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Review Article

Phase space approach to the use of integrator rods and optical arrays in illumination systems

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Abstract

Integrator rods and optical arrays are the most frequently used components in illumination design for homogenizing radiation fields. However, these two standard components are very different in their performance and characteristics. This tutorial is aimed to illustrate the operation principle, basic design rules and the performance of those components. It should guide the optical designer towards the optimum choice for the individual illumination application. To illustrate the functionality of integrator rods and optical arrays simultaneously in angle and position, the concept of phase space is introduced. Here the effect of the homogenizing components can nicely be illustrated in the form of phase space transformations. This offers new insight and a different perspective onto the employment and characteristics of those elements.

Keywords: illumination design; lithography; micro-optics; optical integrators; phase space.

1. Introduction

In imaging optical design, the standard component is a spherical lens – fundamental design types and most aberration theory is built upon this single standard optical component. Illumination design is different. Typically, the light source, the system requirements and the application of illumination systems can vary over a wide range. Therefore, in addition, the employed optical elements will usually be very different. However, if an illumination designer is asked for those elements which are most frequently used to homogenize light distributions, he will most likely list homogenizer rods and optical arrays on top of the list. It is therefore worthwhile to understand the operation principle and the characteristic of those elements for everybody who tries to achieve homogeneous illumination within an optical system.

The history of employing these elements in illumination design is very long, and for integrator rods as well as optical arrays it is difficult to specify the 'inventor' of those components. Hollow reflective triangular or rectangular elements have probably already been known by the ancient Greeks and were reinvented under the label 'kaleidoscope' by Brewster in the beginning of the 19th century [1]. Early use of integrating rods in technical illumination systems is known from Kodak in the early 20th century within projector systems [2]. Later, rods were proposed and used for microscopy illumination [3–5], and lithographic illumination systems [6, 7]. Even in the current laser lithography illumination systems integrator rods are used for homogenization of laser light sources, often in combination with an optical array [8, 9].

A similar picture exists for the use of optical arrays. Paul Cark suggested in 1905 [10] an arrangement of prism or mirrors to 'shuffle' the light. Some years later, he and Henry Gage independently suggested the use of facetted mixing systems to improve the homogeneity of a projector [11, 12]. A double facetted optical array was suggested by Mechau [13] and also by Räntsch and colleagues [14, 15]. In microscopy double facetted integrators are also known as 'Köhler integrators' as each channel provides a Köhler illumination [16]. Since the beginning of microlithography optical arrays are used for lithography illumination systems and mask aligners. Only recently the use of high performance optical arrays allowed the improvement of mask aligners [17], as well the design of lithographic illumination systems with very complex and variable pupil shapes, as, for example, used in modern source-mask optimized lithographic processes [18, 19]. Moreover, variations of optical arrays in terms of gratings and holographic elements have been suggested [20, 21]. Recently, Köhler integrators have also been proposed to improve the performance of complex solar concentrators [22, 23].

2. The concept of phase space

Integrator rods and optical arrays will, in general, affect the spatial light distribution as well as the angular distribution of the interacting radiation field. It is therefore desirable to find an optical system representation, where the effect on ray angles and ray positions can simultaneously be observed and illustrated. The concept of phase space in optics provides such a platform [24, 25]. Phase space methods are well known and extensively used in classical mechanics and quantum mechanics [26], where position and the velocity of a particle define its location in phase space. In optics, the position and velocity

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of a particle are replaced by position and angle of a single ray within an optical system. Thus, ray tracing within optical systems corresponds to phase space trajectories leading to a transformation of radiation distributions in phase space. An analysis of this phase space transformation provides a complete picture of the optical functionality of the optical system, however, from a different perspective as compared to standard ray-tracing pictures. It turns out that in particular for illumination design problems, where the general transport of radiance is important and not the transfer of a spatial pattern, phase space provides an interesting access towards illumination design. However, to be able to follow this concept we first need to cover some basic properties of phase space.

2.1. Phase space volume and flux distribution: etendue and radiance

As we are dealing with illumination problems we need to understand the basic connection of radiometry [27] and phase space [28, 29]. Let us consider a source, or generally a radiation field, located at the origin of the coordinate system as shown in Figure 1. The phase space volume occupied by this source is defined by its spatial and angular extend and thus follows from integration over the relevant area and solid angle. This quantity is called the etendue [30, 31]:

$$Etendue=n^2\iint \cos(\theta)\ dA\ d\Omega$$

Here, n is the index of refraction and θ is the angle between the normal of the differential area dA and the centroid of the differential solid angle $d\Omega$. In phase space, the etendue is conveniently expressed in terms of the projected solid angle du and dv, also containing the refractive index of the medium, thus etendue can be expressed as:

$$Etendue=\iint dx\ dy\ du\ dv$$

Figure 1 illustrates a differential entendue element and its relation to solid angle and projected solid angle. The amount of flux, or optical power $d\phi$, contained in a certain phase space volume element then defines the radiance distribution of the source, i.e., the 'energetic' weight within phase space:

$$L(x,y,u,v)=\frac{d\phi}{dx\ dy\ du\ dv}$$

In general, the radiance distribution [32] is a four-dimensional function of the phase space variables (x, y, u, v). However, as four dimensions are very difficult to visualize we will restrict ourselves within the context of this tutorial to two dimensions. In other words, we will only consider light distributions or ray patterns in the xz-plane. Thus, the radiance L is a function of x and u only. In this case, the angular variable u is associated with $\sin(\theta)$ of a ray relative to optical axis as apparent from Figure 1.

From the radiance distribution $L(x, u)$ the radiant intensity $I(u)$ (flux per projected solid angle) and the irradiance $E(x)$ (flux per area) can be calculated by integration over the spatial and angular dimension. In phase space, this corresponds to a projection of the radiance to the corresponding axis, as illustrated in Figure 2.

For geometrical optical systems it follows from the Lagrange invariant [33], i.e., $du\ dx=const$, that phase space volume or etendue is conserved. As a consequence, as flux is conserved within lossless systems, the radiance itself is also conserved. Therefore, a geometrical optical system, with only refractive and reflective elements, will only be able to redistribute phase space volume elements and its associated flux, but cannot change the total occupied volume, or the flux associated with each phase space element. Thus, as we will see, geometrical optical systems can only lead to a deformation or rearrangement of phase space distributions.

Currently, we need to note that the radiance as described above follows from empiric radiometric definitions. As we are relating this radiance to the concept of phase space, it is important to note that not every radiance distribution is consistent with a rigorous electromagnetic treatment of radiation [33]. A physical correct calculation of the generalized radiance distribution, or Wigner function [34] of an arbitrary light field is rather complex, as it contains all coherence properties of the

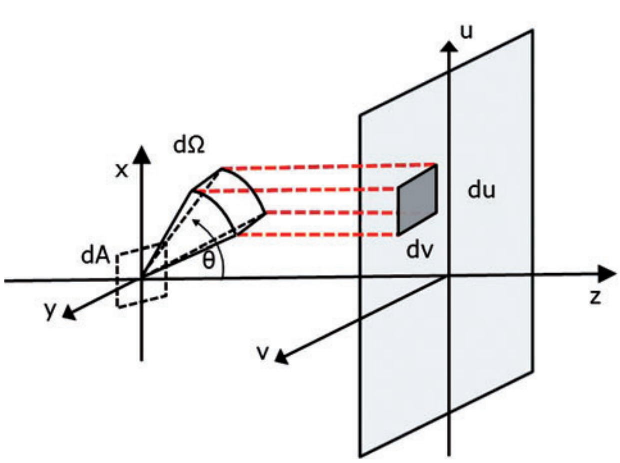

Figure 1 Illustration of a phase space volume element, expressed in solid angle and projected solid angle.

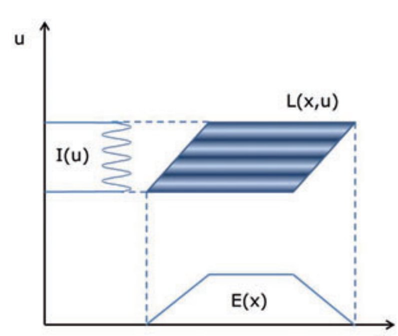

Figure 2 Illustration of a radiance distribution $L(x,u)$ in phase space. The projection of the radiance distribution onto the spatial or angular axis corresponds to the irradiance $E(x)$ and radiant intensity $I(u)$.

source [35]. However, as this is a tutorial, we will not concentrate on the calculation of the correct radiance distribution of a physical source, but are more interested in the propagation of this distribution through an optical system, as this will help us in understanding the optical functionality. Therefore, throughout this paper we will use simplified radiance distributions, even those that might not correspond to real physical sources. For those readers who are more interested in a thorough discussion of the Wigner function and its phase space behavior, we refer to the work of Bastiaans [36], Brenner and Ojeda-Castaneda [37], or the book by Testorf et al. [24].

2.2. Propagation laws for phase space distributions

To study the effect of optical elements within phase space it is necessary to understand the propagation laws for phase space distributions. It generally follows from the superposition principle of linear systems that the propagation of a phase space distribution $F(x, u)$ through an optical system can be expressed as [36]:

$$F_o(x_o,u_o)=\frac{1}{N}\iint K(x_o,u_o,x_i,u_i)F_i(x_i,u_i)dx_i du_i \quad (1)$$

Here, the kernel K describes the systems answer to a δ function in position and in angle. We will now associate these δ functions with a single optical ray and the kernel K with the ray transfer matrix of the optical system, describing the transformation of an input ray with a certain angle and position (x_i,u_i) onto an output ray (x_o,u_o).

Note that this is a severe approximation, as an optical light field, consisting of a single ray and a δ function in angle and position, is highly nonphysical and violates Heisenberg's uncertainty principle. As a consequence, the ray-based approach presented here does not contain any diffraction or interference effects, as it corresponds to the geometrical limit of optics. A more rigorous phase space approach has to be based on the Wigner distribution function [24, 34]. The resulting complex propagation laws will include all coherence, interference and diffraction effect.

Although we will mention some diffraction and interference effects during the discussion of the homogenizing elements, from here on in we will restrict ourselves to the geometrical optical limit of phase space. In this picture, it is sufficient to study the geometrical optical ray transfer matrix of the systems.

In the case of paraxial optical system, the ray transfer matrix is related to the *ABCD* matrix formalism of optical systems [28, 38], as the *ABCD* matrix will transform an input ray of a given position x_i and a given angle u_i to the output position x_o and angle u_o *via* the relation:

$$\begin{pmatrix} x_o \\ u_o \end{pmatrix}=\underbrace{\begin{pmatrix} A & B \\ C & D \end{pmatrix}}_{M}\begin{pmatrix} x_i \\ u_i \end{pmatrix}$$

Therefore, it follows for a lossless first-order optical system that the input-output relation simply follows the *ABCD* formalism and reduces to:

$$F_o(x_o,u_o)=F_i(Ax_i+Bu_i,Cx_i+Cu_i) \quad (2)$$

The most important cases of first-order optical systems treated in this tutorial correspond to a free propagation (M_z) along a distance z, propagation through a thin lens (M_f) of focal length f, and reflection of a plane mirror (M_m) oriented along the z-direction. Therefore, we list the corresponding *ABCD* matrices for those cases, as follows:

$$M_z=\begin{pmatrix} 1 & z \\ 0 & 1 \end{pmatrix}\; M_f=\begin{pmatrix} 1 & 0 \\ \frac{-1}{f} & 1 \end{pmatrix}\; M_m=\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\; M_r=\begin{pmatrix} 0 & f \\ \frac{-1}{f} & 0 \end{pmatrix} \quad (3)$$

As another important arrangement within optical systems is the propagation from the front focal plane of a thin lens to the back focal plane of the same lens, i.e., a Fourier lens, above we also list the corresponding matrix (M_r). This matrix results from sequential application of the free propagation, thin lens and free propagation matrices and corresponds to a rotation in phase space.

As, in particular, for integrator rods the angle u is not necessarily small enough to fulfill a paraxial approach, we note that the exact free space propagation for arbitrary angles corresponds to:

$$F_o(x_o,u_o)=F_i\left(x_i+\frac{u_i z}{\sqrt{1-u_i^2}},u_i\right) \quad (4)$$

which results from the free space trajectory of a ray with arbitrary angle θ, as the above expression containing the square root, just corresponds to the $\tan(\theta)$ of the ray. In a similar way, optical aberrations can be included as additional nonlinear terms in the transport equations [39].

2.3. Illustration of phase space propagation

According to the above principle the propagation of phase space distributions, as, e.g., the radiance distribution $L(x,u)$, can be easily illustrated. Let us first consider a single ray propagating through an optical system. At each z-position inside the optical system the position and the angle of the ray can be calculated from ray tracing (or, for paraxial systems, follows from the *ABCD* matrix formalism). A single ray may be associated with a point in phase space. Therefore, the propagation of the ray through the optical system corresponds to a trajectory in phase space as illustrated in Figure 3.

Therefore, ray tracing through an optical system will yield the corresponding phase space trajectories. Now the phase space distribution is 'attached' to the rays and will follow the rays and the phase space trajectories, mathematically follows from Eq. (1). Thus, the input phase space distribution (or radiance distribution) will be deformed according to the phase space trajectories resulting from ray tracing through the system.

Employing the *ABCD* matrix formalism of Eq. (2) and Eq. (3) or the free space propagation, Eq. (4), we can thus illustrate different transformations corresponding to optical systems that will be important for the understanding of rod integrators and optical arrays in Figure 4.

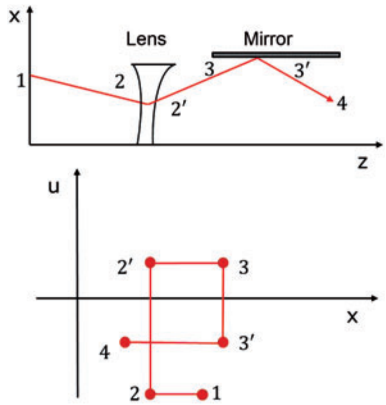

Figure 3 Ray tracing of a single ray through an optical system and the corresponding trajectories in phase space.

From Figure 4, we note that paraxial free propagation and the action of a thin lens will result in a shear of the initial distribution. By contrast, nonparaxial propagation results in a distortion of the distribution for large angles *u*. The propagation from the front to the back focal plane of a lens corresponds to a rotation of the initial distribution. Finally, for propagation in the presence of a mirror at position x_m, as illustrated in Figure 3, the distribution will be back-fold at the mirror to the front of the mirror. As we will see these basic transformations are sufficient to allow a full understanding of complex systems, such as integrator rods and optical arrays.

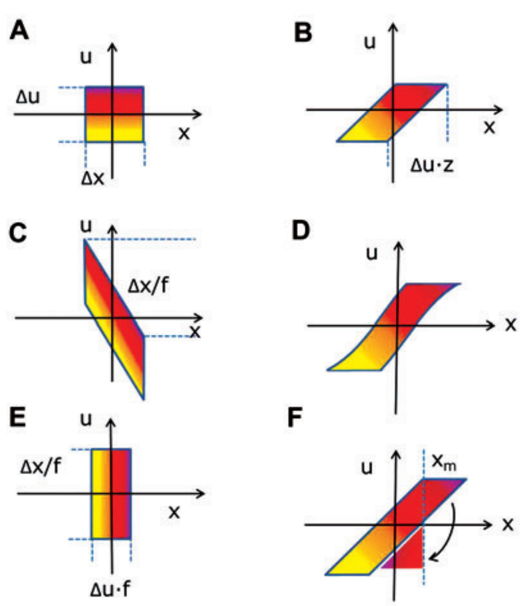

Figure 4 Phase space transformations: (A) initial distribution, (B) free paraxial propagation along a distance *z* along the optical axis, (C) propagation through a thin lens of focal length *f*, (D) free space propagation over a distance *z* for nonparaxial angles, (E) propagation from the front focal plane of a lens of focal length *f* to the back focal plane of the lens, (F) free space propagation in combination with a reflection of a plane mirror parallel to the optical axis.

3. Integrator rods

3.1. Basic operation principle of integrator rods

The operation principle of a rod integrator is based on employing multiple reflections inside a solid or hollow light guide to mix the incoming light distribution. The reflection in solid rods is typically based on total internal reflection (TIR) within the medium, such as fused silica, optical glass or plastic [40], but also hollow light guides with reflective coatings can be used. The cross-section geometry can vary from quadratic, to hexagonal, or round, whereas the shape along the *z*-direction is typically flat or conically, which is then called a tapered light pipe.

The optical functionality is best illustrated by the simple case of a rectangular rod. Let us assume a point source with some angular divergence entering the rod as shown in Figure 5A. As the sides of the rod act as plane mirrors, the rays will mix as they propagate through the rod. To obtain a better understanding of the effect the ray tracing can be unfolded and then corresponds to a free propagation. In Figure 5B, this is illustrated for the propagation of a point source with Gaussian angular distribution. The angular distribution will spread the beam as the light is propagating along the *z*-direction. However, in the presence of the reflecting faces of the rod, the distribution will be back-folded and superimposed at the exit of the rod. Therefore, different portions of the distribution will be added which results in a homogeneous intensity distribution at the exit [41].

The effect of the rod can also, however, be viewed from a different perspective. If we 'look into the rod' from the exit face, as illustrated in Figure 5C, we will see multiple reflections of the entrance distribution. This effect is the 'kaleidoscope' effect, as known from children's toys, creating multiple patterns of the input distribution. Therefore, the

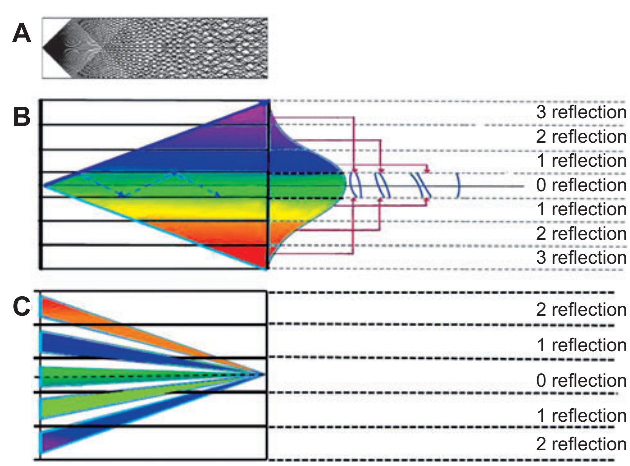

Figure 5 Illustration of the optical functionality of a mixing rod: (A) a ray-tracing picture of an integrator rod, illustrating the effect of homogenization, (B) 'unfolded' light propagation inside the rod and superposition of different parts, due to multiple reflections at the rod faces, (C) view into the rod from one field point, illustrating the facetted angular spectrum resulting from multiple reflections of the entrance face distribution.

angular distribution at each point of the rod exit will be structured, each pattern resembling the intensity distribution at the entrance of the rod.

This already illustrates that the exit light distribution from an integrator rod in the general case will be nontrivial and can only for some geometries be easily derived without extensive simulation. Moreover, angular and spatial distributions are entangled.

To provide a more intuitive understanding of the element, in the following section, we will therefore chose a phase space approach to look at the optical transformation resulting from the rod.

3.2. Illustration of integrator rods in phase space

We will now employ the phase space transformations derived in Section 2 and illustrated in Figure 4 to derive the phase space distribution at the exit of an integrator rod. Note that this final phase space distribution contains all information about the angular and spatial properties of the light exiting the rod.

The phase space transformation of the rod only consists of two basic operations, namely the free propagation inside the rod, as illustrated in Figures 4B and 4D, and the action of the reflecting side walls of the rod, as illustrated in Figure 4E.

In Figure 6, we apply these two basic operations onto the input distribution at the entrance of the rod (Figure 6A). In the absence of the rod side walls this initial distribution will be just subject to free propagation. A free propagation corresponds to a shear in phase space. Thus, the input radiance distribution will be sheared as the light is propagating over the length of the rod. This is illustrated in Figure 6B. If we now include the effect of the reflections at the sides of the rod, the radiance distribution will be folded back and mirrored in angle each time the radiance distribution is reflected. Taking both effects together, the phase space distribution at the exit of the rod corresponds to the sheared, multiple folded, and mirrored free-propagation radiance distribution, as illustrated in Figure 6C.

From Figure 6C, both the homogenizing effect as well as the kaleidoscope effect can be easily understood in one picture. The resulting irradiance distribution, shown at the bottom of Figure 6C, is fairly homogeneous, as it corresponds to the superposition of segmented and shifted parts of the free-propagated phase space distribution. Similarly the angular distribution at the exit of the rod can be understood. At each point x of the rod, the angular distribution is fragmented into multiple segments, corresponding to the multiple reflections of the distribution. By contrast, the integral angular distribution, shown to the left of Figure 6A, of the exiting light is basically unchanged, however, symmetrized due to the even and odd number of reflections at the rod side walls. This symmetrizing effect is nicely visible by the color-coded phase distributions in Figure 6. Although the input angular distribution is coded from blue to red, this order is mixed within the final distribution.

Figure 6 even reveals more complex details of the action of the integrator rod. The nonparaxial angles inside the rod will, in addition to the paraxial shear of the distribution, lead to a slight distortion of the distribution as it propagates inside the rod – compare Eq. (4). This nonlinear behavior is visible in the angular spectrum of the final distribution, as the kaleidoscope patterns will be not equidistant in angle anymore, as the angles become large.

The phase space picture also nicely reveals the nature of the element in phase space. If the input radiance distribution does not completely fill the rod in angle and position, the rod will pattern the accessible phase space, and such homogenize the light. Thus, the phase space is not continuously filled but rather thinned or diluted. Therefore, a rod, as it is based on reflection only, will not increase etendue, but rather dilute it.

3.3. Performance factors and sensitivities of integrator rods

Clearly the homogenization improves with the number of reflections m at the rod faces. This number depends on the ray angle θ relative to the axis of the rod and on the aspect ratio between length L and width b of the rod according to $m=L\tan(\theta)/b$. This implies that to achieve a large number of reflections the light needs to be tightly focused into the rod and numerical apertures of NA=0.5–0.7 at the rod entrance are common. In addition, a long and narrow rod will yield a better homogenization. However, the aspect ratio is restricted

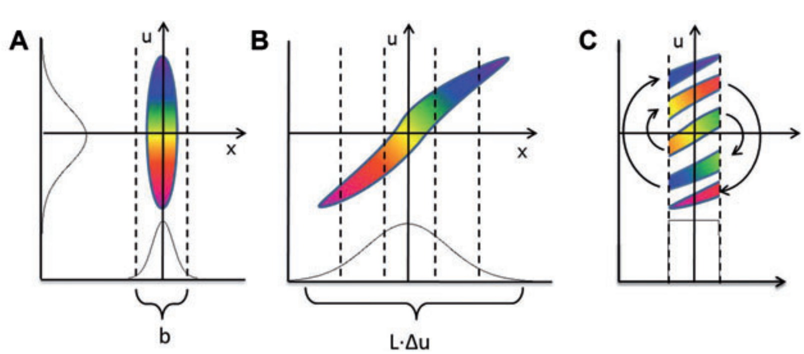

Figure 6 Phase space illustration of the optical transformation introduced by an integrating rod: (A) initial phase space distribution at the entrance to the rod, (B) distribution after free propagation over the length of the rod, (C) final distribution due to reflections at the rod sidewalls.

by fabrication requirements and typically the ratio of diameter to length is in the order of 1:10–1:15. As a consequence, in typical rod homogenizers the maximum number of reflections is approximately 10–15 for the extreme angles.

As the resulting homogeneity at the exit of the rod is determined by the light distribution corresponding to free propagation along the length of the rod, the achievable homogeneity is somewhat dependent on this distribution. For a broad and smooth distribution, homogenization will usually work rather well, if the number of reflections is sufficiently high. Moreover, as the rod will with each reflection invert the corresponding segment of the light distribution, tilts in the input distribution will be averaged out. More critical are large variations of the angular distribution for small angles, as here the number of reflections is low and thus the averaging effect is limited.

4. Optical arrays and Köhler integrators

4.1. Operation principle of optical arrays

In general, an optical array consists of multiple (typically equal) parallel small lenses, or micro-lenses. The incoming light is separated by the individual lens apertures into different channels. The light in each channel is then refracted by the individual lens. All channels are subsequently superimposed at the back focal plane of an integrator lens, as illustrated in Figure 7.

If we consider only one single channel, we notice that each channel forms an afocal telescope or beam expander. The input intensity distribution is thus segmented into patches according to the pitch p of the array, which are then expanded and superimposed at the back focal plane of the integrator lens. Thus, the width w at the target plane is:

$$w = p \times F/f \quad (5)$$

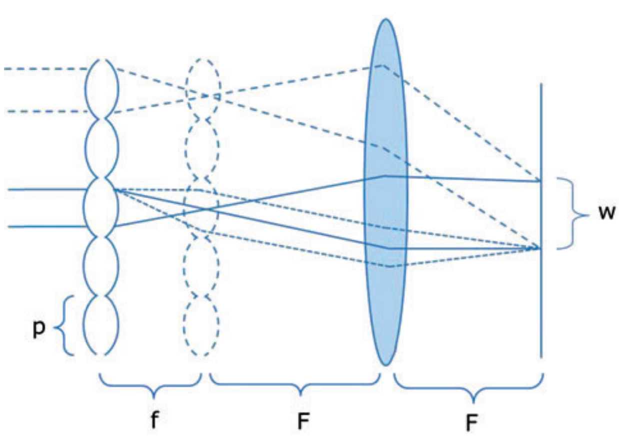

Figure 7 General operation principle of an optical array. The input light distribution is separated by a first optical array, consisting of micro-lenses of pitch p and focal length f into multiple channels, which are superimposed with the help of an integrator lens of focal length F. An additional second array of micro-lenses can be used to image the first array onto the target plane.

where f and F are the focal length of the micro-lens and the integrator lens. However, the aperture stop of each channel is defined by the entrance diameter of each micro-lens and thus is located at the micro-lens array itself. This fact results in sensitivity of this setup towards divergence, or pointing, in front of the array. Angles in front of the array will shift the light distribution at the target plane. Thus, pointing will result in a shift of the distribution and divergence results in a convolution of the distribution with the divergence spectrum.

This disadvantage can be overcome by using an additional second optical array, typically located at the focal plane of the first array. This second array of micro-lenses then acts as a field lens array, and will image the stop and the first array, onto the target plane. As a result, the arrangement will be insensitive to angles in front of the array.

As generally, the far field of the light source is used as an input to the optical array, a double array will image that far field onto the illumination field. Therefore, double optical arrays of this type are also called Köhler integrators, as they provide a 'Köhler' type illumination [16].

4.2. Phase space illustration of optical arrays

Similar to the rod integrator in Section 4, we will now employ the phase space transformations developed in Section 2 and illustrated in Figure 4 to derive the final phase space distribution at the exit of an optical array. To do so, let us first consider step-by-step the phase space transformation performed by one single channel of the array.

Figure 8 illustrates the consecutive operations performed on the input phase space distribution within one channel. Figure 8B shows the phase space distribution right behind the first micro-lens. In paraxial approximation, the micro-lens will shear the input distribution, introducing an angular spectrum of width p/f, where p is the width of one channel and f is the focal length. Propagation to the focal plane of the micro-lens will result in a rotated and sheared distribution, where the residual shear is due to the initial angular spectrum Δu, as illustrated in Figure 8C. By contrast, Figure 8D shows the situation if a second micro-lens is employed, i.e., a double optical array, or a Köhler integrator. In this case, the residual shear is removed, as the second micro-lens introduces a compensating amount of shear into the phase space distribution. Finally, the distribution will be propagated from the front focal length of the integrator lens to the back focal length. This corresponds to a rotation in phase space and scaling with focal length F of the integrator lens, as illustrated in Figure 8E for a single array and in Figure 8F for a double array.

This simple analysis of the phase space transformation induced by an optical array already allows qualitative and quantitative understanding of the system. Following from the basic phase space operations derived in Section 2, we found that the final spatial width of the irradiance distribution is given by $w=p \times F/f$, in accordance with Eq. (5). Additionally, the benefit of a double array is immediately clear, as it allows to remove the residual shear of the distribution, resulting in a smear of the irradiance distribution of the amount $\Delta w = \Delta u \times F$. Thus, a double array will be able to deliver an irradiance distribution with

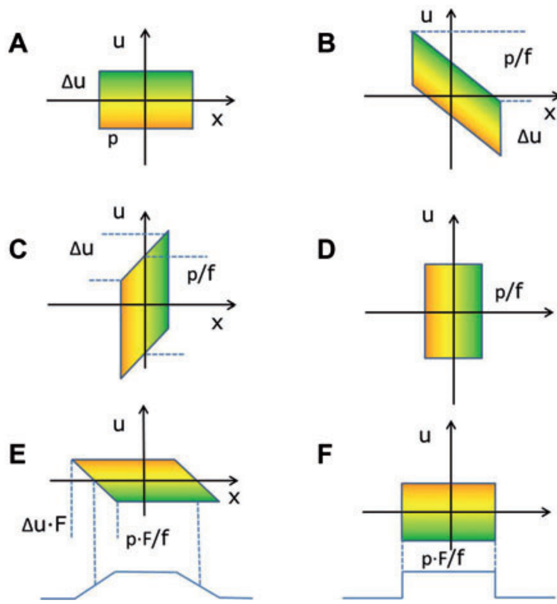

Figure 8 Phase space transformations within a single channel of an optical array: (A) initial distribution, (B) distribution directly behind the first micro-lens, (C) distribution at the focal plane of the first micro-lens, (D) same as before but in the presence of a second micro-lens, (E) final distribution at the back focal plane of the integrator lens for a single micro-lens array, (F) same as before, but for a double micro-lens array.

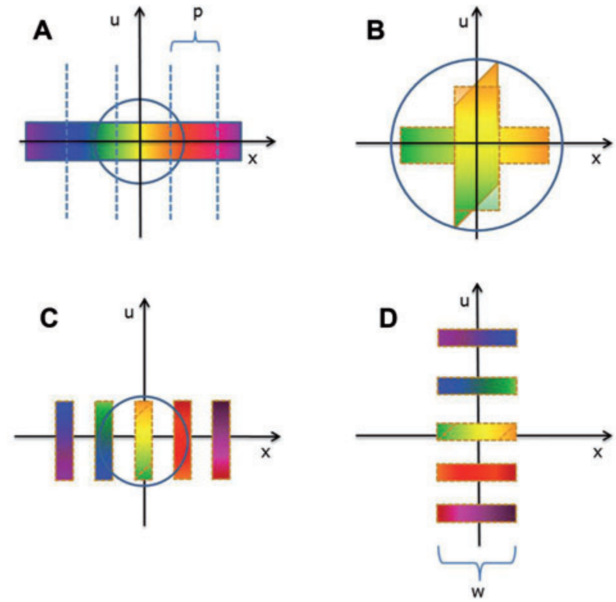

Figure 9 Phase space transformation for an optical array: (A) initial distribution segmented according to the pitch *p* of the array, (B) rotation of the phase space segment within each channel, for a double array (solid) and single array (dotted), (C) phase space distribution in the back focal plane of the array, (D) final phase space distribution at the target plane of width *w*.

a well-defined and sharp width, whereas the edges of the single array distribution are smeared out. This effect is obvious from the resulting spatial irradiance distribution at the exit of the optical array, as illustrated at the bottom of Figure 8.

Having analyzed the transformation of a single channel we can now derive the transformation of the full optical array, as illustrated in Figure 9. The initial distribution in phase space is segmented according to the lateral extend of each channel, as shown in Figure 9A. While propagating to the focal plane of the first micro-lens, each segment of the optical array will then result in a rotation as described above, in Figures 8C and 8D, respectively. This is illustrated in Figure 9B. Note again that a single optical array will exhibit some residual shear of the input distribution, whereas a double array will produce a full rotation due to the action of the second micro-lens. Owing to the periodicity of the arrangement, these effects will repeat itself within each of the channels and thus create a comb-like structure in phase space, as shown in Figure 9C. As a final step, after the array the Fourier (or integrator) lens will introduce a rotation of the complete comb-like distribution in phase space, and finally produce the phase space distribution at the exit, as illustrated in Figure 9D.

This reveals the general homogenization functionality. At the final image plane, the spatial distribution consists of the sum over the phase space segments corresponding to each channel, thus providing homogenization in the spatial domain. By contrast, the angular spectrum at each field point in x will exhibit an identical periodic pattern, each segment corresponding to the angular divergence entering the channel.

Therefore, all features of the integration mechanism in an optical array integrator can be well understood from the phase space approach.

4.3. Performance factors and sensitivities of optical arrays

The performance of the array will improve as the number of channels is increased, thus it is desirable to reduce the size of the micro-lenses. As, generally, the focal length F of the integrator lens is fixed by track length constraints the geometrically introduced divergence angle $\sin(\theta)=p/f$ should be constant as the size of the micro-lens is reduced, to maintain the same illumination field width w. However, there are limitations in the minimum size of one channel. The first limitations arise from fabrication methods, and another limitation is due to diffraction. It should be noted that the geometrical optical treatment presented here does not include diffraction. However, as we are dealing with micro-lenses of some small dimension p diffraction becomes relevant and we need to compare the angles introduced by diffraction to the geometrical induced divergence angle. To be able to neglect divergence effects the pitch should be chosen large enough such that geometrical divergence is dominating, i.e. $p/f>>\lambda/p$. Another related point to consider if optical arrays are used in combination with coherent or partial coherent light (such as lasers) is that the periodicity of the array will lead to periodic diffraction angles of the angular period λ/p due to interference of neighboring channels [42].

In the case of double micro-lens arrays the maximum acceptable divergence into the integrator is limited, as the light focused by the first array needs to remain within the same channel. Therefore, the maximum allowed angle entering the double array is given by $\sin(\theta_{max}) \leq p/f$. The source and collimator arrangement in front of the array thus has to be designed to fulfill this requirement on collimation.

As apparent from Figure 9, an optical integrator will segment an input light distribution into a large number of segments (corresponding to the number of channels) and will superimpose these segments at the image plane. However, as is also apparent from the illustration there are no reflections or other symmetrizing effects involved. As a consequence, within each channel left and right (top and bottom, respectively) will be transferred with the same orientation to the target plane. Therefore, arrays are somewhat sensitive to irradiance tilts in the input distribution. However, in practice and for two-dimensional arrays this is typically not a severe limitation. One way to avoid this sensitivity is to use alternating arrays of positive and negative micro-lenses, as then the image of each second channel will be inverted. Currently, in general, a large variety of sizes and geometries for optical arrays can be manufactured and we refer to the supplier for further information [43].

5. Applications

5.1. Light homogenization

The main application for integrator rods and arrays is to homogenize light distributions. Therefore, both components are widely used in projection optical systems [44]. Here, both components are in general well suited for homogenization; however, they will differ in their performance and will also influence the optical system design, as illustrated in Figure 10.

Typically, the source light distribution (1) needs to be conditioned before entering the integrating device (3). An optical array will require a collimated input beam, thus a collimator system (2) will be employed. For a rod integrator usually the source is re-imaged with the help of a relay system (2) onto the entrance of the rod, also providing the required large numerical aperture into the rod.

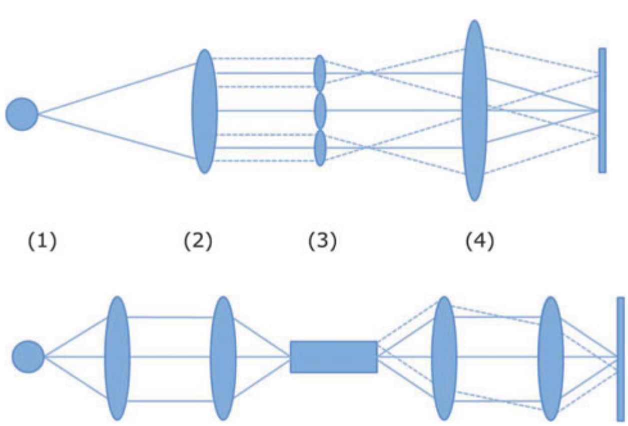

Figure 10 Typical system layout for array integrator and rod systems. Typically the light source (1) needs to be collimated and re-imaged (2). After the integrating element (3) a Fourier lens or relay system (4) is required to produce an accessible illumination field.

Similarly at the exit of the integrating element an adapted optical system is required. For optical arrays a Fourier lens (4) is needed to create the homogeneous illumination at the target plane (which is the back focal length of the Fourier lens). Usually the focal length of the Fourier optics together with the optical array can be chosen such that the image plane has the proper field size. In the case of the rod only very rarely the rod exit face can serve as the final image plane, therefore another relay system is required to re-image the rod-exit onto an accessible target plane, in many cases also adapting the field size.

In general, thus, an array integrator system can be more compact in length as compared to a rod system.

5.2. Color mixing

Both elements can be used as color mixing devices; however, the integrator rod is less sensitive to chromatic effects. It is apparent from the basic operation principle that rods mostly relay on reflection, rather than refraction. In fact, hollow rods are completely free of chromatic effects, as they are pure reflective systems.

In TIR homogenizers the only surfaces introducing chromatic aberrations are the entrance and the exit surface leading to a slight difference in the angular spectrum inside the rod. This difference will lead to a shift of the kaleidoscope pattern with wavelength. Therefore, the angular pattern at the exit of the rod will exhibit a chromatic structure. Besides that there is no major chromatic hit in performance, such that integrator rods are widely used for color mixing. For example, rod systems are used in projector systems for mixing of RGB laser diodes or LEDs to achieve white illumination at the exit of the rod.

By contrast, optical arrays are based on refraction and are as a consequence more affected by chromatic effects. The main chromatic effect will be the dependence of the focal length of the micro-lens on the wavelength. According to Eq. (4), a change in the focal length will affect the width of the illuminated area. Therefore, the main chromatic effect in refractive micro-lens arrays is a color-dependent illumination field width, visible as a colored edge of the illumination field. Besides that the chromatic aberrations will not fundamentally affect the functionality of the array; therefore, for a certain spectral width the mixing in the central area of the illumination field will be unaffected. Thus, it depends on the application and on the spectral range if the described effects, e.g., the colored edge, can be tolerated and arrays are a valid choice for color mixing.

5.3. Lithography systems

Integrator rods as well as optical arrays are also used in most lithography illumination systems and mask aligners. However, the purpose of the elements is not only to homogenize the light distribution but also to 'copy' (integrate) the illumination pupil across the illumination field. This is of crucial importance, as any lithographic process requires equal image transfer at any point in the field, thus intensity and illumination angles, i.e., the pupil, must be invariant.

A typical lithography illumination system concept is in agreement with the general system architecture shown in Figure 10. In a first step the desired illumination pupil, i.e., the desired illumination angles at the mask, is generated. This pupil generation typically is performed within the collimated beam path of element (2) in Figure 10, with the help of complex apertures or optical elements. This pupil shape then needs to be copied across the illumination field to provide: (i) uniform intensity across the field and (ii) an identical illumination pupil across the field. The integrating element (rod or array) thus needs to 'copy' the input pupil across the illumination field. Therefore, it is a requirement that the optical integrator does preserve the shape of the input pupil. This is of particular importance in the case of more complex illumination pupils, as, e.g., used in source mask optimized lithographic processes, where very complex masks and also very complex illumination pupils are used [19].

As explained in Section 3, an integrator rod will due to its operation principle symmetrize the angles, i.e., the pupil, and also lead to some field-dependent pupil segments. By contrast, an optical array integrator will create an exact field invariant copy of the input pupil.

As a consequence, integrator rods are not very well suited for complex, in particular nonsymmetric illumination pupils used in modern source mask optimized lithographic processes [45, 46]. This is one of the reasons why the latest generation of lithography illumination systems does employ arrays instead of rod integrators.

6. Summary

Within this tutorial our attempt was to introduce and apply simple phase space methods to provide insight into the operation principle of mixing rods and optical arrays – the two most frequently employed components in illumination design. To share this insight with non-experts in the field we have tried to avoid complex mathematical discussions but rather concentrated on a more illustrative insight and also on the geometrical optical limit. We aligned this phase space approach with a standard ray-based explanation of the operation principle of the components. It turned out that the phase space perspective can provide a rather efficient understanding of the functionality of those elements, as angular and spatial properties are revealed in one picture. This allows access to even complex illumination systems, as, e.g., used in lithography.

This should be encouraging to the community and is definitely encouraging to us, to extend phase space methods towards the design and understanding of more complex light mixing and concentrating devices, even so this requires a full four-dimensional treatment in phase space. The four-dimensional treatment of complex rod or array geometries was beyond the scope of this tutorial. However, it is still our general belief that phase space is a solid basis for solving illumination design problems, similarly as aberration theory forms a basis for optical imaging design.

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