© 2013 THOSS Media & DE GRUYTER DOI 10.1515/aot-2013-0029 — Adv. Opt. Techn. 2013; 2(5-6): 445–453

Review Article

Fengzhou Fang*, Ying Cheng and Xiaodong Zhang

Design of freeform optics

Abstract: Freeform optics has certain characteristics such as complex surfaces and more degrees to be controlled, so that it is capable of achieving optical mapping relationship, which cannot be achieved by the conventional optical surfaces. There is no applicable method for designing all freeform optics in the optical field until now. The selection of design methods should be based on specific applications. The paper introduces the main optical freeform design methods and their typical applications.

Keywords: design of freeform optics; freeform optics; imaging optics; non-imaging optics.

OCIS codes: 050.0050; 080.0080; 110.0110; 120.0120; 130.0130; 190.0190; 240.0240.

***Corresponding author: Fengzhou Fang,** State Key Laboratory of Precision Measuring Technology and Instruments, Centre of MicroNano Manufacturing Technology, Tianjin University, Tianjin 300072, China, e-mail: fzfang@gmail.com
Ying Cheng and Xiaodong Zhang: State Key Laboratory of Precision Measuring Technology and Instruments, Centre of MicroNano Manufacturing Technology, Tianjin University, Tianjin 300072, China

1 Introduction

With the development of ultraprecision machining technology, it is possible to realize the machining of freeform optics, which has been increasingly applied in various areas such as the aviation and aerospace, bioengineering, environment, and communication. Therefore, the development of designing freeform optical surfaces has become an important area in freeform optics.

Freeform optics is defined as any nonrotationally symmetric surfaces or microarray surfaces [1]. There are three main ways to describe the freeform surfaces, i.e., NURBS [2], XY polynomial [3], and radial basis function representation [4–8]. As optical freefrom surfaces have more freedom than traditional surfaces, there is no universal design method until now; thus, specific methods are needed to be selected based on the applications of the freeform surfaces.

2 Available design methods

The available optical freeform design methods include multiparameter optimization, Wassermann-Wolf differential equation, tailoring method, point-to-point mapping, geometric and variational method, and simultaneous multiple surface method. These methods are briefly introduced below.

2.1 Multiparameter optimization

The multiparameter optimization method is an indirect optical freeform surface design method, which needs to establish an initial surface first. The parameters of freeform surface are set to optimize the surface gradually until the design requirements are met [9]. The optimization time and final results are affected by the selection of the initial surface. It usually has a long design period. This method has been integrated in commercially available optical design softwares.

2.2 Wassermann-Wolf differential equation

The imaging optical surfaces can be developed gradually according to spherical, aspheric, higher-order aspheric, anamorphic aspheric, and freeform surfaces, and the optical systems are from symmetric to nonrotationally symmetric systems. Currently, the imaging systems are designed by optical optimization softwares, but the nonrotationally symmetric systems are still needed to form initial surfaces for optimization.

Wasserman-Wolf differential equation is the basis for the design of imaging optical elements, especially that has an important significance in conformal optical surfaces.

The Wassermann-Wolf differential equation was derived by Wasserman and Wolf in 1949 and used to design two aspheric surfaces for centered system, as shown in Figure 1, basing on the Abbe sine condition to solve a first-order differential equation, and the system could correct axial stigmatism and sine condition [10]. In 1957, Vaskas provided a method to extend the Wassermann-Wolf

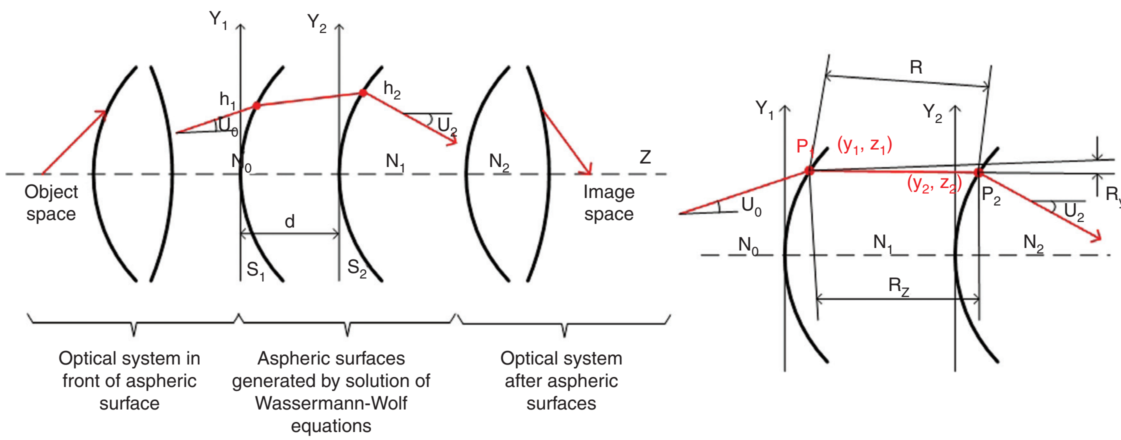


Figure 1 Layout of a typical system [10].

method to a more general situation, in which the two surfaces can be separated by a number of known surfaces [11]. In 2002, Knapp in his doctoral dissertation provided a new approach for the design of correctors for nonrotationally symmetric optical systems, which was similar to that of Wassermann-Wolf's. They implemented it in a commercial software macro called the Generalized Aspheric Design Program (GAP) to validate the new equations [12].

From the above discussion, it can be seen that the calculation of the Wassermann-Wolf method is for discrete light and only can be used in 2D freeform optics design. Although this method was proposed for imaging systems, but because its calculation is for light, it also can be used in non-imaging optics, which is noteworthy.

2.3 Tailoring method

Tailoring method is for illumination design, the shape of the optical surface is obtained by solving a set of partial nonlinear differential equations according to the relationship among the incident light vector, the exit light vector, and the normal vector, and simultaneously according to the restrictions between the light source and the reference surface irradiance distribution [13].

The tailoring method first appeared in 1993; Winston and Ries designed the non-imaging reflector by the establishment of the function about the angle of incidence and target surface illumination distribution [14]. The edge-ray principle is an important theory in tailoring method. In 1994, Davies tested and verified the edge-ray principle by geometrical optics. In the same year, Ries and Rabl verified the edgy ray principle in phase space, which established a foundation for the tailoring method [15]. In 1993, Ries and Winston determined the reflector profile by numerically solving a differential equation. Beyond the angular region in which the power distribution could be strictly controlled, the power dropped to zero in a finite decay range, and this decay range became narrower as the reflector increases in size [16]. In 1994, Rabl and Gordon obtained a solution for extended sources by establishing differntial equation, instead of the point source [17]. In 1996, Jenkins and Winston used a new integral design method based on the edge-ray principle of non-imaging optics, which gave much more compact reflector shapes by eliminating the need of a gap between the source and the reflector profile [18]. In 1996, Ong based his study on the study of Rabl to design tailoring lighting reflectors by partial differential equations for an extended Lambertian source. He defined the method as tailored edge-ray designs (TEDs), and divided it into four topologically distinct classes of such reflectors. The source was a tubular light source [19].

In 2002, Ries and Muschaweck used freeform elements to attain a desired irradiance distrbution by using the tailoring method, as shown in Figure 2 [13].

In 2003, Timinger and Ries et al. used the tailoring method to attain a rectangle distribution, which was used on street lighting, office lighting, and so on, and the uniformity is as good as expected [20]. In 2008, Ding et al. used this approach on LED lighting to achieve uniformity of LED lighting, and the uniformity is up to 90% [21].

However, the exact solution of nonlinear partial differential equation is a problem, which usually used Runge-Kutta to get approximate solution. Subsequently, Oliker proposed geometric and variational method [22–24].

A non-isotropic point source is positioned at the origin of a Cartesian coordinate system O in R^3 and emits rays in a set of directions defined by the aperture Ω given as a closed set on a unit sphere S centered at O. $I(m)$ is

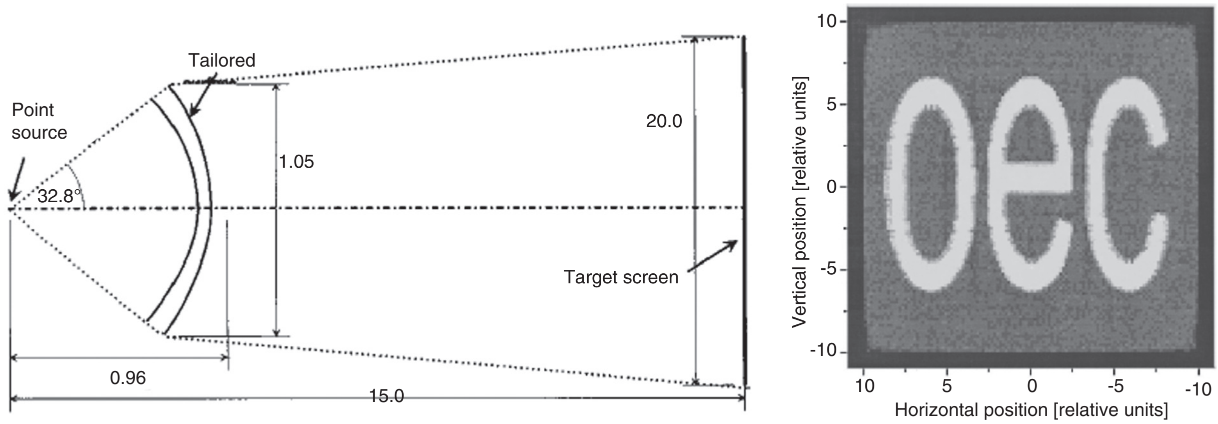


Figure 2 Sketch of the setting and irradiance distribution [13].

the intensity of this source in the direction ($m \in \Omega$). The light ray that emits from the source O in the direction m is perfectly reflected by the surface R at point $r(m)$, and the reflection ray is in the direction $y(m)$ [22]. Thus, the surface R defines a map $\gamma: m \rightarrow y$. This freeform design problem is solved using the variational method [25, 26].

The geometrical optics is combined with calculus of variations, which could solve the beam-shaping problem of the known illumination distribution of target surface and complete the one-reflector or two-reflector design. Variational method is about solving the extremum problem and makes the boundary to be discrete under geometrical optics to obtain a linear solution. Oliker found a set of optimal solutions according to the Monge-Kantorovich quality problems [25, 26].

Figure 3 is a design example of two reflectors using this method. B_1 is the incoming beam, and B_2 is the output beam, which consists of parallel light rays propagating in the same dirction as B_1. The rays in B_1 are reflected by reflector R_1 and then intercepted by reflector R_2 and at last reach the target suface (z=d) [25].

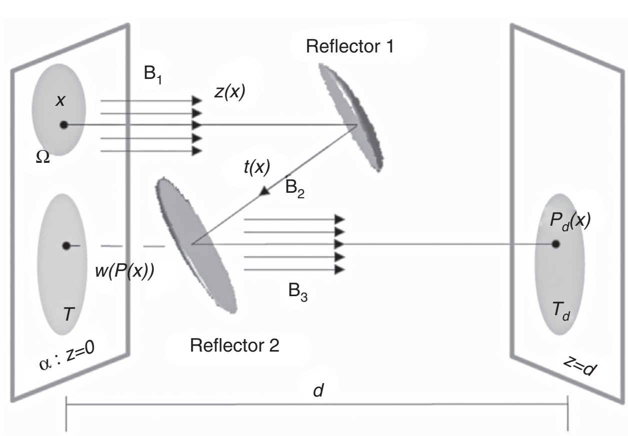


Figure 3 Design of two-reflector optical systems [25].

The approach is based on a rigorous mathematical theory and does not assume rotational or other symmetry of the data. There is restriction of input or output apertures and intensities [22].

From the above discussion, it can be seen that the calculation of the tailoring method is based on energy and can be used in 3D freeform optics design. As the non-imaging optics is concerned with energy collection rate, the method has important applications in non-imaging optics. But the solving process of the method is very complex, and the calculation for extended source is approximate, and the method cannot be used to complete more than two freeform surface designs.

2.4 Mapping method

Mapping method is based on the principle of energy conservation to establish the equation between the lighting energy emitted by the source and the lighting energy obtained by the target to get the point-to-point mapping. Then, this mapping is used to abtain the coordinates and the normal vector of the points on freeform surfaces by iterative solution, finally getting the shape of the lens surface.

Figure 4 is the schematic diagram of the mapping method. The source S is located at the origin of an orthogonal coordinate system; the points on the target plane T for the illumination can be expressed as $t(x, y, z)$. The freeform lens p is located in a spherical coordinate system, that is, $(\theta, \varphi, \rho(\theta, \varphi))$, and the normal vector at points p of the lens is N, I is the vector of the incident light at point p. Figure 4B is the topological mapping from source to target

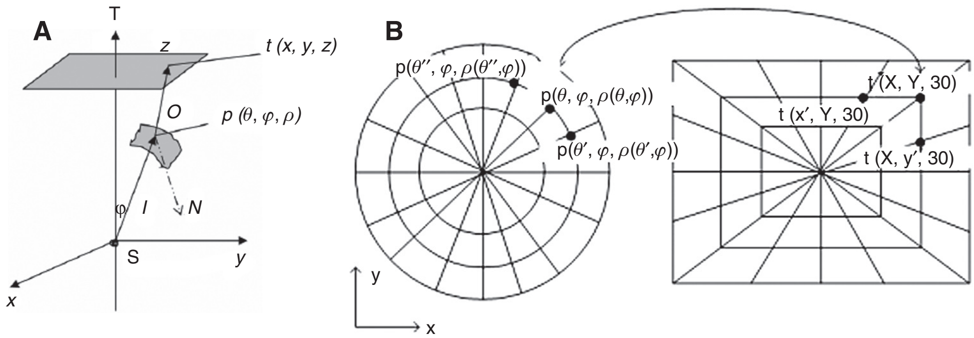


Figure 4 (A) The relationships between the vectors in refraction. (B) The topological mapping from source to target plane [21].

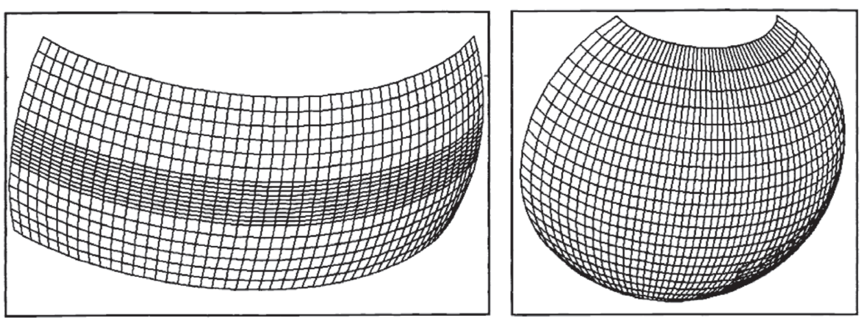


Figure 5 The two girds [27].

plane, which means lights of equal φ would be refracted to the same edge of the rectangle [21].

In 1998, Parkyn proposed the mapping method to design lenses for general illumination tasks. The crux of the method was the specification of the illumination task by a grid on the unit sphere of directions and used extrinsic differential geometry (EDGE) to obtain the normal vectors of the lens to generate a smooth surface. One of grid had cells, which varied in a solid angle such that each encompassed the same luminous flux; the other grid had the same topology, and the number of cells was formed according to the intensity distribution of the source, as shown in the Figure 5 [27].

In 2006, Parkyn and Pelka based their study on the research of Parkyn to apply a new pseudo-rectangular spherical grid to establish a correspondence between the source grid cells and the rectangular cells of a target grid. He obtained the central spines by a linear integration, after that, obtaining the adjacent rows successively in a lawnmower fashion as shown in Figure 6 [28].

In 2010, Fournier proposed a freeform reflector design method based on the mapping of equi-flux grids between a point source and a target, and the maps satisfied integrability condition. The generated reflectors can produce continuous illuminance distributions [29, 30].

In addition, it can also be combined with other design methods, such as combining with the Monte Carlo ray tracing. A LED package was developed to ensure high-performance LED lighting [31]. Combining with optimization calculation, in 2007, a gradient method was proposed to design freeform for forming a specified irradiance on a curved surface, basing on eikonal polynomial to minimize the error functional that represents the difference of the calculated and specified irradiance gradiently [32]. A feedback modification method was introduced based on variable separation mapping in design of freeform optical system with uniform illuminance for LED source. The size of LED source was taken into account, and a smooth

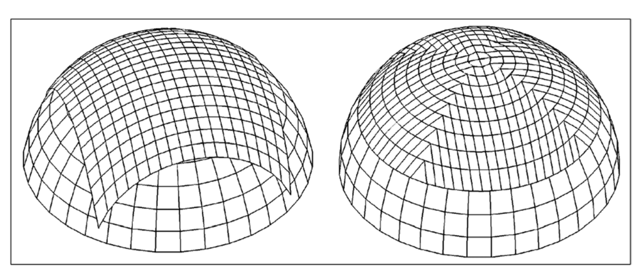


Figure 6 Rectangular grid on shere has uneven coverage (left) vs. pseudo-rectangular grid (right) with polar coverage [28].

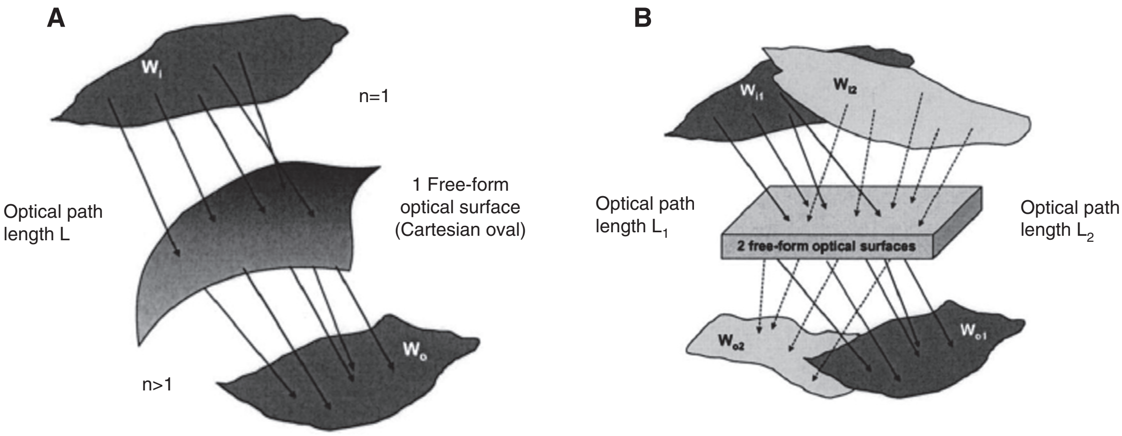


Figure 7 (A) Cartesian oval transforms one input congruence into another output congruence and (B) the SMS method provides two surfaces transformimg two input congruences into output ones [35].

freeform lens with rectangular illuminance distribution is designed. The illuminance uniformity is improved from 18.75% to 81.08% after optimization [33].

The mapping method is based on energy and light, and this method is easy to combine with other methods. However, this method can only be used to calculate one freeform surface. The calculation for extended light source is only approximate.

2.5 Simultaneous multiple surface method

Simultaneous multiple surface (SMS) method was proposed in 1990, which began from the 2D non-imaging optical design. It is an important breakthrough in freeform optics design. The abbreviation SMS comes from the fact that it enables the simultaneous design of multiple optical surfaces. The original idea came from Minano. The first generalization to 3D geometry came from Benitez. Thus, SMS also is called as the Minano-Benitez design method [34].

SMS is compact, efficient, and simple [35] with a principle of ‘bundle-coupling’ and ‘prescribed-irradiance’. ‘Bundle-coupling’ means that input and output bundles are coupled, i.e., any incidence ray entering into the optic device is the exit rays. ‘Prescribed-irradiance’ means one bundle must be included in the other [35], i.e., SMS couples two input wavefronts into two output wavefronts, as shown in Figure 7. The SMS 3D method is a procedure for designing two optical surfaces such that two given normal congruencies, W_{i1}, W_{i2}, are transformed into another two given normal congruencies, W_{o1} and W_{o2}. The computing of SMS resorts to chain and ribs as shown in Figure 8 [35].

The 3D SMS is at present the most powerful direct design method for illumination devices using extended sources [36].

Compared with other methods, SMS is based on the edge principle and the Fermat theorem of light. SMS is not only capable of calculating the point source but also calculating the extended light source. SMS can be used to take the size and angle of the source into calculations.

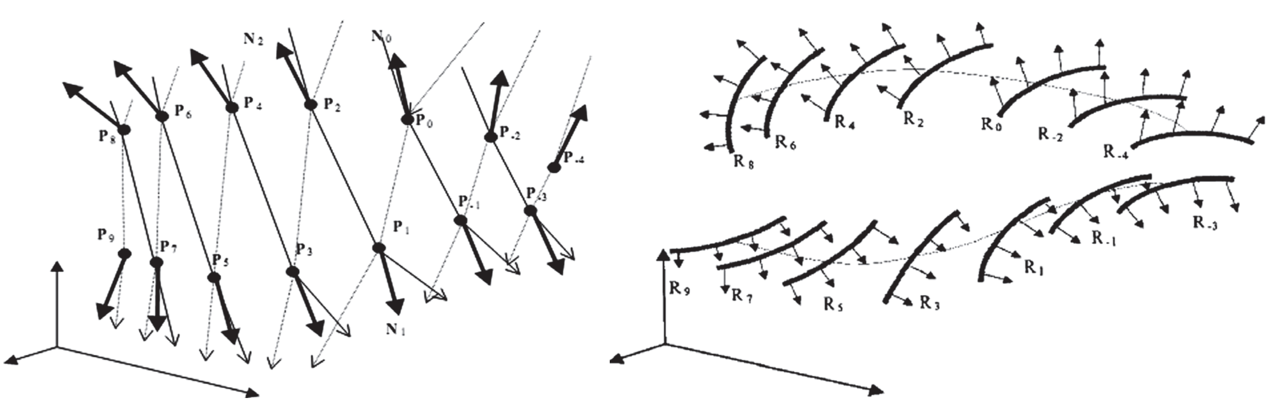


Figure 8 SMS chain and ribs [35].

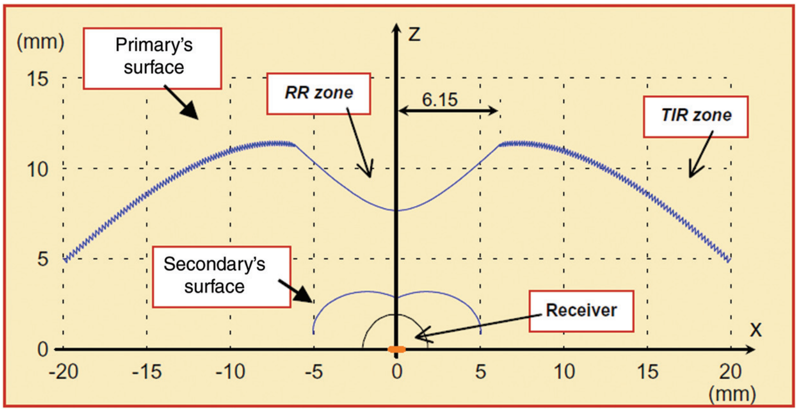


Figure 9 Surface profile of the TIR-R concentrator [37].

SMS is also capable of designing at least two free surfaces once.

SMS can be used in imaging and non-imaging optics. Since 1990, Light Prescriptions Innovators Europe (LPI, Altadena, USA) has designed a variety of concentrators used in CPV, including R(refractive)R [37], X(reflective)R [37], RX [38], RXI [total internal reflection (TIR)] concentrator [38], TIR-R concentrator [37, 39], and so on. Taking the TIR-R concentrator, for example, as shown in Figure 9, it has two parts: the primary surface is TIR surface as a microstructure with infinitesimal flat facets, and the secondary surface is refractive surface. The aim of this design is to avoid the metalized surfaces, which cause reflection losses and are difficult to manufacture, and to optimize the position of the emitter/receiver for the encapsulation, electrical connection, and heat sink. A concentrator with a magnification of 1256× has a theoretical efficiency of 100% (without optical losses) with an acceptance angle of ±1.7° and aspect ratio of 0.34 [37, 39]. Figure 10 shows another 3D freeform device designed by SMS for CPV applications. The concentration is 1000. Its tolerance angle is ±1.5°, and overall efficiency (solar to electric energy) is above 27% [36]. It is believed that the optical freeform technology would be used in nontracking solar concentrator systems to achieve greater economic benefits.

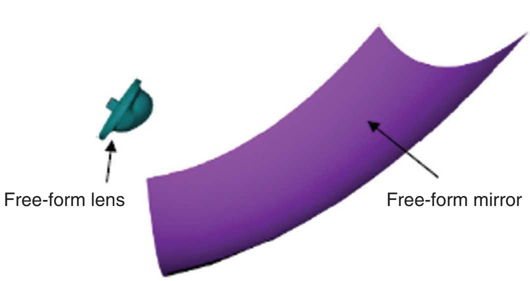


Figure 10 Freeform XR photovoltaic applications [36].

LPI also used 3D SMS to design illumination opticals elements. Figure 11 shows a freeform device (RXI) for a LED, which can perfectly control the bundle of rays issuing from the LED chip corners [36]. Figure 12 shows an example of an RXI with Kohler integration for automotive applications. This strategy allows obtaining intensity patterns quite insensitive to the source (LED) positioning errors and can be used to get uniform illumination on the cell of photovoltaic concentrators for any incidence angle [36]. In addition, LPI used freeform condenser, which designed to overcome the limitations inherent to conventional condensers. The power sent by the freeform

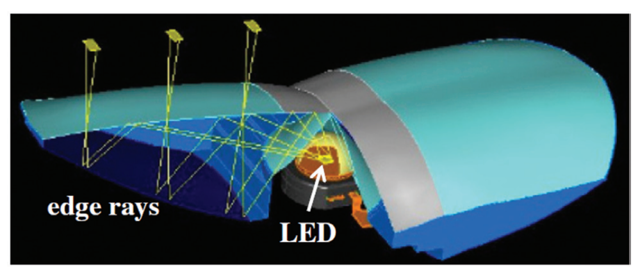


Figure 11 Freeform RXI for an LED [36].

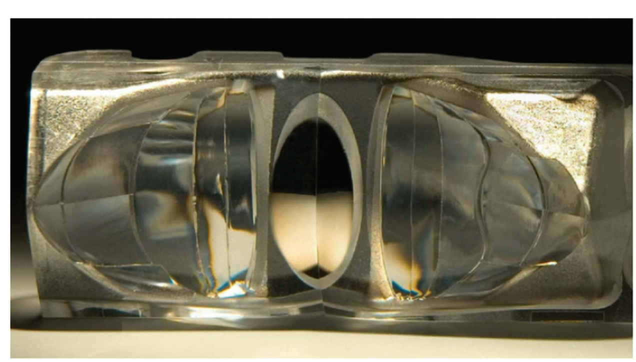


Figure 12 A freeform Kohler integrator RXI for illumination [36].

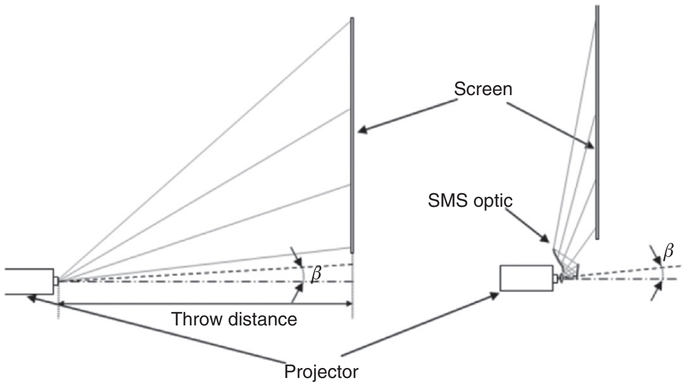


Figure 13 Ultrawide-angle projection mirror with conventional slide projector [41].

condenser is 1.8 times of that by an equivalent elliptical condenser for a 4:1 target aspect ratio and 1.5 times for 16:9 target and for practical values of target etendue [40].

LPI used SMS to design imaging devices. Figure 13 is an example of an SMS imaging device, which is a new video projection optics system. It has short throw distance, high compactness, and wide angle projection [41].

The SMS method is based on light so that the freeform optics for imaging optical system can be realized by discretization of the light. Multiple surface designs can be achieved simultaneously, and also, the extended source can be calculated accurately. The calculation of the SMS method is relatively simple and easy to realize. It can be said that the SMS method is one of the most promising freeform design methods.

3 Conclusions

Freeform optics is a new generation of optics, having more controllable freedom degrees. The freeform optics of imaging system can be employed to simplify the system structure, correct the distortion of large field-of-view, and make the system structure more innovative. The freeform optics of a non-imaging system can be employed to improve the system with a greater efficiency.

The freeform design method should be selected according to the specific applications. The tailoring method and the mapping method can be used for the design of an optical surface with a point source. The mapping method can be used between a point source and a square target. The SMS method can be used for extended source. The Wassermann-Wolf differential equation is used for imaging applications; the tailoring method, mapping, and geometric and variational methods are used for non-imaging optical elements; SMS can be used for both imaging and non-imaging optical designs. At present, the SMS method is one of the most promising freeform design methods.

Acknowledgments: The authors appreciate the supports of the National Basic Research Program of China (973 Program, Grant No. 2011CB706703), the National Natural Science Foundation of China (Grant No. 90923038), and the '111' project by the State Administration of Foreign Experts Affairs and the Ministry of Education of China (Grant No. B07014).

Received May 30, 2013; accepted September 24, 2013

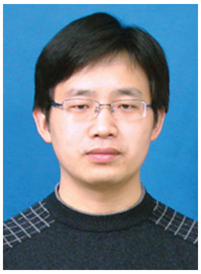
References

[1] K. Garrard, T. Bruegge, J. Hoffman, T. Dow and A. Sohn, in 'Optics and Photonics 2005. International Society for Optics and Photonics' 58740A-58740A-11 (2005).
[2] T. L. Davenport, in 'International Optical Design Conference 2002,' Int. Soc. Opt. Photonics 293–301 (2002).
[3] W. J. Cassarly and M. J. Hayford, in 'International Optical Design Conference 2002,' Int. Soc. Opt. Photonics 258–269 (2002).
[4] O. Cakmakci, G. E. Fasshauer, H. Foroosh, K. P. Thompson and J. P. Rolland, Proc. SPIE 70610D-1, 7061 (2008).
[5] O. Cakmakci, S. Vo, H. Foroosh and J. Rolland, Opt. Lett. 33(11), 1237–1239 (2008).
[6] I. Kaya, O. Cakmakci, K. Thompson and J.P. Rolland, in 'Optical Fabrication and Testing,' Opt. Soc. Am. (2010).
[7] A. K. Chan, C. K. Chui and L. T. Guan, in 'Electronic Imaging'90,' Santa Clara, 11–16 Feb '95. Int. Soc. Opt. Photonics 62–72 (1990).

[8] I. Kaya and J. P. Rolland, in 'Frontiers in Optics.' Opt. Soc. Am. (2010).
[9] P. Ben, Opt. Photonics News 18, 20–25 (2007).
[10] G. D. Wassermann and E. Wolf, Proc. Phys. Soc. B 62, 2 (1949).
[11] E. M. Vaskas. JOSA 47, 669–670 (1957).
[12] D. J. Knapp, Conformal optical design, Ph.D. Dissertation, College of Optical Sciences, the University of Arizona (2002).
[13] H. Ries and J. Muschaweck, JOSA A 19(3), 590–595 (2002).
[14] R. Winston and H. Ries, JOSA A 10, 1902–1908 (1993).
[15] H. Ries and A. Rabl, JOSA A 11, 2627–2632 (1994).
[16] H. R. Ries and R. Winston, JOSA A 11, 1260–1264 (1994).
[17] A. Rabl and J. M. Gordon, Appl. Opt. 33, 6012–6021 (1994).
[18] D. Jenkins and R. Winston, Appl. Optics 35, 1669–1672 (1996).
[19] P. T. Ong, J. M. Gordon and A. Rabl, Appl. Optics 35, 4361–4371 (1996).
[20] A. L. Timinger, J. A. Muschaweck and H. Ries, in 'Optical Science and Technology, SPIE's 48th Annual Meeting,' Int. Soc. Opt. Photonics 128–132 (2003).
[21] Y. Ding, X. Liu, Z. Zheng and P. F. Gu, Opt. Express 16, 12958–12966 (2008).
[22] V. Oliker, Opt. Photonics 2005. Int. Soc. Opt. Photonics 594207–594207-12 (2005).
[23] S. A. Kochengin, V. I. Oliker and O. von Tempski, Inverse Probl. 14, 661 (1998).
[24] V. Oliker, in 'Contract Proceedings 2006,' Int. Soc. Optics Photonics 634211–634211-12 (2006).
[25] T. Glimm and V. Oliker, Indiana Univ. Math. J. 53, 1255–1278 (2004).
[26] T. Glimm and V. Oliker, J. Math. Sci. 117, 4096–4108 (2003).
[27] W. A. Parkyn, in 'SPIE's International Symposium on Optical Science, Engineering, and Instrumentation,' Int. Soc. Opt. Photonics 154–162 (1998).
[28] B. Parkyn and D. Pelka, Opt. Photonics. Int. Soc. Opt. Photonics 633808–633808-7 (2006).
[29] F. R. Fournier, W. J. Cassarly and J. P. Rolland, Opt. Express 18, 5295–5304 (2010).
[30] F. R. Fournier, W. J. Cassarly and J. P. Rolland, in 'International Optical Design Conference,' Opt. Soc. Am. (2010).
[31] K. Wang, F. Chen, Z.Y. Liu, X.B. Luo and S. Liu, Opt. Express 18, 413–425 (2010).
[32] A. A. Belousov, L. L. Doskolovich and S. I. Kharitonov, J. Opt. Technol. 75, 161–165 (2008).
[33] Y. Luo, Z.X. Feng, Y.J. Han and H.T. Li, Opt. Express 18, 9055–9063 (2010).
[34] R. Winston, J. C. Miñano and P. Benítez, in 'Nonimaging Optics' (Academic Press, New York, 2005).
[35] P. Benítez, J. C. Miñano, J. Blen, R. Mohedano, J. Chaves, et al., Opt. Eng. 43, 1489–1502 (2004).
[36] J. C. Minano, P. Benítez and A. Santamaría, Opt. Rev. 16, 99–102 (2009).
[37] J.L. Alvarez, M. Hernandez, P. Benitez and J.C. Miñano, in 'International Symposium on Optical Science and Technology,' Int. Soc. Opt. Photonics 32–42 (2001).
[38] J. C. Minano, J. C. Gonźlez and P. Benítez, Appl. Opt. 34, 7850–7856 (1995).
[39] W. A. Parkyn, P. L. Gleckman and D. G. Pelka, in 'SPIE's 1993 International Symposium on Optics, Imaging, and Instrumentation,' Int. Soc. Opt. Photonics 78–86 (1993).
[40] J.C.Miñano, P. Benitez, J.Blen and A. Santamaria, Opt. Express 16, 20193–20205 (2008).
[41] F. Muñoza, P. Benítezb and J. C. Miñanob, Proc. SPIE 7061, 70610G-1 (2008).

Dr Fengzhou Fang is working as a Professor at Tianjin University. His research interests are in the fields of ultraprecision machining, optical freeform machining, micro/nano machining, and metrology. He is the founding president of the International Society for Nanomanufacturing (ISNM), the editor-in-chief for the International *Journal of Nanomanufacturing* (IJNM), and a fellow of the International Academy for Production Engineering (CIRP).

Ying Cheng is currently a PhD candidate working on non-imaging optics, optical freeform design and manufacturing.



Xiaodong Zhang is working as an Associate Professor in the Centre of MicroNano Manufacturing Technology at Tianjin University. He holds 20 patents and has published more than 40 publications, including one book. His research interests include freeform optics design, ultraprecision machining and metrology, freeform optics manufacture and applications.