Research Article

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Freeform surface descriptions. Part II: Application benchmark

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Abstract: Optical systems can benefit strongly from freeform surfaces; however, the choice of the right representation is not trivial, and many aspects must be considered. Many possibilities to formulate the surface equations in detail are available, but the experience with these newer representations is rather limited. Therefore, in this work, the focus is to investigate the performance of several classical descriptions as well as one extended freeform surface description in their performance in concrete design optimization tasks. There are different influencing factors characterizing the surface representations, the basic shape, the boundary function, the symmetry, a projection factor, as well as the deformation term describing higher order contributions. We discuss some possibilities and the consequences of describing and using these options with success. These surface representations were chosen to evaluate their impact on all these aspects in the design process. As criteria to distinguish the various options, the convergence over the polynomial orders, as well as the quality of the final solutions, is considered. As a result, recommendations for the right choice of freeform surface representations for practical issues in the optimization of optical systems can be given under restrictions of the benchmark assumptions.

Keywords: correction; freeform surface; optical design; optimization; surface representation.

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1 Introduction

Because of the growing computer resources as well as the ongoing progress in technological possibilities to fabricate complicated-shaped surfaces with high optical quality, the additional degrees of freedom created by freeforms have been extensively used in optical design in the last years. There are very special geometries and applications, which are not symmetric by definition of the application concept; typically, such setups benefit significantly from freeform surfaces. Because of the advantageous properties of symmetric systems and the fundamental principles of aberration theory, it makes no sense to use freeform surfaces if there is no other need to break the circular symmetry of the optical system. In very compact systems or obscuration-free mirror systems, a great boost of new superiorly performing optical concepts is observed if freeforms are used. However, the community has learned in the past that many aspects, assumptions, definitions, methods, tools, and algorithms are changing if the traditional symmetry is given up. Therefore, the complete development chain must be modified and adapted to the new technology. In a previous paper [1], we showed some opportunities on how the surfaces can be descried from the mathematical point of view. Based on these possibilities, the next question of an optical designer would be how to select the surface type optimal for an efficient optimization process with a comfortable result. In this article, we present the results of a first assessment of the mathematical options from the design application viewpoint. We are restricting to the issues of the surface representation in the design phase without considering questions of sensitivity, tolerancing, and manufacturing. Therefore, the results are collected in the form of a benchmark, which compares different freeform surface descriptions for optical systems for various applications with different properties. From the viewpoint of practical work and efficiency, there are several criteria for the selection of a special surface representation. The surface representation should allow for a fast ray trace, the parametrization should be flexible with a small number of parameters, and the optimization of the parameters should be robust and converge quickly with a good result in the design process. In particular, the stability of the freeform surface equations in the optimization seems to be a problem if higher orders are included.

Practical results of optical freeform design work are found in literature with a large amount of publications (see, for example, Refs. [2–6]). Typically, the representation of the freeform surface used by the authors is governed by the available options in the commercial design programs. Therefore, a more systematic comparison and critical review of different options is missing until today. To limit the complexity and make the results easier for interpretation, here, only a small number of four sample systems is selected. Because of the complexity of the problem, all examples are restricted to the use of only one freeform surface. We tried to select practical relevant setups with different types of symmetry, including refractive, reflective, and catadioptric systems.

Only the most important surface descriptions are used here. The properties of these representations under the viewpoint of different initial systems, special properties of the systems, sensitivity in optimization, and quality of the final result are compared and discussed. In particular, the influence of the basic shape selection, the basic geometry to be Cartesian or polar, and the importance of the orthogonality are investigated. To obtain a fair and objective comparison, the design task is formulated simply, and the final results typically do not have the best performance that is possible. Furthermore, the algorithm for optimization, which is used, plays a significant role. Here, the implementation of a damped least square algorithm in the commercial tool Zemax™ is used. However, a comparison of the various cases is critical because there are quite many possibilities of parameter combinations, fixation of constraints, and optimization strategies.

In the second section of this publication, the method of the benchmark is explained in detail. An overview of the tested freeform representations shows the approach to cover the most relevant possibilities. The optimization procedure with initial system selection, growing degree of freedoms, as well as the re-optimization of the remaining system data is described to make the process of computation transparent. The third section describes the properties of the selected optical systems and the reasons for these special choices. The systems under investigation are a symmetry-free Yolo-type two-mirror telescope, a plane-symmetric three-mirror anastigmat (TMA), a head-mounted device (HMD) in a recently suggested folded setup [7], and a double-plane-symmetric anamorphic system with remaining straight optical axis. In Section 4, the detailed results of the benchmark calculations are presented and discussed for each system individually and in comparison. Finally, in a conclusion, the major results are summarized, and a recommendation for practical work is formulated.

2 Benchmark method

As described in the first part of this publication [1], many choices for freeform surface descriptions are available. A goal of this investigation is to evaluate the criteria for choosing an appropriate representation for a given task with a freeform system. Therefore, a benchmark with different freeform systems was performed. Hereby, each of the chosen representations is used to optimize the systems over several polynomial orders, and the performance is evaluated. The polynomial order is hereby defined as the sum of the radial and azimuthal order of the polynomial terms. For Cartesian descriptions, the polynomial order is one half of the sum of the order in x and y, due to simple coordinate conversion. Contrary to other investigations [3, 6] were the number of degrees of freedom is equivalent for all representations, we prefer the approach that the polynomial order for all descriptions is equivalent. This leads to the fact that the number of degrees of freedom per surface differs for the various descriptions for each polynomial order.

The optimization algorithm used here for the benchmark is the damped least square (DLS) provided by Zemax™. To investigate the convergence over the polynomial order and specific contribution by each order, the optimization is proceeded stepwise. Starting with the initial system (Figure 1), setting the basic shape and additional system variables (see Table 2), we increase, order by order, the parameter of the deformation terms of the freeform surface (beginning with the fourth) and re-optimized each time with 300 cycles of DLS up to the 14th polynomial order. The used surface parameters are limited

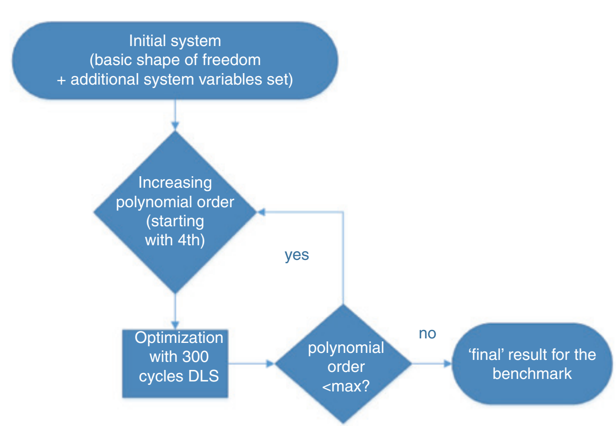


Figure 1: Benchmark process.

to those that contribute to the symmetry of the system. Moreover, the lower-order terms, like tilts and offset, as well as defocus, are not used for the correction. Previous investigations [8] have shown that the convergence speed within one order is mainly dependent on the presence of orthogonality, not on the kind of orthogonality; therefore, we concentrate, here, on the convergence over the orders, so there is improvement of the performance for additional polynomial orders and, therefore, degrees of freedom.

The descriptions chosen for the benchmark can be seen in Table 1. As in part 1, we concentrate on global freeform descriptions. Each of them represents a different combination of none, spatial, and gradient orthogonality and of a (unit) circle or square domain. The Monomials are a special case here: as they are not orthogonalized on a unit grid, they can be used either way. The A-polynomials 2nd kind, introduced in Part 1, are not considered in this benchmark.

Table 1: Investigated representations: (blue) Cartesian-defined descriptions, (red) polar-defined representations.

Orthogonality	**Domain**	
	(Unit) Circle	**(Unit) Square**
None	Monomials	Monomials
Spatial	Zernike Fringe	Chebyshev 2D 1st kind Chebyshev 2D 2nd kind Legendre 2D
Gradient	Q-polynomials	A-polynomials 1st kind

Additionally, the three polar-defined Zernike Fringe, Q-polynomials, and A-polynomials 1st kind (due to the Zernike Fringe set as a basis) are chosen to investigate the impact of the choice of basic shape. Therefore, each of the three descriptions performs the benchmark once with a conic basic shape and once with a biconic. In the case of the Q-polynomials, the 'best-fit-conic' basic shape was extended to 'best-fit-biconic'. The other descriptions used a biconic basic shape for the whole benchmark.

In order to investigate the influence of a different weighting over the domain, the Chebyshev 1st kind, 2nd kind, and Legendre (each in 2D) were chosen for the spatial orthogonal description defined on a unit square. With the Monomials, they represent the descriptions of Cartesian definition.

As our investigations have shown, there is no significant impact of the projection factor on the performance of these systems. This was not addressed here. In the case of systems with stronger inclination of the rays at the boundary, this aspect might be more important.

3 Benchmark systems

For the benchmark, four systems with different symmetry representing typical refractive, reflective, and catadioptric applications of freeform surfaces are investigated (Figure 2): a symmetry-free Yolo-type two-mirror telescope, a plane symmetric TMA, a plane symmetric HMD

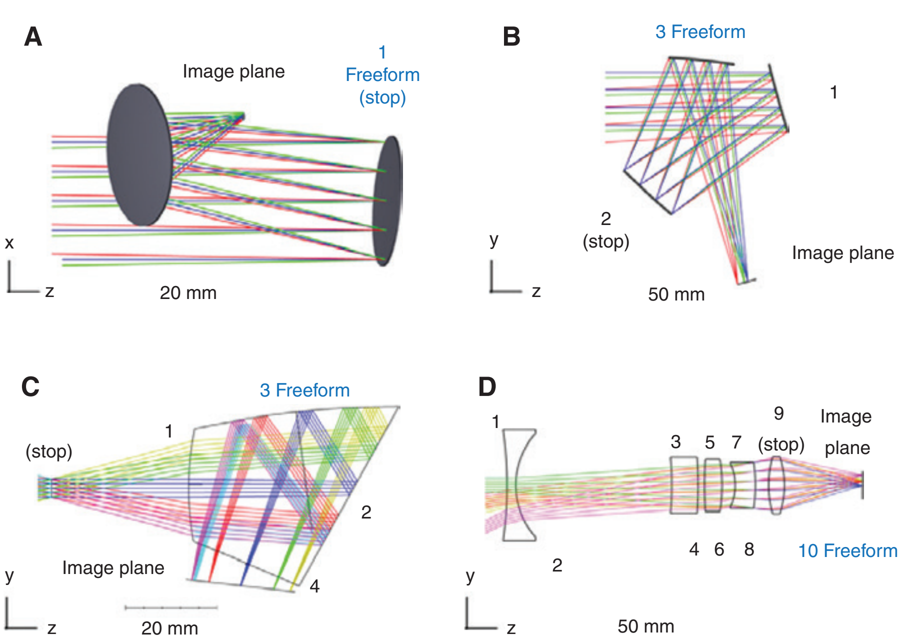


Figure 2: Layouts of investigated systems: (A) Yolo telescope, (B) TMA, (C) HMD, (D) anamorphic system.

in a recently suggested folded setup, and a double-plane-symmetric anamorphic system with straight optical axis. All systems are imaging setups and should be corrected for an improved resolution by one freeform. Because of simplification and comparability with the original design, the correction of distortion was not considered. For all examples, the focal length (EFFL) is kept constant during re-optimization. For simplification, only one wavelength is considered. The norm radius, respectively, the norm width was adapted to the semi-diameter plus a small offset after each cycle to make sure the domain of the surface description and the optical region of interest are equivalent.

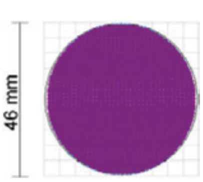
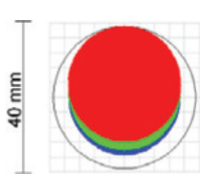
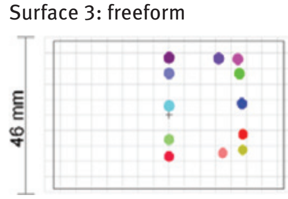
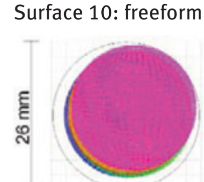
The Yolo telescope (Figure 2A) is based on two tilted mirrors (the first one bending in y and the second one in x), which leads to a symmetry-free system with a small field of view (FOV) and F number (Table 2A). The first mirror is hereby the freeform and the second a conic. The footprint on the freeform surface (Table 2A) is almost perfectly circular. The main criteria for the performance is the resolution (rms spot). In addition to the surface parameters of the freeform, the radius and conic of the second surface as well as the final distance and the tilt of the image plane are used as optimization variables.

The TMA is a reflective plane-symmetric system (Figure 2B). The first and second mirror are spherical, and the third one a freeform. With the small F number and FOV (Table 2B), the separation of the field bundles is very poor, seen on the nearly circular footprint of the freeform (Table 2B). The criteria for optimization are the resolution (rms spot) with small contribution by effective focal length, as well as restrictions for the obscuration. The radii of the first two mirrors plus the final distance are additional variables for the optimization.

The folded HMD (Figure 2C) based on the recently published design by Chen [7] is slightly simplified for the benchmark.

The plane-symmetric catadioptric system has a spherical refractive entrance (first) and exit (fourth) surface and a plane mirror as a second surface. The third surface is the reflective one used as a freeform. The system has a large FOV and F number (Table 2C). The good separation of the field bundles can be seen in the rectangularly shaped footprint on the freeform surface (Table 2C). The optimization criteria are similar to the TMA, namely, the resolution (rms spot) and obscuration restrictions. For the optimization, only the radii of the front and rear surface are additional variables. For comparability with the original design,

Table 2: System data of benchmark-systems: A) Yolo telescope, B) TMA, C) HMD, D) anamorphic system.

	A) Yolo telescope	**B) TMA**	**C) HMD**	**D) Anamorphic system**
Symmetry	–	Plane symmetry	Plane symmetry	Double-plane symmetry
Specification	EPD: 21 mm FOV: 1.5°×1.5° Field points: 9 F-number: 1.9 Wavelenght: 0.55 µm Stop position: surface 1	EPD: 28 mm FOV: 0°×3° Field points: 3 F-number: 2.14 Wavelength: 0.66 µm Stop position: surface 2	EPD: 4 mm FOV: 25°×30° Field points: 12 F-number: 10.89 Wavelength: 0.5775 µm Stop position: before system	EPD: 7.49 mm FOV: 12°×25° Field points: 5 F-number: 4.0 Wavelength: 0.587 mm Stop position: surface 9
Optimization	Criteria: rms spot	Criteria: rms spot + restrictions for obscuration	Criteria: rms spot + restrictions for obscuration	Criteria: rms spot
Surfaces	Surface 1: freeform Surface 2: conic	Surfaces 1 and 2: sphere Surface 3: freeform	Surfaces 1 and 4: sphere Surface 2: plane Surface 3: freeform	Surfaces 1, 3, 5–9: sphere Surfaces 2 and 4: cylinder Surface 10: freeform
Footprint freeform surface				
Additional variables	– Radius and conic parameter of surface 2 – Tilt of the image plane in x and y – Final distance	– Radii of surfaces 1 and 2 – Final distance	– Radii of surfaces 1 and 4	– Radii of surfaces 1–9 – Final distance

EPD, entrance pupil diameter.

distortion was not considered in the merit function. This simplification may distort the final results slightly.

The anamorphic system (Figure 2D) is a double-plane-symmetric refractive freeform system based on a patent by Wartmann [9], which is simplified for the benchmark. The first nine surfaces are spherical, except the second and the fourth, which are cylinders. The last surface in front of the image plane is the freeform. The system has a medium FOV and F number (Table 2D). The poor separation of the field bundles is again seen on the footprint of the last surface (Table 3D). The optimization criterion is similar to the Yolo telescope, namely, the resolution (rms spot). The radii of the nine non-freeform surfaces, as well as the final distance, are additional variables for the optimization.

4 Results

In the following, we present the results of the benchmark for the four investigated systems. The performance values in the figures are in all cases the averaged spot radius of the selected field points (merit function).

4.1 Yolo telescope

The Yolo-type telescope has, due to the lack of symmetry, the full set of parameters for the optimization. In the case of the polar descriptions (Zernike Fringe, Q-polynomials and A-polynomials 1st), these are 60 in total up to the 14th order (second-order terms are excluded); for the Cartesian descriptions, these are 33.

As seen in Figure 3A, with these asymmetric degrees of freedom, the system can be improved up to a factor of 25 (biconic → biconic + A-polynomials 1st). Specifically, for the fourth-order, the Monomials and Chebyshev 2nd kind are not able to correct in the same way as the Zernike Fringe, Q-polynomial, or A-polynomials 1st. However, with higher orders and a higher number of degrees of freedom, the different descriptions are in a similar range, so the Monomials and Chebyshev 2nd kind could compensate the disadvantage within one order, and very high orders (12th to 14th) correct in a better way than the Zernike Fringe or Q-polynomial set. The A-polynomial 1st kind, on the other hand, seems to have a slight advantage over the other descriptions for lower and middle orders. Specifically, higher orders improved much better than the similar Zernike Fringe and Q-polynomial, so that the final result is similar to one from Chebyshev 2nd kind and differ by a factor of around 1.5 compared to the one from Zernike Fringe, for example.

The freeform contribution by the deformation terms (seen in Table 3A) by each description investigated for these systems shows that the general shape is almost identical for Zernike Fringe, Q-polynomial, and A-polynomials 1st kind, including the absolute values of the sag. So during the optimization, these descriptions followed, probably, the same path to this minima, with the A-polynomials 1st kind benefitting from better higher-order contributions, which leads to small differences in shape (right and left boundary) and a better final result. Looking at the freeform sag-contribution by the deformation terms (seen in Table 3A) for the monomials and Chebyshev 2nd, the result is less clear. Some similarities in shape can be seen (outer boundary), but the center parts have neither a resemblance with each other nor with the other three descriptions. So, despite the similar result for the optimization and resolution for all orders up to the 14th order (except the 12th), the descriptions found paths to achieve this, which end up in very different shapes. Additionally, in the case of the Chebyshev 2nd, the overall sag is a factor of 5 smaller than for the other descriptions (−0.03 −0.01 to −0.1 to 0.1 for the Zernike Fringe, for example).

Another aspect, the choice of basic shape investigated with the Zernike Fringe, Q-polynomial, and A-polynomials description, can be seen in Figure 3B. It is clearly shown that the benefit of a more sophisticated basic shape can lead to an improvement by a factor of 2 (with the same number of freeform parameters, but two additional for the biconic). This can be explained with the dominating xy-coupling terms, due to the tilts of the mirrors in x and y. Moreover, the results for the different orders show that the same correction can be obtained by a conic with 60 additional freeform parameters (14th order) or a biconic with 21 additional freeform parameters (eighth order). The impact of almost 40 parameters can, here, be compensated completely by the additional two biconic parameters.

Additionally the influence of the weighting of the description over the domain on the performance with the Monomials, Chebyshev 1st kind, Chebyshev 2nd kind, and Legendre polynomials was investigated. The result is shown in Figure 3C. Up to the 10th order, only minor differences occur, but with the next two orders, the descriptions separate into two groups: one with the Chebyshev 1st and Monomials, the second with the Chebyshev 2nd and Legendre polynomials. As discussed before, the freeform sag contribution (seen in Table 3A) by the deformation terms do not indicate the separation so clearly as they differ in shape (specifically the center) and absolute value (factor 5). The four descriptions seem to have found different

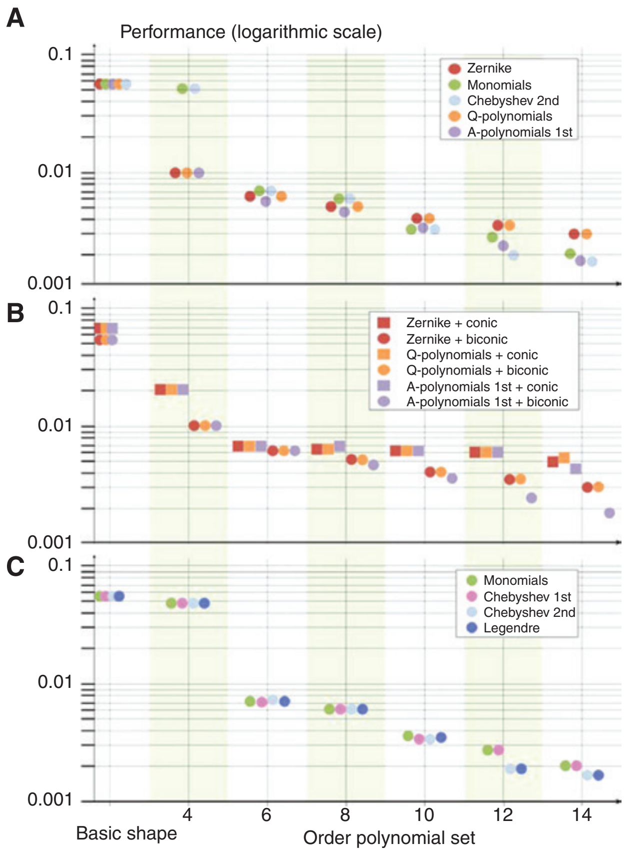


Figure 3: Results for Yolo telecscope: (A) performance of the main representations (all with biconic basic shape), (B) impact of different basic shapes for the performance of polar descriptions, (C) comparison of Cartesian descriptions (all with biconic basic shape).

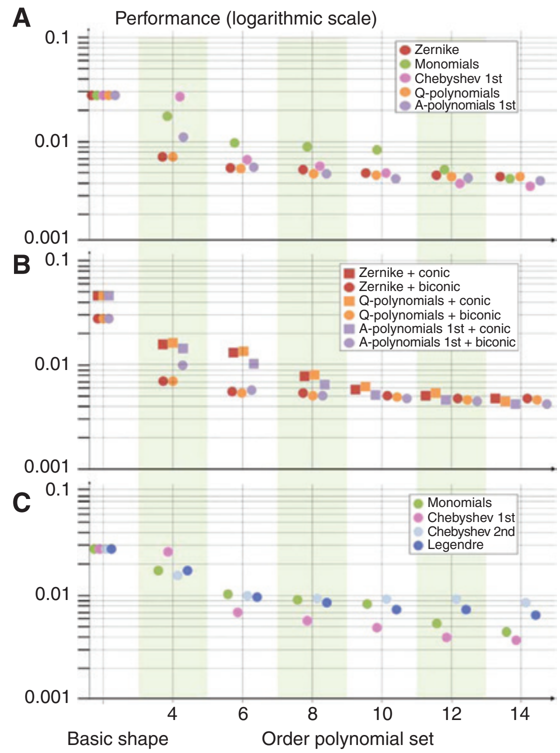


Figure 4: Results for TMA: (A) performance of the main representations (all with biconic basic shape), (B) impact of different basic shapes for the performance of polar descriptions, (C) comparison of cartesian descriptions (all with biconic basic shape).

minima with similar final results in which the two descriptions with generally higher values at the boundary than in the center (Legendre and Chebyshev 2nd kind) tend to correct the system slightly better.

4.2 Three-mirror anastigmat

The TMA is a plane-symmetric system, which reduces the number of additional degrees of freedom by the deformation terms to 33 in the case of Zernike Fringe, Q-polynomial, and A-polynomial 1st kind and 18 for the Monomials, Chebyshev 1st, Chebyshev 2nd, and Legendre (excluding the second order).

The improvement, as seen in Figure 4A, by the freeform surface is a factor of 8 compared to a biconic. The final result of the optimization for the different descriptions is hereby very similar. However, from the fourth order, specifically the descriptions with a rectangular domain lack performance, but with the sixth order, this is compensated. The Monomials, as the one description without any orthogonality, cannot compensate this lack for the first 10 orders, but with the 12th and 14th order reach the same result as the others. A look on the freeform sag contribution by the deformation terms of the freeform surface (seen in Table 3B) shows that the shape for the Zernike Fringe, Q-polynomial, and A-polynomials 1st kind is almost identical with slight differences in the absolute values. Again, the A-polynomials are slightly better in correction for the final order. On the other hand, the shape defined by the Monomials and Chebyshev 1st has a dominating y dependence with almost no x dependence seen. Although similar in shape, the absolute values differ by a factor of 3. Compared to, e.g. the Zernike Fringe, there are similarities for the outer boundary, but again, they differ for the center part. So, the shape found by each group during the optimization leads to the same final result for resolution, independent of its differences.

Contrary to the previous system, the impact of the extended basic shape (Figure 4B) is only seen for low orders and can be equally compensated by each of the polynomial sets with the 10th order. So, the astigmatism correction is of such low order that the conic plus the deformation terms up to the investigated order are enough to represent that correction.

The influence of the weighting over the domain, as seen in Figure 4C, shows again that the Chebyshev 1st kind lack specifically the fourth order of impact, but for higher orders, the impact is much higher than for the Legendre or Chebyshev 2nd kind. In the case of the Legendre polynomials, the improvement after the sixth order is minimal, but still better than for the Chebyshev 2nd kind. The final results (14th order) of both (each with 18 freeform parameter) is similar to the result of the Chebyshev 1st with only fourth- and sixth-order deformation terms (four parameters). Although the Monomials are in the range of the Chebyshev 2nd kind until the 10th order, they improve to the range of the Chebyshev 1st within the following two orders. This difference in final performance by a factor of 2 (Chebyshev 1st kind/Monomials to Legendre/Chebyshev 2nd kind) is not indicated by shape or absolute values of the sag contribution of the asymmetric deformation terms (seen in Table 3B), but may occur due to a higher-order contribution to the correction.

4.3 Head-mounted device

Like the TMA, the HMD is a plane-symmetric system, which leads to 18 freeform parameters for the Monomials, Chebyshev 1st, Chebyshev 2nd, and Legendre, and 33 for the Zernike Fringe, Q-polynomial, and A-polynomials 1st kind with an optimization up to the 14th order.

The improvement by a freeform in this system was a factor of 45 (biconic to Q-polynomial with biconic). Comparing the different descriptions (Figure 5A), a similar behavior for the Zernike Fringe and Q-polynomial can be seen for the lower orders. However, with higher orders, the Q-polynomial improves better than the other two. The freeform sag contribution by these three (Table 3C) shows a very similar shape, but a clear difference in the absolute values (−0.42 to 0.58 for the Q-polynomial to −1.13 to 2.25 for the A-polynomials 1st kind) and specifically in the lower outer region of the domain. Comparing these results with the Chebyshev 1st kind and Monomials, it is clearly seen that the Monomials lack the ability for correction in this specific system over the complete optimization. The results in each order are around a factor 5 worse than the best performing description in this order. The freeform sag contribution (Table 3C) of the Monomials shows a much different shape and have a factor 5 higher sag values for the HMD than the other descriptions. Why the Monomials are not able to represent the needed correction is not clear, but it might be a problem of orthogonality. The shape of the Chebyshev 1st kind, on the other hand, has similarities with the shapes of the Q-polynomial, specifically in

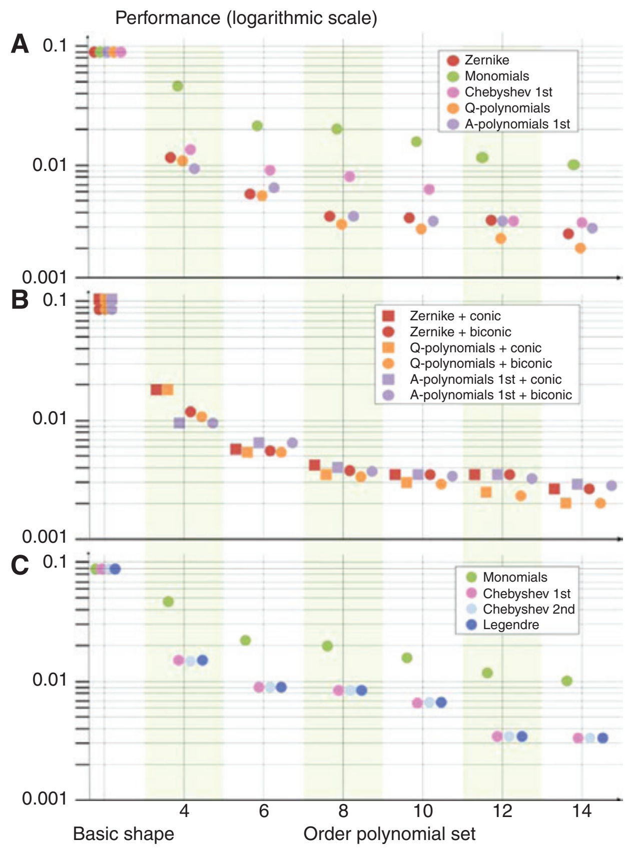


Figure 5: Results for HMD: (A) performance of the main representations (all with biconic basic shape), (B) impact of different basic shapes for the performance of polar descriptions, (C) comparison of cartesian descriptions (all with biconic basic shape).

the outer region of the boundary, but again differs in the center. So, despite the lack of correction ability for lower orders, it results finally in a comparable result to the A-polynomials 1st kind and end up with similarities in the shape, although the path during the optimization was a different one. The influence of the basic shape for the performance (Figure 5B) can be seen in the lower orders, as well as for the final result. Between the sixth and 10th order, the difference is negligible. Nevertheless, the combination of biconic basic shape and high-order freeform terms leads to a difference of a factor of 1.5 for all three representations.

The weighting over the domain, as seen in Figure 5C, has no visible impact on the performance in this HMD design. The three descriptions Chebyshev 1st kind, Chebyshev 2nd kind, and Legendre have similar performance values over the complete run and a similar shape of the final freeform sag contribution. The difference in the absolute values and center of the surface (Table 3C) is based in the different weighting, but does not affect the performance for this system.

4.4 Anamorphic system

The anamorphic system is a special benchmark system due to the double plane symmetry. This leads to a reduction in the contributing terms up to nine for the Monomials, Chebyshev 1st kind, Chebyshev 2nd kind, and Legendre. Additionally, due the specific structure of the polynomial set, every second order (sixth, 10th, and 14th) has no contribution at all. For the Zernike Fringe, Q-polynomial and A-polynomials 1st kind, the number is reduced to 18 with every order being present.

The improvement by the freeform surface in this system is only a factor of 4 (biconic to A-polynomials 1st kind with biconic). The lack of contribution by every second order is also seen in the performance (Figure 6A) of the Chebyshev 2nd kind and Monomials. Interestingly, although the final performance is similar, the freeform sag contribution (Table 3D) is not only different in shape but also in absolute values. A resemblance in the surfaces can be seen for the outer boundary, but the center is completely different, which means with the minimal number of degrees of freedom, the two descriptions choose two different paths in the optimization, with similar performance but different surfaces. The performance of the system can be improved better by Zernike Fringe, Q-polynomial, and A-polynomials 1st kind over the orders. However, the Q-polynomial lacks the correction ability here compared with the Zernike, which results in a slightly worse result over the orders. On the other hand, the A-polynomials 1st kind has slightly better results for higher orders. The difference of the final performance is relatively small, like the difference of the freeform sag contribution between the three descriptions, which is based mainly on higher-order contribution.

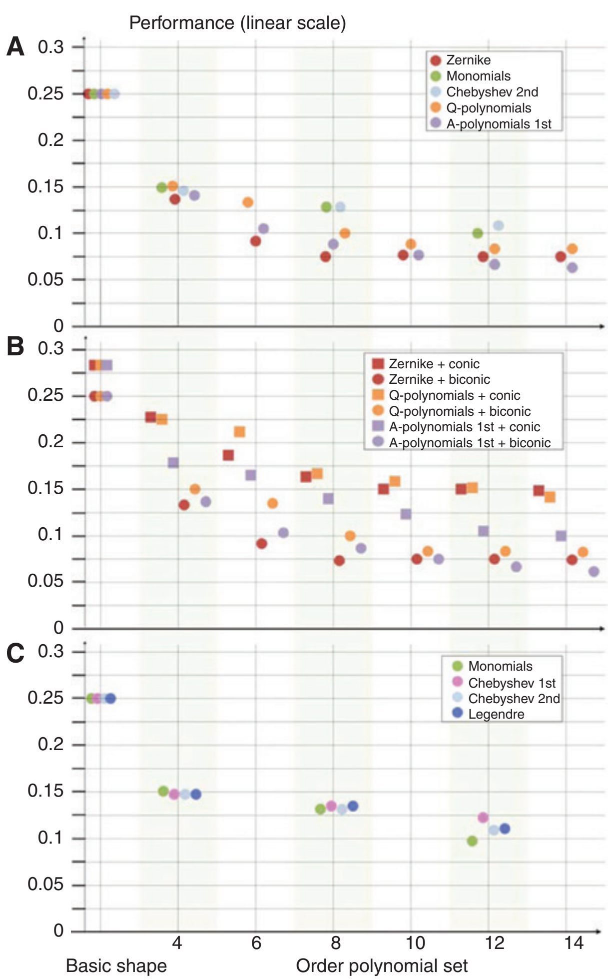


Figure 6: Anamorphic system: (A) performance of the main representations (all with biconic basic shape), (B) impact of different basic shapes for the performance of polar descriptions, (C) comparison of Cartesian descriptions (all with biconic basic shape).

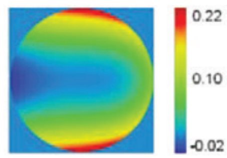
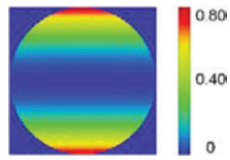
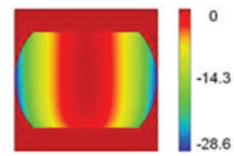
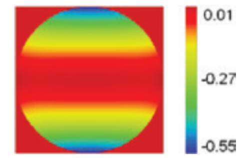
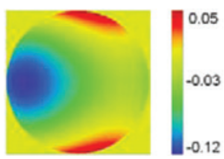
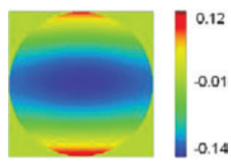
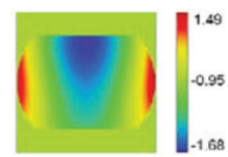
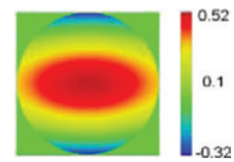
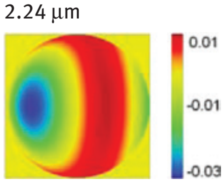
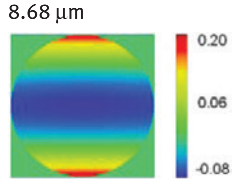
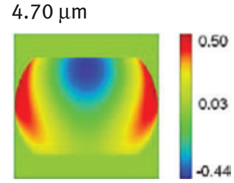
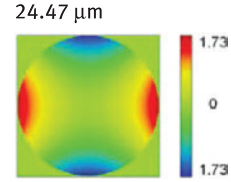
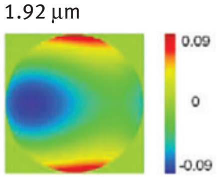
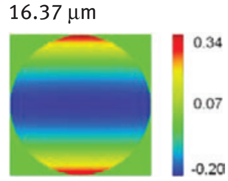
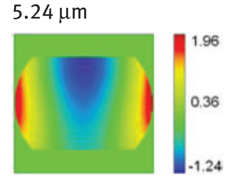
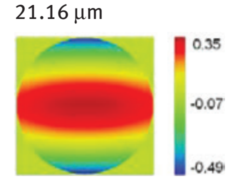
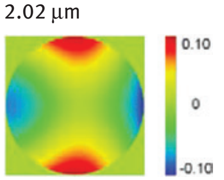
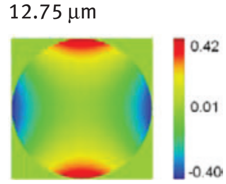
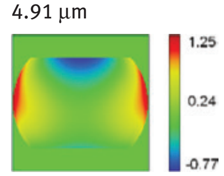
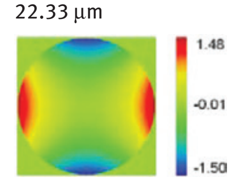
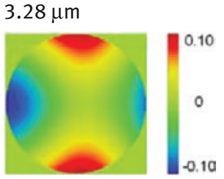
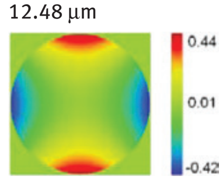
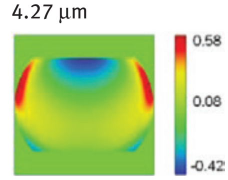
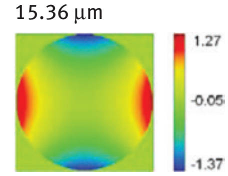
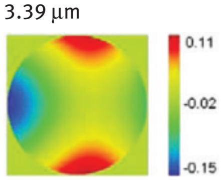
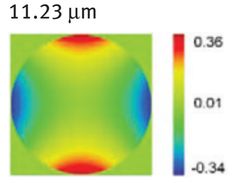
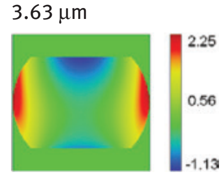
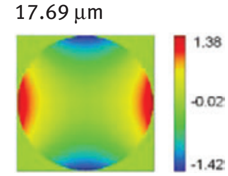
The impact of the basic shape on this double-plane symmetric system can be seen in Figure 6B. Over the orders and for all three descriptions, the difference is a factor of 2, which means that the correction for the system contains very high-order astigmatism, which cannot be compensated by the polynomial setup to the 14th order.

As discussed before, the performance of the Monomials, Chebyshev 1st kind, Chebyshev 2nd kind, and Legendre lacks the ability of correction due to the extremely reduced number of parameters and orders. The freeform sag contribution of Chebyshev 1st and Legendre have, hereby, more resemblance with the Monomials, contrary to the Chebyshev 2nd, which have much more similarities with the Zernike Fringe, Q-polynomial, and A-polynomials 1st kind. Despite the reduced degrees of freedom, the descriptions tend toward different paths, and specifically, the Chebyshev 2nd end up with the similar shape like the Zernike Fringe, with a completely different path to this minimum.

4.5 Comparison

The systems show an improvement from a factor of 4 up to 45 when using a freeform for the correction, depending on the system. Moreover, the differences in the final results by the different descriptions correspond to the averaged rms spot radius over the fields for the system (Table 3). A comparison of the freeform sag contribution by each description between the investigated systems is not reasonable, as this is highly dependent on the overall system specification and other surface parameters.

Table 3: Averaged rms spot radius over all fields (ASRF), peak-valley of sag contribution by basic shape (PVBS), and sag contribution by the deformation terms for freeform surface of each system by each description with biconic basic shape (sag values given in mm).

	A) Yolo	B) TMA	C) HMD	D) Anamorphic system
Improvent factor	25	8	45	4
Airy radius	0.88 µm	4.22 µm	7.14 µm	1.29 µm
Monomials				
PVBS	0.42 mm	1.34 mm	24.1 mm	0.79 mm
ASRF	3.28 µm	7.26 µm	10.60 µm	20.79 µm
Chebyshev 1st kind				
PVBS	0.45 mm	1.28 mm	1.95 mm	0.81 mm
PVBS	2.24 µm	8.68 µm	4.70 µm	24.47 µm
Chebyshev 2nd kind				
PVBS	0.45 mm	1.15 mm	2.69 mm	2.01 mm
ASRF	1.92 µm	16.37 µm	5.24 µm	21.16 µm
Legendre				
PVBS	0.45 mm	1.22 mm	2.00 mm	0.67 mm
ASRF	2.02 µm	12.75 µm	4.91 µm	22.33 µm
Zernike Fringe				
PVBS	0.23 mm	1.08 mm	2.85 mm	3.59 mm
ASRF	3.28 µm	12.48 µm	4.27 µm	15.36 µm
Q-polynomial				
PVBS	0.22 mm	1.04 mm	3.92 mm	3.86 mm
ASRF	3.39 µm	11.23 µm	3.63 µm	17.69 µm
A-polynomials 1st kind				
PVBS	0.28 mm	1.01 mm	2.76 mm	3.21 mm
ASRF	2.30 µm	10.23 µm	4.31 µm	11.69 µm

5 Conclusion

We investigated in this paper four different reflective, refractive, and catadioptric systems representing typical applications for freeform surfaces with different symmetry, FOVs, F-numbers, and separation of field bundles, as well as different domains for the freeform surfaces. Out of the broad range of freeform descriptions we presented in part I of the publication [1], we chose seven to represent the possible combinations of domain and orthogonality, as well as different weightings over the domain.

Generally, it can be said that freeform surfaces can improve the system tremendously with only one sophisticated surface. Crucial for the correction of the aberration in this system and final performance is how good the description is able to represent the needed correction and how to establish the initial system. In reality, questions regarding sensitivity and manufacturability are also important and must be considered.

The investigated systems, diverse as they were, showed a very similar behavior: the outer boundary was comparable for all descriptions, and it has a sufficient correction impact. For a better correction, the center part of the surface was of more importance. Generally spoken, the effect of the domain on the results is minimal. On the other hand, the effect of the defining grid for the description is of much higher influence. Additionally, the higher-order contributions by the individual representations made the final difference. Nevertheless, the variation of the different descriptions was not as strong as might be expected. Except for the Monomials in one case, the worst result was less than a factor of 2 than the best. Additionally, the aperture of the freeform surface and description did not influence the results at all. The impact of the orthogonality was only partly seen in the result with faster convergence over the orders by some of the sets. The greater impact, nevertheless, can be seen in the convergence over the 300 cycles within one order. The convergence speed is much higher for orthogonal polynomial sets than for non-orthogonal (Monomials), as shown in a previous publication [8]. Another important aspect is the basic shape, which to some extent has an even greater impact on the final performance than the choice of description itself. With systems suffering from astigmatism, a more sophisticated basic shape like a biconic can reduce the number of additional needed freeform parameters to a third to represent the same system or improve the system by a factor of 2 with the same number of freeform terms.

This leads to the question, ‘How many representations do I really need?’

This question is not easy to answer. Thinking simply on the final result from each of the descriptions we presented should be sufficient. Nevertheless, there are more detailed aspects to consider. The Chebyshev 1st/2nd kind and Legendre have only about half of the parameter number of the Zernike Fringe set, for example, and can reach, in most cases, a similar result. Contrary to this, when it comes to higher-order correction, the Zernike Fringe, Q-polynomial, or A-polynomial 1st kind are more beneficial.

For data exchange and manufacturing, descriptions with a polynomial structure are preferred. However, as the projection factor for Q-polynomial and A-polynomials 1st kind causes these descriptions to not be real polynomials any more, this can be a disadvantage for later processes. On the other hand, the projection factor offers the opportunity of tolerance directly with the coefficients, which is a huge advantage.

Deciding on a sufficient description is, therefore, highly dependent on the further process with the system, but to cope with a high number of possibilities, the following descriptions are highly recommended:

– Zernike Fringe with biconic basic shape: spatial orthogonal description with fast convergence, simple data transfer for manufacturing, easy access to aberrations (implemented in most of the commercial optical design software).
– Q-polynomial or A-polynomials 1st kind with biconic basic shape: gradient orthogonal description with fast convergence, for tolerancing: easy access to tolerancing with the coefficients (not yet implemented in most of the commercial optical design software).
– Chebyshev 1st kind with biconic basic shape: spatial orthogonal description with small number of parameters, easy data transfer for manufacturing (not broadly implemented in most of the commercial optical design software).

We could reduce the number of necessary descriptions to these three representations. Nevertheless, a prediction of the ‘best shape’ of correction in the form of the optimal number, position, and even the order of correction of the freeform surfaces is still not known and needs to be investigated. With the extension of the existing aberration theory above the sixth order, it may be possible to solve this problem in the future.

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