_{T_1}}$	

depreciation, and amortization (EBITDA). To determine the optimal economic duration in weeks, we need to solve a constrained maximization problem with producers' profit as the objective function. The constraints are defined in terms of productivity, mortality, and the expected price³.

Given producer total profit for a single production period (T) defined by equation (1), and the estimated data from partial budgeting (e.g., egg grading information, variable production cost, number of production weeks, productivity, mortality, and covariance) during any production period (T) for a given egg producer, one can define the algebraic representation of the producer profit maximization problem as follows:

$$\text{Maximize } \pi_T = \left(\sum_{g=1}^G p_g \bar{e}_{gT} - \bar{c}_T \right) (\bar{\phi}_T (1 - \bar{\tau}_T) - \text{cov}(\phi_T, \tau_T)) NT - C_0$$

Subject to :

Productivity constraints :

$$\begin{aligned} \bar{\phi}_T &= \frac{7}{12} [(aT^b \exp(-cT + dT^{0.5})) / 100] \\ \bar{\phi}_T &\leq \frac{7}{12} \end{aligned}$$

Mortality constraints :

$$\begin{aligned} \bar{\tau}_T &= \alpha_0 + \alpha_1 T \\ \bar{\tau}_T &\leq 1 \end{aligned}$$

Price constraints :

$$\begin{aligned} \bar{p}_g &= \sum_{g=1}^G p_g \bar{e}_{gT} = \beta_0 + \beta_1 T \\ \bar{p}_g &\geq \bar{c}_T \\ \bar{p}_g &\leq 2.26. \end{aligned}$$

where g indexes different egg grades: jumbo/extra/large (J), medium (M), small (S), peewee (P), cracked (C), dirty (D), and rejected (R). \bar{e}_{gT} refers to the average proportion of eggs by grade during the total production period (T).

ENDOGENOUS VARIABLES:

T : total laying cycle length/production period (number of weeks)

ϕ_T : average hen-day productivity during period (T) (number of dz/hen/week)

τ_T : average cumulative mortality rate (in %) during period (T)

p_g : expected producer price per dz eggs (\$ /dz)

EXOGENOUS VARIABLES:

e_{JT} : average proportion of eggs graded Jumbo Extra Large (% of dz > 56 g)

e_{MT} : average proportion of eggs graded Medium (% of dz > 49 g)

e_{ST} : average proportion of eggs graded Small (% of dz > 42)

e_{PT} : average proportion of eggs graded Peewee (% of dz < 42 g)

e_{CT} : average proportion of eggs graded Cracked (% of dz)

e_{DT} : average proportion of eggs Dirty (% of dz)

e_{RT} : average proportion of eggs Rejected (% of dz)

$\text{cov}(\phi_T, \tau_T)$: covariance between productivity and mortality during period (T)

C_T : average time-varying production cost per dozen eggs (\$ /dz)

T_0 : total time-invariant production cost (\$)

PARAMETERS:

P_{JT} : producer price per dozens of eggs graded Jumbo Extra Large (\$ /dz)

P_{MT} : producer price per dozen eggs graded Medium (\$ /dz)

P_{ST} : producer price per dozen eggs graded Small (\$ /dz)

P_{PT} : producer price per dozen eggs graded Peewee (\$ /dz)

P_{CT} : producer price per dozen eggs graded Cracked (\$ /dz)

P_{RT} : producer price per dozen eggs Rejected (\$ /dz)

3 The functional forms used to determine productivity, mortality, and expected price that in the equations of the optimal model are from the literature or from our own fit using software programs such as R and @Risk. When the literature contained more than one alternative, such as productivity model, we choose the one that best fit our data series.

N : total number of hens housed
 $a = 2.993$; $b=4.303$; $c=-0.095$; $d=-2.564$
 $\alpha_0 = 0.007$; $\alpha_1=0.0007$; $\beta_0=1.850$; $\beta_1=0.006$

Note that although the average proportions of eggs by grade (\bar{e}_{gT}) are exogenous in the optimization model, they are indirectly a function of T since these proportions are used to calculate the expected producer price (\bar{p}_{gT}), which is a function of T . Similarly, the time-varying production costs, as defined, are exogenous since the average cost per dozen moves approximately in a constant relationship with added weeks of laying cycle lengths past 52 weeks.

To solve the maximization problem and obtain the optimal economic number of weeks (T), we developed a GAMS program using data from an aviary/free-run system producer. The script and data are available upon request.

5 A case study: an aviary/free-run housing system

As indicated previously, although our model structure could be used with different housing systems, for the purpose of this article, our model was developed using real farm data for the extended lay cycle collected between 2020 and 2022 from two large aviary/free-run farms located in Quebec. The 51-week

laying cycle data were collected over the same period from a much larger set of Quebec aviary farms. This choice was motivated by data availability and the fact that use of the aviaries/free-run housing type has increased by 33% since 2017, with approximately 11% of laying hens in Canada in 2021 residing in a housing of this type (Egg farmers, 2021, p.15).

5.1 Descriptive statistics

Table 3 presents descriptive statistics for flock characteristics and performance variables estimated from the partial budgeting analysis for our two analytical scenarios. Both scenarios are for a rather large (by Canadian standards) egg farm that houses 43,350 Lohmann LSL-LITE white hens in a multi-level aviary/free-run housing system. In this housing system, hens have access to a barn floor where they may move freely. Our results indicate that extending the laying cycle from 52 to 65 weeks slightly increases the average feed consumption per hen from 0.732 kg/week to 0.736 kg/week, and also the cumulative mortality rate from 4.023% to 5.089%. Similarly, the average proportions of jumbo, extra, and large eggs, increased, as expected, from 78.11% to 81.37%. The proportions of cracked eggs (from 0.38% to 0.51%), dirty eggs (from 0.12% to 0.14%), and rejected eggs (from 0.26% to 0.27%) also increased. In terms of decrease, we note a decrease in average feed cost (from \$0.588 to \$0.582 per kg) and

TABLE 3 Descriptive statistics for performance variables estimated from the partial budgeting analysis.

	Unit	Benchmark	Alternative
Flock characteristics			
Length of the production cycle	Number of weeks	51 weeks	64 weeks
Flock size	Number of hens housed	43,350	43,350
The strain of the bird		White hen	White hen
Housing system		Aviary	Aviary
Performance variables			
Average feed consumption	kg/hen/week	0.732	0.736
Average feed cost	\$/kg	0.588	0.582
Average hen productivity ($\bar{\phi}_T$)	Number of dz/hen/week	0.543	0.540
Cumulative mortality rate (τ_T)	Percentage (%)	4.023	5.089
Average cumulative mortality rate ($\bar{\tau}_T$)	Percentage/week	2.403	2.755
Covariance (productivity, cumulative mortality)		0.00019	0.00010
Jumbo, extra, and large eggs (\bar{e}_{JT})	Percentage of eggs >56 g (%)	78.11	81.37
Medium eggs (\bar{e}_{MT})	Percentage of eggs >49 g (%)	15.21	12.90
Small eggs (\bar{e}_{ST})	Percentage of eggs >42 g (%)	4.47	3.63
Peewee eggs (\bar{e}_{PT})	Percentage of eggs <42 g (%)	1.44	1.17
Cracked eggs (\bar{e}_{CT})	Percentage (%)	0.39	0.51

(Continued)

TABLE 3 Continued

	Unit	Benchmark	Alternative
Dirty eggs ($\bar{\epsilon}_{D_r}$)	Percentage (%)	0.12	0.14
Rejected eggs ($\bar{\epsilon}_{R_r}$)	Percentage (%)	0.26	0.27
Estimated producer unit profit	\$/dz	0.8296	0.8943

average hen productivity (from 0.543 to 0.540 dozen eggs per week). The average proportion of medium eggs (from 15.21% to 12.90%), small eggs (from 4.47% to 3.63%), and peewees eggs (from 1.44% to 1.17%) also decreased.

5.2 Results of partial budgeting and its implications for the egg supply chain

The results related to the change in cost and revenue variables and the net change in producer total profit associated with longer laying cycles are reported in Table 4. Overall, our results indicate that extending the laying cycle from 52 to 65 weeks will result in a positive net change in producer net income, which means that the positive financial change (increase in revenues and reduction of costs) associated with a longer laying cycle is greater than the negative financial change (decrease in revenues and/or increase in costs). In particular, for our aviary case study, we found that extending the laying cycle from 52 weeks (51 weeks of production and 1 week of cleaning) to 65 weeks (64 weeks of production and 1 week of cleaning) over a production period of 5 years would increase producer net income (EBITDA) on an annual basis from \$971,824 to \$1,028,496, which corresponds to an increase of 5.83%. Note that since extending the laying cycle does not impact amortization and financing costs, the change in net EBITDA corresponds to profit before taxes.

We find that a longer laying cycle increases the number of eggs graded jumbo/extra/large (2.62%) and decreases the cost of pullets (20.00%), feed (0.55%), capture (20.88%), and cleaning (20.00%)⁴, which offset the fact that larger proportions of cracked eggs and dirty eggs are produced (30.26% and 19.97%, respectively), and the additional costs of electricity (0.39%), and maintenance and repair (0.39%). The direct implications of these results for hatcheries and pullet growers are a decrease in demand. In the presence of supply management, where the economic profit is on average zero, this implies that egg farmers will reduce the farm price of eggs by approximately 6% (on average). This should translate into lower prices for consumers. In addition, fewer resources will be required per egg produced, which, based on the literature, should translate into positive environmental benefits.

⁴ Note that the revenues from culled hens are marginal and are therefore integrated with cleaning cost (i.e., reducing cleaning cost).

5.3 Optimization results: optimal laying cycle length, productivity, and mortality

To determine the optimal economic duration of a laying cycle for hens housed in an aviary system, we solved a constrained maximization problem with producer profit as the objective function. The optimization results were obtained under some assumptions on functional forms for⁵ productivity (non-linear function in time, suggested by McNally (1971)), mortality (linear function in time), and expected price (linear function in time). The results are reported in Table 5. We found with our data that a laying cycle of 71 weeks would be economically optimal, with an average productivity of 6.7 eggs per hen per week and a cumulative mortality rate of 5.53%. Although we cannot generalize these specific results to the Canadian egg industry as a whole, these results show that the actual production cycle length (52 weeks) is most likely suboptimal, particularly in aviary systems.

6 Conclusion

Over the past few decades, the Canadian egg industry has improved its performance through genetic improvements, improved feeds, light management, and new technologies such as artificial ventilation systems, light and climate control, automated feeding, egg collection, and manure removal. In fact, laying hens can now produce 500 eggs in an 80-week laying cycle following significant genetic and management improvements (Gwendolyn, 2018; Dekalb, 2023).

This poses a challenge to the *status quo*, which is a laying cycle of 52 weeks in Canada (including 1 week of cleaning) even if the hen laying rate is over 92% when they are culled. Although numerous reasons can explain Canadian egg farmers' hesitations, we identify the absence of economic impact studies and the particularity of supply management as the most important ones.

The objective of this article was to analyze the economic impacts of longer egg-laying cycles in Canada. The main results allowed us to conclude that extending the laying cycle beyond 52

⁵ Those functional forms are integrated in the equations of the optimal model and are from the literature or from our own fit using software programs such as R and @Risk. When the literature contained more than one alternative, such as productivity model, we choose the one that best fit our data series.

TABLE 4 Comparative results of a 65-week production cycle versus 52 weeks over a 1-year period.

Increases in income			Decreases in income		
<i>Added income due to change</i>			<i>Added costs due to change</i>		
Jumbo/extra/large eggs (eggs >56 g)		\$54 426,81	Electricity cost		\$108,54
			Labor cost		\$675,00
			Maintenance and repair cost		\$108,54
			The cost associated with more cracked eggs		\$2 908,67
			The cost associated with more dirty eggs		\$478,28
<i>Total increase</i>		\$54 426,81	<i>Total increase</i>		\$4 279,04
<i>Reduced costs due to change</i>			<i>Reduced income due to change</i>		
Pullet cost		\$72 429,57	Medium eggs (eggs >49 g)		\$60 122,71
Feed cost		\$5 102,63	Small eggs (eggs >42 g)		\$17 198,29
Capture cost		\$3 866,10	Peewee eggs (eggs <42 g)		\$1 557,05
Cleaning cost		\$4 004,00			
<i>Total decrease</i>		\$85 402,30	<i>Total decrease</i>		\$78 878,06
Increase in income		\$139 829,12	Decrease in income		\$83 157,10
Change in net income		\$56 672,02			
Percentage change in net income		5,83%			

TABLE 5 Optimization results, aviary/free-run farms.

	Unit	Reference value	Optimal value
Length of the production cycle	Number of weeks	52	71
Average hen productivity	Number of dz/hen/week	0.543	0.5632
The average cumulative mortality rate	%	2.40	5.53
Expected producer price	\$/dz	2.159	2.26

weeks will result in a positive net change in total profit for egg producers, meaning that the positive financial changes associated with a longer laying cycle outweigh the negative.

However, in the context of supply management, this implies that most of the financial gain at the farm will be transferred to graders (buyers of farm eggs) and passed on partially to consumers. This does not generate a great incentive for Canadian egg farmers to move to longer laying cycles. However, lowering egg prices contributes to what is often referred to as the social contract that allows supply management to exist. Moreover, although this was not calculated for the purpose of this article, extending laying cycles has the potential to reduce the shell egg imports that are above Canadian trade requirements, which would increase domestic production to the benefit of egg farmers in Canada. More importantly, fewer resources will be required per egg produced, which, based on the literature, should translate into positive environmental benefits.

Although our results are based on data from aviary sites, according to our interpretation of the literature, other types of housing systems should yield similar or better results. Nevertheless, further research is needed so that the models can be improved through a higher volume of data. Similarly, given the impact of different housing systems on numerous health and production variables in the literature, the developed models should be used to compare the impact of different housing systems on the economic results of longer laying cycles.

Data availability statement

The raw data supporting the conclusions of this article will be made available by the authors, without undue reservation.

Author contributions

OT: Data curation, software, formal analysis, investigation, methodology, visualization, and writing—original draft. MD: Conceptualization, funding acquisition, resources, project administration, supervision, validation, and writing—review and editing. All authors contributed to the article and approved the submitted version.

Funding

This research project is funded by Egg Farmers of Canada (EFC), Egg farmers of Alberta-Result Driven Agriculture Research (RDAR). The funding sources had no involvement in study design, collection, analysis, interpretation of data, report writing, or in the decision to submit this article for publication.

Acknowledgments

We would like to thank the egg farmers who generously shared their farm data and expertise with us.

References

- Aerni, V., Brinkhof, M., Wechsler, B., Oester, H., and Fröhlich, E. (2005). Productivity and mortality of laying hens in aviaries: a systematic review. *World's Poultry Sci. J.* 61, 130–142. doi: 10.1079/WPS200450
- Bain, M. M., Nys, Y., and Dunn, I. C. (2016). Increasing persistency in lay and stabilising egg quality in longer laying cycles. what are the challenges? *Br. Poultry Sci.* 57, 330–338. doi: 10.1080/00071668.2016.1161727
- Bergeron, S., Doyon, M., Saulais, L., and Labrecque, J. (2019). Using insights from behavioral economics to nudge individuals towards healthier choices when eating out: a restaurant experiment. *Food Qual. Preference* 73, 56–64. doi: 10.1016/j.foodqual.2018.12.001
- Dekalb (2023). *How to feed layers for a longer production cycle and high performance*. Available at: <https://www.dekalb-poultry.com/en/news/how-feed-layers-longer-production-cycle-and-high-performance/>. (Accessed January 14, 2023)
- Doyon, M., Bergeron, S., Saulais, L., Labonté, M.-È., and Provencher, V. (2023). Do consumers value welfare and environmental attributes in egg production similarly in fresh eggs and prepared meals? *Animals* 13, 324. doi: 10.3390/ani13030324
- Egg farmers (2021). *Annual Report 2021*. (Ottawa, Canada). Available at <https://www.eggfarmers.ca/resource/annual-reports/>. (Accessed February 20, 2022)
- Fraser, A. (1994). "A comparison of eggshell structure from birds housed in conventional battery cages and in a modified free range system," in *Proceedings of the 9th European Poultry Conference*, Glasgow, UK, August 7–12, 1994. 151–152.
- Freidberg, S. E. (2008). The triumph of the egg. comparative studies in society and history 50, 400–423. doi: 10.1017/S0010417508000182
- Gwendolyn, J. (2018). *Persistency in lay - achieving 500 eggs in 100 weeks*. Available at: <https://www.pancosma.com/laying-persistency-500-eggs-in-a-single-laying-cycle-in-100-weeks/>. (Accessed March 11, 2023)
- Hartman, R. (1958). *Keeping chickens in cages: A description of the outdoor individual cage system of poultry management as developed mainly in Southern California*. (Palo Alto, California: Pacific Poultryman)
- ISA (2000). *ISA brown management guide*. Available at: <https://www.joiceandhill.co.uk/en/free-range-laying-hens/isa-brown-laying-hens/>. (Accessed January 20, 2023)
- Leenstra, F., Maurer, V., Bestman, M., van Sambeek, F., Zeltner, E., Reuvekamp, B., et al. (2012). Performance of commercial laying hen genotypes on free range and organic farms in Switzerland, France and the Netherlands. *Br. Poultry Sci.* 53, 282–290. doi: 10.1080/00071668.2012.703774
- Matthews, W. A., and Sumner, D. A. (2015). Effects of housing system on the costs of commercial egg production. *Poultry Sci.* 94, 552–557. doi: 10.3382/ps/peu011
- McNally, D. (1971). 315. note: mathematical model for poultry egg production. *Biometrics*, 735–738.
- Mench, J., Van Tienhoven, A., Marsh, J., McCormick, C., Cunningham, D., and Baker, R. (1986). Effects of cage and floor pen management on behavior, production, and physiological stress responses of laying hens. *Poultry Sci.* 65, 1058–1069. doi: 10.3382/ps.0651058
- National Research Council. (1994). *Nutrient Requirements of Poultry: Ninth Revised Edition, 1994* (Washington, DC: The National Academies Press). doi: 10.17226/2114.
- Pelletier, N. (2018). Changes in the life cycle environmental footprint of egg production in Canada from 1962 to 2012. *J. Cleaner Production* 176, 1144–1153. doi: 10.1016/j.jclepro.2017.11.212
- Pelletier, N., Doyon, M., Muirhead, B., Widowski, T., Nurse-Gupta, J., and Hunniford, M. (2018). Sustainability in the Canadian egg industry—learning from the past, navigating the present, planning for the future. *Sustainability* 10, 3524. doi: 10.3390/su10103524
- Pelletier, N., Ibarburu, M., and Xin, H. (2014). Comparison of the environmental footprint of the egg industry in the united states in 1960 and 2010. *Poultry Sci.* 93, 241–255. doi: 10.3382/ps.2013-03390
- Samiullah, S., Omar, A. S., Roberts, J., and Chousalkar, K. (2017). Effect of production system and flock age on eggshell and egg internal quality measurements. *Poultry Sci.* 96, 246–258. doi: 10.3382/ps/pew289
- Sharma, B. (2010). Poultry production, management and bio-security measures. *J. Agric. Environ.* 11, 120–125. doi: 10.3126/aej.v11i0.3659
- Siegel, P., Blair, M., Gross, W., Meldrum, B., Larsen, C., Boa-Amponsem, K., et al. (2006). Poultry performance as influenced by age of dam, genetic line, and dietary vitamin e. *Poultry Sci.* 85, 939–942. doi: 10.1093/ps/85.5.939
- Silversides, F., Korver, D., and Budgell, K. (2006). Effect of strain of layer and age at photostimulation on egg production, egg quality, and bone strength. *Poultry Sci.* 85, 1136–1144. doi: 10.1093/ps/85.7.1136
- Sözcü, A., İpek, A., Oguz, Z., Gunnarsson, S., and Riber, A. B. (2021). Comparison of performance, egg quality, and yolk fatty acid profile in two

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Turkish genotypes (atak-s and atabey) in a free-range system. *Animals* 11 (5), 1458.

Statistics Canada, (2022). *Table 32-10-0119-01 production and disposition of eggs, annual*. Available at <https://doi.org/10.25318/3210011901-eng>.

TierZucht, L. (2023). *HY-LINE NORTH AMERICA*. Available at: <https://www.hyline.com/filesimages/Hy-Line-Products/Hy-Line-Product-PDFs/NA/Leghorn%20Guide%20English.pdf>. (Accessed January 14, 2023)

Travel, A., Nys, Y., and Bain, M. (2011). "Effect of hen age, moult, laying environment and egg storage on egg quality," in *Improving the safety and quality of eggs and egg products* (Woodhead Publishing), 300–329.

Van Den Brand, H., Parmentier, H., and Kemp, A. (2004). Effects of housing system (outdoor vs cages) and age of laying hens on egg characteristics. *Br. Poultry Sci.* 45, 745–752. doi: 10.1080/00071660400014283

Vits, A., Weitzenbürger, D., Hamann, H., and Distl, O. (2005). Production, egg quality, bone strength, claw length, and keel bone deformities of laying hens housed in furnished cages with different group sizes. *Poultry Sci.* 84, 1511–1519. doi: 10.1093/ps/84.10.1511

Weeks, C. A., Lambton, S. L., and Williams, A. G. (2016). Implications for welfare, productivity and sustainability of the variation in reported levels of mortality for laying hen flocks kept in different housing systems: a meta-analysis of ten studies. *PLoS One* 11, e0146394. doi: 10.1371/journal.pone.0146394

Appendix

This appendix presents the mathematical proof for the functional form of equation (1). To derive the mathematical expression of this equation, let us assume that within a laying cycle of length (T), an egg farmer housed N hens at the beginning of the laying cycle and produced Q_T dozen eggs at average time-varying cost per dozen egg (\bar{c}_T) and total time-invariant cost (\bar{C}_0). Let us assume that each egg produced during the laying cycle is graded as jumbo/extra/large (J) or medium (M); small (S), peewee (P), cracked (C), dirty (D), and rejected (R). Let \bar{e}_{gT} denotes the average proportion of eggs by grade during the entire production period (T) and p_g defines the producer price of egg graded g. Given this information, one can define the producer total profit during the entire production period (T) as follows:

$$\pi_T = \left(\sum_{g=1}^G p_g \bar{e}_{gT} - \bar{c}_T \right) Q_T - C_0 \quad (4)$$

Let us assume that egg producers record production information (average proportion of eggs by grade, average laying rate, and cumulative mortality rate of hens) on a weekly (t) basis. Given that the weekly revenues of farmer depend on variables such as the quality and size of eggs, the laying rate, and the mortality rate of hens, one can determine the total number of dozen eggs produced during the entire laying cycle as follows:

$$\begin{aligned} Q_T &= \sum_{t=1}^T \bar{\phi}_t (1 - \bar{\tau}_t) N \\ &= \left(\sum_{t=1}^T \bar{\phi}_t - \sum_{t=1}^T \bar{\phi}_t \bar{\tau}_t \right) N \\ &= \left(T \bar{\phi}_T - \sum_{t=1}^T \bar{\phi}_t \bar{\tau}_t \right) N \\ &= \left(\bar{\phi}_T (1 - \bar{\tau}_T) - \text{cov}(\bar{\phi}_T, \bar{\tau}_T) \right) NT \end{aligned} \quad (5)$$

where $\text{cov}(\bar{\phi}_T, \bar{\tau}_T) = \frac{1}{T} \sum_{t=1}^T \bar{\phi}_t \bar{\tau}_t - \bar{\phi}_T \bar{\tau}_T$ refers to the covariance between productivity and mortality during the laying cycle (T). Given the expression of the total dozen eggs produced, the profit equation (4) can be rewritten as follows:

$$\pi_T = \left(\sum_{g=1}^G p_g \bar{e}_{gT} - \bar{c}_T \right) \left(\bar{\phi}_T (1 - \bar{\tau}_T) - \text{cov}(\bar{\phi}_T, \bar{\tau}_T) \right) NT - C_0 \quad (6)$$