

## Letter

So Ito\*, Ryo Aihara, Woo Jae Kim, Yuki Shimizu and Wei Gao

# Three-axis vibration measurement by using a grating-interferometric vibrometer

**Abstract:** Three-axis vibration measurement of the linear air-bearing stage demonstrated by utilizing the method of the multi-axis laser interferometer is discussed. In order to detect the X-Y-Z directional vibration of the positioning stage table simultaneously, two-dimensional XY gratings are utilized as the scale grating and the reference grating of the interferometer. The X- and Y-directional positive and negative first-order diffracted beams are superimposed to generate the interference signals on the photodetectors. When the multi-axis vibration is applied to the scale grating, the intensities of the interference signals in X- and Y-directions varied corresponding to the vibration of the scale grating. Consequently, three-axis vibrations of the scale grating can be calculated by processing the X- and Y-directional positive and negative first-order interference signals. In this paper, a three-axis vibrometer based on the grating-interferometer has been developed for measurement of the positioning stage table vibration. The detection method of the vibration based on the Doppler frequency shift has been demonstrated. As an application of the multi-axis grating-interferometric vibrometer, multi-axis vibrations of the linear air-bearing stage are measured by using the developed vibrometer.

**Keywords:** air-bearing linear stage; grating interferometer; multi-axis measurement; surface encoder; vibrometer.

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## 1 Introduction

Precision planar positioning stages are one of the important key components for measurement apparatuses and machining tools, so those positioning stages have been widely used in various fields such as precision machining, semiconductor manufacturing and precision measurement [1, 2]. With regard to the ultra-precision positioning stages that have sub-micrometric positioning accuracy and nanometric displacement resolution, an air-bearing slide is generally employed to realize the smooth motion without the friction of the sliding part [3, 4]. However, irregular vibration of the stage table is often caused due to the turbulent flow of the compressed air [5]. Therefore, the vibration measurement and evaluation of the working table will be required for the ultra-precision positioning stages. With respect to the movement of the positioning planar stage, the displacement of the stage table can usually be measured by the displacement sensors represented by a linear encoder, laser interferometer, and capacitive displacement sensor. However, since the measurement directions of those displacement sensors are limited in general to one-direction, it is necessary to use multiple sensors for measuring irregular multi-axis vibration of the positioning stage. Therefore, the use of multiple sensors causes complication and enlargement of the measurement system, and makes difficult the measurement of the positioning stages.

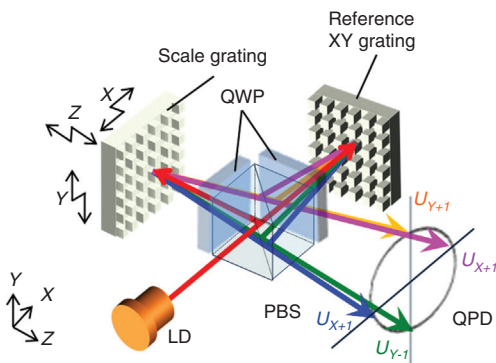
As for the detection of the multi-axis movement of the stage table, multi-axis grating interferometric encoders have been developed for the displacement measurement of the multi-axis motions by using one sensor head. By using the two-dimensional XY-grating, the X- and Y-directional positive and negative first-order diffracted beams can also be utilized for the detection of the X-Y-Z three-directional displacement. In this study, a multi-axis vibrometer is developed based on the grating-interferometer techniques for the vibration measurement. In order to measure the vibration of the air-bearing linear stage, a bandwidth of several kilohertz order is required [5]. The detection system of the grating-interferometer has been improved to realize the bandwidth which is required for

the vibration measurement. By utilizing the proposed grating interferometric multi-axis vibrometer, the vibration measurement of the linear air-bearing stage is carried out. The multi-axis vibrations of the working table of the air-bearing linear stage can be detected by the developed vibrometer.

## 2 Principle of grating-interferometric vibrometer

The multi-axis vibrometer has been developed based on the configuration of the multi-axis grating-interferometer [6, 7], and the configuration of the optical system used in this vibrometer is almost same as the conventional interferometer which is described in Reference [7]. Figure 1 shows the basic schematic of the multi-axis vibrometer. The sensor head, which is used for the detection of the interference signal, consists of a laser diode (LD), polarizing beam splitter (PBS), quadrant photodiode (QPD) and quarter-wave plates (QWPs). Two-dimensional XY-gratings are employed as the scale grating and the reference grating of the interferometer. The incident laser beam is split into two by the PBS, and the beams are irradiated on the two-dimensional XY gratings of the scale grating and the reference grating

As described in Reference [7], X- and Y-directional motion of the scale grating can be detected by the interferences of the first-order diffracted beams. Each of the directional positive and negative first-order diffraction beams are reflected from the gratings with diffraction angle of  $\theta$ . The diffracted beams which are reflected on the two-dimensional gratings are superimposed upon the surface of the QPD to generate the interference signal. The intensity of the interference light can be measured by each channel of the QPD. When the scale grating is moving in X-Y-Z directions with a certain velocity, the frequencies of



**Figure 1** Basic schematic of the multi-axis vibrometer using the XY grating for 3-axis velocity measurement.

X- and Y-directional positive and negative first order diffraction beams are modulated by the Doppler effect [8] according to the velocity of the grating. The relationship between the Doppler frequency shift in X-direction  $\Delta f_x$  and the velocity of the scale grating in X-direction  $u_x(t)$  can be expressed as follows:

$$\Delta f_x = \frac{\lambda}{cg} \cdot u_x(t) \cdot f_i = \frac{u_x(t)}{c} \sin\theta \cdot f_i \quad (1)$$

where,  $f_i$  is the frequency of the incident beam,  $c$  is the velocity of light,  $g$  is a grating pitch and  $\lambda$  is a wavelength of light source. As can be seen Equation 1,  $u_x(t)$  can be obtained from the X-directional positive and negative first order diffraction beams according to the modulation by the Doppler effects. Therefore, the modulated frequencies of X-directional positive ( $f_{x+1}$ ) and negative ( $f_{x-1}$ ) first order diffraction beams can be expressed as follows:

$$f_{x+1} = f_i + \Delta f_x = f_i + \frac{1}{g} \cdot u_x(t) \quad (2)$$

$$f_{x-1} = f_i - \Delta f_x = f_i - \frac{1}{g} \cdot u_x(t) \quad (3)$$

Similar to Eqs. (2) and (3), the modulated light frequencies ( $f_{y+1}$ ,  $f_{y-1}$ ) of Y-directional first-order diffraction beams can be expressed as follows:

$$f_{y+1} = f_i + \Delta f_y = f_i + \frac{1}{g} \cdot u_y(t) \quad (4)$$

$$f_{y-1} = f_i - \Delta f_y = f_i - \frac{1}{g} \cdot u_y(t) \quad (5)$$

where,  $\Delta f_y$  can be expressed as follows:

$$\Delta f_y = \frac{\lambda}{cg} \cdot u_y(t) \cdot f_i = \frac{u_y}{c} \sin\theta \cdot f_i \quad (6)$$

$u_y(t)$  is the velocity of the scale grating in the Y-direction.

On the other hand, the vibration frequencies of X- and Y-directional first-order diffraction beams are also modulated by not only XY-directional motion but also the Z-directional vibration of the scale grating because the length of the optical path is varied. The spurious velocity  $u'_z(t)$  which is the velocity of the scale grating in Z-direction to affect the frequency modulation of the first order diffraction beam can be shown in the following equation:

$$u'_z(t) = 2u_z(t)(1 + \cos\theta) \quad (7)$$

where,  $u_z(t)$  indicates the velocity of the scale grating in Z-direction. The Doppler effect of X- and Y-directional positive and negative first-order diffraction beams which

are considering the vibration in Z-direction can be calculated as follows:

$$\Delta f_{x,z} = \frac{u_z(t)}{c} \cdot f_i = \frac{2u_z(t)}{\lambda} f_i (1 + \cos\theta) \quad (8)$$

Therefore, the modulated frequencies  $f_{x\pm 1,Z}$  and  $f_{y\pm 1,Z}$  in X-directional first-order diffraction beams which are considering the vibration in Z-direction can be expressed as follows:

$$f_{x+1,z} = f_i + \Delta f_x - \Delta f_{x,z} = f_i + \frac{1}{g} u_x(t) - \frac{2}{\lambda} (1 + \cos\theta) \cdot u_z(t) \quad (9)$$

$$f_{x-1,z} = f_i - \Delta f_x - \Delta f_{x,z} = f_i - \frac{1}{g} u_x(t) - \frac{2}{\lambda} (1 + \cos\theta) \cdot u_z(t) \quad (10)$$

$$f_{y+1,z} = f_i + \Delta f_y - \Delta f_{y,z} = f_i + \frac{1}{g} u_y(t) - \frac{2}{\lambda} (1 + \cos\theta) \cdot u_z(t) \quad (11)$$

$$f_{y-1,z} = f_i - \Delta f_y - \Delta f_{y,z} = f_i - \frac{1}{g} u_y(t) - \frac{2}{\lambda} (1 + \cos\theta) \cdot u_z(t) \quad (12)$$

As a result, the intensity of the interference lights on the QPD, which are X- and Y-directional positive and negative first-order interference signal, can be calculated as follows:

$$I_{x+1} = 2E_0^2 \left\{ 1 + \cos \left[ 2\pi t \left( \frac{u_x(t)}{g} + \frac{2u_z(t)}{\lambda} (1 + \cos\theta) \right) \right] \right\} \quad (13)$$

$$I_{x-1} = 2E_0^2 \left\{ 1 + \cos \left[ 2\pi t \left( \frac{-u_x(t)}{g} + \frac{2u_z(t)}{\lambda} (1 + \cos\theta) \right) \right] \right\} \quad (14)$$

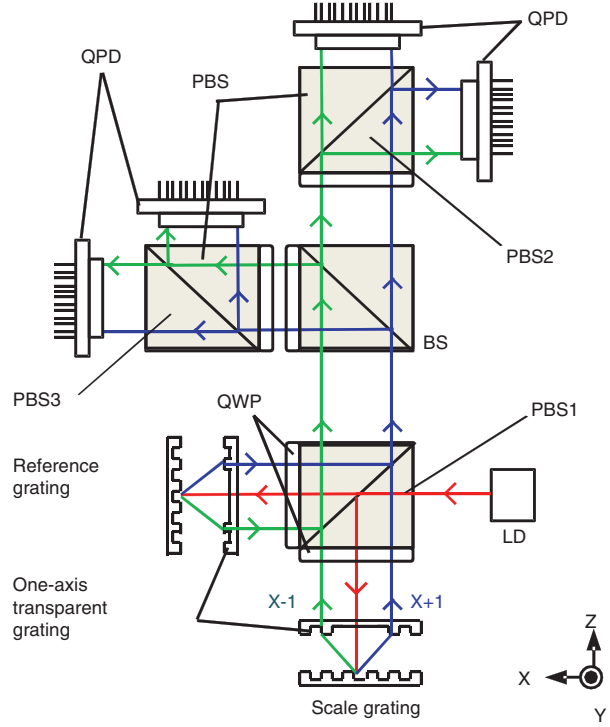
$$I_{y+1} = 2E_0^2 \left\{ 1 + \cos \left[ 2\pi t \left( \frac{u_y(t)}{g} + \frac{2u_z(t)}{\lambda} (1 + \cos\theta) \right) \right] \right\} \quad (15)$$

$$I_{y-1} = 2E_0^2 \left\{ 1 + \cos \left[ 2\pi t \left( \frac{-u_y(t)}{g} + \frac{2u_z(t)}{\lambda} (1 + \cos\theta) \right) \right] \right\} \quad (16)$$

where,  $E_0$  is the intensity of the light source. Consequently, the X-Y-Z directional velocities can be calculated by processing the X and Y-directional positive and negative first-order interference signals.

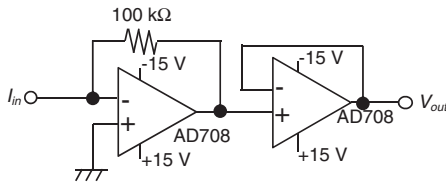
### 3 Experimental result and discussions

Figure 2 shows the optical system of the three-axis grating-interferometric vibrometer. The configuration of the optical system has been constructed based on the



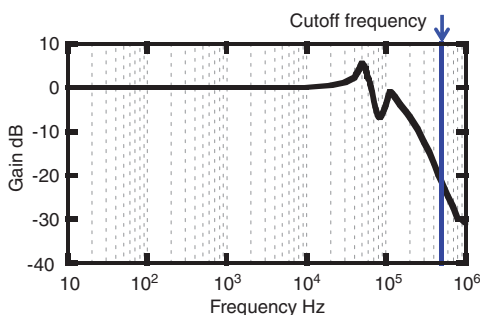
**Figure 2** Optical layout of three-axis grating interferometric vibrometer using the two-dimensional grating for detection of three-axis vibrations.

multi-axis encoder used in the previous researches [6, 7]. The pitch of the two-dimensional grating is  $1.0 \mu\text{m}$ . An LD (ML1412R-01, Mitsubishi Electric Corporation, Tokyo, Japan) with a wavelength of  $685 \text{ nm}$  is used as the light source of the interferometer. The incident laser beam emitted from the LD is split into p-polarized light and s-polarized light by PBS1 (Edmond Optics, Barrington, USA). These incident beams are converted into circularly polarized light by passing through the QWP (Edmond Optics, Barrington, USA). Two one-axis transmission diffraction gratings are mounted between the PBS1 and two XY-gratings in order to bend the first-order diffraction beams. By passing through the 1-axis transmission diffraction grating, the first-order diffracted beams can be parallel with respect to the incident laser beam [9]. Since the polarization direction of the diffraction beam is perpendicular to that of the incident beam, two beams cannot generate the interference signal at the PBS. Hence, as described in the Reference [7], the polarization directions of four diffraction beams are changed by the QWPs in order to distinguish the displacement of the direction. As a result, the diffracted beams are superimposed to generate interference on the QPDs (S1336-18BK, Hamamatsu PKK, Hamamatsu, Japan). The displacement direction of the scale grating can be calculated from a pair of the intensity



**Figure 3** Current-to-voltage conversion circuits.

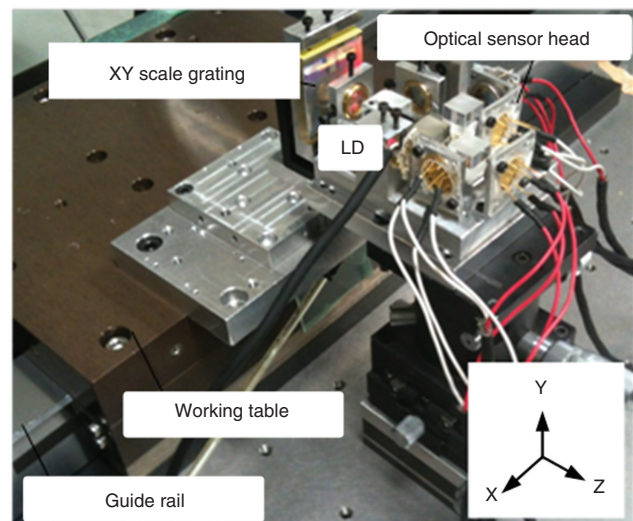
variation of the interference signal [6, 7]. With regard to the vibration measurement in Z-direction, zero order light is employed, and the vibration in Z-direction can also be detected by using the method of the conventional interferometric vibrometer. The interference signals detected by the QPDs are converted to the voltage signal by a current-to-voltage (I-V) conversion circuit. The bandwidth of the conventional grating-interferometer, which is described in Reference [7], was at most 1 kHz because its aim was the displacement measurement of the scale grating. However, from the viewpoints of the vibration measurement, it is necessary to improve the bandwidth of the detection system. Figure 3 shows the configuration of I-V conversion circuit. Since the cutoff frequency of the QPD used in this study is 25 MHz, the bandwidth of the interferometric vibrometer is mainly confined by the bandwidth of the I-V conversion circuit. In order to increase the bandwidth of the I-V conversion circuit, an operational amplifier with high frequency response has been employed. Figure 4 shows the frequency response characteristics of the I-V conversion circuit. Sinusoidal wave voltage was applied to the I-V conversion circuit by changing the input frequency. The vertical axis in Figure 4 indicates the root-mean-square (rms) value of the circuit output, and those values are normalized by the rms value of 10 Hz. As can be seen in this figure, the ripple can be observed in the vicinity of 50 kHz, and the gain of the circuit is reduced in higher frequency region. Therefore, the detection system



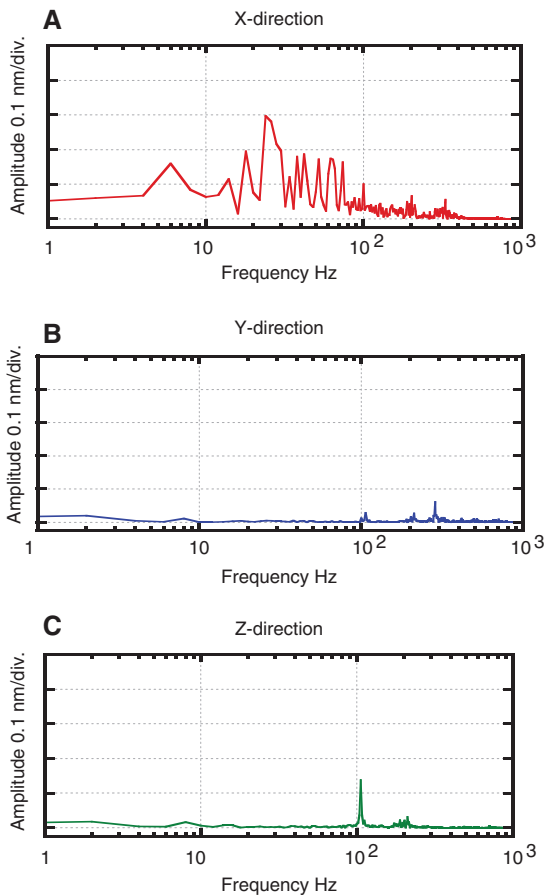
**Figure 4** Frequency response characteristics of I-V conversion circuit.

has sufficient bandwidth to measure the vibration of several kilohertz order. Relationship between the developed interferometer output and the displacement of the scale grating has already investigated in References [6] and [7], and the characteristics with respect to known frequency vibration and cross-talk between each axis have been discussed in Reference [10].

The three-axis vibrations of the air-bearing linear stage were measured by utilizing the developed three-axis grating interferometric vibrometer. A scale grating was set on the working table of the linear air-bearing stage (TAAT30L-19, NTN Corporation, Japan) as shown in Figure 5. Before supplying the compressed air to the air-bearing of the linear stage, the vibration of the working table was measured in the stationary state by the multi-axis interferometric vibrometer. Three-axis vibrations of the scale grating were measured simultaneously. Output signal of the optical sensor head was recorded by a digital storage data logger (Omniace III RA2300, Nippon Avionics, Tokyo, Japan). The sampling frequency was set to 10 kHz. The result of the vibration measurements were converted to the frequency spectrum by fast Fourier transforms (FFT) in order to identify the frequency peak of the vibration. Figure 6 shows the frequency spectrum of the working table vibration without supplying the compressed air. Figure 6(A-C), shows the frequency spectrum of the working table in X-, Y- and Z-direction, respectively. As can be seen in Figure 6, the vibration in X-direction was larger than that of Y- and Z-direction. It was considered that the vibration in X-direction would become larger in comparison with other axial-directions because the moving direction of



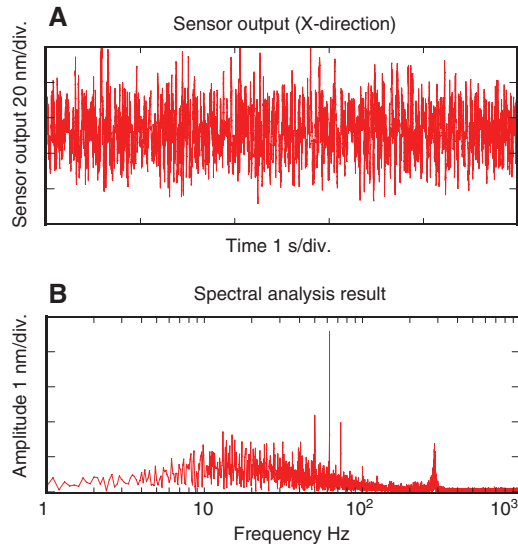
**Figure 5** Experimental configuration of three-axis vibration measurement.



**Figure 6** Frequency spectrum of the working table without supplying the compressed air.  
 (A) X-direction.  
 (B) Y-direction.  
 (C) Z-direction.

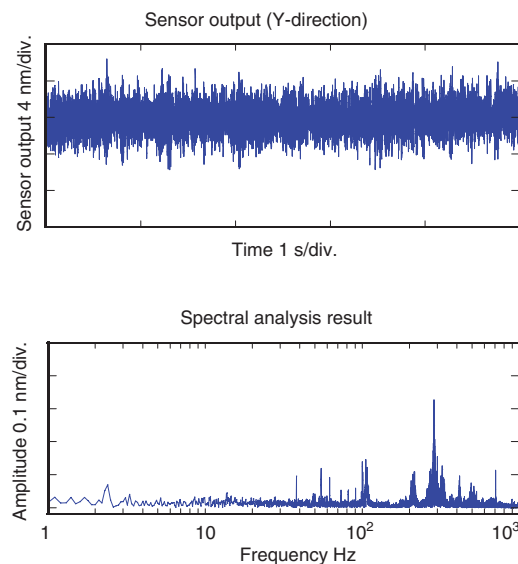
the working table was in X-direction. Next, an air pressure of 0.3 MPa was applied between the working table and the guide rail of the linear air-bearing stage. Sampling frequency was set to 10 kHz, and the measurement time was 10 s. Figure 7(A) shows the sensor output of the scale grating vibration in X-direction. Peak-to-valley (PV) value of the amplitude was 85 nm. Figure 7(B) shows the spectral analysis result of the sensor output in the X-direction. Many low frequency components can be observed in the spectral analysis of the sensor output in the X-direction. As mentioned above, X-direction in this experiment was consistent with the movement direction of the working table on the linear stage.

Since the stage table can be displaced smoothly during the application of compressed air, the vibration of the scale grating was increased. In particular, a frequency of 60 Hz was noticed as the frequency peak of the main X-directional vibration. A similar frequency

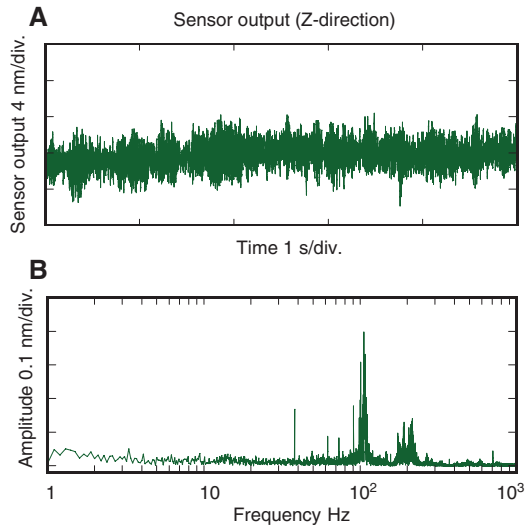


**Figure 7** Sensor output in X-direction.  
 (A) Sensor output (X-direction).  
 (B) Spectral analysis result.

peak could not be observed, as shown in Figure 6(A), when the compressed air was not supplied to the air-bearing. Therefore, it is assumed that the frequency peak of 60 Hz is caused by the floating of the working table. Furthermore, the frequency of 200 Hz was noted in Figure 7(B). Figure 8 shows the measurement result of the Y-directional sensor output. The PV value of the sensor output amplitude was 14 nm. The frequency spectrum of Y-directional



**Figure 8** Sensor output in Y-direction.  
 (A) Sensor output (Y-direction).  
 (B) Spectral analysis result.



**Figure 9** Sensor output in Z-direction.

(A) Sensor output (Z-direction).

(B) Spectral analysis result.

vibration is shown in Figure 8(B). The frequency peaks are observed in the vicinity of 100, 200 and 300 Hz. Although those frequency peaks can be seen in Figure 6(B), their amplitude in the stationary state is very small in comparison with Figure 8(B). With respect to the Z-direction, Figures 9(A) and (B) show the sensor output and frequency spectrum, respectively. The PV value of the amplitude in Z-direction was 10 nm. The frequency peak of 100 and 200 Hz can be seen in Figure 9(B), and those frequency peaks are also appeared in the stationary state shown in Figure 6(C). As shown in Figures 8 and 9, the amplitude of the frequency peaks was increased after applying the compressed air. Therefore, the vibration of the working table was caused by the air pressure of the air-bearing sensor. In consequence, three-axis vibration of the linear air-bearing stage could be measured simultaneously by utilizing the developed three-axis grating interferometric vibrometer.

## 4 Conclusion

The grating-interferometric vibrometer, which uses two-dimensional XY gratings, has been developed for the measurement of the three-axis vibration of the linear air-bearing stage. The measurement principle based on the Doppler effect was described. By improving the bandwidth of the detection system of the interferometer within the kilohertz-order, the grating interferometer can be utilized for the measurement of the multi-axis vibration. The vibration measurement using the grating interferometric multi-axis vibrometer was clarified that the vibration of the working table was induced by the air pressure applied for the air-bearing stage. Therefore, this grating interferometric multi-axis vibrometer will contribute towards improving ultra-precision positioning.

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