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# A new spectrally-invariant approach to the remote sensing of inhomogeneous clouds

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Current operational satellite retrievals of cloud optical and microphysical properties go back to the Nakajima-King technique developed at the end of the 1980 s. This technique is based on library calculations for plane-parallel homogeneous clouds. It often works well for overcast skies but leads to substantial errors for inhomogeneous and broken cloud fields where the plane-parallel-geometry assumption is no longer valid. The basic concept of a new technique was first introduced in nuclear physics to quantify the critical conditions of reactors. Based on the eigenvalues of the radiative transfer equation, their approach provides a powerful means to parameterize the structure of 3D media. This parameterization was later successfully applied to relate surface reflectance spectra to 3D canopy structure and is known as the spectrally-invariant approximation. The proposed approach adapts this technique to the remote sensing of cloud properties such as droplet single scattering albedo and the average number of scatterings (which are the fundamental parameters in radiative transfer theory), with an emphasis on guantifying the associated errors and uncertainties. This retrieval is free from the plane-parallel homogeneous cloud assumption.

#### KEYWORDS

clouds, remote sensing, number of scatterings, single scattering albedo, spectral-invariant

#### **1** Introduction

Clouds cover roughly two thirds of the globe. Reflecting and absorbing solar and thermal radiation, they strongly affect the Earth's energy (Rossow et al., 2002). The way this budget is modulated depends, among other factors, on the cloud particle size distribution. Since clouds contribute the largest uncertainty to estimates of the Earth's changing energy budget, accurate retrievals of particle size from satellite observations is one of the most critical goals in cloud remote sensing. Indeed, cloud particle size has not only a significant influence on cloud response to aerosol modification (Várnai and Marshak, 2002; Oreopoulos and Platnick, 2008; Spencer et al., 2019), but it is also a key parameter for understanding aerosol-cloud-precipitation interactions (e.g., Albrecht, 1989; Twomey, 2007).

The current approach to simultaneously estimating cloud optical depth  $\tau$  (vertical integral of cloud extinction coefficient over cloud physical thickness) and the effective particle radius  $r_{\text{eff}}$  (the ratio of the 3rd to the 2nd moment of particle size distribution) goes back to Twomey and Seton (1980). Their method was further developed and popularized by



Nakajima and King (1990). Since then, this approach has been thoroughly studied in both theory and practice (see Platnick et al., 2003; Platnick et al., 2017 and references therein) and has been applied to a variety of observations, for example to Advanced Very High Resolution Radiometer (AVHRR) and Moderate Resolution Imaging Spectroradiometer (MODIS) data. The approach uses reflectances from two narrow spectral bands: one in the visible (or near-infrared) region where cloud absorption is negligible, and the other in a near-infrared region where solar radiation is slightly absorbed by cloud particles. When the two reflectance measurements are combined, both  $\tau$  and  $r_{\rm eff}$  can be determined (Nakajima and King, 1990).

The retrievals are based on library calculations for plane-parallel homogeneous clouds (Platnick et al., 2003). The underlying assumption of plane-parallel clouds and the use of a one-dimensional (1D) radiative transfer forward model can introduce substantial errors for retrievals applied to inhomogeneous cloud scenes (e.g., Loeb and Coakley, 1998; Zuidema and Evans, 1998; Stephens and Kummerow, 2007; Evans et al., 2008; Di Girolamo et al., 2010; Wolters et al., 2010). Left and middle panels in Figure 1, reproduced from Zhang and Platnick (2011), show the global monthly means of effective radii retrieved using a visible band in combination with the 2.1  $\mu$ m and 3.7  $\mu$ m MODIS bands, respectively.

Figure 1 shows the dramatic difference between  $r_{\rm eff}$  retrieved from 2.1 µm to 3.7 µm. As demonstrated in Zhang and Platnick (2011), 2.1 µm is affected more by inhomogeneous cloud structure than 3.7 µm, which remains relatively more stable. The difference is well correlated with cloud fraction (CF): lower CF leads to higher values of  $r_{\rm eff}$  being retrieved using 2.1 µm. Moreover, the error in  $r_{\rm eff}$ retrieved from 2.1 µm strongly increases with the cloud horizontal heterogeneity index ( $H_{\sigma}$ ) introduced by Liang et al. (2009). This suggests the need for physically meaningful structural variables to account for cloud heterogeneity in new retrieval techniques (e.g., Fu et al., 2019).

The use of the plane-parallel assumption in droplet effective radius retrievals can result in substantial errors in inhomogeneous or broken cloud regions. Here we propose a new and completely different retrieval approach that has the potential of filling the gap in current retrieval capabilities for inhomogeneous clouds. This technique comes from nuclear reactor physics and was developed in 1960 s (e.g., Bell and Glasstone, 1970). We propose to adapt the technique for space-based cloud remote sensing. It is not based on the 1D assumption and can be applied to any inhomogeneous and broken cloud fields. Obviously, the proposed technique has its own strengths and weaknesses that will be also addressed here.

The outline of the paper is as follows. Section 2 discusses the operationally retrieved cloud droplet effective radius and cloud optical depth and then Section 3 examines the ratio of cloud reflectance over single scattering albedo. Next, Section 4 states the two main hypotheses for interpreting satellite observations. Section 5 discusses retrievals of single scattering albedo and number of scatterings. Finally, Sections 6, 7 summarize our theoretical research questions and provide a few concluding remarks.

# 2 Cloud droplet effective radius and cloud optical depth

Figure 2 illustrates the relationship between the operationally retrieved values of droplet effective radius  $r_{\rm eff}$  and cloud optical depth  $\tau$  for all 29,148 data points of water clouds in a MODIS Aqua cloud product granule over the South Pacific Ocean. After considering all data points (grey dots), we selected from them the 1794 data points with the fixed value of ~0.48 for the ratio of 0.87 µm and 2.13 µm reflectances (blue dots). Out of those, we selected 103 data points where the scattering angle between solar and viewing directions is ~99° (red dots). We note that the values of 0.48° and 99° were chosen arbitrarily, as representative values that occur abundantly in the sample MODIS granule shown in Figure 2. As we see, the red points lie along a curve that has a roughly hyperbolic shape, suggesting that the product of  $r_{\rm eff}$  and  $\tau$  is nearly constant.

# 3 Ratio of cloud reflectance over single scattering albedo

For a sample cloudy pixel in the left panel of Figure 2, Figure 3 illustrates the ratio of the observed MODIS Top-of-Atmosphere (TOA) reflectance  $I_{\lambda}$  over the spectral single scattering albedo  $\omega_{0\lambda}$  (calculated based on the retrieved effective radius  $r_{\rm eff}$  of 13 µm) plotted vs. the reflectance  $I_{\lambda}(\Omega, \Omega_0)$  at four MODIS bands ( $\lambda = 0.86, 1.65, 2.13,$  and 3.75 µm). Here  $\Omega$  and  $\Omega_0$  are solar and viewing directions, respectively. A linear fit indicates that the relationship



FIGURE 2

Left: Aqua MODIS retrieved effective radius vs. cloud optical depth for water clouds over ocean Right: The granule (MYD021KM.A2013264.0945.2013265162742) and its location showed as a red dot on a map. Grey (29,148 data points): all  $r_{eff}$  vs.  $\tau$ . Blue (1794 data points):  $l_{\lambda=2.13}/l_{\lambda=0.87} = 0.48 \pm 0.01$ , all solar and viewing angles. Red (103 data points) for the fixed scattering angle of 99°  $\pm$  1°.



were calculated from Mie theory using the refwat code included in the publicly available LibRadtran software package (Mayer and Kylling, 2005; Emde et al., 2016). The refwat code is based on the results in Segelstein (1981). The ratio  $l_\lambda/\omega_{0\lambda}$  vs.  $l_\lambda$  is plotted for four MODIS bands: 0.86, 1.65, 2.13, and 3.75 µm. We can see that the dots follow a strong linear relationship with a slope of p = 0.943 (q = 0.011); the regression coefficient *R* is 0.9999.

$$I_{\lambda}(\Omega, \Omega_0) / \omega_{0\lambda} = p I_{\lambda}(\Omega, \Omega_0) + q(\Omega, \Omega_0)$$
(1)

holds with a slope p = 0.943 and an offset q = 0.011. Equation 1 can be rearranged as

$$I_{\lambda}(\Omega, \Omega_0) = \omega_{0\lambda} q(\Omega, \Omega_0) / (1 - p\omega_{0\lambda})$$
(2)

Analyses of successive order of scattering suggest that Eq. 2 represents the exact solution of the 3D radiative transfer equation with non-reflecting boundaries such as a black underlying surface (Huang et al., 2007; Knyazikhin et al., 2011; Yang et al., 2017). Here p is the recollision probability, defined as the probability that a photon scattered in the medium will collide in the medium again. The variable  $q(\Omega, \Omega_0) = \rho(\Omega)i_0(\Omega_0)$  quantifies escape events:  $\rho(\Omega)$  is the probability that a scattered photon will escape the medium in a given (up- or downward) direction  $\Omega$  and  $i_0(\Omega_0)$  is the interceptance, defined as the portion of photons in the incident beam coming from direction  $\Omega_0$  that collide with cloud droplets for the first time. The interceptance  $i_0$  is strongly sensitive to 3D medium structure and is related to direct transmittance: their sum is equal to 1. The fraction of intercepted photons initiates the process of photon-medium interactions while the recollision probability determines the number of scattering events as results of multiple interactions: the higher p, the more interactions the photons undergo. On average, an intercepted photon will have

$$N_{\rm scat} = 1/(1 - p\omega_{0\lambda}) \tag{3}$$

interactions (Marshak et al., 2011).

Indeed, in the literature of vegetation canopy remote sensing, Eq. 2 is usually written as (e.g., see Knyazikhin et al., 2013; Yang et al., 2017)

$$I_{\lambda}(\Omega,\Omega_{0}) = DASF(\Omega,\Omega_{0})W_{\lambda} = \left[\frac{\rho(\Omega)i_{0}(\Omega_{0})}{1-p}\right] \left[\omega_{0\lambda}\frac{1-p}{1-p\omega_{0\lambda}}\right],$$
(4)

Where  $DASF(\Omega,\Omega_0) = \rho(\Omega)i_0(\Omega_0)/(1-p)$  is the Directional Area Scattering Factor and  $W_{\lambda} = \omega_{0\lambda}(1-p)/(1-p\omega_{0\lambda})$  is the scattering coefficient of a radiative active layer (Knyazikhin et al., 2013). Note, that in Eq. (4) the extinction coefficient and the scattering phase function, normalized by single scattering albedo, do not depend on  $\lambda$ . As a result, DASF becomes independent of wavelength while  $W_{\lambda}$  is



almost independent of the sun-view geometry but varies with wavelength.

# 4 Two hypotheses for interpreting the observations

#### 4.1 Hypothesis 1

This hypothesis helps interpreting Figure 2 by relating the observed behavior to cloud absorption. It reads:

All red points in Figure 2 are impacted by the same level of absorption, which can be estimated as a product of the average number of scatterings and the single scattering co-albedo averaged along all photon paths contributing to the observed reflectances.

For an "intuitive explanation," we neglect dependency of the phase functions on wavelength. This implies that photons follow the same path (trajectory) in both absorbing and non-absorbing spectral channels. In this case, the relative effect of absorption can be estimated as the product of the (wavelength-dependent) average number of scatterings  $N_{\text{scat}}$  per intercepted photon and the (also wavelength-dependent) single scattering co-albedo 1- $\omega_0$ :

$$A = (I_{\text{nonabs}} - I_{\text{abs}}) / I_{\text{nonabs}} = 1 - I_{\text{abs}} / I_{\text{nonabs}} = N_{\text{scat}} * (1 - \omega_0).$$
(5)

Note that the absorption (coefficient)  $A = 1-W_{\lambda}$ , i.e. sum of absorption and scattering coefficients is equal to 1. Figure 4 schematically illustrates the above relationship on a  $1-\omega_0$  vs.  $N_{\text{scat}}$  plane for different level of absorption.



Single scattering co-albedo  $1\text{-}\omega_{0\lambda}$  and its approximation as 2/3  $\kappa_\lambda$   $r_{eff}.$  Water droplets with  $r_{eff}$  = 10  $\mu m$  are used.

Of course, MODIS provides neither the total number of scatterings nor the single scattering co-albedo. However, cloud optical depth can be related to the number of scatterings. For example, for conservative scattering in a plane-parallel homogeneous layer, the diffusion approximation relates (see, Marshak et al., 1995; Davis and Marshak, 2002) cloud optical depth to the average number of scatterings as

$$N_{\rm scat} \sim \tau \text{ (for reflected radiation)}$$
 (6)

and

$$N_{\rm scat} \sim (1-g)\tau^2$$
 (for transmitted radiation) (7)

where the tilde sign (~) indicates proportionality. The coefficients of proportionality in Eqs 6, 7 depend on the "extrapolation length" (expressed in transport mean free paths as  $1/[(1-g)\sigma]$  where g is the asymmetry factor of scattering particles (typically  $\approx 0.85$ ) and  $\sigma$  is the extinction coefficient); it is around 2 for reflected photons and 0.5 for transmitted ones (e.g., Marshak and Davis, 2005, pg. 556).

On the other hand, the single scattering co-albedo is proportional to the effective radius (Twomey and Bohren, 1980):

$$1 - \omega_{0\lambda} \approx c \, k_{\lambda} \, r_{\rm eff} \tag{8}$$

where  $\kappa\lambda$  is the bulk absorption coefficient ( $4\pi$  multiplied by the ratio of the imaginary part of the refractive index to wavelength  $\lambda$ ). As shown in Figure 5, coefficient *c* in front of  $\kappa_{\lambda}$  can be well approximated by 2/3 (Frank Evans, personal communication).

Following Eqs 5–8, we can state that for a given level of absorption and solar and viewing geometry, the product of cloud optical depth  $\tau$  and droplet effective radius  $r_{\rm eff}$  can be well approximated by a constant. Thus a plot of  $r_{\rm eff}$  vs.  $\tau$  will have a hyperbolic shape as we observed in Figure 2. Naturally, dynamical/microphysical processes are important in shaping the specific (hyperbolic-shaped)  $r_{\rm eff}$  vs.  $\tau$  relationship.



#### FIGURE 6

Illustration of spectrally-invariant approximation. (left) The ratio of spectral nadir radiance over  $\omega_0$  as a function of nadir radiance. Radiances at three MODIS channels are highlighted. Slopes *p* for all wavelengths (red) and just for three MODIS channels: 0.87, 1.64, and 2.13 µm (grey) are provided (modified from Figure 7 of Marshak et al., 2011). (right) SBDART-calculated spectral nadir radiance between 0.4 µm and 2.4 µm, plotted at 0.01 µm resolution. Cloud properties:  $\tau_{cloud} = 10$ ,  $r_{eff} = 16$  µm. Aerosol properties: rural aerosol,  $\tau_{aer} = 0.2$ , relative humidity is 80%. Solar Zenith Angle (SZA) is 45°. Strong water vapor absorption bands are excluded and replaced by "doggy legs".

#### 4.2 Hypothesis 2

The hypothesis interprets Figure 2 as the spectrally invariant approximation. It reads:

For cloudy atmospheres, the ratio of spectral radiance over spectral single scattering albedo vs. spectral radiance is wavelength independent.

Using the ratio  $I_{abs}/I_{nonabs}$  alone is not sufficient to find both  $\omega_0$ and  $N_{scat}$  from Eq. 3. One more piece of information is needed. This can come from the spectrally invariant assumption (Knyazikhin et al., 2005; Marshak et al., 2011) expressed as Eq. 1, where  $I_{\lambda}$  and  $\omega_{0\lambda}$ are the wavelength-dependent radiance and single scattering albedo, while *p* and *q* are the spectrally-invariant (wavelength-independent) recollision and directional escape probabilities, respectively.

While  $\omega_{0\lambda}$  is a well-known parameter in atmospheric radiation, p and q are less known and thus require some explanation. They were first introduced and developed in nuclear reactor physics (Bell and Glasstone, 1970, p. 115-125). The reciprocal of the product of  $\omega_{0\lambda}$  and p describes the criticality condition, i.e., a situation when more than one neutron is emitted per collision (Case and Zweifel, 1967). In photon transport, p becomes the conditional probability that a scattered photon will interact with the medium again (recollision probability) and q is the conditional probability that a scattered photon will leave the medium in a given direction without further scattering (directional escape probability). The product of the recollision probability p and the single scattering albedo  $\omega_{0\lambda}$  approximates the maximum eigenvalue of the radiative transfer equation (see the Appendix in Marshak et al., 2011; Knyazikhin et al., 2005, pg. 634) and can be calculated as a function of  $r_{\rm eff}$  using methods developed earlier in reactor physics.

To illustrate the validity of Eq. 1, we used the Santa Barbara DISORT Atmospheric Radiative Transfer (SBDART) code (Ricchiazzi et al., 1998). Figure 6 shows the results of SBDART calculations with a spectral resolution of 0.01 µm for a cloudy



atmosphere with a cloud optical depth of 10 and an aerosol optical depth of 0.2. The three highlighted data points correspond to MODIS channels at 0.87, 1.64, and 2.13  $\mu$ m.

In addition to the above plots, Dr. Zhibo Zhang tested the spectral invariant assumption with a Large-Eddy Simulation (LES) model (e.g., Stevens et al., 1999) and 3D radiative transfer

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calculations of cloud reflectance at four wavelengths (0.87, 1.64, 2.13, and 3.7  $\mu$ m) and at a variety of viewing directions. Testing the validity of Eq. 1, he found a good fit by a straight line with a regression coefficient close to unity (http://userpages.umbc.edu/~zzbatmos/research/I3RC\_test\_3.html).

#### 5 Retrieval of single scattering albedo and number of scatterings

To complete our retrieval approach, we need to estimate the slope *p* from Eq. 1. For this, we combine radiances at MODIS nonabsorbing (say, 0.87 µm) and water absorbing channels (1.64, 2.13, and 3.75 µm) using Twomey and Bohren's (1980) approximation (4), which relates  $1-\omega_{0\lambda}$  at these channels to  $r_{\rm eff}$  as shown in Figure 5. For any given p, the number of scatterings  $N_{\text{scat}}$  is related to the single scattering albedo  $\omega_0$  through Eq. 3 (see also Eq. 9 in Marshak et al., 2011). If we then add to Figure 4 the curve of  $N_{\text{scat}}$  as a function of  $\omega_0$ , the two independent sets of curves will intersect (see Figure 7). For each spectral reflectance  $I_{\lambda}$  of a cloudy pixel, this intersection provides the two fundamental radiative transfer parameters: the number of scatterings  $N_{\text{scat}}$  and the single scattering albedo  $\omega_0$ averaged over all photon paths contributing to the observations. Naturally, both  $N_{\text{scat}}$  and  $\omega_0$  are functions of wavelength  $\lambda$ . Note that using  $\omega_0$  instead of  $r_{\rm eff}$  will allow a more flexible interpretation of droplet size if we need to adjust the shape of the particle size distribution (whereas  $r_{\rm eff}$  is tied to a specific distribution-shape).

Here we briefly summarize Figure 7, which is the key plot explaining the proposed approach. For each cloudy pixel that is associated with a fixed solar-viewing geometry, the applicable thin line is selected based on the ratio as  $A = 1 - I_{\lambda=2.13}/I_{\lambda=0.87}$  (see Eq. 5), while the applicable thick line is specified by Eq. 1 for the spectrally-invariant slope *p* (the recollision probability) obtained by performing a regression based on Eq. 2 to determine what *p* and *q* values work best for a set of radiances observed at multiple wavelengths at the same location, with the  $\omega_0$  values at different wavelengths being related through Eq. 8.

# 6 Discussion and theoretical research questions

First, let us point out that the plane-parallel assumption has been neither stated nor used in the above approach. Indeed, the fundamental radiative transfer theory behind this approach is based on Eqs 1, 2, which are valid for any 3D scattering and absorbing medium (Várnai and Marshak, 2002; Várnai and Marshak, 2009; Cahalan et al., 2005; Marshak et al., 2006). Even so, both the number of scatterings  $N_{\rm scat}$  and the single scattering albedo  $\omega_0$  (averaged over the photon path) can be obtained for each wavelength  $\lambda$  (note that passive 1D reflectance-based cloud retrievals are fundamentally retrieving a 1D  $\omega_0$ ). Our main assumption here is that the "absorbing wavelength photons" follow the same path as the non-absorbing ones. In other words, we assumed that the solution of the radiative transfer equation,  $I_{\lambda}$ , depends on  $\{\tau_{\lambda}, P_{\lambda}\}$  (where  $P_{\lambda}$  is the spectral phase function) in a way that does not change with wavelength, and so the wavelength dependence of  $I_{\lambda}$  comes only from the  $\omega_{0\lambda}$  spectra (Marshak et al., 2012).

We note that a such separation of variables is natural for radiative transfer in leaf canopies (Knyazikhin et al., 2011), where the scattering objects are much larger than the wavelength of solar radiation—and the dependence on  $\tau$  and p is determined by canopy structure (Schull et al., 2007), while the dependence on  $\omega_0$  comes entirely from leaf physiology (Knyazikhin et al., 2013). For atmospheric radiative transfer this assumption *is not met*, since the size of scattering objects (air molecules, aerosol and cloud particles) is comparable to (or smaller than) the wavelength of solar radiation. However, in cloudy atmospheres these assumptions can be *met approximately* for a wide range of wavelengths (Marshak et al., 2011).

As a proof of concept, Figure 8 illustrates for several cloud optical depths the impact of neglecting the wavelength-dependence of scattering phase functions. As expected, the impact decreases as the optical depth increases: For example, for  $\lambda = 2.13 \mu m$  and  $\tau = 5$ , the errors are about 8%–12%, while for  $\tau = 40$  they are reduced to 3%–8%, depending on the Solar Zenith Angle (SZA).

Finally, we note that 1D radiative transfer theory predicts that the reflectance functionally depends on the scaled optical depth  $\tau$  (1g) (e.g., van de Hulst, 1980). Assuming that g is known a priori, the impact of phase function variations can be mitigated and the assumption of wavelength-independent phase functions can be relaxed to a certain degree. Quick preliminary calculations showed that the difference between the pair of diamond symbols for  $\tau = 5$  in the left panel of Figure 8 corresponds to a roughly 2 µm change in  $r_{\rm eff}$ . The difference is even smaller in case of larger optical depths (middle and right panels in Figure 8). This bias can be much smaller than the one due to the assumption of plane-parallel geometry in case of strongly inhomogeneous clouds.

The above approach has been partially developed only for a black surface. However, if we know the reflectivity of the underlying surface, the surface albedo effects can be adequately removed (Stephens and Kummerow, 2007). We are familiar with the decomposition of the reflected radiation into the "black surface" problem and the additional radiative field due to the interaction between the underlying surface and the medium (Knyazikhin and Marshak, 2000; Knyazikhin et al., 2005).

While the value of retrieving  $\omega_0$  (and hence  $r_{\text{eff}}$ ) is clear and may warrant using the proposed approach even for just this purpose, one may wonder, what information about cloud properties does the retrieved number of scatterings convey? First of all, for non-absorbing scattering  $(\omega_0 = 1)$  in a plane-parallel medium, one can relate the average number of scatterings  $N_{\text{scat}}$  to cloud optical depth  $\tau$  (and scattering asymmetry factor g) using Monte Carlo simulations or the diffusion approximation (Eqs 6, 7; see also Figure 3 in Kato and Marshak (2009), which relates  $N_{\rm scat}$  to  $\tau$  for different viewing zenith angles). For absorbing wavelengths  $(\omega_0 < 1)$ , the more complex formulae also include the single scattering albedo  $\omega_0$  (Platnick, 2001a; Marshak and Davis, 2005, pg. 559). Thus, under the common plane-parallel assumption, we get cloud optical depth  $\tau$  and, combining  $\tau$  with  $r_{\text{eff}}$  we can also get liquid water path as done by current operational retrievals of, for example, MODIS (e.g., Platnick et al., 2003). Second, the average number of scatterings  $N_{\rm scat}$ helps estimating horizontal photon transport; as horizontal transport can be expressed as a function of  $N_{\text{scat}} \omega_0$  and g (Platnick, 2001b). Indeed, the root-mean-square displacement of reflected and transmitted photons can be estimated from diffusion theory if the average number of scatterings is known (Platnick, 2001a; Platnick, 2001b; Marshak and Davis, 2005, pg. 560-561). We note that inconsistencies between the



retrieved  $N_{\text{scat}}$  values and the optical depths retrieved independently using the 1D Nakajima-King approach may help identify the cases where the current operational cloud products are significantly impacted by heterogeneity effects. Finally, the average number of scatterings is closely related to the average photon path length thus it is a useful quantity in its own right. We note that the average number of scatterings obey reciprocity (Platnick, 2001b), thus all results are valid upon exchange of solar and viewing directions.

## 7 Summary

In this paper we propose to retrieve two fundamental radiative transfer parameters: single scattering albedo,  $\omega_0$ , and number of scatterings,  $N_{\text{scat}}$ . This retrieval does not require the assumption of plane-parallel geometry. The traditional cloud microphysical properties like optical depth,  $\tau$ , and droplet size,  $r_{\text{eff}}$  can be easily obtained from these parameters.

Eliminating the 1D assumption does not come for free here. We assume that the photons' trajectory is the same in absorbing and non-absorbing channels. This assumption is strong and creates a bias in  $\tau$  and  $r_{\rm eff}$ . However, in many cases, the bias appears likely to be (much) smaller than the biases that the assumption of 1D geometry brings for strongly inhomogeneous clouds.

In addition, the retrieved number of scatterings and single scattering albedo convey broad information about cloud properties. For example,  $\omega_0$  allows a more flexible interpretation of droplet size in cases of different shapes of particle size distribution, while  $N_{scat}$  allows to infer the impact of photon horizontal transport.

## Data availability statement

The original contributions presented in the study are included in the article, further inquiries can be directed to the corresponding author.

## Author contributions

AM: Writing-original draft, Writing-review and editing. YK: Writing-review and editing. TV: Writing-review and editing.

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# Conflict of interest

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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