Supplementary Material

1 [S1] ANALYSIS OF CUBIC EQUATION

A detailed analysis of the third degree polynomial

$$g_1(\lambda^*) = C_3\lambda^{*3} + C_2\lambda^{*2} + C_1\lambda^* + C_0 = 0 \tag{S1}$$

is conducted to determine the number of positive endemic equilibria exhibited by model system (1). The procedure by Descartes, referred as Descarte's rule of signs [1] is applied on cubic equation (S1) to determine the possible number of positive endemic equilibria. The number of positive endemic equilibria and the value of the control reproduction number can be used to predict the type of bifurcation structure likely to be observed when the endemic equilibria is plotted against the control reproduction $\mathcal{R}_c$. For instance, see the sixth and seventh column of Table S1.

Table S1. Represents possible number of positive endemic equilibria for model system (1).

Value of $\mathcal{R}_c$	C_3	C_2	C_1	C_0	Number of positive endemic equilibria	Type of bifurcation
$\mathcal{R}_c = 1$	> 0	< 0	< 0	0	1	Backward bifurcation
	> 0	> 0	< 0	0	1	Backward bifurcation
	> 0	< 0	> 0	0	2	Hysteresis effect
$\mathcal{R}_c < 1$	> 0	< 0	< 0	> 0	2	Backward bifurcation
	> 0	> 0	< 0	> 0	2	Backward bifurcation
	> 0	< 0	> 0	> 0	2	Backward bifurcation
$\mathcal{R}_c > 1$	> 0	> 0	> 0	< 0	1	Forward bifurcation
	> 0	> 0	< 0	< 0	1	Forward bifurcation
	> 0	< 0	< 0	< 0	1	Forward bifurcation
	> 0	< 0	> 0	< 0	3 or 1	Forward hysteresis effect or forward bifurcation

2 [S2]FURTHER EXPLANATION OF THE INEQUALITIES (27)-(30)

It is important to stress that the last term added in equation (26) to obtain inequality (27) is assumed to be negative or equal to zero. That whenever the following inequalities are satisfied

$$\left(\frac{V_p^{**}V}{V_pV^{**}} - 1\right) \leq 0,\ \left(1 - \frac{V_p}{V_p^{**}}\right) \geq 0,\ 0 \leq V \leq V^{**}, 0 \leq V_p \leq V_p^{**}, \quad \text{and}$$
$$\left(\frac{V_p^{**}V}{V_pV^{**}} - 1\right) > 0,\ \left(1 - \frac{V_p}{V_p^{**}}\right) < 0,\ V > V^{**},\ V_p > V_p^{**}$$

we obtain

$$\left(\frac{V_p^{**}V}{V_pV^{**}} - 1\right)\left(1 - \frac{V_p}{V_p^{**}}\right) \leq 0, V > 0,\ V_p > 0.$$

In a similar manner the inequalities (28)-(30) can be easily obtained.

REFERENCES

[1]Bruce Anderson, Jeffrey Jackson, and Meera Sitharam. Descartes' rule of signs revisited. *The American Mathematical Monthly*, 105(5):447–451, 1998.