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Spatial reasoning in context: bridging cognitive and educational perspectives of spatial-mathematics relations

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Spatial reasoning is ingrained in daily life, such as when locating our keys or parking our car. At a broad level, spatial reasoning describes the ability to mentally represent and transform objects and their relations. Spatial reasoning is comprised of distinct, yet related, spatial skills, most of which have strong links with mathematics achievement. Subsequently, understanding the ways spatial reasoning connects with mathematics has the potential to support achievement in school. However, current research practices have failed to translate into practical outcomes for students. To date, research has often focused on decontextualized spatial skills, measured by psychometric tests, to generalize about broader models of spatial reasoning. However, spatial reasoning goes beyond test performance. In this theoretical review, I have sought to find the points of connection between the fields of cognitive psychology, often based in the lab, and mathematics education, situated within classrooms, and discussed ways to connect this currently siloed work for greater impact on classroom practice. The paper addresses the emergence of spatial research from its historical roots in intelligence testing and the influence these conceptualizations have had on contemporary methodologies. It goes on to discuss how these research traditions may be limiting our ability to understand the mechanisms linking spatial reasoning and mathematics. The paper argues for a broader view of research problems and methodologies in spatial cognition research to facilitate the translation of research to meaningful contexts in pedagogy and learning.

KEYWORDS

spatial reasoning, mathematics education, spatial skills, spatial cognition, classroom practice

1 Introduction

Spatial reasoning is broadly defined as the ability to mentally represent, organize, and transform objects and their relations (Linn and Petersen, 1985; Carroll, 1993; Hegarty and Waller, 2005; Newcombe and Shipley, 2015). The overarching concept of spatial reasoning encompasses a range of separate, yet related, skills that have been identified through intelligence testing and now form the basis of many investigations into their role in education and engagement with our highly spatial world (McGee, 1979; Hegarty and Waller, 2005; Mix and Cheng, 2012). Spatial skills are strongly related to success in Science, Technology, Engineering, and Mathematics (STEM) educational pursuits and career choice (Wai et al., 2009; Tian et al., 2022). As a result, these skills have received increased attention across education and cognitive psychology fields over the last half a century (Newcombe, 2010; Woolcott et al., 2020).

Spatial reasoning is an intrinsic part of daily life: for example, when arranging objects in the home; planting a garden; interpreting data, and for activities that involve awareness of large-scale space such as navigating familiar or new environments (Hegarty, 2010; Newcombe, 2017; Newcombe et al., 2022; Hawes et al., 2023). However, there are inconsistencies and gaps between cognitive models of spatial skills and their applications in everyday practice (Gagnier and Fisher, 2020; Woolcott et al., 2020; Coutrot et al., 2022; Harris et al., 2022; Newcombe et al., 2022). For example, although often considered to possess poor spatial skills based on cognitive test performance (Levine et al., 2005; Wai et al., 2009; Jirout and Newcombe, 2015), individuals classified as disadvantaged in terms of socioeconomic status and geographic isolation have been found to possess superior navigation and spatial location skills (Coutrot et al., 2022; Harris et al., 2022; Lowrie et al., 2022). Most gaps emerge in terms of the translation of experimental findings into classroom practice where despite evidence for the influence of spatial skills on STEM understandings, they do not form part of standard STEM instruction (Davis and Spatial Reasoning Study Group, 2015; Gagnier and Fisher, 2020; Hawes et al., 2023).

This theoretical review explores possible reasons for these inconsistencies and gaps across two key fields where significant strides have been made in spatial research, yet the bodies of work remain largely siloed (Bruce et al., 2017; Lowrie et al., 2020; Harris, 2021): mathematics education and cognitive psychology. These two fields have documented historical trends towards diverse research methodologies, with cognitive psychology tending to focus on experimental rigor and mathematics education focused on situated behavioral research (Bruce et al., 2017; Mix and Levine, 2018; Resnick and Stieff, 2024). By synthesizing literature across these domains, this paper argues for a broader view of research problems and methodologies in spatial reasoning studies and recognizes the importance of the translation of research to meaningful contexts in pedagogy and learning. We end with a conclusion focused on practical ways to reduce these gaps and explore possibilities for collaborative multi-dimensional research.

2 Where do the gaps in translation emerge?

The authors of the National Research Council (NRC) *Learning to Think Spatially* report (NRC; 2006) argued that "our goal must be to foster a generation of students (1) who have the habit of mind of thinking spatially, (2) who can practice spatial thinking in an informed way, and (3) who adopt a critical stance to spatial thinking" (p. 3). Not long after the NRC report was released, Wai et al. (2009) published the results of the Project Talent 50-year longitudinal study which revealed the benefits of strong spatial skills in high school for future STEM education success and career choice. Since that time, many studies have explored spatial-mathematics relations [see meta-analyses by Xie et al. (2020) and Atit et al. (2022) for a thorough review] and the affordances of spatial training for improving mathematics achievement (Hawes et al., 2022). Researchers are now confident that spatial skills are inherently trainable (Uttal et al., 2013; Montello et al., 2014), with training showing some transfer to other STEM skills (Wright et al., 2008; Cheng and Mix, 2014; Lowrie et al., 2017a, 2019, 2021; Gilligan-Lee et al., 2020; Mix et al., 2021; Adams et al., 2022; Hawes et al., 2022). Furthermore, the benefits of strong spatial skills for mathematics competence and achievement are widely reported (Xie et al., 2020; Atit et al., 2022).

Despite the amassed volume of research around spatial reasoning and its connections to mathematics, there are still critical theoretical and practical gaps around:

- Operational definitions and measurement of spatial reasoning and skills that limit, rather than expand, theory development (Mix and Levine, 2018; Harris, 2021).
- Understanding the mechanisms that support the transfer of spatial skills to mathematics content knowledge (Gagnier and Fisher, 2020; Lowrie et al., 2020; Hawes et al., 2022; Resnick and Stieff, 2024).
- The translation between cognitive models of spatial-mathematics relations and meaningful impact on educational practice (Gagnier and Fisher, 2020; Harris et al., 2022, 2023; Newcombe et al., 2022).
- Practically positioning spatial learning at the forefront of educational practice (National Research Council, 2006; Doerschuk et al., 2016; Gagnier and Fisher, 2020; Gilligan-Lee et al., 2022; Tian et al., 2022).

2.1 Operationalizing and measuring spatial reasoning

A challenge when connecting across academic disciplines is aligning the operational definitions that shape research in the different fields. Definitions of spatial reasoning are vast and often lines are blurred even within a single discipline (Mix and Levine, 2018; Lowrie et al., 2020; Harris, 2021). Furthermore, spatial reasoning is not a new construct, and its utility and operationalization has evolved over time and contexts (Thurstone, 1950; McGee, 1979; Bishop, 1980; Carroll, 1993; Hegarty and Waller, 2005). In the following sections I contrast some ways that spatial reasoning has been operationalized across mathematics education and cognitive psychology which may contribute to the gaps summarized above. The discussion has implications for closely related fields, such as educational and developmental psychology, mathematics cognition, and other STEM fields, however, including all relevant disciplines is beyond the scope of this paper.

2.1.1 Historical context

The extensive history and evolution of spatial research in psychology has its roots in intelligence testing at the beginning of the 20th century (for comprehensive accounts see Hegarty and Waller, 2005; Buckley et al., 2018). Beginning with the notion of general intelligence (*g*, Spearman, 1904), researchers eventually discriminated between verbal and spatial-mechanical intelligence factors (Thurstone, 1938, 1950). Further factor-analytic studies categorized a range of separate, yet related, spatial skills based on a variety of different tests (e.g., mental rotation, spatial visualization, spatial perception; Smith, 1964; Ekstrom et al., 1976; McGee, 1979). The exact structure of spatial skills varied between studies and by the beginning of the 21st century no consensus had been reached (Linn and Petersen, 1985; Carroll, 1993; Hegarty and Waller, 2005). To this day, the way spatial skills are defined and measured continues to evolve (Buckley et al., 2018).

The foundations of spatial testing in mechanical fields (McGee, 1979; Clements and Battista, 1992) and the relation between high spatial ability and pursuits of STEM careers (Wai et al., 2009) provide ecological validity for the link between spatial skills and STEM disciplines. However, work to capitalize on this relationship did not begin in earnest until the 21st century with initial intervention studies using largely decontextualized spatial tasks, such as repetitive training of mentally rotating 2D images, to improve these skills for broader mathematics implications (Cheng and Mix, 2014; Hawes et al., 2015, 2023). These studies produced promising results, but the nature of the transfer and replicability were something of a mystery (Hawes et al., 2015; Stieff and Uttal, 2015).

2.1.1.1 How mathematics education adopted psychological theory

Early work in mathematics education relating to spatial reasoning was largely based on work by psychologists such as Thurstone (1938), Smith (1964), Piaget and Inhelder (1967), McGee (1979), Bishop (1980), Woolcott et al. (2020), and Thom et al. (2021). Gutiérrez (1996) argued that although much of the published work around spatial reasoning throughout the 20th century was in psychology, visualization has always formed an intrinsic part of mathematics, and in particular geometry.

In his 1996 paper, Gutiérrez considered the terms visualization and spatial thinking synonymous, incorporating both the mental processes and use of problem-solving tools (such as diagrams) under this visualization umbrella (and argues for the same in Gutiérrez et al., 2018). However, psychological definitions of visualization are traditionally positioned within more narrow definitions, for example “visualization requires that the figure be mentally restructured into components for manipulation” (Ekstrom et al., 1976, p. 173). Similar operational differences are found across a range of spatial and mathematics concepts, such as symmetry, perspective-taking, spatial visualization, geometric reasoning (Ramful et al., 2015; Bruce et al., 2017; Mix and Levine, 2018; Harris, 2021).

The contrasting definitions employed within the fields are indicative of broader differences, bound within research goals and methodologies, that may contribute to gaps in knowledge (Bruce et al., 2017; Mix and Levine, 2018; Lowrie et al., 2020; Resnick et al., 2020; Harris, 2021; Maresch and Sorby, 2021). Mathematics education researchers often operate from a top–down perspective, focusing on the spatial maneuvers present in mathematical contexts (Ramful et al., 2017; Woolcott et al., 2020). Consequently, spatial development and observation is wide-ranging, individualistic, and at times incidental, rather than intentional (Davis and Spatial Reasoning Study Group, 2015; Lowrie and Logan, 2018).

By contrast, in cognitive psychology it is traditionally a goal to isolate individual components of spatial reasoning and target these, often-decontextualized, skills either through training or modelling of relationships between constructs (i.e., bottom-up; Resnick and Stieff, 2024). In this way cognitive psychologists have built theoretical models with the aim to be generalized to broader contexts (Linn and Petersen, 1985; Uttal et al., 2013; Newcombe and Shipley, 2015). Historically, these research traditions have existed in parallel, contributing to a growing body of evidence in each field but with limited access to the work in the respective opposite field (Harris, 2021; Resnick and Stieff, 2024).

2.1.2 Contemporary conceptualizations in mathematics education

Although early conceptualizations of spatial reasoning in mathematics education emerged from the psychological literature, the reciprocal relationship seems mostly to have stopped there. Current practice in mathematics education does not acknowledge the range of spatial skills identified in the cognitive psychology literature and the affordances of these different skills for different mathematics tasks (Cutting, 2021; Harris et al., 2021a,b). Instead, spatial reasoning is often operationalized through mathematics curriculum and assessment.

According to some current curriculum standards (e.g., the Australian Curriculum Assessment and Reporting Authority [ACARA]; Ontario Ministry of Education [OME]), spatial reasoning is a foundational component of mathematics which incorporates an awareness of space, the ability to imagine objects and relations, and to use this information to reason and problem-solve (Ontario Ministry of Education, 2020; Woolcott et al., 2020; Australian Curriculum, Assessment and Reporting Authority, 2023). Foundational stages of mathematical development models are undeniably spatial (Clements and Battista, 1992; Mulligan et al., 2018), for example the basic visualization level in the hierarchical van Hiele (1986) theory or the recursive nature of Pirie and Kieren’s (1994) theory for the growth of mathematical understanding. However, under a curriculum lens, spatial reasoning is often categorized as such simply because tasks sit within a geometry strand (Ontario Ministry of Education, 2020; Woolcott et al., 2020; Downton and Livy, 2021; Australian Curriculum, Assessment and Reporting Authority, 2023).

As a cohort, educators have lower than average spatial skills compared with the general population (Verdine et al., 2017; Atit et al., 2018), and therefore spatial opportunities do not always translate to spatialized instruction (Gilligan-Lee et al., 2022; Hawes et al., 2023). Even for educators with high spatial skills, it is uncommon for experts to rely on their spatial skills when completing tasks, even those considered highly spatial (Uttal and Cohen, 2012; Resnick and Shipley, 2013; Atit et al., 2020). Although there are countries with explicit mention of spatial reasoning in their curriculum (e.g., Australia and Canada), other countries (e.g., England and the United States) rely on inherently spatial content areas, like shape and measurement to promote spatial reasoning (Gilligan-Lee et al., 2022). Without explicit spatial curriculum to support less spatially minded educators, opportunities to foster student spatial development within standard teaching practice are often missed (Davis and Spatial Reasoning Study Group, 2015; Lowrie and Logan, 2018; Bates et al., 2022; Gilligan-Lee et al., 2022).

Another way spatial reasoning is identified in mathematics education is through tasks within numeracy assessment that have a perceived reliance on visualization (Australian Curriculum, Assessment and Reporting Authority, 2016; Logan and Lowrie, 2017; Ramful et al., 2017; Seah and Horne, 2020). No distinction is made between the tendency to conjure up visual images and the ability to apply spatial skills (Bishop, 1980; Presmeg, 2008; Ramful et al., 2015). Whilst positively associated with mathematics achievement, mental imagery can hinder successful problem-solving in cases where too much detail can make it difficult to extract task-relevant information (Hegarty and Kozhevnikov, 1999). Kozhevnikov et al. (2010) found a direct trade-off between one's ability to visualize objects and visualize spatially, where the former relates to the detail within an object and the latter the spatial configuration of the object(s). This understanding of spatial relations is critical for success across a range of mathematics tasks (Hegarty and Kozhevnikov, 1999; Mix et al., 2016; Harris et al., 2021a). Therefore, relying on task intent alone may be counterproductive to spatial research and assessment in instances where students do not have the requisite spatial skills to perform spatial-based mathematics tasks proficiently (e.g., Casey et al., 2011; Lowrie, 2020; Harris et al., 2023).

One of the biggest challenges of the behavioral nature of educational research is that a considerable body of research remains localized within classrooms or contexts, with little regard for transforming the field as a whole (Woolcott et al., 2020; Grootenboer and Peter-Koop, 2024). Observational case studies (e.g., Patahuddin et al., 2020), and design research (e.g., Cutting, 2021) provide opportunities to explore authentic applications of spatial reasoning and in skilled hands these opportunities can be leveraged to support student learning (Woolcott et al., 2020). However, potential benefits occur on a case-by-case basis for a privileged few, and curriculum reform remains largely at the mercy of policymakers and curriculum designers, rather than evolving from an established evidence-based (Lowrie, 2024).

2.1.3 Contemporary conceptualizations in cognitive psychology

The current prevailing theoretical model used to characterize spatial reasoning in cognitive psychology is a two-by-two typology based on the cognitive processes the skills invoke (Uttal et al., 2013; Newcombe and Shipley, 2015). This typology differentiates static and dynamic spatial skills which refer to the imagined movement, or lack of, in a referent object(s), and intrinsic and extrinsic spatial skills, which refer to changes in the form of an object, or the relations between objects, respectively. Conceptually, this typology covers a broad range of spatial tasks; however, to date there is still no compelling empirical application of this classification system in educational settings (Mix et al., 2018; Jung et al., 2020; Hodgkiss et al., 2021).

Primarily, cognitive psychology researchers have focused on identifying and characterizing individual spatial skills to generalize about broader models of spatial reasoning. Methods have included factor analytic approaches using psychometric test batteries (e.g., Mix et al., 2016; Gilligan et al., 2019; Johnson et al., 2022), behavioral performance comparisons (e.g., Hegarty and Waller, 2004), and neurological observations (e.g., Zacks et al., 2000; Zacks and Michelon, 2005; Hawes et al., 2019a). These methodologies have all largely, and purposefully, remained removed from content areas (such as mathematics) and applications of skills beyond the lab (Resnick and Stieff, 2024).

While practically important for experimentation, the tightly controlled nature of lab-based research leads to questions about connections to real-world application and significance (Fan, 2001; Bruce et al., 2017; Golinkoff et al., 2017; Gagnier and Fisher, 2020; Lowrie et al., 2020). The removal of potentially confounding variables allows for more controlled and rigorous experimental designs, but in doing so it is possible that the results become less generalizable beyond the lab (Bronfenbrenner, 1977; Resnick and Stieff, 2024). For example, effect sizes noticeably decrease when replication occurs outside of the controlled lab environment or psychometric measures are embedded in real-world learning and assessment (Hawes et al., 2015; Green and Newcombe, 2020; Harris et al., 2022; Newcombe et al., 2022).

Although current approaches in cognitive psychology have laid the theoretical groundwork around spatial-mathematical relations, there is limited translation of experimental research into classroom practice as findings tend to remain inaccessible to policymakers and impractical for implementation by educators (Gagnier and Fisher, 2020; Lowrie, 2024). Golinkoff et al.'s (2017) reflections express the need to extend findings beyond the research lab:

> …unless we embrace the complexity of real-world environments and seek to better understand children's lives where they happen, our research will be peripheral to the pressing problems children face. (p. 1405)

2.2 Translation between traditional tests and real-world applications

Measuring spatial skills often results in broad assumptions that students are reasoning spatially (Kozhevnikov and Hegarty, 2001). However, without measures of strategy or evidence for the ways students are representing problems, such assumptions remain speculative. Hegarty (2018) reported that both global mental rotation (i.e., keeping a referent object intact) and analytic strategies were effective in completing 3D mental rotation comparison tasks. When strategy measures are included, there is evidence that participants perform spatial tasks in a variety of ways, many different to the intentions of the tests (Hegarty and Waller, 2005; Hegarty, 2018).

In a sample of middle school students completing an embodied perspective taking task, Harris et al. (2022) reported a pattern of increasing error as a function of landmarks' increasing deviation from students' position. This pattern mirrored error patterns reported for paper-based tasks which use embodied spatial strategies (Kozhevnikov and Hegarty, 2001), yet there was no relation between students' performance on the paper-based and embodied tasks. In fact, a handful of grade 5 children were able to complete the Spatial Orientation Test (Hegarty and Waller, 2004) with remarkable accuracy but were unable to demonstrate these spatial skills by locating a familiar park or train station in their local area (Harris et al., 2022). The authors concluded that while the decontextualized (paper-based) and embodied spatial orientation measures appeared to be tapping into similar cognitive skills, current measures were helpful but not sufficient to characterize the complexity of spatial reasoning in real-world environments.

It is possible that an isolated focus on testing in relational and training studies contributes to the gap that impedes the translation between theoretical literature and practical applications. For example, in studies exploring relations between mathematics and spatial skills, significant differences in spatial test performance contribute to an advantage ranging from less than 1 and up to 2 points on mathematics assessments (e.g., Gilligan et al., 2019; Harris et al., 2021a). On a related note, in intervention studies, as yet there is no way to know if transfer occurs due to improvements in spatial cognition or an increased tendency to employ spatial strategies (Hawes et al., 2022). Although spatial skills are a valid and reliable indicator of spatial reasoning and development, the question of what we can uncover through testing alone is critical in the larger discussion around the mechanisms connecting spatial skills with mathematics as we consider what is a meaningful impact in the classroom.

2.2.1 What can individual differences tell us about barriers to translation?

Through the exploration of individual differences in cognitive testing, patterns have emerged that suggest some individuals are more disadvantaged than others in terms of spatial skills (Linn and Petersen, 1985; Levine et al., 2005; Wai et al., 2009; Möhring et al., 2021). Although gender differences are widely reported in spatial research, strategy training can ameliorate these differences in testing scenarios (Hegarty, 2018). Training female participants in analytical strategies that are commonly employed by males removes historical gender differences (Boone and Hegarty, 2017).

Another confounding factor in spatial research relates to socioeconomic status (SES) because of its influence on test performance (e.g., Levine et al., 2005; Wai et al., 2009; Möhring et al., 2021). In terms of SES, there is often no differentiation between mid- and high-SES categories, but a perceived advantage for both over disadvantaged individuals (Levine et al., 2005; Wai et al., 2009; Jirout and Newcombe, 2015; Johnson et al., 2022). Disadvantage can take many forms and is often assessed through economic and education measures (Levine et al., 2005; Casey et al., 2011; Frick, 2019; Gilligan et al., 2019). This process is designed to ensure equity; however it only serves to highlight inequality in education. A significant contribution from the field of mathematics education can be found in work with disadvantaged communities using non-traditional spatial tasks, such as the use of symmetry and proportional reasoning amongst basket weavers in Papua New Guinea (Owens, 2020) or the highly tuned navigational skills demonstrated in Indigenous communities (Watson-Verran and Chambers, 1989; Lowrie et al., 2022). In these studies, disadvantage stops becoming a factor and in fact we see strengths in the practical and meaningful applications of spatial skills.

In the Uttal et al. (2013) meta-analysis of spatial training studies, studies from countries lowest on the Human Development Index were removed from the meta-analysis because the effect sizes of the interventions were too large and would have impacted the main analysis. What we can conclude from these studies is that (1) poor performance on spatial tests may, in some instances, be an issue with testing, rather than a skill deficit, and (2) populations who suffer the most in terms of inequity, of education and living conditions, may benefit considerably from the work currently underway in psychology labs, if only there was a functional avenue to translate these findings into practice.

3 Mechanisms that link spatial reasoning to mathematics

The discussion thus far has revolved largely around the operationalization and measurement of spatial reasoning, and the differences in research methodology that result; however, these issues offer little more than scholarly debate without grounding in their practical connections with mathematics understanding. Decisions made by researchers regarding operational definitions, measurement, and methodology are nonetheless critical as they underpin the broader question that frequently emerges over the nature of the link between spatial skills and mathematics, particularly in the search for mechanisms that enable transfer to take place (Mix and Cheng, 2012; Hawes and Ansari, 2020; Lowrie et al., 2020).

One view within mathematics education is that spatial skills are likely the foundational steps and tools in the process towards deeper conceptual understanding and they facilitate growth in mathematical understanding as content becomes more complex and abstract (Clements and Battista, 1992; Pirie and Kieren, 1994; Lowrie and Kay, 2001). The continuous and dynamic relationship between spatial and mathematics skills at different developmental timepoints has been mirrored in cognitive psychology studies (e.g., Gunderson et al., 2012; Mix et al., 2016; Hawes et al., 2019a,b). This suggests that while a strong focus on spatial skills in early years education is critical for supporting mathematics development (Gunderson et al., 2012; Levine et al., 2018; Mulligan et al., 2018; Rittle-Johnson et al., 2019; Resnick and Lowrie, 2023), as content becomes increasingly complex, the exclusion of continued spatial skill development is to the detriment of students (Harris et al., 2023).

A range of spatial skills have been identified that are important for mathematics performance, at varying stages of the lifespan (e.g., Young et al., 2018; Mix, 2019; Harris et al., 2021a). Work with adults has shown that these relationships do not end with the school years (Wei et al., 2012; Thompson et al., 2013). However, fundamental questions remain concerning what specific spatial skills are important for mathematics and where in the mathematics curriculum educators could be focusing their attention for maximum impact.

3.1 Object-based spatial skills

Two object-based spatial skills that frequently appear in mathematics research are mental rotation and spatial visualization. While mental rotation is often targeted because of its clear definition and alignment to geometric transformations (Shepard and Metzler, 1971; Vandenberg and Kuse, 1978; Voyer, 2011; Bruce and Hawes, 2015; Battista et al., 2018), spatial visualization is considered a "catch-all" phrase for complex spatial maneuvers that occur within the boundaries of an object (Linn and Petersen, 1985; Ramful et al., 2015; Lowrie et al., 2020). Although these two skills are often combined within one factor (e.g., Mix et al., 2018), reasons to separate these object-based skills are both functional based on the task demands and links to mathematics (Sanchez and Wiley, 2017; Harris et al., 2021b) and drawn from factor analytic literature (Linn and Petersen, 1985; Carroll, 1993).

3.1.1 Mental rotation

Mental rotation is a cognitive spatial skill in its own right and also intrinsic to mathematics content such as transformational geometry (Bruce and Hawes, 2015; Mulligan et al., 2018; Australian Curriculum, Assessment and Reporting Authority, 2023). Mental rotation tasks present opportunities for exploring the impact of different constraints which result in a clear complexity progression, moving from 2 to 3 dimensions, and across single or multiple axes (Voyer, 2011; Bruce and Hawes, 2015). This progression aligns with curriculum progression in geometry as the focus moves from the properties of shapes and their measurement to transformations on a cartesian plane (Clements and Battista, 1992).

Increasing attention has recently been paid to mental rotation in the field of mental mathematics such as number calculations (Thompson et al., 2013; Cheng and Mix, 2014; Bruce and Hawes, 2015). Cheng and Mix (2014) trained 6–8-year-old children in 2D mental rotation for 40 min and found improvements in their ability to complete missing term problems (e.g., 4 + __ = 12). They posited possible reasons for this transfer such as an enhanced ability to rotate equations into more conventional formats or an improvement in visuospatial working memory.

Other hypotheses have been put forward for the link between mental rotation and number problems that reflect the different task demands and difficulty levels of 2D and 3D mental rotation tasks. For example, in 2D mental rotation training with young children, it is possible their discrimination capabilities are improving in synchrony with rotation skills which would help with decoding mathematics symbols (Young et al., 2018; Lowrie et al., 2019). Similar discrimination needs to occur with 3D stimuli however relevant features also need to be extracted such as the direction of stimuli components when making judgements about rotation versus reflection (Young et al., 2018). Mental rotation remains a key focus of research in both cognitive psychology and mathematics education domains, however the mechanisms linking training of mental rotation skills and mathematics outcomes remain theoretical.

3.1.2 Spatial visualization

Categorization of spatial visualization is not as straight forward (Ramful et al., 2015; Sezen Yüksel, 2017). For example, a 2D to 3D conversion task (e.g., Surface Development Test) and a mental folding task (e.g., Paper Folding Test) may both require dynamic spatial maneuvers, visuospatial working memory capacity, and fall in the same category of a test battery (e.g., Ekstrom et al., 1976), but the nature of the transformations are different (Harris et al., 2013; Sezen Yüksel, 2017).

The different tasks that exemplify spatial visualization allow us to infer links to different types of mathematical thinking. For example, mental folding relates to the multiplicative and algebraic ways of thinking that align to the mapping of folds to parts (Empson and Turner, 2006) and also the ability to hold and manipulate information through multi-stage mental processes (Hawes and Ansari, 2020). By contrast, 2D to 3D conversion tasks like the Surface Development Test are more closely aligned to geometric thinking and interrogating the structure of 3D solids (Seah and Horne, 2020). On a conceptual level, the cognitive processes associated with symmetry may support understanding of operations in terms of equivalence on either side of an "equal" sign (Verdine et al., 2017). It may be this diversity that has made spatial visualization such an effective intervention tool due to the wide range of tasks it encompasses, flexibility of thinking it promotes, and the broad range of mathematical implications (Levine et al., 2016; Lowrie et al., 2019; Hawes and Ansari, 2020; Hawes et al., 2022).

Despite promising results emerging between spatial skill development and mathematics (Gunderson et al., 2012; Hawes et al., 2022), no spatial silver bullet has emerged that can be used to improve mathematics broadly. However, different types of spatial skills have implications for different mathematics tasks at different time points (Casey et al., 2011; Young et al., 2018). Importantly, we need to acknowledge the different kinds of spatial encoding across different skills which have potentially different functions in mathematics. For example, once initially encoded, the greatest spatial demand in mental rotation tasks is to update and compare the orientation of the object, much like during geometric transformations (Shepard and Metzler, 1971; Battista et al., 2018). By contrast, in spatial visualization tasks the form of the object (i.e., the encoding) requires updating while simultaneously performing ongoing, complex spatial transformations (Wright et al., 2008), which holds a similar function to mental computation (Hawes and Ansari, 2020). Resnick and Stieff (2024) suggest that a practical approach may be to build spatial skills across different settings, thereby producing more generalizable skills with greater opportunities for transfer to mathematics understanding.

3.2 Large-scale spatial skills

Existing studies in mathematics rarely address spatial skills beyond object-based skills, leaving only speculation as to the role large-scale spatial skills, such as spatial orientation or perspective-taking, play in spatial-mathematics relations (Newcombe, 2018). Recent work has indicated that spatial orientation plays an influential role in mathematics performance for some content areas (e.g., Mix et al., 2016; Frick, 2019; Harris et al., 2021a). Frick (2019) found that spatial orientation skills at kindergarten predicted grade 2 performance in geometry, magnitude, and quantity measures but not arithmetic. By contrast, Harris et al. (2021a) found the role of spatial orientation important for both geometry and number problems, but the strength of this relationship was influenced by factors such as gender, age, and mathematics content, where the relationship was strongest in primary school, amongst female participants, and in geometry.

This recent work suggests that broad theoretical models should not overlook the role of large-scale spatial skills, such as spatial orientation, in spatial-mathematics relations (Mix et al., 2016; Frick, 2019; Harris et al., 2021a,b). In some instances, the influence of large-scale skills on mathematics performance exceeds object-based spatial skills (Harris et al., 2021a,b). Although longitudinal and cross-sectional studies indicate that early perspective-taking skills predict simultaneous and future performance on mathematics measures (Mix et al., 2016; Frick, 2019; Harris et al., 2021a,b) further work is needed to examine the multi-directional nature of this relationship. It is possible that advanced mathematics related to cartesian coordinate systems, if appropriately grounded in students' knowledge, may help develop large-scale spatial skills in a way not previously explored.

The influential nature of large-scale spatial skills may be embedded in certain mathematical contexts, such as different orthogonal views of 3D structures (Ramful et al., 2017), or exist at a more global level in the relations between objects (Harris et al., 2021a). A theoretical question for future research is how these large-scale spatial skills, such as perspective-taking, relate to mathematics understanding. Newcombe (2018) proposed that on the surface these skills do not have a clear link to mathematics content knowledge. A working theory for the relationship relates to the nature of the perspective-taking tasks themselves (Harris et al., 2021a). That is, whether the skills being assessed are purely about different perspectives, or whether there is also an influence related to spatial relations between objects. When learning new environments there is a heavy reliance on landmarks and identifiable features (Siegel and White, 1975), similarly at the root of many mathematical concepts is the idea of relations between objects (Mix and Cheng, 2012; Mix, 2019). Perhaps a connection between these different tasks is the spatial relations, beyond the spatial skills alone. This returns us to the question of strategy. If spatial tasks are being completed using a combination of spatial skills, visuospatial memory, visual discrimination, and relational encoding, then it makes sense that similar strategies could transfer to mathematics tasks (Hawes et al., 2022).

3.3 The question of geometry

One of the key discrepancies between the fields of mathematics education research and cognitive psychology concerns the attention paid to spatial skills in geometry. Mathematics education researchers argue that geometry is inherently spatial, and without spatial skills it is near impossible to perform the transformations required for geometric problem-solving (Clements and Battista, 1992; Mulligan, 2015; Battista et al., 2018; Downton and Livy, 2021; Thom et al., 2021). In fact, some mathematics education researchers go so far as to use the terms geometric reasoning and spatial reasoning interchangeably (Gutiérrez, 1996; Downton and Livy, 2021). On the other hand, cognitive psychologists argue that despite the spatial representations present in geometry curricula, the intricacies of mathematical conventions and operational knowledge make spatial skills alone insufficient to support geometric reasoning (Newcombe, 2018).

In their theoretical review, Hawes and Ansari (2020) focused solely on numerical skills, excluding geometry and measurement because of the inherent link to spatial processes. Geometry is frequently excluded from theoretical models within cognitive psychology because of the difficulty extracting the mathematical conventions integral to the content from the spatial characteristics within the representations (such as depictions of space and shape; Casey et al., 2011; Newcombe, 2018). While this is true of many complex geometric problems (e.g., area and 2D representations of 3D shapes), students tend to rely on rote-learning formulae when solving geometric proofs, rather than capitalizing on the spatial affordances of such tasks, often to their detriment (Fujita et al., 2020; Harris et al., 2023). Furthermore, foundational geometric skills rely heavily on the ability to imagine and perform spatial transformations (Battista et al., 2018).

A number of studies have explored the influence of spatial skills on performance across mathematics content that can be categorized as either formula- or spatial-based (e.g., Casey et al., 2011; Hawes et al., 2019b; Harris et al., 2021a,b). In the primary cohort of a study by Harris et al. (2021a), the proportion of variance in mathematics accounted for by spatial skills was equivalent for geometry and measurement, and number sense. Hawes et al. (2019b) found spatial skills were the only significant factor for predicting geometry performance in a sample of primary children, at the exclusion of executive function and numerical ability. For numeration content, spatial skills were significant alongside numerical ability in predicting performance (Hawes et al., 2019b). However, in the secondary cohort of the Harris et al. (2021a) study, despite a general downwards shift in the strength of the relationships, spatial skills accounted for a greater proportion of the variance in geometry-measurement compared with number sense. In a follow-up study focused on spatial skills at an item level Harris et al. (2021b) reported that measurement items in secondary school were significantly correlated with spatial skills, even more than items that focused on degrees and direction of 2D rotation. Casey et al. (2011) focused on the relationship between spatial skills and measurement tasks, separated into formula-based or spatial-conceptual items. The majority of their analysis explored the impact of community income and gender on this relationship; however it is noteworthy that spatial skills were significantly correlated (at an alpha level of $p < 0.001$) with both categories of measurement tasks ($r = 0.44$ and 0.50 respectively).

Although the operational knowledge required for completing many geometry and measurement tasks is critical (e.g., formula for calculations or interpretation of graphs), these curriculum areas remain important for unpacking spatial-mathematics relations (Harris et al., 2023). When considering the growing field of spatial interventions, larger mathematics training effects have been found in spatial training studies that were closely aligned (i.e., between spatial transformation training [visualization] and geometric transformations; Hawes et al., 2023). Although there are still links between less overtly spatial elements of mathematics, content areas such as geometry provide an opportunity to integrate spatial learning into mathematics classrooms without increasing teacher load (Lowrie and Logan, 2018; Thom et al., 2021; Harris et al., 2023; Hawes et al., 2023).

3.4 Linking spatial research and classroom mathematics

One approach to consider is to narrow the focus towards skills aligned to the mathematics curriculum (Hawes et al., 2023). This is not to say that we focus on curriculum only, but rather to the skills that underpin mathematics curriculum demands. Part of this recommendation involves expanding our ideas of what is spatial reasoning within mathematics curriculum (Mulligan, 2015; Woolcott et al., 2020). Shape and geometry provide many explicit opportunities to explore spatial concepts (Gilligan-Lee et al., 2022), however there are many inherently spatial elements of a mathematics curriculum that provide avenues to build connections between spatial reasoning and mathematics understanding. For example, Cutting (2021) reported significant growth in fraction understanding amongst 6 and 7 years old by incorporating spatial skills (i.e., spatial visualization and structuring) and tools (i.e., gesture and representation) into a carefully designed unit of work.

Drawing on the knowledge that has emerged from cognitive psychology to design targeted intervention and curriculum reform can provide pathways to integrate spatial instruction without overburdening teachers. For example, in the longitudinal work by Mix et al. (2016), early mathematics performance was linked to object-based spatial skills such as mental rotation, while later mathematics performance was more strongly associated with visuospatial working memory. Furthermore, skills that are established precursors to spatial and mathematical development, such as mental transformation (Gunderson et al., 2012) and patterning (Mulligan et al., 2018, 2020; Rittle-Johnson et al., 2019) should become an integral part of early years' learning.

Early years curriculum could embed spatial skill development into classroom practice (e.g., through concrete experiences that promote spatial thinking; Jirout and Newcombe, 2015; Levine et al., 2018;

Möhring et al., 2021). This would lay the spatial foundations, identified by psychologists and valued by mathematics educators (e.g., Pirie and Kieren, 1994; Presmeg, 2008), to link developing spatial skills with emergent mathematical concepts such as geometric structure, algebra and number. Children's pre-number spatial skills are more advanced than previously thought (Larkin et al., 2022) and they can be further enhanced with targeted spatial intervention embedded in play-based learning in the early years (Resnick and Lowrie, 2023).

The results of the meta-analysis by Hawes et al. (2022) revealed that with age, intervention effect sizes grow, meaning that spatial training becomes *more* important not less as mathematics becomes more complex. In the upper years of school, students can be supported in their mathematical development with embedded spatial skills that focus on linking spatial and mathematical concepts. This approach has been successfully implemented with grade 8 students by including pictorial and concrete materials before the introduction of symbolic representations (i.e., formulae) without compromising on delivery of content knowledge (Lowrie et al., 2021; Harris et al., 2023). The key is to focus on critical spatial skills in the early years and continue to embed spatial thinking into mathematics instruction throughout schooling.

4 Practical implications

In terms of spatial reasoning, the goals of mathematics education and cognitive psychology are aligned, that is, to harness spatial reasoning as a means of fostering mathematics development. I propose that by systematically and thoughtfully considering both mathematics education and cognitive psychology fields, we can start to uncover how these factors influence transfer (Mix and Levine, 2018; Gagnier and Fisher, 2020; Hawes and Ansari, 2020; Lowrie et al., 2020; Resnick et al., 2020; Hawes et al., 2022).

In a rapidly evolving 21st century, the demands on students are growing (Lowrie et al., 2017b; Ramful and Patahuddin, 2021). Therefore, we cannot assume that training isolated skills alone will be enough to help students achieve big goals, such as supporting the critical STEM pipeline (Doerschuk et al., 2016; Tian et al., 2022). Instead, I propose that in future we target our approach by contextualizing interventions in two ways: (1) embedding training into pedagogy to support students to make the connections between their developing spatial skills and curriculum content (Lowrie et al., 2017a, 2018, 2019, 2021); and (2) ensuring research becomes more translation-focused by integrating studies into classroom environments (Gagnier and Fisher, 2020). This is particularly important in primary schools where teachers may lack the skills and confidence to successfully embed spatial teaching into practice (Davis and Spatial Reasoning Study Group, 2015; Atit et al., 2018; Gagnier and Fisher, 2020; Bates et al., 2022; Gagnier et al., 2022; Hawes et al., 2023).

4.1 Designing interventions for enduring impact

Intervention studies have highlighted the potential causal relationship between spatial and mathematics skills, where developing spatial skills has led to improvements in mathematics performance (Cheng and Mix, 2014; Hawes et al., 2017, 2022; Lowrie et al., 2017a, 2019, 2021; Gilligan-Lee et al., 2020; Mix et al., 2021; Resnick and Lowrie, 2023). Despite success across a variety of intervention paradigms (Hawes et al., 2022), there is still no clear mechanism that allows transfer to take place between spatial training and mathematics performance and most importantly, the intervention structure that will lead to long-term benefits for students (Gagnier and Fisher, 2020; Lowrie et al., 2020; Resnick and Stieff, 2024).

It is important to consider long term intervention effects beyond the life of research studies if we are to have a meaningful impact on student learning. For example, in a training study by Levine et al. (2018) although significant gains were found in their mental transformation study from pre- to post-test, retest (a week later) showed no further signs of improvement. Although it is promising that students showed no loss of learning gains, the learning appeared to be limited to the training session. However, follow up studies like these are rare (Hawes et al., 2022) and should form a greater part of our research if we are to understand the true impact of spatial training. Furthermore, what does it mean if training impacts immediate post-test but does not impact ongoing learning? Hawes et al. (2022) hypothesized that one reason behind the impacts of spatial training may be in the priming of spatial strategies when completing tasks after training. Future studies need to expand the existing spatial training paradigm to explore changes to student understanding and representation.

Gagnier and Fisher (2020) proposed a model of integrating scientific findings into classroom practice to account for what they term the "black box of translation" (p. 1). They acknowledge that a significant obstacle to overcome is building teacher confidence so that they can support students to build their spatial skills (Gagnier et al., 2022). While teachers are often confident in their ability to deliver curriculum content, a lack of spatial curriculum or pre-teacher training in spatial reasoning may contribute to lower confidence in developing student's spatial skills through curriculum delivery (Lowrie and Logan, 2018; Bates et al., 2022; Gagnier et al., 2022; Gilligan-Lee et al., 2022).

In a content-heavy curriculum, it is not always practical to achieve a direct transfer between psychological lab-based studies and classroom practice (Gagnier and Fisher, 2020). Even the affordances of spatialized curriculum currently being explored through mathematics education research (e.g., Mulligan et al., 2020; Patahuddin et al., 2020; Pollitt et al., 2020; Cutting, 2021) are not easily translated into classroom practice, despite evidence for its efficacy (Resnick et al., 2020; Lowrie, 2024). Educators are well-placed to develop spatial skills through classroom opportunities. However, without explicit spatial curriculum, and limited to no instruction in spatial skills during pre-service education (unlike mathematics or literacy more generally), these opportunities often go unrealized (Davis and Spatial Reasoning Study Group, 2015; Lowrie and Logan, 2018; Gilligan-Lee et al., 2022).

4.1.1 Pedagogy models for fostering spatial reasoning in mathematics

One possibility for building teacher agency and developing interventions that go beyond spatial skill drills is through pedagogy that supports the link between skills development and conceptual understanding (Lowrie et al., 2018; Lowrie, 2024). Pedagogy provides multiple pathways to translation by (1) empowering teachers to adapt to the needs and context of their students (Lowrie et al., 2018), (2) creating explicit links between conceptual development and procedural knowledge (Harris et al., 2023), and (3) ensuring isolated skills development does not remain isolated as it is embedded in practice, with greater implications for transfer and translation (Hawes et al., 2023).

One example of spatial pedagogy is the Experience-Language-Pictorial-Symbolic-Application framework (ELPSA; Lowrie et al., 2018). The ELPSA framework grounds learning in students' pre-existing experiences (E) before building on key language and terminology (L). Mathematical concepts are then introduced in concrete forms, which include graphics and physical materials (P) before moving to more abstract, symbolic notations (S). Finally, when students have a firm grasp of a concept, new applications are presented for students to apply and extend their knowledge (A). These applications then become the experiences that launch the next iteration of a learning cycle. This model is designed to encourage students to move freely between stages as need demands. Initially developed as a model for mathematical concept development (Lowrie and Patahuddin, 2015), Lowrie et al. (2018) adapted the model as a framework for delivering spatial interventions (Lowrie et al., 2019, 2021). The ELPSA model provides a pedagogical structure that considers the cyclical nature of the learning process and the explicit links between pictorial (i.e., spatial) and symbolic (i.e., abstract) stages of learning critical for spatial reasoning.

Another complementary perspective to consider is the role of spatial reasoning in the growth of mathematical understanding (Pirie and Kieren, 1994). Mathematical concept development is not a linear process and is supported by the ability to create mental (i.e., spatial) representations throughout the learning cycle. The recursive nature of mathematical development means that although content knowledge builds sequentially, the mechanisms needed to fold back to earlier strategies, such as the relational and schematic representations characteristic of spatial reasoning, make the links between spatial skills and mathematics critical across development. That is, spatial skills are not simply a starting point or stepping-stone to content knowledge, but rather an ongoing, crucial part of the mathematics learning cycle (Pirie and Kieren, 1989). At each recursion of the learning cycle, visualizations and actions form the basis for abstraction in more advanced mathematical reasoning (Pirie and Kieren, 1989, 1994). Therefore, interventions that build spatial skills hold potential for supporting mathematics development throughout the education years (National Research Council, 2006; Hawes et al., 2022, 2023).

4.2 Taking a broader view of spatial research

In separate fields through intentional use of language, methodologies, epistemologies, publication expectations, and research traditions we are limited by the small pieces that are illuminated from our individual perspective. It is only through sharing knowledge that we can begin to see the whole picture. In this paper, I have tried to focus on points of overlap and potential avenues for translation from theory to practice.

Both mathematics education and cognitive psychology acknowledge a set of spatial skills that support mathematical understanding; however, the way we define and assess these skills can impact our findings (Lowrie et al., 2020; Resnick et al., 2020; Resnick and Stieff, 2024). In both fields we talk about individual differences, whether through tightly controlled isolation of differences in psychology (Mix and Levine, 2018; Resnick and Stieff, 2024) or acknowledging the experiences of individual learning trajectories in education (Mix and Levine, 2018; Confrey, 2019). Critically, how we approach context, through parsing it out or embedding, impacts our findings (Harris et al., 2022; Resnick and Stieff, 2024).

Spatial reasoning is so intrinsically linked to our everyday experience that isolating the individual skills cannot provide complete answers about spatial-mathematics relations. By removing the context in which spatial skills are developed and applied, it is possible that the way these skills are enacted changes as well (Boone et al., 2019; Newcombe et al., 2022). Through student-centered approaches to research, we have an opportunity to explore how student thinking and strategies correspond with what is expected based on performance and relationships modelled by data (Golinkoff et al., 2017; Mix and Levine, 2018).

Finally, there are still parts of the system that are important driving factors not currently being considered, such as lived experiences (Coutrot et al., 2022; Harris et al., 2022), the tendency to apply spatial thinking, regardless of skill level (Kim and Bednarz, 2013; Peterson et al., 2020), and the tools and representations indicative of spatial thinking but not measured by current tests (Newcombe, 2018; Logan et al., 2022). These factors are influential and should be incorporated to support student learning (National Research Council, 2006). An important goal is to translate the emerging knowledge from research to tangible, accessible practices for educators (Gagnier et al., 2022; Lowrie, 2024). Spatial reasoning provides a unique opportunity among cognitive skills to ground learning in life experiences and then connect emerging knowledge to conceptual ideas in meaningful ways. The nature of the resulting skills then provides long term benefits beyond education (Wai et al., 2009; Tian et al., 2022).

5 Concluding comments

The emergence of spatial reasoning as a critical field of enquiry in mathematics education necessitates a more integrated research perspective on the role of spatial reasoning in mathematics learning and instruction. Although large strides have been made in terms of theoretical positing of spatial reasoning, and its importance for mathematics, a large gap remains in the translation of research findings to classroom practice. Throughout this paper I have argued for a broader view of research problems and methodologies in spatial reasoning studies and the importance of the application of research to meaningful contexts in pedagogy and learning. A continued siloed approach is detrimental to advancing the field and continues to disadvantage the most vulnerable students.

We are at a critical juncture where possibilities for collaborative, multi-dimensional research are opening up and would ensure the field advances in theoretical and practical ways. Groundbreaking publications such as the Project TALENT results reported by Wai et al. (2009) and the meta-analysis of training studies published by Uttal et al. (2013) gave us insights into the potential power of spatial reasoning as a vehicle for developing 21st century skills. With increasing attention and focus on the need for a strong STEM pipeline, we need to empower educators and students with the tools at our disposal in the research lab. This can only happen when we look beyond our traditional research paradigms to the context in which spatial skills are applied in and out of the classroom.

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Conflict of interest

The author declares that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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References

Adams, J., Resnick, I., and Lowrie, T. (2022). Supporting senior high-school students' measurement and geometry performance: does spatial training transfer to mathematics achievement? *Math. Educ. Res. J.* 35, 879–900. doi: 10.1007/s13394-022-00416-y

Atit, K., Miller, D. I., Newcombe, N. S., and Uttal, D. H. (2018). Teachers' spatial skills across disciplines and education levels: exploring nationally representative data. *Arch. Sci. Psychol.* 6, 130–137. doi: 10.1037/arc0000041

Atit, K., Power, J. R., Pigott, T., Lee, J., Geer, E. A., Uttal, D. H., et al. (2022). Examining the relations between spatial skills and mathematical performance: a meta-analysis. *Psychon. Bull. Rev.* 29, 699–720. doi: 10.3758/s13423-021-02012-w

Atit, K., Uttal, D. H., and Stieff, M. (2020). Situating space: using a discipline-focused lens to examine spatial thinking skills. *Cognit. Res.* 5, 1–16. doi: 10.1186/s41235-020-00210-z

Australian Curriculum, Assessment and Reporting Authority (2016). NAPLAN: Numeracy. Available at: https://www.nap.edu.au/naplan/numeracy

Australian Curriculum, Assessment and Reporting Authority (2023). Understand this learning area: mathematics. Available at: https://v9.australiancurriculum.edu.au/teacher-resources/understand-this-learning-area/mathematics

Bates, K. E., Williams, A. Y., Gilligan-Lee, K. A., Gripton, C., Lancaster, A., Williams, H., et al. (2022). Practitioner's perspectives on spatial reasoning in educational practice from birth to 7 years. *Br. J. Educ. Psychol.* 93, 571–590. doi: 10.1111/bjep.12579

Battista, M. T., Frazee, L. M., and Winer, M. L. (2018). "Analyzing the relation between spatial and geometric reasoning for elementary and middle school students" in *Visualizing mathematics: the role of spatial reasoning in mathematical thought*. eds. K. S. Mix and M. T. Battista (Cham, Switzerland: Springer), 195–228.

Bishop, A. J. (1980). Spatial abilities and mathematics education: a review. *Educ. Stud. Math.* 11, 257–269. doi: 10.1007/BF00697739

Boone, A. P., and Hegarty, M. (2017). Sex differences in mental rotation tasks: not just in the mental rotation process! *J. Exp. Psychol. Learn. Mem. Cogn.* 43, 1005–1019. doi: 10.1037/xlm0000370

Boone, A. P., Maghen, B., and Hegarty, M. (2019). Instructions matter: individual differences in navigation strategy and ability. *Mem. Cogn.* 47, 1401–1414. doi: 10.3758/s13421-019-00941-5

Bronfenbrenner, U. (1977). Toward an experimental ecology of human development. *Am. Psychol.* 32, 513–531. doi: 10.1037/0003-066X.32.7.513

Bruce, C. D., Davis, B., Sinclair, N., McGarvey, L., Hallowell, D., Drefs, M., et al. (2017). Understanding gaps in research networks: using "spatial reasoning" as a window into the importance of networked educational research. *Educ. Stud. Math.* 95, 143–161. doi: 10.1007/s10649-016-9743-2

Bruce, C. D., and Hawes, Z. (2015). The role of 2D and 3D mental rotation in mathematics for young children: what is it? Why does it matter? And what can we do about it? ZDM. *Math. Educ.* 47, 331–343. doi: 10.1007/s11858-014-0637-4

Buckley, J., Seery, N., and Canty, D. (2018). A heuristic framework of spatial ability: a review and synthesis of spatial factor literature to support its translation into STEM education. *Educ. Psychol. Rev.* 30, 947–972. doi: 10.1007/s10648-018-9432-z

Carroll, J. B. (1993). *Human cognitive abilities: a survey of factor-analytic studies* Cambridge University Press.

Casey, B. M., Dearing, E., Vasilyeva, M., Ganley, C. M., and Tine, M. (2011). Spatial and numerical predictors of measurement performance: the moderating effects of community income and gender. *J. Educ. Psychol.* 103, 296–311. doi: 10.1037/a0022516

Cheng, Y.-L., and Mix, K. S. (2014). Spatial training improves children's mathematics ability. *J. Cogn. Dev.* 15, 2–11. doi: 10.1080/15248372.2012.725186

Clements, D. H., and Battista, M. T. (1992). "Geometry and spatial reasoning" in *Handbook of research on mathematics teaching and learning*. ed. D. A. Grouws (New York: Macmillan), 420–464.

Confrey, J. (2019). A synthesis of research on learning trajectories/progressions in mathematics. Commissioned for the OECD 2030 learning framework, OECD mathematics curriculum document analysis project workshop. Available at: www.oecd.org/education/2030.

Coutrot, A., Manley, E., Goodroe, S., Gahnstrom, C., Filomena, G., Yesiltepe, D., et al. (2022). Entropy of city street networks linked to future spatial navigation ability. *Nature* 604, 104–110. doi: 10.1038/s41586-022-04486-7

Cutting, C. (2021). "Spatial reasoning and the development of early fraction understanding" in *Excellence in mathematics education: Foundations and pathways (proceedings of the 43rd annual conference of the mathematics education research Group of Australasia)*. eds. Y. H. Leong, B. Kaur, B. H. Choy, J. B. W. Yeo and S. L. Chin (Singapore: MERGA), 171–178.

Davis, B.Spatial Reasoning Study Group (2015). *Spatial reasoning in the early years: principles, assertions, and speculations*. New York: Routledge.

Doerschuk, P., Bahrim, C., Daniel, J., Kruger, J., Mann, J., and Martin, C. (2016). Closing the gaps and filling the STEM pipeline: a multidisciplinary approach. *J. Sci. Educ. Technol.* 25, 682–695. doi: 10.1007/s10956-016-9622-8

Downton, A., and Livy, S. (2021). Insights into students' geometric reasoning relating to prisms. *Int. J. Sci. Math. Educ.* 20, 1543–1571. doi: 10.1007/s10763-021-10219-5

Ekstrom, R. B., French, J. W., Harman, H. H., and Dermen, D. (1976). *Kit of factor-referenced cognitive tests*. Princeton, New Jersey: Educational Testing Service.

Empson, S. B., and Turner, E. (2006). The emergence of multiplicative thinking in children's solutions to paper folding tasks. *J Mathematical Behav* 25, 46–56. doi: 10.1016/j.jmathb.2005.11.001

Fan, X. (2001). Statistical significance and effect size in education research: two sides of a coin. *J. Educ. Res.* 94, 275–282. doi: 10.1080/00220670109598763

Frick, A. (2019). Spatial transformation abilities and their relation to later mathematics performance. *Psychol. Res.* 83, 1465–1484. doi: 10.1007/s00426-018-1008-5

Fujita, T., Kondo, Y., Kumakura, H., Kunimune, S., and Jones, K. (2020). Spatial reasoning skills about 2D representations of 3D geometrical shapes in grades 4 to 9. *Math. Educ. Res. J.* 32, 235–255. doi: 10.1007/s13394-020-00335-w

Gagnier, K. M., and Fisher, K. R. (2020). Unpacking the black box of translation: a framework for infusing spatial thinking into curricula. *Cognit. Res.* 5, 1–19. doi: 10.1186/s41235-020-00222-9

Gagnier, K. M., Holochwost, S. J., and Fisher, K. R. (2022). Spatial thinking in science, technology, engineering, and mathematics: elementary teachers' beliefs, perceptions, and self-efficacy. *J. Res. Sci. Teach.* 59, 95–126. doi: 10.1002/tea.21722

Gilligan, K. A., Hodgkiss, A., Thomas, M. S. C., and Farran, E. K. (2019). The developmental relations between spatial cognition and mathematics in primary school children. *Dev. Sci.* 22:e12786. doi: 10.1111/desc.12786

Gilligan-Lee, K. A., Hawes, Z. C., and Mix, K. S. (2022). Spatial thinking as the missing piece in mathematics curricula. Npj science of. *Learning* 7, 10–14. doi: 10.1038/s41539-022-00128-9

Gilligan-Lee, K. A., Thomas, M. S. C., and Farran, E. K. (2020). First demonstration of effective spatial training for near transfer to spatial performance and far transfer to a range of mathematics skills at 8 years. *Dev. Sci.* 23:e12909. doi: 10.1111/desc.12909

Golinkoff, R. M., Hirsh-Pasek, K., Grob, R., and Schlesinger, M. (2017). "Oh, the places You'll go" by bringing developmental science into the world! *Child Dev.* 88, 1403–1408. doi: 10.1111/cdev.12929

Green, C. S., and Newcombe, N. S. (2020). Cognitive training: how evidence, controversies, and challenges inform education policy. *Policy Insights Behav. Brain Sci.* 7, 80–86. doi: 10.1177/2372732219870202

Grootenboer, P., and Peter-Koop, A. (2024). "Practice theory in mathematics education research" in *Fourth international handbook of mathematics education: controversial issues in mathematics education*. eds. M. A. Clements, B. Kaur, T. Lowrie, V. Mesa and J. Prytz (Cham, Switzerland: Springer)

Gunderson, E. A., Ramirez, G., Beilock, S. L., and Levine, S. C. (2012). The relation between spatial skill and early number knowledge: the role of the linear number line. *Dev. Psychol.* 48, 1229–1241. doi: 10.1037/a0027433

Gutiérrez, A. (1996). "Visualization in 3-dimensional geometry: in search of a framework" in *Proceedings of the 20th international conference for the psychology of mathematics education*. eds. L. Puig and A. Gutierrez, vol. *1* (Valencia, Spain: International Group for the Psychology of Mathematics Education), 3–19.

Gutiérrez, A., Ramírez, R., Benedicto, C., Beltrán-Meneu, M. J., and Jaime, A. (2018). "Visualization abilities and complexity of reasoning in mathematically gifted students' collaborative solutions to a visualization task: a networked analysis" in *Visualizing mathematics: The role of spatial reasoning in mathematical thought*. eds. K. S. Mix and M. T. Battista (Cham, Switzerland: Springer), 309–340.

Harris, D. (2021). "Spatial ability, skills, reasoning or thinking: what does it mean for mathematics?" in *Excellence in mathematics education: Foundations and pathways (proceedings of the 43rd annual conference of the mathematics education research Group of Australasia)*. eds. Y. H. Leong, B. Kaur, B. H. Choy, J. B. W. Yeo and L. F. Wong (Singapore: MERGA), 219–226.

Harris, J., Hirsh-Pasek, K., and Newcombe, N. S. (2013). Understanding spatial transformations: similarities and differences between mental rotation and mental folding. *Cogn. Process.* 14, 105–115. doi: 10.1007/s10339-013-0544-6

Harris, D., Logan, T., and Lowrie, T. (2021b). Unpacking mathematical-spatial relations: problem-solving in static and interactive tasks. *Math. Educ. Res. J.* 33, 495–511. doi: 10.1007/s13394-020-00316-z

Harris, D., Logan, T., and Lowrie, T. (2022). Authentic perspective-taking: looking beyond abstract spatial skills to the influence of culture and environment. *Learn. Cult. Soc. Interact.* 33:100611. doi: 10.1016/j.lcsi.2022.100611

Harris, D., Logan, T., and Lowrie, T. (2023). Spatial visualization and measurement of area: a case study in spatialized mathematics instruction. *J. Mathematical Behav.* 70:101038. doi: 10.1016/j.jmathb.2023.101038

Harris, D., Lowrie, T., Logan, T., and Hegarty, M. (2021a). Spatial reasoning, mathematics, and gender: do spatial constructs differ in their contribution to performance? *Br. J. Educ. Psychol.* 91, 409–441. doi: 10.1111/bjep.12371

Hawes, Z., and Ansari, D. (2020). What explains the relationship between spatial and mathematical skills? A review of evidence from brain and behavior. *Psychon. Bull. Rev.* 27, 465–482. doi: 10.3758/s13423-019-01694-7

Hawes, Z., Gilligan-Lee, K. A., and Mix, K. S. (2022). Effects of spatial training on mathematics performance: a meta-analysis. *Dev. Psychol.* 58, 112–137. doi: 10.1037/dev0001281

Hawes, Z., Gilligan-Lee, K. A., and Mix, K. S. (2023). "Infusing spatial thinking into middle school mathematics: what, why, and how?" in *Mathematical cognition and understanding*. eds. K. M. Robinson, A. Dubé and D. Kostopoulos (Cham, Switzerland: Springer Nature Publishing), 13–33.

Hawes, Z., Moss, J., Caswell, B., Naqvi, S., and MacKinnon, S. (2017). Enhancing children's spatial and numerical skills through a dynamic spatial approach to early geometry instruction: effects of a 32-week intervention. *Cogn. Instr.* 35, 236–264. doi: 10.1080/07370008.2017.1323902

Hawes, Z., Moss, J., Caswell, B., and Poliszczuk, D. (2015). Effects of mental rotation training on children's spatial and mathematics performance: a randomized controlled study. *Trends Neurosci. Educ.* 4, 60–68. doi: 10.1016/j.tine.2015.05.001

Hawes, Z., Moss, J., Caswell, B., Seo, J., and Ansari, D. (2019b). Relations between numerical, spatial, and executive function skills and mathematics achievement: a latent-variable approach. *Cogn. Psychol.* 109, 68–90. doi: 10.1016/j.cogpsych.2018.12.002

Hawes, Z., Sokolowski, H. M., Ononye, C. B., and Ansari, D. (2019a). Neural underpinnings of numerical and spatial cognition: an fMRI meta-analysis of brain regions associated with symbolic number, arithmetic, and mental rotation. *Neurosci. Biobehav. Rev.* 103, 316–336. doi: 10.1016/j.neubiorev.2019.05.007

Hegarty, M. (2010). Components of spatial intelligence. *Psychol. Learn. Motiv.* 52, 265–297. doi: 10.1016/S0079-7421(10)52007-3

Hegarty, M. (2018). Ability and sex differences in spatial thinking: what does the mental rotation test really measure? *Psychon. Bull. Rev.* 25, 1212–1219. doi: 10.3758/s13423-017-1347-z

Hegarty, M., and Kozhevnikov, M. (1999). Types of visual-spatial representations and mathematical problem solving. *J. Educ. Psychol.* 91, 684–689. doi: 10.1037/0022-0663.91.4.684

Hegarty, M., and Waller, D. (2004). A dissociation between mental rotation and perspective-taking spatial abilities. *Intelligence* 32, 175–191. doi: 10.1016/j.intell.2003.12.001

Hegarty, M., and Waller, D. (2005). "Individual differences in spatial abilities" in *Handbook of visuospatial thinking*. eds. P. Shah and A. Miyake (New York: Cambridge University Press), 121–169.

Hodgkiss, A., Gilligan-Lee, K. A., Thomas, M. S. C., Tolmie, A. K., and Farran, E. K. (2021). The developmental trajectories of spatial skills in middle childhood. *Br. J. Dev. Psychol.* 39, 566–583. doi: 10.1111/bjdp.12380

Jirout, J. J., and Newcombe, N. S. (2015). Building blocks for developing spatial skills: evidence from a large, representative U.S. Sample. *Psychol. Sci.* 26, 302–310. doi: 10.1177/0956797614563338

Johnson, T., Burgoyne, A. P., Mix, K. S., Young, C. J., and Levine, S. C. (2022). Spatial and mathematics skills: similarities and differences related to age, SES, and gender. *Cognition* 218:104918. doi: 10.1016/j.cognition.2021.104918

Jung, S., Meinhardt, A., Braeuning, D., Roesch, S., Cornu, V., Pazouki, T., et al. (2020). Hierarchical development of early visual-spatial abilities – a taxonomy based assessment using the MaGrid app. *Front. Psychol.* 11:article 871. doi: 10.3389/fpsyg.2020.00871

Kim, M., and Bednarz, B. (2013). Effects of a GIS course on self-assessment of spatial habits of mind (SHOM). *J. Geogr.* 112, 165–177. doi: 10.1080/00221341.2012.684356

Kozhevnikov, M., Blazhenkova, O., and Becker, M. (2010). Trade-off in object versus spatial visualization abilities: restriction in the development of visual-processing resources. *Psychon. Bull. Rev.* 17, 29–35. doi: 10.3758/PBR.17.1.29

Kozhevnikov, M., and Hegarty, M. (2001). A dissociation between object manipulation spatial ability and spatial orientation ability. *Mem. Cogn.* 29, 745–756. doi: 10.3758/BF03200477

Larkin, K., Resnick, I., and Lowrie, T. (2022). *Preschool children's repeating patterning skills: evidence of their capability from a large scale, naturalistic, Australia wide study* Mathematical Thinking and Learning. doi: 10.1080/10986065.2022.2056320

Levine, S. C., Foley, A., Lourenco, S., Ehrlich, E., and Ratliff, K. (2016). Sex differences in spatial cognition: advancing the conversation. *Cogn. Sci.* 7, 127–155. doi: 10.1002/wcs.1380

Levine, S. C., Goldin-Meadow, S., Carlson, M. T., and Hemani-Lopez, N. (2018). Mental transformation skill in young children: the role of concrete and abstract motor training. *Cogn. Sci.* 42, 1207–1228. doi: 10.1111/cogs.12603

Levine, S. C., Vasilyeva, M., Lourenco, S. F., Newcombe, N. S., and Huttenlocher, J. (2005). Socioeconomic status modifies the sex difference in spatial skill. *Psychol. Sci.* 16, 841–845. doi: 10.1037/bul0000191

Linn, M. C., and Petersen, A. C. (1985). Emergence and characterization of sex differences in spatial ability: a meta-analysis. *Child Dev.* 56, 1479–1498. doi: 10.2307/1130467

Logan, T., and Lowrie, T. (2017). Gender perspectives on spatial tasks in a national assessment: a secondary data analysis. *Res. Mathematics Educ.* 19, 199–216. doi: 10.1080/14794802.2017.1334577

Logan, T., Lowrie, T., Resnick, I., and Harris, D. (2022). Spatialising the pedagogy: Directions for future research [conference presentation]. In Mathematics education research Group of Australasia 44th annual conference, Launceston, Tasmania

Lowrie, T. (2020). The utility of diagrams in elementary problem-solving. *Cogn. Dev.* 55:100921. doi: 10.1016/j.cogdev.2020.100921

Lowrie, T. (2024). "Coming apart at every nail: where to now for the theory-practice dream?" in *Fourth international handbook of mathematics education: controversial issues in mathematics education*. eds. M. A. Clements, B. Kaur, T. Lowrie, V. Mesa and J. Prytz (Cham, Switzerland: Springer)

Lowrie, T., Downes, N., and Leonard, S. (2017b). STEM education for all young Australians: a bright spots learning hub foundation paper. Available at: https://www.socialventures.com.au/assets/STEM-education-for-all-young-Australians-Smaller.pdf

Lowrie, T., Harris, D., Logan, T., and Hegarty, M. (2021). The impact of a spatial intervention program on students' spatial reasoning and mathematics performance. *J. Exp. Educ.* 89, 259–277. doi: 10.1080/00220973.2019.1684869

Lowrie, T., Jorgensen, R., Logan, T., and Harris, D. (2022). Culture and geography: how do primary students map their local environment? *Aust. Educ. Res.* 49, 261–284. doi: 10.1007/s13384-021-00440-0

Lowrie, T., and Kay, R. (2001). Relationship between visual and nonvisual solution methods and difficulty in elementary mathematics. *J. Educ. Res.* 94, 248–255. doi: 10.1080/00220670109598758

Lowrie, T., and Logan, T. (2018). "The interaction between spatial reasoning constructs and mathematics understandings in elementary classrooms" in *Visualizing mathematics: the role of spatial reasoning in mathematical thought*. eds. K. S. Mix and M. T. Battista (Cham, Switzerland: Springer), 253–276.

Lowrie, T., Logan, T., Harris, D., and Hegarty, M. (2018). The impact of an intervention program on students' spatial reasoning: student engagement through mathematics-enhanced learning activities. *Cognit. Res.* 3, 50–10. doi: 10.1186/s41235-018-0147-y

Lowrie, T., Logan, T., and Hegarty, M. (2019). The influence of spatial visualization training on students' spatial reasoning and mathematics performance. *J. Cogn. Dev.* 20, 729–751. doi: 10.1080/15248372.2019.1653298

Lowrie, T., Logan, T., and Ramful, A. (2017a). Visuospatial training improves elementary students' mathematics performance. *Br. J. Educ. Psychol.* 87, 170–186. doi: 10.1111/bjep.12142

Lowrie, T., and Patahuddin, S. M. (2015). ELPSA as a lesson design framework. *J. Mathematics Educ.* 6, 1–15. doi: 10.22342/jme.6.2.2166.77-92

Lowrie, T., Resnick, I., Harris, D., and Logan, T. (2020). In search of the mechanisms that enable transfer from spatial reasoning to mathematics understanding. *Math. Educ. Res. J.* 32, 175–188. doi: 10.1007/s13394-020-00336-9

Maresch, G., and Sorby, S. (2021). Perspectives on spatial thinking. *J. Geometry Graphics* 25, 271–293.

McGee, M. G. (1979). Human spatial abilities: psychometric studies and environmental, genetic, hormonal, and neurological influences. *Psychol. Bull.* 86, 889–918. doi: 10.1037/0033-2909.86.5.889

Mix, K. S. (2019). Why are spatial skill and mathematics related? *Child Dev. Perspect.* 13, 121–126. doi: 10.1111/cdep.12323

Mix, K. S., and Cheng, Y.-L. (2012). The relation between space and math: developmental and educational implications. *Adv. Child Dev. Behav.* 42, 197–243. doi: 10.1016/B978-0-12-394388-0.00006-X

Mix, K. S., Hambrick, D. Z., Satyam, V. R., Burgoyne, A. P., and Levine, S. C. (2018). The latent structure of spatial skill: a test of the 2 × 2 typology. *Cognition* 180, 268–278. doi: 10.1016/j.cognition.2018.07.012

Mix, K. S., and Levine, S. C. (2018). "Part II commentary 2: disparities and opportunities: plotting a new course for research on spatial visualization and mathematics" in *Visualizing mathematics: the role of spatial reasoning in mathematical thought*. eds. K. S. Mix and M. T. Battista (Cham, Switzerland: Springer), 347–353.

Mix, K. S., Levine, S. C., Cheng, Y., Stockton, J. D., and Bower, C. (2021). Effects of spatial training on mathematics in first and sixth grade children. *J. Educ. Psychol.* 113, 304–314. doi: 10.1037/edu0000494

Mix, K. S., Levine, S. C., Cheng, Y., Young, C., Hambrick, D. Z., Ping, R., et al. (2016). Separate but correlated: the latent structure of space and mathematics across development. *J. Exp. Psychol. Gen.* 145, 1206–1227. doi: 10.1037/xge0000182

Möhring, W., Ribner, A. D., Segerer, R., Libertus, M. E., Kahl, T., Troesch, L. M., et al. (2021). Developmental trajectories of children's spatial skills: influencing variables and associations with later mathematical thinking. *Learn. Instr.* 75:101515. doi: 10.1016/j.learninstruc.2021.101515

Montello, D. R., Grossner, K., and Janelle, D. G. (Eds.) (2014). *Space in mind: concepts for spatial learning and education*. Cambridge, Massachusetts: MIT Press.

Mulligan, J. (2015). Looking within and beyond the geometry curriculum: connecting spatial reasoning to mathematics learning. ZDM. *Math. Educ.* 47, 511–517. doi: 10.1007/s11858-015-0696-1

Mulligan, J., Woolcott, G., Busatoo, S., Lai, J., Mitchelmore, M., and Davis, B. (2020). Evaluating the impact of a spatial reasoning mathematics program (SRMP) intervention in the primary school. *Math. Educ. Res. J.* 32, 285–305. doi: 10.1007/s13394-020-00324-z

Mulligan, J., Woolcott, G., Mitchelmore, M., and Davis, B. (2018). Connecting mathematics learning through spatial reasoning. *Math. Educ. Res. J.* 30, 77–87. doi: 10.1007/s13394-017-0210-x

National Research Council (2006). *Learning to think spatially: GIS as a support system in the K—12 curriculum*. Washington DC: National Academy Press.

Newcombe, N. S. (2010). Picture this: increasing math and science learning by improving spatial thinking. *Am. Educ.* 34, 29–43.

Newcombe, N. S. (2017). *Harnessing spatial thinking to support STEM learning.* OECD education working paper, no. 161, OECD publishing: Paris.

Newcombe, N. S. (2018). "Part II commentary 3: linking spatial and mathematical thinking: the search for mechanism" in *Visualizing mathematics: the role of spatial reasoning in mathematical thought*. eds. K. S. Mix and M. T. Battista (Cham, Switzerland: Springer), 355–360.

Newcombe, N. S., Hegarty, M., and Uttal, D. (2022). Building a cognitive science of human variation: individual differences in spatial navigation. *Top. Cogn. Sci.* 15, 6–14. doi: 10.1111/tops.12626

Newcombe, N. S., and Shipley, T. F. (2015). "Thinking about spatial thinking: new typology, new assessments" in *Studying visual and spatial reasoning for design creativity* (Dordrecht, The Netherlands: Springer), 179–192.

Ontario Ministry of Education, (2020). Mathematics curriculum – 2020. Available at: https://www.dcp.edu.gov.on.ca/en/curriculum/elementary-mathematics/context/the-strands-in-the-mathematics-curriculum#strand-e-spatial-sense.

Owens, K. (2020). Transforming the established perceptions of visuospatial reasoning: integrating an ecocultural perspective. *Math. Educ. Res. J.* 32, 257–283. doi: 10.1007/s13394-020-00332-z

Patahuddin, S. M., Rokhmah, S., and Ramful, A. (2020). What does teaching of spatial visualisation skills incur: an exploration through the visualise-predict-check heuristic. *Math. Educ. Res. J.* 32, 307–329. doi: 10.1007/s13394-020-00321-2

Peterson, E. G., Weinberger, A. B., Uttal, D. H., Kolvoord, B., and Green, A. E. (2020). Spatial activity participation in childhood and adolescence: consistency and relations to spatial thinking in adolescence. *Cognit. Res.* 5, 1–13. doi: 10.1186/s41235-020-00239-0

Piaget, J., and Inhelder, B. (1967). *The child's conception of space* Routledge & Kegan Paul.

Pirie, S. E. B., and Kieren, T. E. (1989). A recursive theory of mathematical understanding. *Learn Mathematics* 9, 7–11.

Pirie, S. E. B., and Kieren, T. E. (1994). Growth in mathematical understanding: how can we characterise it and how can we represent it? *Educ. Stud. Math.* 26, 165–190. doi: 10.1007/BF01273662

Pollitt, R., Cohrssen, C., and Seah, W. T. (2020). Assessing spatial reasoning during play: educator observations, assessment and curriculum planning. *Math. Educ. Res. J.* 32, 331–363. doi: 10.1007/s13394-020-00337-8

Presmeg, N. (2008). "Spatial abilities research as a foundation for visualization in teaching and learning mathematics" in *Critical issues in mathematics education: Major contributions of Alan Bishop*. eds. P. Clarkson and N. Presmeg (New York: Springer), 83–95.

Ramful, A., Ho, S. Y., and Lowrie, T. (2015). Visual and analytical strategies in spatial visualisation: perspectives from bilateral symmetry and reflection. *Math. Educ. Res. J.* 27, 443–470. doi: 10.1007/s13394-015-0144-0

Ramful, A., Lowrie, T., and Logan, T. (2017). Measurement of spatial ability: construction and validation of the spatial reasoning instrument for middle school students. *J. Psychoeduc. Assess.* 35, 709–727. doi: 10.1177/0734282916659207

Ramful, A., and Patahuddin, S. M. (2021). "The fourth industrial revolution: implications for school mathematics" in *Teaching and learning in the 21st century: Embracing the fourth industrial revolution*. ed. J. Naidoo (Leiden, The Netherlands: Brill).

Resnick, I., Harris, D., Logan, T., and Lowrie, T. (2020). The relation between mathematics achievement and spatial reasoning. *Math. Educ. Res. J.* 32, 171–174. doi: 10.1007/s13394-020-00338-7

Resnick, I., and Lowrie, T. (2023). Spatial games improve spatial reasoning and numeracy in preschool: findings from a large scale nationally representative randomized control trial. *J. Res. Math. Educ.* 54, 295–316. doi: 10.5951/jresematheduc-2022-0051

Resnick, I., and Shipley, T. F. (2013). Breaking new ground in the mind: an initial study of mental brittle transformation and mental rigid rotation in science experts. *Cogn. Process.* 14, 143–152. doi: 10.1007/s10339-013-0548-2

Resnick, I., and Stieff, M. (2024). "Characterizing cognition and context within mathematics education" in *Fourth international handbook of mathematics education: Controversial issues in mathematics education*. eds. M. A. Clements, B. Kaur, T. Lowrie, V. Mesa and J. Prytz (Cham, Switzerland: Springer).

Rittle-Johnson, B., Zippert, E. L., and Boice, K. L. (2019). The roles of patterning and spatial skills in early mathematics development. *Early Child. Res. Q.* 46, 166–178. doi: 10.1016/j.ecresq.2018.03.006

Sanchez, C. A., and Wiley, J. (2017). "Dynamic visuospatial ability and learning from dynamic visualizations" in *Learning from dynamic visualization: Innovation in research and application*. eds. R. Lowe and R. Ploetzner (Cham, Switzerland: Springer), 155–176.

Seah, R., and Horne, M. (2020). The influence of spatial reasoning on analysing about measurement situations. *Math. Educ. Res. J.* 32, 365–386. doi: 10.1007/s13394-020-00327-w

Sezen Yüksel, N. (2017). "Measuring spatial visualization: test development study" in *Visual-spatial ability in STEM education*. ed. M. S. Khine (Cham, Switzerland: Springer), 59–84.

Shepard, R. N., and Metzler, J. (1971). Mental rotation of three-dimensional objects. *Science* 171, 701–703. doi: 10.1126/science.171.3972.701

Siegel, A. W., and White, S. H. (1975). The development of spatial representations of large-scale environments. *Adv. Child Dev. Behav.* 10, 9–55. doi: 10.1016/S0065-2407(08)60007-

Smith, I. M. (1964). *Spatial ability: Its educational and social significance*. London, UK: Knapp.

Spearman, C. (1904). General intelligence, objectively determined and measured. *Am. J. Psychol.* 15, 201–292. doi: 10.2307/1412107

Stieff, M., and Uttal, D. (2015). How much can spatial training improve STEM achievement? *Educ. Psychol. Rev.* 27, 607–615. doi: 10.1007/s10648-015-9304-8

Thom, J. S., McGarvey, L. M., and Lineham, N. D. (2021). "A recursive theory of mathematical understanding" in *Excellence in mathematics education: Foundations and pathways (proceedings of the 43rd annual conference of the mathematics education* *research Group of Australasia)*. eds. Y. H. Leong, B. Kaur, B. H. Choy, J. B. W. Yeo and S. L. Chin (Singapore: MERGA), 385–392.

Thompson, J. M., Nuerk, H.-C., Moeller, K., and Cohen Kadosh, R. (2013). The link between mental rotation ability and basic numerical representations. *Acta Psychol.* 144, 324–331. doi: 10.1016/j.actpsy.2013.05.009

Thurstone, L. L. (1938). *Primary mental abilities*. Chicago: Chicago University Press.

Thurstone, L. L. (1950). "Some primary abilities in visual thinking" in *Psychometric laboratory research report*, vol. *59* (Chicago: University of Chicago Press).

Tian, J., Ren, K., Newcombe, N. S., Weinraub, M., Vandell, D. L., and Gunderson, E. A. (2022). Tracing the origins of the STEM gender gap: the contribution of childhood spatial skills. *Dev. Sci.* 26:e13302. doi: 10.1111/desc.13302

Uttal, D. H., and Cohen, C. A. (2012). Chapter four - spatial thinking and STEM education: when, why, and how? *Psychol. Learn. Motiv.* 57, 147–181. doi: 10.1016/B978-0-12-394293-7.00004-2

Uttal, D. H., Meadow, N. G., Tipton, E., Hand, L. L., Alden, A. R., Warren, C., et al. (2013). The malleability of spatial skills: a meta-analysis of training studies. *Psychol. Bull.* 139, 352–402. doi: 10.1037/a0028446

Van Hiele, P. M. (1986). *Structure and insight: a theory of mathematics education* Academic Press.

Vandenberg, S. G., and Kuse, A. R. (1978). Mental rotations, a group test of three-dimensional spatial visualization. *Percept. Mot. Skills* 47, 599–604. doi: 10.2466/pms.1978.47.2.59

Verdine, B. N., Golinkoff, R. M., Hirsh-Pasek, K., and Newcombe, N. S. (2017). I. Spatial skills, their development, and their links to mathematics: spatial skills, their development, and their links to mathematics. *Monogr. Soc. Res. Child Dev.* 82, 7–30. doi: 10.1111/mono.12280

Voyer, D. (2011). Time limits and gender differences on paper-and-pencil tests of mental rotation: a meta-analysis. *Psychon. Bull. Rev.* 18, 267–277. doi: 10.3758/s13423-010-0042-0

Wai, J., Lubinski, D., and Benbow, C. P. (2009). Spatial ability for STEM domains: aligning over 50 years of cumulative psychological knowledge solidifies its importance. *J. Educ. Psychol.* 101, 817–835. doi: 10.1037/a0016127

Watson-Verran, H., and Chambers, D. (1989). *Singing the land, signing the land*. Melbourne, Australia: Deakin University Press.

Wei, W., Yuan, H., Chen, C., and Zhou, X. (2012). Cognitive correlates of performance in advanced mathematics. *Br. J. Educ. Psychol.* 82, 157–181. doi: 10.1111/j.2044-8279.2011.02049.x

Woolcott, G., Logan, T., Marshman, M., Ramful, A., Whannell, R., and Lowrie, T. (2020). "The re- emergence of spatial reasoning within primary years mathematics education" in *Research in mathematics education in Australasia 2016–2019*. eds. J. Way, C. Attard, J. Anderson, J. Bobis, H. McMaster and K. Cartwright (Singapore: Springer), 245–268.

Wright, R., Thompson, W. L., Ganis, G., Newcombe, N. S., and Kosslyn, S. M. (2008). Training generalized spatial skills. *Psychon. Bull. Rev.* 15, 763–771. doi: 10.3758/PBR.15.4.763

Xie, F., Zhang, L., Chen, X., and Xin, Z. (2020). Is spatial ability related to mathematical ability: a meta-analysis. *Educ. Psychol. Rev.* 32, 113–155. doi: 10.1007/s10648-019-09496-y

Young, C., Levine, S. C., and Mix, K. S. (2018). "What processes underlie the relations between spatial skill and mathematics?" in *Visualizing mathematics: The role of spatial reasoning in mathematical thought*. eds. K. S. Mix and M. T. Battista (Cham, Switzerland: Springer), 117–148.

Zacks, J. M., and Michelon, P. (2005). Transformations of visuospatial images. *Behav. Cogn. Neurosci. Rev.* 4, 96–118. doi: 10.1177/1534582305281085

Zacks, J. M., Mires, J., Tversky, B., and Hazeltine, E. (2000). Mental spatial transformations of objects and perspective. *Spat. Cogn. Comput.* 2, 315–332. doi: 10.1023/A:1015584100204