Supplementary Material

# Carbon price pathways in integrated assessment models

Figure 1 shows the carbon price pathway of scenarios from fifteen integrated assessment models taken from the AR5 IPCC scenario database Version 1.0.1. (<https://secure.iiasa.ac.at/web-apps/ene/AR5DB>). All pathways are normalized to a value of 1 in 2020 and reflect 2 °C pathways (either 450 ppm or 2.6 W/m2), or, if not available for that model, 550 ppm or 3.7 W/m2. The scenarios were developed in different projects (EMF22, EMF27, AMPERE, LIMITS, and AME). The figure shows that of the fifteen models, nine show a carbon price pathway strongly similar to Hotelling. Dashed lines show pathways that do not show a clear Hotelling pathway.

**Figure 1. Carbon price pathways of scenarios from integrated assessment models. Values are normalized to 1 in 2020.**

# Analytical proof on how learning representation affects the carbon price path

We first provide the analytical proof that the Hotelling rule is optimal with learning over time. For simplicity, we first assume a situation with no learning at all. Using Pontryagin’s Maximum Principle, we can solve the optimal control problem posed in Eq. 8 and 9 in the main text. This technique requires the usage of a Hamiltonian, defined as:

where , the costate variable of the optimal control problem, represents the shadow costs of emissions at time . A necessary condition of optimality originating from Pontryagin’s Principle is that  should be a local optimum of the Hamiltonian:

where we used the Fundamental Theorem of Calculus together with the chain rule. Note that , which allows us to simplify the equation to:

.

Assuming now that , and that as well, we can conclude that

Depending on the sign of  we can determine using this general result if the optimal carbon price grows at exponential rate  (confirming the Hotelling rule), faster (when ) or slower (when ).

The sign of  can be determined using the adjoint equation of Pontryagin’s Principle. Since we use the cumulative emissions  as state variable, the adjoint equation becomes

However, we assumed a constant MAC. In particular, all the terms in the Hamiltonian are independent of the state variable . This implies that the right-hand side of the previous equation is zero, implying that

In other words, the costate variable  is constant. For clarity, we call it now . Combining this with Equation *p*(*t*) above, we obtain

Since we have proved that the optimal carbon price path follows an exponential growth with growth rate equal to the interest rate, we have consequently confirmed the optimality of the Hotelling rule for a constant MAC (no learning). If the MAC curve is reduced purely dependent on time, the Hamiltonian is still independent of the state variable , leading to the same result: a confirmation of the Hotelling rule. The main difference is that due to technological learning, the initial carbon price  will be lower, while the growth rate stays the same.

The above derivation is valid for any general MAC curve until Eq. 4: . In the *learning by doing* experiment, we assumed a functional form of the MAC based on van Vuuren (2007). Here, we use a more general form of learning by doing.

Let  be the cumulative abatement, defined as the cumulative baseline emissions minus the cumulative emissions:

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The MAC curve is now, in its most general form, given by , where we assume that

for all , meaning that higher cumulative emissions reduce the abatement costs. We can now calculate the adjoint equation , which now, contrary to the previous section, is not zero anymore:

As shown in Van der Wijst (2018), by using the Fundamental Theorem of Calculus, the chain rule and the product rule, this expression becomes:

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By noting that , the last two terms cancel, giving the much shorter expression:

By assumption, was strictly negative for all , which means that in the most general *Learning by doing* case

or, in other words, that the carbon tax grows strictly slower than the exponential Hotelling price path.

The situation analysed by Goulder and Mathai (2000) is in many aspects similar to the one looked at here: a social planner has to reduce emissions such that the CO2 concentration stays below a chosen threshold, while minimizing the total associated costs. However, instead of using the carbon price as control variable, they use the abatement  as control variable, and instead of cumulative carbon emissions as state variable they use carbon dioxide concentration as state variable. Both differences are not significant, because i) there are direct connections between carbon price and abatement level through the MAC curve, and ii) cumulative carbon emissions and carbon dioxide concentrations are closely related.

The main difference resides in their treatment of accumulated knowledge. While we assume that increased cumulative abatement lowers the MAC, and therefore the abatement costs, Goulder and Mathai model knowledge as a separate knowledge variable . The associated abatement cost curve  is then dependent on both the abatement level and the level of knowledge accumulation.

The system now consists of a two-dimensional state variable (CO2 concentration and knowledge):

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The knowledge growth function is chosen such that and .

As a consequence of the two-dimensional state variable, using Pontryagin’s principle requires two costate variables. In fact, the Hamiltonian of the above system becomes:

By introducing the new current-value costate variables and and moving to a current value Hamiltonian , we obtain the expression introduced in Goulder and Mathai (2000):

where  is the costate variable for the CO2 concentration and  for the knowledge. Using the same steps as in SI.2. and SI.3., it follows that the adjoint equation for the concentration state variable implies that

or, in other words,  when assuming that the CO2 reduction parameter is negligible.  Goulder and Mathai argue that  represents the shadow value of a small additional amount of CO2, since it is the costate variable associated with the change in CO2 amount. They further argue that the optimal carbon tax is equal to this shadow value of additional CO2, which would prove the Hotelling rule for learning by doing. This is exactly our main objection: this argument does not use the shadow value of improved knowledge, , which should also be taken into account since it is associated to a state variable. The adjoint equation associated to this variable is equal to:

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However, we argue that this adjoint equation cannot be ignored and that the carbon price should definitely take into account the increased knowledge.

The same setting can be obtained by directly incorporating knowledge accumulation in the abatement cost function, like we have done throughout this paper. However, since we describe the same setting using only one state variable, the costate variable will not yield a purely exponentially increasing function. Therefore, we argue that Goulder and Mathai’s result that the Hotelling rule is optimal for learning by doing is an artefact of modelling knowledge accumulation in a separate variable.

This ambiguity can be solved by defining the carbon price as the inverse of the marginal abatement cost curve, instead of by defining the carbon price as shadow value of increased CO2 emissions.

**References**

Goulder, L.H., Mathai, K., 2000. Optimal CO2 Abatement in the Presence of Induced Technological Change. Journal of Environmental Economics and Management 39, 1-38.

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Van der Wijst, K.-I., 2018. Optimal policy for carbon pricing: Challenging the Hotelling rule and dissecting mitigation cost uncertainties, <https://dspace.library.uu.nl/handle/1874/372378>.