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Research Article

Hung-Lin Hsieh, Ju-Yi Lee* and Yu-Che Chung

Wavelength-modulated heterodyne grating shearing interferometry for precise displacement measurement

Abstract: A wavelength-modulated heterodyne grating shearing interferometry using a birefringent crystal is proposed for two-dimensional displacement measurement. There is a difference in the optical path lengths of the p- and s- polarizations of the light beam in the birefringent crystal because of the double refraction caused by the birefringence. By passing through the unequal-path-length optical configuration, the wavelength-modulated light beam is converted into a heterodyne light beam having two frequencies. The modulated heterodyne light beam is further combined with grating-shearing interferometry based on the quasi-common-optical-path (QCOP) design concept. According to the working principle and the Jones calculation, the displacement information of a moving grating can be obtained by means of the optical phase variation resulting from the grating. Theoretical analysis shows that the measurement sensitivity of the proposed method is about 0.134°/nm. The experimental results indicate that the resolution is about 10 nm for the centimetric-level measurement range.

Keywords: birefringence; diffraction gratings; heterodyne; interferometry.

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1 Introduction

Optical heterodyne interferometry has shown its important position in nanotechnology, semiconductor technology, nano-positioning, nono-imaging, and industry [1–4]. This displacement measuring technique is widely used, because it can provide not only a wide measurement range but also simultaneously offers high measurement resolution. Moreover, in comparison to homodyne type interferometry, heterodyne interferometry demonstrates an improved anti-noise ability [3, 4].

The measurement principles used in the heterodyne interferometer depend on the two different optical frequencies used in the light source. Normally, the reference and measurement light beams have their own frequencies. The displacement information can be determined by means of sensing the phase difference variation between these two beams. Different heterodyne interferometers have been proposed and developed. However, the introduction of optical modulators, such as AOM (acoustic-optical modulator) or EOM (electric-optical modulator) [5, 6], makes the set-up of the two types of heterodyne interferometers more expensive, even if these two modulation techniques are useful ways to measure the displacement.

Laser diodes (LD) are an important light source [7–9] used in the recently booming semiconductor industry. The luminous power and wavelength of the LD can be modulated by controlling the injection current, and this feature has been used instead of the heterodyne frequency shift architecture. A heterodyne interferometric design with a frequency-modulated LD for measuring surface topography was proposed by Chen in 1988 [7]. This was the beginning of using wavelength- or frequency-modulated techniques in heterodyne interferometry.

Most heterodyne interferometric methods for displacement measurement are based on the Michelson type configuration [10, 11], since this configuration is simple and can be performed easily. However, since Michelson type interferometry utilizes the non-common optical path configuration, disturbance of the stability of the wavelength and environmental factors will cause measurement error. Grating interferometry is proposed to reduce the effect of this type of measurement error [12–16]. Moreover, a grating interferometer using the design concept of the common optical path configuration can efficiently eliminate the effects of environmental factors.

In this study, a heterodyne grating interferometer based on the wavelength modulated technique is proposed for displacement measurement. The novel heterodyne light source is composed of a birefringent crystal and a tunable laser diode. The wavelength-modulated light beam is converted into a heterodyne light beam with two different frequencies after passing through a birefringent crystal. By combining the heterodyne light beam with the grating shearing interferometry, a two-dimensional displacement measurement configuration is constructed. The optical phase variation resulting from the grating displacement can be carried on the heterodyne light beam and then further demodulated by the lock-in amplifier (LIA). The grating displacements in two dimensions are obtained from the phase changes in each corresponding direction. The theory of our proposed wavelength-modulated heterodyne grating shearing interferometry is described in detail below. Displacement experiments are performed to demonstrate the advantages of the proposed method.

2 Measurement theory

2.1 Wavelength-modulated heterodyne light

As shown in Figure 1, a laser beam with a central wavelength of λ_c is emitted from the LD. The driving current with triangular waveforms is used to modulate the wavelength of the LD and can be formulated as follows:

$$\lambda(t)=\lambda_c+\Delta\lambda t/T, \tag{1}$$

where $\Delta\lambda$ stands for the modulation depth, t and T represent the time and modulation period, respectively, and $\Delta\lambda$ is far less than λ_c.

After passing through a polarizer with 45° polarization status, the laser beam enters a birefringent crystal (BC) with a length of l. The optical axis (OA) of the BC is parallel to the plane of incidence, and Δn stands for the difference in the refractive index between the ordinary ray and extraordinary ray. As the light passes through a measurement system (MS, such as the interferometer), the physical quantities (such as displacement) of the measurement system will be carried on the light beam. The intensity of the light beam passing through the measurement system can be expressed as [11]

$$I_t=1+\cos(2\pi\Delta nl/\lambda(t)+)\phi, \tag{2}$$

where ϕ represents the optical phase coming from the measurement system. Substituting the modulated-wavelength $\lambda_c(t)$ into Eq. (2), the intensity of the light beam can be further written as

$$I_t=1+\cos(2\pi\Delta nl/\lambda_c\text{-}\omega t+\phi), \tag{3}$$

where $\omega=2\pi\Delta nl\Delta\lambda/(\lambda_c^2T)$ means the modulation heterodyne frequency. This is the heterodyne light obtained by using the design concept of the wavelength modulation and the birefringent crystal.

2.2 Quasi-common-optical-path (QCOP) grating shearing interferometry for displacement measurement

A schematic diagram of the proposed wavelength-modulated heterodyne grating shearing interferometry is shown in Figure 2. The wavelength-modulated heterodyne beam

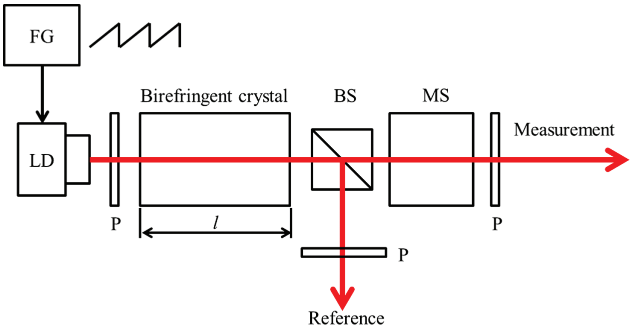

Figure 1 Schematic diagram of the architecture of the heterodyne light source (FG: function generator; LD: laser diode, P: polarizer; BS: beam-splitter; MS: measurement system).

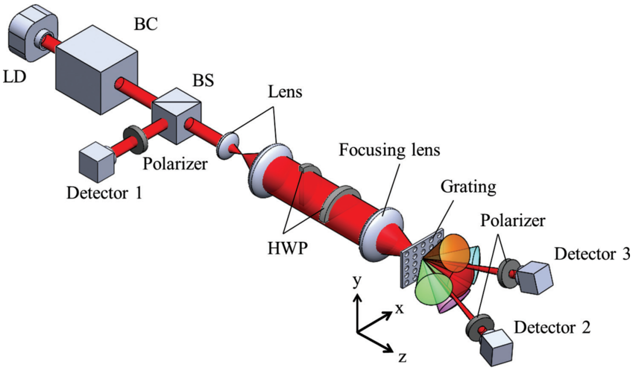

Figure 2 Optical configuration of the wavelength-modulated heterodyne grating shearing interferometer (LD: Laser diode; BC: birefringent crystal; BS: Beam splitter; HWP: Half-wave-plate).

is expanded after passing through the beam expander. Two semi-circular half-wave-plates (HWPs) for which the fast axes are at 45° are inserted. After passing through the two HWPs, the expanded wavelength-modulated heterodyne beam is divided into 4 parts A, B, C and D [15]. The heterodyne light beam is further focused on a 2D holographic grating by a lens and then diffracted. By choosing a suitable focusing lens and grating pitch, the Zero (0) order beam will partially overlap with the +1st and -1st order diffracted beams. The beam distribution is shown in detail on the right-hand side of Figure 3. For example, the interference between part A in the -1st order and part B in the zero order diffraction beams ($A_{x\text{-}1}$ and B_0) is used to determine the *x* displacement. In the same way, the interference between A_0 and C_{y1} is used for the y displacement. Moreover, after passing through two polarizers and two photo-detectors (Detector 3 and Detector 2) respectively receive the two corresponding light interference signals. Interference signals I_x and I_y measured by Detector 3 and Detector 2 can be written as [15]

$$I_x=1+\cos(\omega t+\phi_x),\ I_y=1+\cos(\omega t+\phi_y), \tag{4}$$

where ϕ_x and ϕ_y stand for the corresponding phase differences between the +1st (or -1st) and zero order diffraction beams in the *x* and *y* directions, respectively. A polarizer and a detector (Detector 1) are used to measure the intensity of the reference light beam and can be expressed as

$$I_r=1+\cos(\omega t), \tag{5}$$

Then, all three intensity signals I_x, I_y (measurement signals) and I_r (reference signal) are sent into the LIA, and we can obtain the amount of phase change ϕ_x (or ϕ_y) in the *x* (or *y*) direction. The displacement (*d*) of the grating can be determined by means of the measured phase difference (ϕ_x or ϕ_y). The relationship between the displacement and measured phase difference variation is given by

$$d=\Phi\Lambda/2\pi, \tag{6}$$

where Λ means the grating period, and $\Phi=\phi_x$ (or ϕ_y).

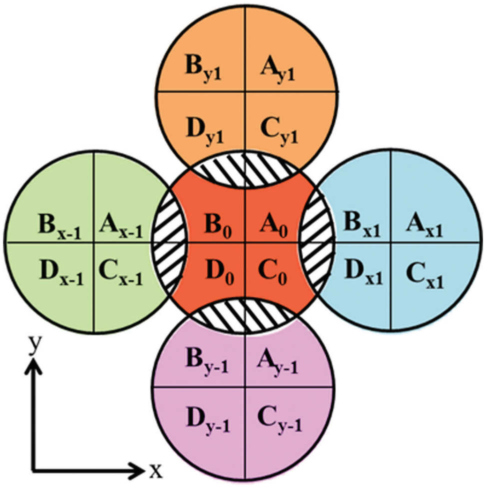

Figure 3 Diagram of the overlapped area for the corresponding direction.

3 Experimental results and discussion

In order to demonstrate that our presented wavelength-modulated heterodyne grating interferometer is capable of measuring displacement in two dimensions, a commercial PZT stage (model: P-611.3S, Physik Instrumente (PI), Karlsbad, Germany) was operated in a closed-loop configuration and the experiment results obtained by the internal sensor and our method were compared. The function generator (model: AFG3022B, Function generator: Tektronix Inc., Beaverton, USA) was used to generate sinusoidal waves for driving the PZT stage. In addition, the tunable LD (model: LDC205B, TED200, Thorlabs, Dachau, Germany) was driven by a tooth waveform current and controlled at 9.8°C. A two-dimensional grating with pitch 2.67 μm was placed on the PZT stage. The detailed information of the experimental conditions and parameters is listed in Table 1.

The PZT stage was made to perform a sinusoidal motion with an amplitude of 45 μm in *x* and *y* directions, respectively. An internal strain gauge mounted on the PZT stage was also utilized for comparison with the measurement results obtained using our proposed method. In order to conveniently distinguish between the measurement results obtained with our method and the results obtained by the internal strain gauge, the experimental results are plotted with 1 second-delay in Figures 4 and 5.

Table 1 Experimental condition.

Parameter	**Model or description**
Birefringent crystal	YVO_4, 30 mm (*l*)
Modulation period (*T*)	1 ms
Modulation depth (Δλ)	0.04 nm
Modulation refractive index (Δ*n*)	0.2225
Wavelength (λ)	638 nm
Heterodyne frequency (*ω*)	1 KHz
Lock-in Amplifier	SR850
Photo-detector	PDA36A
Filtering frequency (LIA, SR850, Stanford Research Systems, Sunnyvale, USA)	0.833 Hz
Maximal sampling rate (DAQ card, PCI-6110, Nation Instruments, Austin, USA)	100 Hz
Maximal measurable motion speed	9.7 μm/s

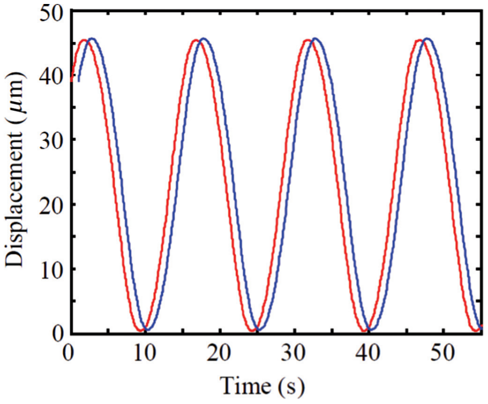

Figure 4 Sinusoidal wave movement in the *x* axis (45 μm).

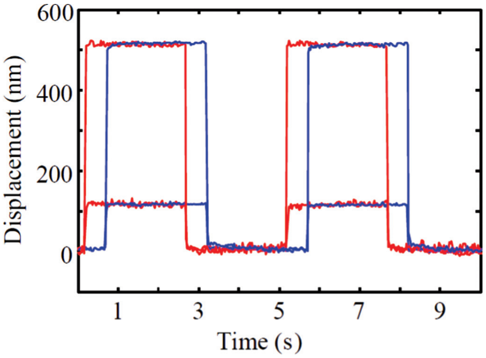

Figure 6 Step movement in the *x* axis (500 nm and 120 nm).

As shown in Figure 4, the experimental results in the *x* direction obtained by the two methods are similar, with peak to peak amplitudes of 45.020 µm (strain gauge, blue, straight line) and 45.031 µm (interferometer: red, straight line). It can be seen that the displacement measurement results obtained using the internal strain gauge and with our method correspond well with each other. This demonstrates that our proposed method has the same ability to sense displacement as a commercial strain gauge. The experimental results in the *y* direction obtained by the two methods are shown in Figure 5. Clearly, the shapes of the two measurement curves are the same, which confirms that our proposed method is just as capable of measuring displacement in the *y* direction as the commercial sensor. Moreover, in order to demonstrate that our proposed interferometer is also capable of sensing short stroke displacement with different motion behaviors, the PZT stage was driven with a stepping waveform motion with traveling distances of 500 nm and 120 nm in the *x* and *y* directions. The stepping waveform displacement experimental results are shown in Figures 6 and 7 (with 0.5 second-delay). Clearly, it can be seen that the short stroke displacement measurement results obtained using the internal strain gauge (strain gauge, blue, straight line) and our interferometer (interferometer: red, straight line) are almost the same. This demonstrates that our proposed interferometer has the ability to measure a small stroke displacement with a resolution comparable to that of a commercial sensor.

In addition, the PZT stage was held stationary and the phase variation for the 200-s test was measured so that the system stability could be accurately defined. Figure 8 shows the experimental results. It can be seen that the system stability of the proposed method is about 20 nm over 200 s. The results shown on the Figure 8 clearly illustrate how much less sensitive to the surrounding disturbance is the proposed interferometer. This demonstrates that our system has high measurement stability. In fact, we found that the drift of 20 nm was caused by the environment disturbance resulting from the thermal drift of the laser or the air conditioner.

The measurement resolution can be defined as the minimum measured amount of displacement of the system. A grating pitch and phase resolution of the LIA used in our current setup are about 2.67 µm and 0.01°,

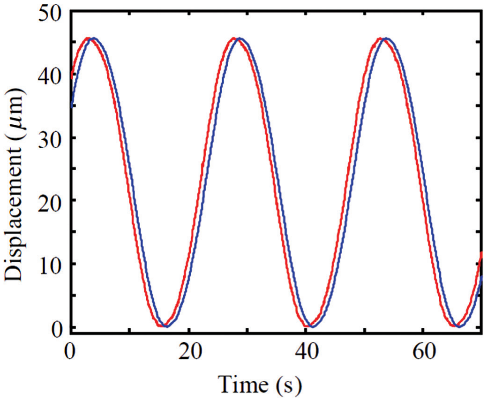

Figure 5 Sinusoidal wave movement in the *y* axis (45 μm).

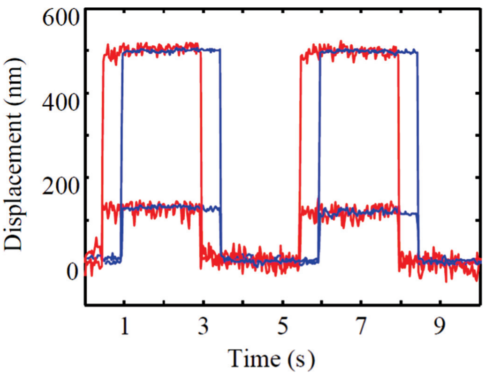

Figure 7 Step movement in the *y* axis (500 nm and 120 nm).

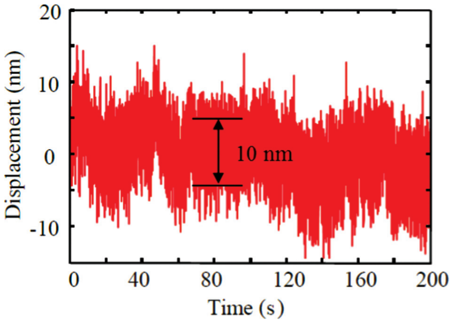

Figure 8 Measurement results of stability (200 s).

according to the measurement principle, the theoretical measurement resolution of our system is 0.074 nm. However, the actual measurement resolution of the system is about 10 nm if electrical noise is considered. This result can also be observed in Figure 8.

Since the proposed method uses the modulation principle of heterodyne interferometry, we need to concentrate on the error effects coming from this interferometric technique. Actually, the non-linear periodic error of the QCOP heterodyne interferometer has been analyzed in our previous work [15]. Here we follow a similar method to analyze the nonlinear periodic error in the proposed interferometer. The error sources mainly come from misalignment of the HWP and polarizer. To estimate the total error resulting from the misalignment effects, the expression of the interference amplitude can be written as

$$\mathrm{E}=\mathrm{P}(\theta_{\mathrm{p}})\cdot\left[\mathrm{HWP}(45^{\circ}+\varepsilon)\begin{pmatrix}\mathrm{e}^{\mathrm{i}\omega\mathrm{t}/2}\\ \mathrm{e}^{-\mathrm{i}\omega\mathrm{t}/2}\end{pmatrix}+\begin{pmatrix}\mathrm{e}^{-\mathrm{i}\omega\mathrm{t}/2}\\ \mathrm{e}^{\mathrm{i}\omega\mathrm{t}}\end{pmatrix}e^{i\phi_{q1}}\right], \tag{7}$$

where θ_{p} and ε stand for the azimuth angles of the polarizer and the HWP, respectively, ϕ_q represents the optical phase coming from the measurement system in the q direction (q=x or y). The interference signal can be written as follows:

$$\mathrm{I}=|\mathrm{E}|^2\propto AC\cos(\omega t+\Phi') \tag{8}$$

where Φ' is the deformed phase. The phase error of our method can be obtained by

$$\Delta\Phi=\Phi'(\theta_{\mathrm{p}},\varepsilon)\text{-}\phi_q, \tag{9}$$

where q=x or y.

The minimum adjustment angles of the polarizers and HWP in our current setup are about 5′ and 25″. Therefore the non-linear periodic error is about 1° for every grating pitch. In our experiments, we found the real nonlinear error to be about 10 nm for every grating pitch in displacement.

4 Conclusions

We proposed a heterodyne grating interferometer design based on the birefringent crystal and wavelength-modulated light source. The heterodyne light source is generated by using a tunable laser diode (LD) and passes through the birefringent crystal. There is a difference in the optical path lengths of the p- and s-polarizations of the light beam in the birefringent crystal as a consequence of the double refraction caused by the birefringence. A heterodyne light source having two frequencies can be obtained by passing through the unequal-path-length optical configuration. The modulated heterodyne light beam is further combined with a grating-shearing interferometer based on the design concept of the quasi common optical path configuration.

According to the working principles and the Jones calculation, the displacement information of a moving grating can be obtained by means of the optical phase variation resulting from the grating. The theoretical measurement resolution and sensitivity of the proposed method are about 0.074 nm and 0.134°/nm, respectively. The actual resolution is demonstrated to be about 10 nm if the electrical noise is considered.

The proposed method not only provides high measurement resolution and good stability, but can also achieve high enough precision to meet the demands of a nano-positioning stage in the field of two-axis application.

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Author contributions: H-L H. and J-Y L. designed the optical configuration. H-L H. and Y-C C. set up the experimental system and carried out the experiments. J-Y L. and Y-C C analyzed the acquired data. J-Y L. and H-L H. derived and estimated the error influence. H-L H., J-Y L. and Y-C C. prepared the original manuscript.

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