DE GRUYTER Adv. Opt. Techn. 2016; 5(5-6): 461–469

Tutorial

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Aspherics in spectacle lenses

DOI 10.1515/aot-2016-0025
Received April 15, 2016; accepted July 22, 2016; previously published online September 2, 2016

Abstract: A review of the use of aspherics in the last decades, understood in a broad sense as encompassing single-vision lenses with conicoid surfaces and free-form and progressive addition lenses (PALs) as well, is provided. The appearance of conicoid surfaces to correct aphakia and later to provide thinner and more aesthetically appealing plus lenses and the introduction of PALs and free-form surfaces have shaped the advances in spectacle lenses in the last three decades. This document basically considers the main target optical aberrations, the idiosyncrasy of single lenses for correction of refractive errors and the restrictions and particularities of PAL design and their links to science vision and perception.

Keywords: conicoid surfaces; ophthalmic lenses; opthalmic optics; free-form lenses; progressive lenses.

1 Introduction

The use of aspheric and free-form surfaces in the design of spectacle lenses has evolved strongly in the last three decades. These lenses are of widespread use in ophthalmic optics. In this work we will review the specificities of this type of lenses when applied to the correction of refractive errors.

In ophthalmic optics the majority of correction aids (spectacle lenses) are composed of a single lens placed in front of the eye, typically sustained by a frame. Moreover, the processing of the image they produce is a direct input to the retina that is processed by the brain. Different images are produced when gazing through different parts of the lens. Two aspects dramatically change the way the lenses are conceived and used.

2 Some definitions and scope of the work

Aspheric lenses are many times referred to as being composed of conicoid surfaces or variations thereof (see for example, ISO 10110 part 12 [1]). In the following we will talk about aspheric lenses in a wide sense adopting the literal consideration that an aspheric lens is any lens that is not composed of spherical surface(s).

This disputable consideration leads us to a vaster and more interesting range of surfaces and lenses that are used in spectacle lenses: the free-form lenses that encompass aspheric designs, progressive addition lenses (PALs) and a set of designs intended for occupational purposes.

In this document we will concentrate on spectacle lenses, dismissing intra-ocular and contact lenses that are ‘anchored’ to the eye, following its movement, and constitute a different category of correction aids. The very relevant facts that spectacle lenses hang in front of the eye and, thus, the geometry of the wavefront leaving them varies according to the angle of gaze makes their study a singular one.

3 Relevant optical aberrations in spectacle lenses

Ophthalmic optics use single lenses to correct the main refractive errors, i.e. myopia, hyperopia, presbyopia, phoriae and other minor deviations of emmetropia.

This, together with the fact that, for spectacle lenses, the constant rotation of the eyes to explore the environment with saccades makes the system work off axis, has a strong impact on the correction that spectacles can provide to the wearer.

In particular, all second-order aberrations will have a negligible impact on the correction since in a single lens, modifying its geometry to account for this level of

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aberrations cannot be done locally without affecting the higher-order aberrations. The movement of the eye makes it very difficult to correct for this level of aberrations since correcting for a particular line of sight would be inconsistent with adjacent corrections.

This leaves us with four main aberrations to account for:

- Oblique astigmatism (OA)
- Curvature of field (or power error) (PE)
- Chromatic aberration
- Distortion

The perceivable effect of the first two is blur, which has a strong impact in the discomfort of the user since sharp vision is what spectacle lenses are intended for.

Chromatic aberration manifests itself as a separation of colours when rotating the eye towards periphery, which actually does not produce blur. Chromatic aberration depends strongly on the dispersion of light thanks to the prismatic effect at each point of the lens and the dispersion of the material expressed through its Abbe number.

Since the prismatic effect depends primarily on the power of the lens, very little can be done in the design of the geometry to reduce or control this aberration except by selecting a substrate with a high Abbe number. The modifications needed to reduce the prismatic effect typically modify OA and PE in ways that are incompatible with a good distribution of aberrations, especially in PALs.

Distortion is a consequence of the variation of PE of the lens towards the periphery. An increase of magnification towards the periphery leads to the pincushion distortion, whereas the contrary leads to barrel distortion, typical of positive and negative lenses, respectively.

This optical aberration can be reduced, and, in fact, it is effectively reduced in aspheric designs as a side effect of the reduction of astigmatism usually by flattening the curvature of the front side. Trying to reduce distortion beyond that creates much more unwanted OA.

In any case, distortion does not produce blur and the brain can adapt quickly to it so that distorted images are perceived as ‘normal’ and even the bending of objects in the periphery of the lens is quickly discarded by the brain.

These four aberrations are not independent of each other, which makes simultaneous correction of all of them impossible. As said before, chromatic aberration and distortion have a comparatively low impact in user comfort. Consequently, they are typically discarded when designing spectacle lenses.

So we have been left with OA and curvature of field. In order to discuss them, we need to recall even in a simplified form how they appear.

Let us consider a spherical wavefront that comes from a point in object space. When the eye rotates away from the optical centre, the wavefront that comes out from the lens crosses it obliquely and no longer has a spherical shape but locally becomes a toroidal one, thus splitting the focus into a spread of focuses that lie in the line that joins the two extreme sagittal and tangential focus, with image of the object through the orthogonal maximum and minimum curvatures of the wavefront depicted as C1 and C2 in Figure 1.

The difference between the sagittal and tangential focuses is called OA, whereas the difference between the average of the two and the frontal power at the axis is the PE or curvature of field.

The focusing system of the brain modifies the shape of the eye lens in order to place the least confusion circle (depicted as an ellipse in Figure 1) in the retina to obtain the least confused image.

These two aberrations, whose appearance basically produces blur in the retina, are actually the only two that are directly taken into account in spectacle lens design and hence the ones that are the key drivers in the quest for correction of refractive errors.

4 The Tscherning ellipse

For single-vision lenses, not any combination of front and back surface that provides a specific optical power provides the minimal aberrations. The best form lenses are those that provide some kind of minimisation of the OA and curvature of field. Using third-order approximation, Tscherning [2] found that the best combination of front and back power for a lens of specified frontal power is essentially of quadratic nature. Whitwell showed that the solution could be depicted as an ellipse in a diagram showing the front surface on the y axis and frontal power in the x axis.

In this context, ‘best form’ meant having zero OA when the eye is rotated 30°. It is easy to compute different Tscherning ellipses for different refractive indices and also for different optimisations. Instead of imposing that OA is 0 at 30°, one can make a combination of OA and field curvature. It can also be computed for far distance vision

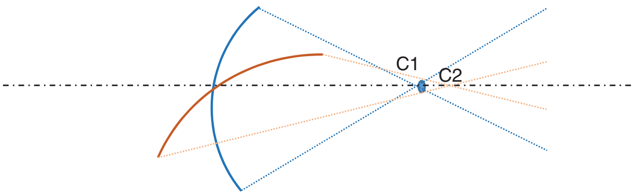
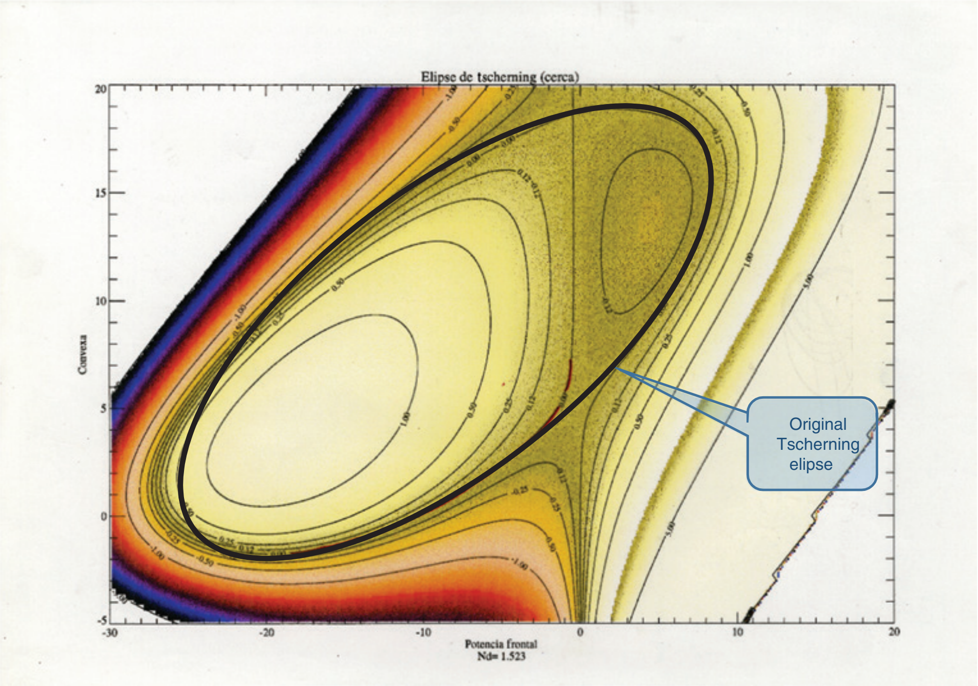


Figure 1: Sagittal and tangential focus, circle of least confusion.

or for near vision, just by changing the position of the object in the computation.

Nowadays, with the use of computers, we can calculate the optical aberrations using exact ray tracing instead of approximations. For example, in Figures 2 and 3 you can see the exact computation of the Oblique astigmatism for any combination of Front Power and front base curve.



Original Tscherning elipse

Figure 2: Tscherning ellipse and the exact computation. Isolines show the OA at 30° for any given power and front base curve combination computed for Nd=1.523 and object distance=-3D. (Copyright INDO Horizons).

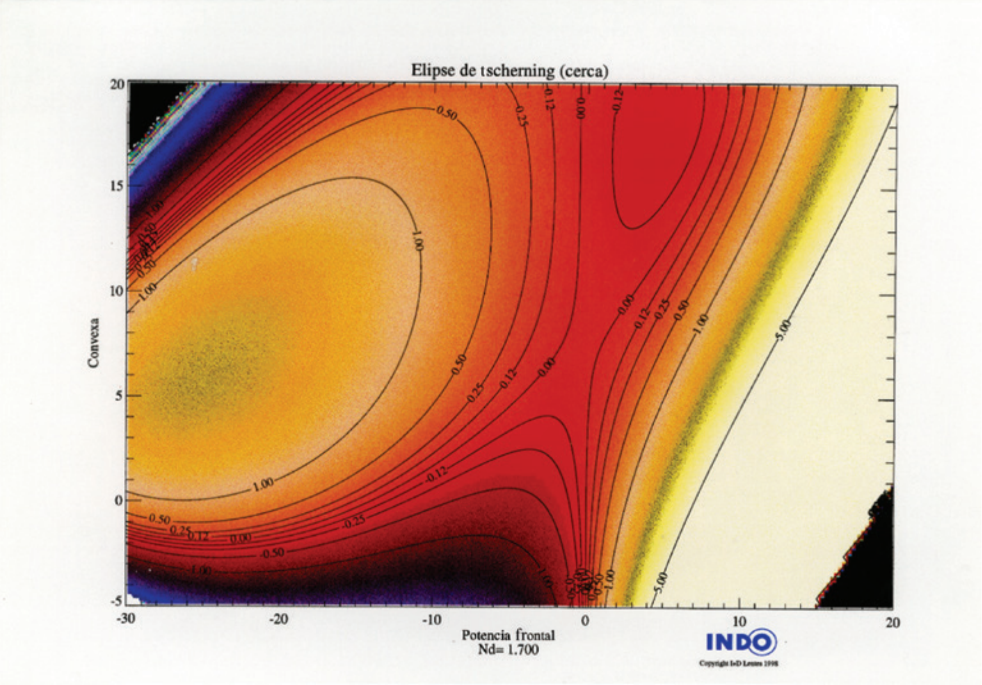


Figure 3: Depiction of OA at 30° for any combination of power and front base curve for Nd=1.7 and object distance=-3D. (Copyright INDO Horizons).

The important issue about Tscherning ellipses or their digital counterparts is the fact that there is no solution for OA beyond approximately +7 D and -23 D of power for Nd=1.523. For other indices the negative part can go beyond -30 D and the positive branch is even more limited having no practical solution from +3 D.

The other remarkable issue is that there are two possible solutions over the range covered by the ellipse; the shallower part of it is called the Ostwalt form (and it is the one that is used for practical purposes), while the Wollaston form that corresponds to the upper part of the ellipse is usually discarded because of its high curvature.

5 The role of aspherics in ophthalmic optics

Aspherical lenses first appeared in ophthalmic optics as a solution for aphakia. This condition occurs when the lens that is responsible for focusing the eye is lacking or is removed.

Intentional removal of the lens was the typical solution for cataract (the opacification of the lens due to age and other environmental factors) before the widespread use of intraocular lenses.

Without the lens, the eye becomes strongly hypermetropic (around 10–12 D) and thus needs a strong positive correction.

Unfortunately, as the Tscherning ellipses show, there is no single-vision lens made of spherical surfaces of that power range that could be free of unwanted OA.

The only solution for this problem is the use of aspherical surfaces with the appropriate eccentricity Q (or asphericity as 1-Q as typically called in the industry) that creates a surface astigmatism that counteracts that of the wavefront, rendering lenses with high power that have minimal OA and/or PE.

Consequently, the use of aspheric lenses was a must as soon as aphakia was used to solve cataracts.

In the 1980s it was still quite usual to leave aphakic eyes as the result of cataract surgery. This led to a development of the market for strong plus power single-vision lenses.

6 Blended lenticulars

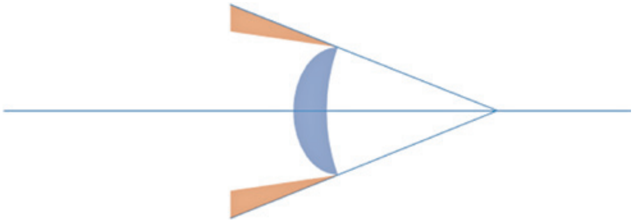


Figure 4: Ring scotoma in a strong positive lens.

Aphakic correction with such high positive lenses leads to a major problem: ring scotoma (see Figure 4) and a secondary problem: very thick lenses. This occurs due to the high convergence of rays in a strong positive lens that creates an annular conus where no ray can reach the eye, giving the sensation to the wearer of looking through a pipe. This is also known as ‘jack in the box’ effect [3].

The solution of both problems came with the so called ‘blended lenticular lenses’ (see Figure 5). In them the optically effective diameter of the lens is reduced to the equivalent of a field of vision of about 60° (30° at each side of the optical axis), which is the limit angle of vision where the majority of persons have already rotated the head to avoid strong elongation and contraction of the eyeball muscles.

This ‘shell’ is then taken closer to the back side to reduce centre thickness, and the rest of the surface is ‘blended’ with the edge with a smooth surface that maintains continuity up to the second derivative with the shell. This mathematical condition ensures that there is no discontinuity in the curvature along the blending surface, and, if it is made to finish with the same tangent of the backside curve, the ring scotoma is completely removed.

This blending nonetheless has a high degree of unwanted aberrations since the main curvatures at each point of the blending zone are very different in magnitude, leading to a high level of surface astigmatism that creates in turn a high level of optical astigmatism.

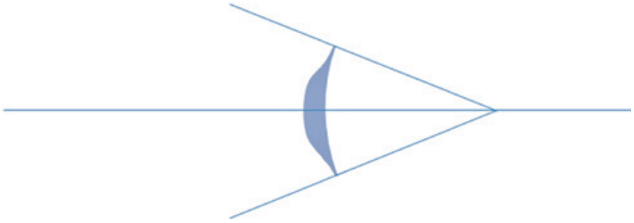


Figure 5: Blended lenticular: elimination of the ring scotoma by creating a lenticular part optically active surrounded by a blending surface that co nnects the optical part with the edge of the lens. Making the blending twice continuous derivable and matching the derivatives of the two surfaces at the blending points guarantees eliminating the ring scotoma. Appropriate positioning and diameter of the lenticular part also reduces thickness.

7 Aspherics for lower prescription lenses

Beyond the correction of aphakia, aspherical lenses had also a place in the range of powers contained by the Tscherning ellipse. Here the need was not of optical nature (which could be covered by spherical lenses) but of an aesthetic one.

The Ostwalt form, being the shallower among the two, had still some high curvature base curves, especially for plus lenses. This leads to high cambered lenses that were deemed aesthetically unpleasant. The solution is obviously to reduce the curvature, escaping from the ellipse and thus entering into zones where OA and CF are significant.

The solution again for this optical behaviour is to use aspherical (typically conicoid) surfaces to reduce the unwanted astigmatism and return to similar or better values for OA and/or CF. The eccentricity needed for that correction can easily be found by simple mathematical optimisation processes.

A side effect of making plus lenses aspherical on their front side is that this curve ends up flattened, allowing for a smaller centre thickness with the same edge thickness. Distortion and magnification are also reduced as a bonus from applying asphericity to the front surface.

This solution works well for positive lenses. In the negative range the use of aspheric surfaces is justified only by the correction of aberrations in very strong myopia (>10 D approximately) since the lens base curvature is already shallow enough within the Ostwalt branch for regular prescription lenses (about 80% of the population lies within the +6 -6 D range).

For lower than 10 D the ellipse enters in the realm of biconcave lenses that are also aesthetically unpleasant, requiring the use of aspheric surfaces if we want to have convex front side curves, but this is a minimal part of the myopic users. Moreover, in negative lenses the gain in centre thickness is non-existent and that of edge thickness is also negligible.

Consequently, there is no aesthetical nor optical justification of asphericity for regular prescription negative lenses. Despite this, marketing considerations have given birth in the past to complete positive and negative ranges of aspherical lenses within the ophthalmic optics industry.

8 Progressive addition lenses

In the literature we can find excellent descriptions of the intricacies of PAL design and the technologies involved in its manufacturing like in Meister [4] and Meister and Fischer [5]. Nevertheless, we consider that an overview of PALs is interesting in the scope of this article.

Following our wide definition of aspherical lenses, PALs are a kind of nonspherical lenses that are constituted in one or all of their two surfaces by free-form shapes intended for the correction of presbyopia alone or in conjunction with the other refractive errors.

Presbyopia is a physiological condition of the eye, highly correlated with age, in which the lens responsible for accommodation (i.e. focusing) to different distances loses gradually its ability to do so (see, for example, Rochester [6]). This begins very early in life but it is around the age of 45 that the loss of accommodation begins to prevent reading at the typical distance (around 40 cm of object distance), and hence presbyopia becomes apparent and requires correction.

The addition is the plus power that has to be added to the normal prescription of the wearer to help him or her to overcome the loss of accommodation and see properly at near distance again.

The most effective correction for presbyopia nowadays is the use of PALs. In them one or the two surfaces of the lens are designed in such a way that wavefront emerging from the lens has different power depending on the line of sight.

This means that there is an upper part of the lens that produces a close to spherical wavefront that is usable to see clearly at distant objects; there is a lower part of the lens whose wavefront is usable for objects located in the near (reading distance) space, and there is a transition zone between the two (called the corridor) that can be used to focus any distance in-between.

The fact that the wavefront escaping the lens has far and near portions with different power and a transition between the two implies that OA accumulates in the lateral periphery of the lens as per the Minkwitz [7] theorem. This leads to OA and Power maps like the following.

The Minkwitz theorem states that for any symmetrical refracting surface S whose transition line (the corridor) is composed of umbilical points, the surface astigmatism A_s varies in the orthogonal direction to the line of progression twice the variation in power at the same point. The validity of this statement is a limit when the distance ζ, orthogonal to the variation of power $Dm(s)$, tends towards 0:

$$\lim_{\zeta \to \pm 0} \frac{\partial A_s(s,\zeta)}{\partial \zeta} = \pm 2 \left| \frac{dDm(s)}{ds} \right|$$

For practical purposes this means that the higher the addition the narrower the corridor and the higher the lateral aberrations and that for a given addition, larger corridors become naturally wider corridors and vice versa. The theorem pinpoints the paramount relevance of the design of the corridor as a starting point influencing all the design of the progressive lens.

9 Optical aberrations and vision science

The aberrations that have by far most influence in the performance of a PAL are, as mentioned before, the same as in ophthalmic optics and for the same reasons: OA and curvature of field.

Both of them cause blur, OA because of its intrinsic blurring nature and power because if used to see at the wrong distance the result is again blur that not always can be refocused or accommodated.

If we approach PALs with a purely optical standpoint, the conclusion would be that they are intrinsically aberrant lenses. If you look at Figure 6 you can see the important variations of OA and Power. In fact, they only work in combination with the cognitive function of the brain and the intrinsic limitations of the eye.

It is well known that the distribution of cones and rods in the retina is not regular. In the fovea there is a high concentration of cones and almost no rods, whereas outside, in the peripheral retina, rods are prevalent and cones fade away abruptly.

The fact that typically several rods integrate their many signals into a single ganglion cell whereas every cone typically sends its signal to one single ganglion cell makes the vision through the fovea (plenty of cones) sharper than that of the peripheral retina (plenty of rods that integrate their signals).

The result is foveal vision (also called central vision), sharp kind of vision subtending a narrow angle of around 3° departing from the fovea. Peripheral vision covers the rest of the field of vision but is much less sharp and intrinsically is a blurry type of vision due to the integration of rod signals that reduces the resolution of this part of the retina.

Foveal vision being the only way to see sharply, the eye moves in saccades to explore the space and help the brain construct a model of our surroundings.

PAL designs take profit of these features of the eye by placing aberration-free images in the far, near and transition zones. Lateral zones host the unavoidable optical aberrations (as per the Minkwitz theorem) but are conceived to be used in peripheral vision.

An accustomed wearer looks through aberration-free areas with foveal vision. In that conditions the image crossing lateral aberrated areas results in a blurred image that falls in the peripheral retina. Since this part or the retina is intrinsically unable to distinguish sharp images from blurred ones, these images become perfectly tolerable by the visual system, after an adaptation period for newcomers (see, for example, Pope [8]).

But blur is not the only effect at stake in PALs. Irregular magnification due to certain distribution of the aberrations produces a perception of waviness when the head moves or when the entire body moves. This motion makes the images of the objects cross the different zones of the lens, changing its perceived size modified by the differences in power and magnification.

This sensation is sooner or later compensated by the brain. Accumulated experience in the lab suggests that this adaptation period depends on the design and the

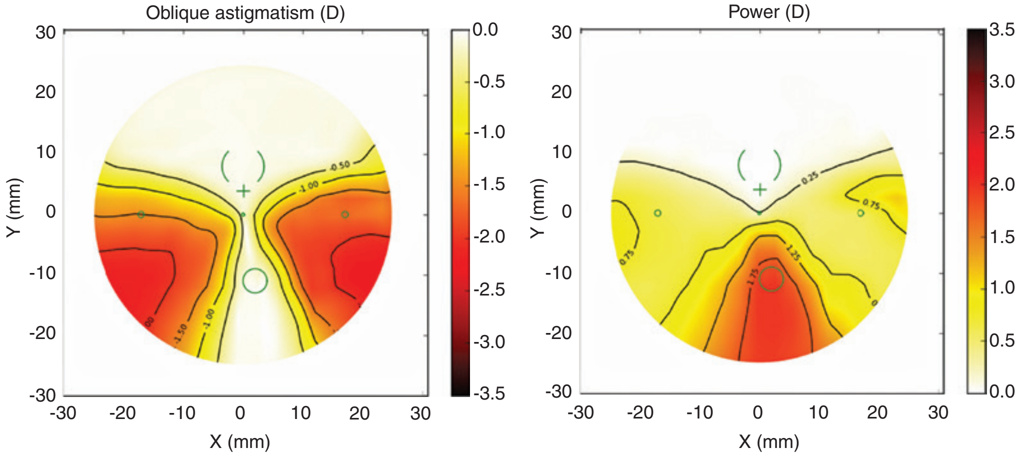


Figure 6: Maps of oblique astigmatism and power error. Isolines are of equal astigmatism or power in each case (source: Indo Horizons Innovation Department).

sensitivity of that person to that particular distribution of aberrations.

But the variety of people characteristics (optical, physiological and perceptual) makes it very difficult to extract strong correlations linking the many factors involved and the satisfaction and fit of the wearer.

Another feature that affects progressive lenses is the potential prism imbalance between corresponding zones when fixating an object and seeing it through each lens.

All these features and perceptual effects, together with the post processing of images that the brain does, mean that the distribution of power, aberrations and prismatic effects is a major part of the design of progressives lenses. This constitutes the art of using sophisticated mathematical techniques to produce wavefronts that can positively impact user perception and satisfaction.

10 Design of PALs

Very few literature can be found on the right way to design and optimise this type of lenses since most of the groups making research in this area are part of companies producing and selling PALs. Consequently, the publications that are not generic are basically patents.

Nevertheless, there are some articles on the techniques underlying the surface formulation [9], optimisation [10] and adaptation and user preferences [11]. In the following we intend to give an overview of the design of PALs.

Basically, designing a PAL involves deciding the location and size of the distance, near and transition zones and distributing wisely the unwanted aberrations and prismatic effects.

The first task affects essentially the length and behaviour of the umbilical (or near to umbilical) main meridian line that links the far distance vision zone with the near distance vision zone constituting the corridor.

Three parameters are key:

- the corridor length along with
- the inset of it inwards to the nose to follow the natural convergence of the eyes when looking near and
- the way in which it provides the variation of the power.

The combination of these three parameters is one of the definitory treats of any design since here the Minkwitz theorem applies fully.

Once this is decided the next task is distributing the unavoidable aberrations using a mathematical optimisation algorithm. Typically, the mathematical formulation that depicts the surface is the B-spline or non uniform rational B-splines formulation; see for example, Dürsteler [12].

Usually, the optimisation algorithm tries to minimise a merit function that in its simplest form takes the following expression:

$$f=\sum_{i}^{n}(a_i\text{-}a_{ri})^2\cdot w_i^2$$

where a_i is the feature (in this case AO for example) measured at location *i*, a_{ri} is the target value for *a* at point I, and w_i is a weight that usually correlates to the relevance of the feature at this point within the general layout of the surface.

Since typically *n* can be of the order of a thousand or more points, the optimisation algorithm tries to find the deepest valley (the minimum) in a landscape of an *n*-dimensional space.

We will not enter into the intricacies of optimisation, which is a field of knowledge in itself, but it should be enough to elicit that some of the solutions lead to local minima, and in some cases the algorithm does not converge, i.e. does not find a solution.

The latter typically occurs because the merit function depicts a situation that is unrealistic. For example, asking a distribution of astigmatism that is incompatible with that requested for the power.

Of paramount importance for the optimisation of the aberrated zones is the composition of the merit function, since defining what the targets are for each point and what features or magnitudes to include leads to different results.

In particular, merit functions can encompass AO alone, combinations of AO and CF, gradients of either one of them, prismatic effects and/or combinations thereof. In fact, in theory you could add features like the weight or even the cost, but such sophistication is usually beyond the scope of PAL design.

Most typically, the merit function contains only values for AO, CF and weights for each point.

Each designer group has their own proprietary software and their own philosophy on how to distribute aberrations and how to design the corridor, i.e. on how to design PALs. In order to understand the many variations, you can look at Sheedy [13].

Most of the ultimate methodology that is used to create those designs only sees the light in patents since each group retains most of the know-how as trade secrets. Unlike in other disciplines, the research groups active in PAL development are located mainly in the business units of lens producers instead on the research groups of universities.

But whatever would be the design philosophy, the key in accepting a design for the market is the clinical studies those groups perform in order to know what designs create satisfaction in the wearer and what can be acceptable in the market, beyond what is seen in the iso-astigmatism and iso-power charts.

Again, the typical studies shown by the companies are those delivering marketing value more than scientific one, but it is possible to find some (few) interesting articles on clinical assessment. One of them, sponsored by one of the main players in the ophthalmic optics business, is Han et al. [14].

The assessment of PALs is done consequently by comparison of its surface shape (for example, Raasch and Bullimore [15]) looking at the OA and CF isolines, comparing height of the surface, the profile of their main meridian and other features of their surface. Alternatively, it can be done with the same features by transmission of a wavefront through both surfaces instead of only the progressive one, depicting the optical behaviour of the lens as a whole.

11 Free-form surfaces

By the end of the 1990s most lens production was done starting with a semi-finished blank produced by some of the big manufacturers of the time. This blank had the front side of the embryonic lens preformed and could be either spherical, aspherical or progressive.

The rotational symmetry of the aspherical lenses made it possible to produce glass lenses either by cutting them with relatively simple machines that, in order to define the shape, used a pantographic template in the early stages, or directly by computerized numerical control (CNC) milling in the late times.

Polishing of aspherics was done with soft pads to follow the shape of the front curve, especially when making blended aspherics.

The final lens containing the prescription of the wearer was ground in the back side of the blank with sphero-torical surfaces using simple mechanical milling machines. Polishing of this side was done using a high quantity of pre-formed metal moulds (up to 5.000 depending on the production of the factory) with accurate torical surfaces that were fitted with abrasive pads and rubbed with oscillating machines against the milled surface.

By that time some German machine producers, led by Schneider GmbH &Co and soon followed by LOH and other companies, began to introduce CNC milling machines and flexible polishing systems that completely changed the way aspherics and PALs were made.

The blanks were no longer needed to host a complex pre-made surface. Instead, it was only needed to have a simple spherical surface since all the complexity was ground and polished in the back side of the blank using free-form surfaces, typically B-splines and related mathematical surfaces.

The appearance of digital manufacturing in the ophthalmic optics arena unleashed many of the restrictions that were imposed by the traditional technologies, and thus, especially in the field of PALs, the concept of personalisation arose.

Free-form technology constituted a revolution in itself since not only did it allow designers to take into account the specific characteristics of the individual end user in their designs but it also reduced greatly the complexity of the prescription laboratory (the RxLab) by eliminating thousands of metal moulds and reducing the stock variety of semi-finished blanks to a mere dozen of them.

Moreover, standard designs were enhanced by having variable corridor lengths instead of having only two or three available. Limited edition and occupational designs were also possible since there was no longer a need to create a big inversion in blank stockage.

12 Summary

Aspherics, in the wide sense adopted in this article, are a whole category of ophthalmic optics lenses that depart strongly from their fine optics cousins due to the fact that their outcome is processed as part of the of the visual system and thus is subjected to a series of perceptive transformations that influence its final performance at the wearer side.

The dominant aberrations to correct in this field are basically OA and curvature of field with enhanced distortion as a side effect of some designs. Chromatic aberration is linked to the material and is usually out of the correction beyond the selection of a good Abbe number material.

Aphakia and the aesthetics of positive lenses of relatively low power have been the drivers behind the use of single-vision forms of aspherics.

PALs are a very successful and convenient solution for presbyopia and constitute an extremely sophisticated form of aspherics.

Unlike single-vision aspherics that have few degrees of freedom, PALs can be generated in many ways with quite different distribution of aberrations depending on the merit function selected, their weights and the decisions taken about the shape of far and near zones and especially about the corridor.

Thanks to the flexibility of the visual system, the brain ends up adapting to different solutions with more or less ease and time, even if they are not perceptually optimal for that particular wearer.

References

[1] ISO 10110-12:2007 Optics and photonics – Preparation of drawings for optical elements and Systems – Part 12 Aspheric surfaces (International Organization for Standardization, Geneva, Switzerland).

[2] M. Tscherning, “Verres de Lunettes” en. En F. e. Lagrange, Enciclopedie Française d’ophtalmologie (pág. Vol 3) (Octave Doin, Paris, 1904).

[3] M. Jalie, Ophthalmic Lenses & Dispensing (Butterworth-Heinemann, Woburn, MA, 1999).

[4] D. J. Meister, Clin. Exp. Optometry 91, 240–250 (2008).

[5] D. J. Meister and S. W. Fischer, Clin. Exp. Optometry 91, 251–264 (2008).

[6] U. O. Rochester, Presbyopia. Retrieved from http://www.cvs.rochester.edu/yoonlab/research/pa.html (2013).

[7] G. Minkwitz, Opt. Acta 10, 223–227 (1963).

[8] D. R. Pope, Progressive Addition Lenses: History, Design, Wearer Satisfaction and Trends in ‘Vision Science and its Applications’, 35 (OSA Santa FE, NM, USA). Retrieved from https://www.osapublishing.org/abstract.cfm?uri=VSIA-2000-NW9 (2000).

[9] B. Otero, J. M. Cela and E. Fontdecaba, Different surface models for progressive addition lenses and their effect in parallelization. Proceedings of Applied Simulation and Modelling. September 3rd–5th, Marbella (2003).

[10] G. Casanellas Peñalver and J. Castro, Modelling and Optimization of Progressive Lenses. Retrieved from https://www.euro-online.org/conf/euro28/treat_abstract?paperid=1759 (2016).

[11] J. S. Solaz, R. Porcar-Seder, B. Mateo, M. J. Such, J. C. Dürsteler, et al., Int. J. Ind. Ergonom. 38, 1–8 (2008).

[12] J. C. Dürsteler, Sistema de diseño de lentes progresivas asistido por ordenador (PhD thesis, Universitat politécnica de Barcelona, Barcelona, 1992).

[13] J. O. Sheedy, Optometry 75, 1–20 (2004).

[14] S. C. Han, A. D. Graham and M. C. Lin, Optometry Vision Sci. 88, 234–243 (2011).

[15] T. Raasch and M. A. Bullimore, Optometry Vision Sci. 90, 565–575 (2013).